Level – 2	Daily Tutorial Sheet - 11

141.(B) $f(p, q) = \int_0^{\pi/2} \cos^p x \cos qx \, dx$

$$f(p-1, q-1) = \int_0^{\pi/2} \cos^{p-1} x . \cos(qx - x) dx$$

$$f(p-1, q-1) = \int_0^{\pi/2} \frac{\cos^p x}{\cos x} (\cos qx \times \cos x + \sin qx \times \sin x) dx$$

$$= \int_0^{\pi/2} \cos^p x \cos qx \, dx + \int_0^{\pi/2} \underbrace{\cos^{p-1} x \sin x}_{II} \underbrace{\sin qx}_{I} \, dx$$

Apply by parts taking $\sin qx$ as I[st] part and remaining as II[nd] part

$$= f(p, q) - \left. \frac{\sin qx \cos^p x}{p} \right|_0^{\pi/2} + \frac{q}{p} \int_0^{\pi/2} \cos qx \cos^p x \, dx \quad \Rightarrow \quad f(p-1, q-1) = f(p, q) - 0 + \frac{q}{p} f(p, q)$$

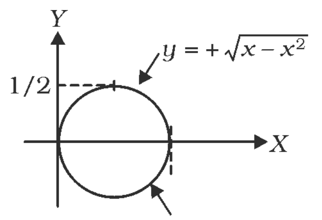
142.(B) $y = \sin^{-1} x \pm \sqrt{x - x^2}$

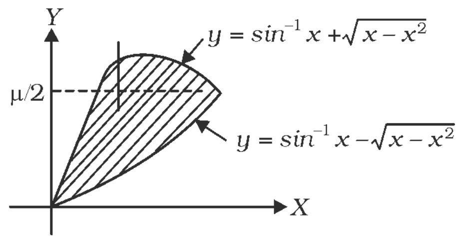
Define only for $x \in (0, 1)$.

$$A = 2\int \sqrt{x - x^2} dx, \qquad = 2\int_0^1 \sqrt{\frac{1}{4} - \left(x - \frac{1}{2}\right)^2} dx$$

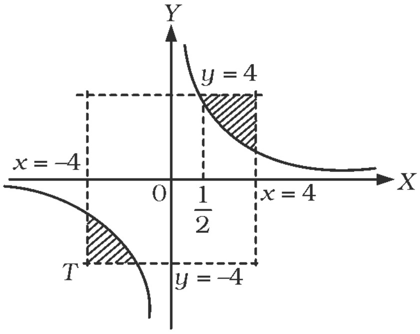
$$= 2\left[\frac{x - \frac{1}{2}}{2} \sqrt{x - x^2} + \frac{1}{8} \sin^{-1} \frac{x - \frac{1}{2}}{\frac{1}{2}} \right]_0^1, = \left(x - \frac{1}{2}\right)\sqrt{x - x^2} + \frac{1}{4}\sin^{-1}(2x - 1) \Big|_0^1$$

$$= \frac{1}{4}\left[\sin^{-1} 1 - \sin^{-1}(-1)\right] = \frac{1}{4}\left[\frac{\pi}{2} + \frac{\pi}{2}\right] = \frac{\pi}{4}$$





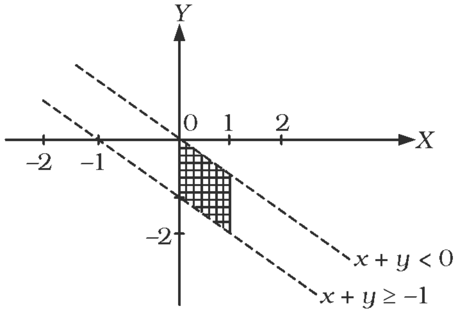
143.(A) $A = 2\int_{1/2}^{4}\left(4 - \frac{2}{x}\right)dx = 4(7 - \ln 8).$



144.(A) $[x + y + 1] = [x] \;\; \Rightarrow \;\; [x + y] = [x] - 1$

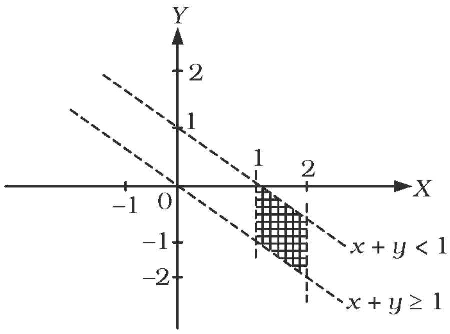
Case 1 :

Consider $x \in (0, 1) \;\Rightarrow\; [x + y] = -1 \;\Rightarrow\; -1 \le x + y < 0 \quad \{\& \; x \in (0, 1)\}$



Case 2 :

Consider $x \in (1, 2) \;\Rightarrow\; [x + y] = 1 - 1 = 0$



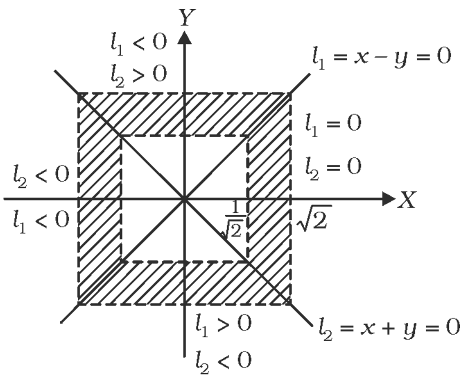
Area enclosed : 1 sq. + 1 sq. = 2 sq.

145.(C) $l_1 > l_2 > 0 \qquad \frac{1}{\sqrt{2}} \le x \le \sqrt{2}$

$l_1 < 0, l_2 > 0 \qquad \frac{1}{\sqrt{2}} \le y \le \sqrt{2}$

$\left.\begin{matrix} l_1 < 0 \\ l_2 \le 0 \end{matrix}\right\} \rightarrow \qquad -\sqrt{2} \le x \le \frac{-1}{\sqrt{2}}$

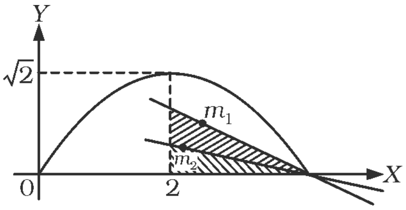
$\left.\begin{matrix} l_1 \ge 0 \\ l_2 \le 0 \end{matrix}\right\} \rightarrow \qquad -\sqrt{2} \le y \le \frac{-1}{\sqrt{2}}$



Area $= (2\sqrt{2})^2 - \left(\frac{2}{\sqrt{2}}\right)^2 = 8 - 2 = 6.$

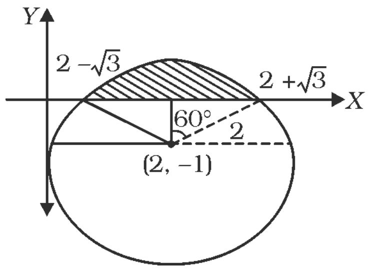
146.(A) Equation of line. $\frac{y-0}{x-4} = m \quad \Rightarrow \quad y = m(x-4)$

$$\int_2^4 \left[\sqrt{2}\sin\left(\frac{\pi}{4}x\right) - m_1(x-4)\right]dx = \int_2^4 m_2(x-4)dx \quad \Rightarrow \quad \frac{4\sqrt{2}}{\pi} + 2m_1 = -2m_2 \quad \Rightarrow \quad m_1 + m_2 = \frac{-2\sqrt{2}}{\pi}.$$



147.(C) $\frac{x(x^2+2)}{100} > 0 \; \forall \; x > 0$ and $0 \le \frac{x(x^2+2)}{100} \le 1 \; \forall \; x \in (0, 4)$

$$A = \pi(2)^2 \times \frac{120}{360} - 2 \times \frac{1}{2} \times \sqrt{3} = \frac{4\pi - 3\sqrt{3}}{3}$$



148.(A) $d(P.AB) = y$; $d(P, BC) = 6 - x$

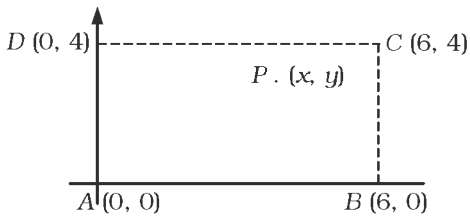
$d(P.CD) = (4 - y)$; $d(P.AD) = x$

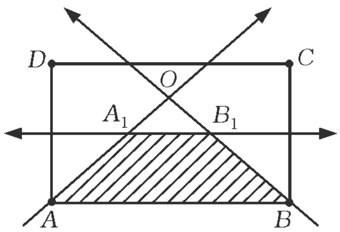
$y \le x$ $\quad y \le 6 - x$ $\quad y \le 4 - y$

$y - x \le 0$ $\quad x + y \le 6$ $\quad y \le 2$

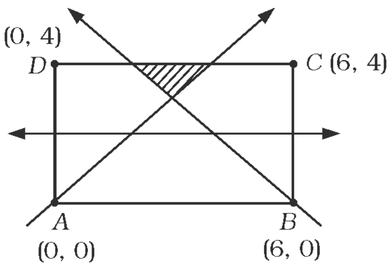
$A_1 = \frac{1}{2} \times 2 \times 1 = 1 = ar\,\Delta OA_1B_1$

Area required $= \frac{1}{2} \times 6 \times 3 - A_1 = 8$





149.(B) $y \ge x \quad x + y \ge 6 \quad y \ge 2.$



Area = 1 sq. unit.

150.(B) $f'(x) = f(x) + \int_0^1 f(x)dx = f(x) + k \qquad k = \int_0^1 f(x)dx;$

$\frac{dy}{dx} = y + k \quad \Rightarrow \int \frac{dy}{y+k} = \int dx \quad \Rightarrow \quad \ln|y+k| = x + c \Rightarrow y + k = e^{x+c} \qquad f(0) = 1$

$\Rightarrow \qquad 1 + k = e^c \qquad \dots \text{(i)}$

$$k = \int_0^1 [e^{x+c} - k]dx = e^{x+c} - kx \Big|_0^1 = e^c(e-1) - k$$

$\Rightarrow \qquad 2k = e^c(e-1); \qquad \dots \text{(ii)}$

From (i) and (ii)

$e^c - 1 = \frac{e^c}{2}(e-1) \Rightarrow y = \frac{2e^x}{3-e} + \frac{1-e}{3-e}$