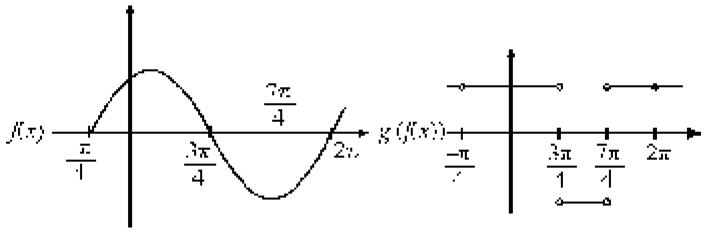
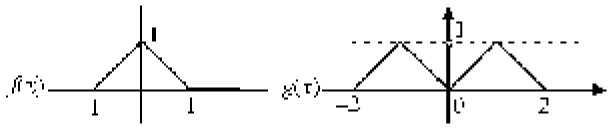
Level – 2	Daily Tutorial Sheet - 10

131.(B) $f(x) = sinx + cosx$

$$\int_{-\pi/4}^{2\pi} g(f(x))dx = (1\times\pi) + (-1)(\pi) + (1)\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$$



132.(A) $I = \int_{-3}^{3} g(x)dx = 2 \times (1\text{sq. unit}) = 2$



133.(B) $I = \int_0^1 \cot^{-1}\left[\frac{1-x(1-x)}{x+(1-x)}\right]dx = \int_0^1 \tan^{-1}\left[\frac{x+(1-x)}{1-x\ (1-x)}\right]dx = \int_0^1 \tan^{-1}x + \int_0^1 \tan^{-1}(1-x)dx = 2\int_0^1 \tan^{-1}xdx$

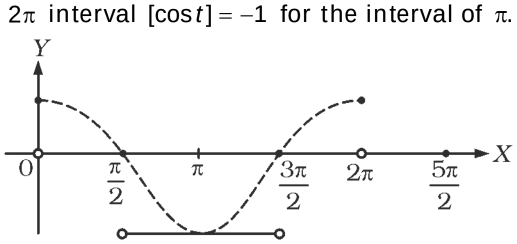
134.(D) $\frac{d}{dx}(F(x)) = \frac{e^{\sin x}}{x}\quad x > 0$

$\Rightarrow \qquad F(x) = \int \frac{e^{\sin x}}{x}dx$; Replace $x \longrightarrow x^3 \Rightarrow dx \longrightarrow 3x^2$

$\Rightarrow \qquad F(x^3) = \int \frac{3e^{\sin x^3}}{x}dx$; Put limits ; $x = 4$; 1

$\Rightarrow \qquad F(64) - F(1) = \int_1^4 \frac{3e^{\sin x^3}}{x}dx \Rightarrow k$ can be 64

135.(A) In each 2π interval $[\cos t] = -1$ for the interval of π.



$\therefore \qquad \int_0^x [\cos t]dt = -n\pi.$

136.(B) Put $t-b=-z \quad \Rightarrow \quad \int_1^0 \frac{e^{z-b}}{-(z+1)}(-dz)=\int_1^0 \frac{e^{-b}e^z dz}{(z+1)}=-ae^{-b}$

137.(D) $I=\int_0^{\pi}\frac{\sin\left(n+\frac{1}{2}\right)x}{\sin\frac{x}{2}}.dx$

$$I=\int_0^{\pi}\frac{\sin\left(n+\frac{1}{2}\right)x-\sin\frac{x}{2}}{\sin\frac{x}{2}}dx+\int_0^{\pi}1dx$$

$$I=\int_0^{\pi}\frac{2\sin\frac{nx}{2}\cos\left(\frac{n+1}{2}\right)x}{\sin\frac{x}{2}}dx+\pi$$

Comparing $\dfrac{2\sin\frac{nx}{2}\cos\left(\frac{n+1}{2}\right)x}{\sin\frac{x}{2}}$ with $\dfrac{\sin\left(\frac{nd}{2}\right)}{\sin\left(\frac{d}{2}\right)}\cos\left(a+(n-1)\frac{d}{2}\right)$

We get : $\frac{nd}{2}=\frac{nx}{2} \Rightarrow d=x$ and $n=n$

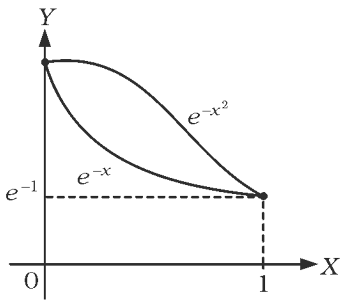
$a+(n-1)\frac{x}{2}=(n+1)\frac{x}{2} \Rightarrow a=x$

So, $\dfrac{\sin\frac{nx}{2}\cos\left(\frac{n+1}{2}\right)x}{\sin\frac{x}{2}}=\cos x+\cos 2x+\cos 3x+\ldots$

$$\Rightarrow \qquad I=\int_0^{\pi}2\cdot[\cos x+\cos 2x+\cos 3x+\ldots]+\pi$$

$$\Rightarrow \qquad I=2\left[\sin x+\frac{\sin 2x}{2}+\ldots..\right]_0^{\pi}+\pi \qquad \Rightarrow \qquad I=0+\pi=\pi$$

138.(D) $e^{-x^2}>e^{-x}\ \forall x\in(0,1)$



$$\Rightarrow \qquad e^{-x^2}\cos^2 x>e^{-x}\cos^2 x \text{ and } e^{-x^2}>e^{-x^2}\cos^2 x$$

$$\Rightarrow \qquad \int_0^1 e^{-x^2}dx>\int_0^1 e^{-x^2}\cos^2 x\,dx>\int_0^1 e^{-x}\cos^2 x\,dx$$

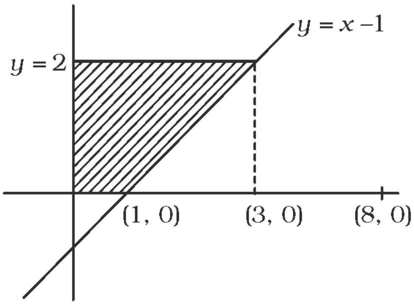
$$\Rightarrow \qquad I_1>I_3>I_2$$

139.(C) $\int_1^2 e^{x^2} dx = a$. Put $x^2 = t \Rightarrow \int_1^4 \frac{e^t}{\sqrt{t}} dt = 2a$ ... (i)

$I = \int_e^{e^4} \sqrt{\ln x}\, dx$. Put $\ln x = t \Rightarrow I = \int_1^4 \sqrt{t}\, e^t\, dt = \sqrt{t}\, e^t \Big|_1^4 - \frac{1}{2}\int_1^4 \frac{e^t}{\sqrt{t}} dt = 2e^4 - e - \frac{1}{2} \times 2a$ [use (i)]

$= 2e^4 - e - a$

140.(C) $y = \left[\frac{x^2}{64} + 2\right] = 2\ \forall\, x \in (0, 8)$



Area $= 3 \times 2 - \frac{1}{2} \times 2 \times 2 = 4$.