| Level – 2 | Daily Tutorial Sheet - 13 |
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**161.(D)** $I = \int_{\cos\theta}^{\sec\theta} f(x)\,dx = -\int_{\cos\theta}^{\sec\theta} \frac{f(1/x)}{x^2}\,dx$ **.**

Put $\frac{1}{x} = t$ to get : $I = \int_{\sec\theta}^{\cos\theta} f(t)\,dt = \int_{\sec\theta}^{\cos\theta} f(x)\,dx = -I \quad \Rightarrow \quad 2I = 0 \quad \Rightarrow \quad I = 0$

**162.(B)** We have, $I = \int_1^2 \left( x^{\left[x^2\right]} + \left[x^2\right]^x \right) dx = \int_1^{\sqrt{2}} (x+1)\,dx + \int_{\sqrt{2}}^{\sqrt{3}} \left(x^2 + 2^x\right) dx + \int_{\sqrt{3}}^{2} \left(x^3 + 3^x\right) dx$

$= \left( \frac{x^2}{2} + x \right)_1^{\sqrt{2}} + \left( \frac{x^3}{3} + \frac{2^x}{\log 2} \right)_{\sqrt{2}}^{\sqrt{3}} + \left( \frac{x^4}{4} + \frac{3^x}{\log 3} \right)_{\sqrt{3}}^{2} = \frac{5}{4} + \sqrt{3} + \frac{\sqrt{2}}{3} + \frac{1}{\log 2}(2^{\sqrt{3}} - 2^{\sqrt{2}}) + \frac{1}{\log 3}(3^2 - 3^{\sqrt{3}})$

**163.(D)** As, $f(x) = \begin{vmatrix} \cos x & e^{x^2} & 2x\cos^2\frac{x}{2} \\ x^2 & \sec x & \sin x + x^3 \\ 1 & 2 & x + \tan x \end{vmatrix} \quad \Rightarrow \quad f(-x) = -f(x) \quad \Rightarrow \quad f(x)$ is odd

$\Rightarrow \quad f'(x)$ is even $\quad \Rightarrow \quad f''(x)$ is odd

Thus, $f(x) + f''(x)$ is odd function let, $\phi(x) = \left(x^2+1\right) . \left\{ f(x) + f'(x) \right\}$

$\Rightarrow \quad \phi(-x) = -\phi(x)$ i.e. $\phi(x)$ is odd $\quad \Rightarrow \quad \int_{-\pi/2}^{\pi/2} \phi(x)\,dx = 0$

**164.(A)** Let $I = \int_0^2 \frac{dx}{\left(17 + 8x - 4x^2\right)\left[e^{-6(1-x)} + 1\right]} \qquad \left[ \because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx \right]$

Adding, we get : $2I = \int_0^2 \frac{1}{17 + 8x - 4x^2} \left( \frac{1}{e^{6(1-x)} + 1} + \frac{1}{e^{-6(1-x)} + 1} \right) dx$

$= \int_0^2 \frac{1}{17 + 8x - 4x^2}\,dx = \frac{1}{4}\int_0^2 \frac{dx}{x^2 - 2x - 17/4} = -\frac{1}{4} \times \frac{1}{2 \times \frac{\sqrt{21}}{2}} \left[ \log \left| \frac{x - 1 - \frac{\sqrt{21}}{2}}{x - 1 + \frac{\sqrt{21}}{2}} \right| \right]_0^2 = -\frac{1}{4\sqrt{21}} \left[ \log \left| \frac{\sqrt{21} - 2}{2 + \sqrt{21}} \right| \right]$

**165.(A)** $\int_0^{2[x+14]} \left\{ \frac{x}{2} \right\} dx = \int_0^{[x]} [x + 14]\,dx \quad \Rightarrow \quad \int_0^{28 + 2[x]} \left\{ \frac{x}{2} \right\} dx = \int_0^{\{x\}} \left(14 + [x]\right) dx$

$\Rightarrow \quad \int_0^{28} \left\{ \frac{x}{2} \right\} dx + \int_{28}^{28 + 2[x]} \left\{ \frac{x}{2} \right\} dx = \left(14 + [x]\right)\{x\} \quad \Rightarrow \quad \int_0^2 \left\{ \frac{x}{2} \right\} dx + \int_0^{2[x]} \left\{ \frac{x}{2} \right\} dx = \left(14 + [x]\right)\{x\}$

{Using $\int_0^{nT} f(x)dx = n\int_0^{T} f(x)dx$ and $\int_a^{a+nT} f(x)dx = \int_0^{nT} f(x)dx$ where T is period of $f(x)$}

$\Rightarrow \quad 14+[x]=(14+[x])\{x\} \quad \Rightarrow \quad (14+[x])(1-\{x\})=0 \quad \Rightarrow \quad [x]=-14 \Rightarrow x\in[-14,-13)$

**166.(B)** $f(x)$ is above the $x$-axis or on the $x$-axis for all $x\in[2,10]$. If $f(x)$ is greater than zero for any sub interval of [4, 8], then $\int_4^8 f(x)dx$ must be greater than zero. But $\int_4^8 f(x)dx = 0 \Rightarrow f(x)=0\ \forall\ x\in[4,8]$

$\Rightarrow \quad f(6)=0$.

**167.(B)** We have $f(x)f(y)+2 = f(x)+f(y)+f(xy)$

Putting $x$ = 1 and $y$ = 1 then $f(1)\ f(1)+2=3f(1)$

we get : $f(1)=1, 2$

$f(1)\neq 1$ $\quad$ ($\because$ $f(0)=1$ and function is injective) then $f(1)=2$

Replacing $y$ by $\frac{1}{x}$ in (i) then $f(x)f\left(\frac{1}{x}\right)+2 = f(x)+f\left(\frac{1}{x}\right)+f(1) \Rightarrow f(x)f\left(\frac{1}{x}\right)=f(x)+f\left(\frac{1}{x}\right)$

Hence $f(x)$ is of the type $\Rightarrow \quad f(x)=1\pm x^n \quad \because \quad f(1)=2$

$\therefore \quad f(x)=1+x^n$ and $f'(x)=nx^{x-1} \Rightarrow f'(x)=n=2$

$f(x)=1+x^2$

$\therefore \quad 3\int f(x)dx - x(f(x)+2) = 3\int(1+x^2)dx - x(1+x^2+2) = 3\left(x+\frac{x^3}{3}\right)-x(3+x^2)+c = c =$ constant

**168.(AB)** $I_n=\int_0^1 (1+x^2)^{-n}dx$ ; $I_n=\left[(1+x^2)^{-n}.x\right]_0^1 + n\int_0^1 \frac{2x^2}{(1+x^2)^{n+1}}dx = 2^{-n}+2n\int_0^1\frac{(x^2+1)-1}{(x^2+1)^{n+1}}dx$

$I_2 = 2^{-n}+2n[I_n-I_{n+1}] \quad \Rightarrow \quad 2nI_{n+1}=2^{-n}+(2n-1)I_n$

Now put $n=1$

$2I_2 = 2^{-1}+I_1=\frac{1}{2}+\int_0^1\frac{1}{1+x^2}dx = \frac{1}{2}+\left(\tan^{-1}x\right)_0^1 \quad \Rightarrow \quad I_2=\frac{1}{4}+\frac{\pi}{8}$

**169.(B)**

**170.(B)** $\int_1^2 \frac{(x^2-1)dx}{x^3\sqrt{2x^4-2x^2+1}} = \int_1^2 \frac{x(x^2-1)dx}{x^4\sqrt{2x^4-2x^2+1}}$

Let $x^2=t \Rightarrow x\,dx = dt/2$

$=\frac{1}{2}\int_1^4\frac{(t-1)dt}{t^2\sqrt{2t^2-2t+1}} = \frac{1}{2}\int_1^4\frac{t-1}{t^3\sqrt{2-\frac{2}{t}+\frac{1}{t^2}}}dt = \frac{1}{2}\int_1^4\frac{\frac{1}{t^2}-\frac{1}{t^3}}{\sqrt{2-\frac{2}{t}+\frac{1}{t^2}}}dt$

[Let $2-\frac{2}{t}+\frac{1}{t^2}=z^2$] $=\frac{1}{2}\int_1^{5/4}\frac{z\,dz}{\sqrt{z^2}}=\frac{1}{2}\int_1^{5/4}dz=\frac{1}{8}=\frac{U}{V} \Rightarrow (1000)\frac{U}{V}=\frac{1000}{8}=125$