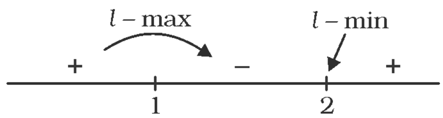
Level – 1	Daily Tutorial Sheet - 7

91.(C) $f'(x) = x(x-1)(x-2)$

$f(2) = \frac{-1}{4}$ $f(3) = 2$ $f(1) = 0$



$f_{\min} \equiv \min[f(1), f(2), f(3)] = \frac{-1}{4} \forall x \in [1, 3]$

$f_{\max} \equiv \max[f(1), f(3)] = 2 \forall x \in [1, 3]$

Range $\equiv \left[\frac{-1}{4}, 2\right]$

92.(A) $\left.\frac{dy}{dx}\right|_{x=1} = \left.\frac{3x^2}{\sqrt{1+x^6}} - \frac{2x}{\sqrt{1+x^4}}\right|_{x=1} = \frac{1}{\sqrt{2}}$ Equation of tangent $\sqrt{2}y + 1 = x$.

93.(B) $F'(x) = 2x\sin x\cos x - 2x\sin x\cos x = 0$

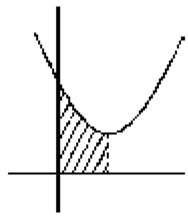
$F(x) = F\left(\frac{\pi}{4}\right) = \int_0^{1/2}[\sin^{-1}\sqrt{t} + \cos^{-1}\sqrt{t}]dt = \int_0^{1/2}\frac{\pi}{2}dt = \frac{\pi}{4}$

94.(D) $I = \int_{-1/2}^{1/2}\sqrt{\left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2}\,dx = \int_{-1/2}^{1/2}\sqrt{\left(\frac{x+1}{x-1} - \frac{x-1}{x+1}\right)^2}\,dx = \int_{-1/2}^{1/2}\left|\frac{x+1}{x-1} - \frac{x-1}{x+1}\right|dx$

$\Rightarrow \quad I = \int_{-1/2}^{1/2}\left|\frac{4x}{x^2-1}\right|dx = 2\int_0^{1/2}\left|\frac{4x}{x^2-1}\right|dx = 2\int_0^{1/2}\frac{4x}{1-x^2}dx = 4\ell n\left(\frac{4}{3}\right)$

95.(D) $f(0) = c$; $f(1) = a + b + c$; $f(-1) = a - b + c$

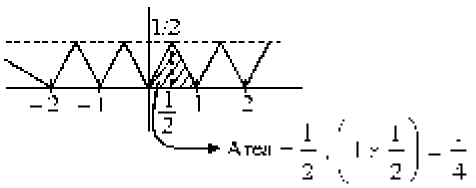
$a = \frac{f(1)}{2} + \frac{f(-1)}{2} - f(0)$; $b = \frac{f(1)}{2} - \frac{f(-1)}{2}$; $c = f(0)$



$\Rightarrow \int_0^1 (ax^2 + bx + c)\,dx = \left[\frac{ax^3}{3} + \frac{bx^2}{2} + cx\right]_0^1 = \frac{a}{3} + \frac{b}{2} + c = \frac{1}{12}[5f(1) - f(-1) + 8f(0)]$

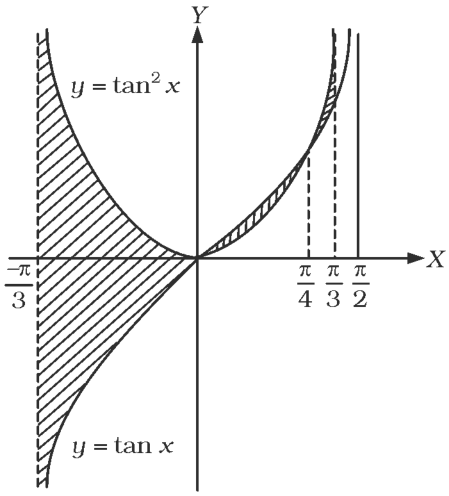
96.(B) $I_1 = \int_{1-\cos^2 t}^{1+\cos^2 t} xf(x(2-x))dx = \int_{1-\cos^2 t}^{1+\cos^2 t}(2-x)f[(2-x)x]dx \Rightarrow I_1 = \frac{1}{2}\int_{1-\cos^2 t}^{1+\cos^2 t} 2f(x(2-x))dx = I_2$

97.(B) $\int_{-2}^{2}\min(\{x\}, \{-x\})dx \quad \Rightarrow \quad$ Total area $= 4 \times \frac{1}{4} = 1$



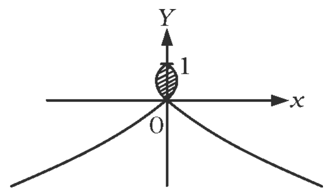
98.(C) $\int_{-\pi/3}^{\pi/3}|\tan^2 x - \tan x|\,dx = \int_{-\pi/4}^{0}(\tan^2 x - \tan x)dx + \int_0^{\pi/4}(\tan x - \tan^2 x)dx + \int_{\pi/4}^{\pi/3}(\tan^2 x - \tan x)dx$

As $\int(\tan^2 x - \tan x)dx = (\tan x - x - \log|\sec x|)$



Required area $= -\frac{\pi}{6} + \log(2) + 2(\sqrt{3} - 1)$

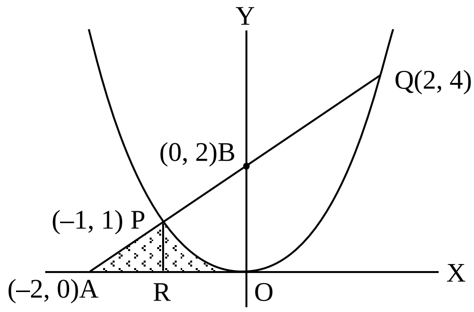
99.(A) $x = \pm y\sqrt{1-y}$. Defined for $y \le 1$.



$A = \int_0^1 2y\sqrt{1-y}\, dy$ put $1 - y = t^2$

$\Rightarrow \quad A = -\int_1^0 4t^2(1-t^2)dt = \int_0^1 (4t^2 - 4t^4)dt = \left.\frac{4t^3}{3} - \frac{4t^5}{t}\right|_0^1 = \frac{8}{15}.$

100.(B) The parabola and line meet in points $P(-1, 1)$ and $Q(2, 4)$.



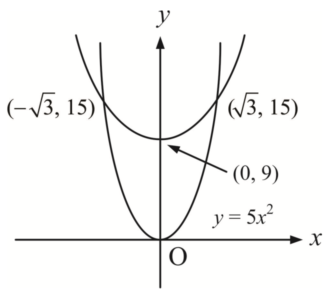
The line cuts the axes in $(-2, 0)$ and $B(0, 2)$.

i.e. area *APR* + area *PRO*

$= \int_{-2}^{-1} \underset{L}{y\, dx} + \int_{-1}^{0} \underset{P}{y\, dx} = \int_{-2}^{-1} (x+2)dx + \int_{-1}^{0} x^2 dx$

$= \left[\frac{x^2}{2} + 2x\right]_{-2}^{-1} + \left[\frac{1}{3}x^3\right]_{-1}^{0} = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ sq. units

101.(C) Eliminating y, we get : $54x^2 - (2x^2 + 9) = 0$ or $3x^2 = 9$ $x = -\sqrt{3}, \sqrt{3}$



The two parabolas are $x^2 = \frac{1}{2}y$, $x^2 = \frac{1}{2}(y - 9)$ and clearly from the figure second parabola is upper and first is lower.

$\therefore$ Required area $= 2\int_0^{\sqrt{3}} (y_1 - y_2)dx$ ... (i)

$= 2\int_0^{\sqrt{3}} \left[(2x^2 + 9) - 5x^2\right] \cdot dx = 2\int_0^{\sqrt{3}} (9 - 3x^2) \cdot dx = 12\sqrt{3}$

102.(D) $A = \int_2^4 y\, dx = \int_2^4 \left(1 + \frac{8}{x^2}\right)dx = \left[x - \frac{8}{x}\right]_2^4 = 4$

Also $\quad A_1 = \int_2^a y\,dx = \frac{1}{2}A = 2 \quad \left[\text{as} \left[x - \frac{8}{x}\right]_2^a = 2\right] \quad$ or $\quad (a-2) - 8\left(\frac{1}{a} - \frac{1}{2}\right) = 2$ or $a - \frac{8}{a} = 0$

$\therefore \quad a^2 - 8 = 0 \qquad \therefore \quad a = 2\sqrt{2}$

103.(B) $y^2 = 4b(b-x)$ or $y^2 = -4b(x-b)$

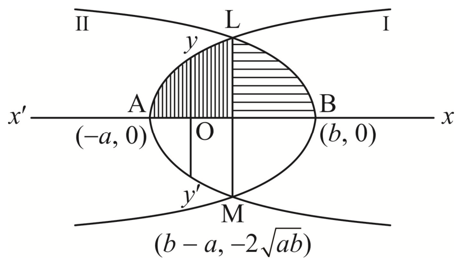
Eliminating y^2, we get : x co-ordinate of their point of intersection as $x = b - a$. $y = \pm 2\sqrt{ab}$

The vertices of the parabolas are $(-a, 0), (0, b)$ and are as shown in the figure.

$$A = 2\left[\int_{-a}^{b-a} y_1\,dx + \int_{b-a}^{b} y_2\,dx\right] = 2\left[2\sqrt{a}\int_{-a}^{b-a}\sqrt{x+a}\,dx + \int_{b-a}^{b} 2\sqrt{b}\,\sqrt{b-x}\,dx\right]$$

$$4\sqrt{2}\,.\,\frac{2}{3}\left[(x+a)^{3/2}\right]_{-a}^{b-a} + 4\sqrt{b}\,.\left(-\frac{2}{3}\right)\left[(b-x)^{3/2}\right]_{b-a}^{b}$$

$$= \frac{8}{3}\sqrt{ab}\,.\,a + \frac{8}{3}\sqrt{ab}\,.\,b = \frac{8}{3}\sqrt{ab}\,(a+b)$$

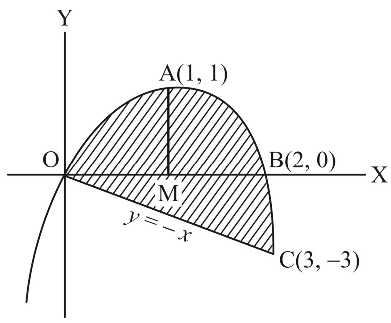


104.(C) $y = 2x - x^2$ is $(x-1)^2 = -(y-1)$

It represents a parabola with vertex at (1, 1).

$$A = \left|\int_0^3 (y_1 - y_2)\,dx\right|$$

$$= \left|\int_0^3 (2x - x^2) + x\,dx\right| = \left|\left(\frac{3x^2}{2} - x^3\right)\right| = \frac{9}{2}$$



105.(B) $\frac{dy}{dx} = \frac{d}{dx}\left[\int_0^x \sqrt{\sin x}\,dx\right] = \sqrt{\sin x} \Rightarrow \frac{dy}{dx}\Big|_{x=\pi/2} = \sqrt{\sin \pi/2} = 1$