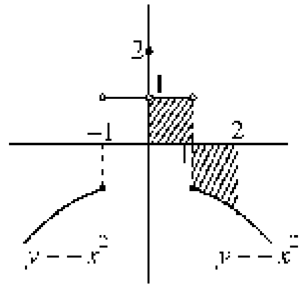
Level – 2	Daily Tutorial Sheet - 9

121.(A) $f(x)$ is an even function.



$$\Rightarrow \qquad I = 2\int_0^2 f(x)dx = 2\int_0^1 1\,.\,dx + 2\int_1^2 -x^2 dx$$

$$= 2-2\left|\frac{x^3}{3}\right|_1^2 = \frac{6-2\,(8-1)}{3} = \frac{-8}{3}$$

122.(B) $I = \int_0^1 \cos\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right)dx = \int_0^1 \cos\left(\cos^{-1}\left(\frac{1-\left(\frac{1+x}{1-x}\right)}{1+\frac{1+x}{1-x}}\right)\right)dx = \int_0^1 \cos\left(\cos^{-1}\left(-\frac{2x}{2}\right)\right)dx$

$$= -\int_0^1 x dx = \left[-\frac{x^2}{2}\right]_0^1 = -\frac{1}{2}$$

123.(B) $I = \int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}\,dx = \int_0^2 \left(\left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}\right)+\left(\frac{\sin^2 x}{\left[\frac{-x}{\pi}\right]+\frac{1}{2}}\right)\right)dx = \int_0^2 \left(\left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}\right)+\left(\frac{\sin^2 x}{-\left[\frac{x}{\pi}\right]-1+\frac{1}{2}}\right)\right)dx = 0$

124.(A) $I = \int_{-1}^{0} [x(0)+1]dx + \int_0^1 [x(1)+1]dx = \int_{-1}^{0} 1dx + \int_0^1 ([x]+1)dx$ $[\because \sin\pi x \in [-1,0]$ and $\sin\pi x \in [0,1]]$

$$= [1]_{-1}^{0} + [1]_0^1 = 2$$

125.(B) $I = \int_{-1}^{1} x\left(\frac{e^x+1}{e^x-1}\right)dx = \int_0^1 \left[x\left(\frac{e^x+1}{e^x-1}\right)+\left(-x\frac{(e^{-x}+1)}{(e^{-x}-1)}\right)\right]dx = \int_0^1 x\,\frac{(e^x+1)}{(e^x-1)}\,.\,2dx = 2\lambda$

126.(A) $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_a^{2a} f(x)dx = \lambda + \int_0^a f(2a-x)\,dx = \lambda + \mu$

127.(C) $I = \int_0^{\pi} \frac{\sin 2Kx}{\sin x}dx = \int_0^{\pi} \frac{\sin 2K(\pi-x)}{\sin(\pi-x)}\,dx = -\int_0^{\pi} \frac{\sin 2\mathrm{K}x}{\sin x}\,dx = -I \Rightarrow I = 0$

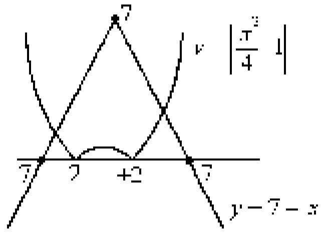
128.(A) $I = \int_0^{\pi} \frac{x^2 \sin x\, dx}{(2x-\pi)(1+\cos^2 x)}$

$$\Rightarrow \qquad I = \int_0^{\pi} \frac{(\pi-x)^2 \sin(\pi-x)\,dx}{(2\pi-2x-\pi)\,(1+\cos^2(\pi-x))} = \int_0^{\pi} \frac{(x^2-2x\pi+\pi^2)\sin x\;dx}{(\pi-2x)(1+\cos^2 x)}$$

$\Rightarrow \quad I = \dfrac{1}{2}\displaystyle\int_0^{\pi} \dfrac{\sin x\ .\ (2\pi x - \pi^2)}{(2x-\pi)(1+\cos^2 x)}dx = \dfrac{1}{2}\int_0^{\pi}\dfrac{\pi \sin x}{1+\cos^2 x}dx$ [Put $cosx = t \ \Rightarrow\ -sinx\ dx = dt$]

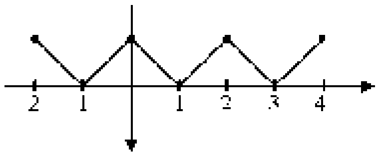
$\Rightarrow \quad I = \dfrac{1}{2}\displaystyle\int_1^{-1}\dfrac{\pi}{1+t^2}(-dt) = \dfrac{\pi}{2}\left[-\tan^{-1} t\right]_1^{-1} = \dfrac{\pi}{2}\ .\ 2\tan^{-1}(1) = \dfrac{\pi^2}{4}$

129.(D) $A = 2\left[\displaystyle\int_0^2\left(7 - x + \dfrac{x^2}{4} - 1\right)dx + \int_2^4\left(7 - x - \dfrac{x^2}{4} + 1\right)\right]dx$



= 32 Sq. units

130.(D) $f(x) = \begin{cases} |\{x\}| & [x] \in odd \\ |\{x\} - 1| & [x] \in even \end{cases}$



Area $= 6 \times \left(\dfrac{1}{2} sq.units\right) = 3$