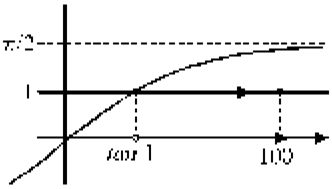
Level – 1	Daily Tutorial Sheet - 3

31.(C) $I = \int_{-a}^{a} (p\tan^3 x + q\cos^2 x + r\sin x)\,dx$

$$= \int_{o}^{a}\left([p\tan^3 x + q\cos^2 x + r\sin x] + [p\tan^3(-x) + q\cos^2(-x) + r\sin(-x)]\right)dx = 2\int_0^a q\cos^2 x\,dx$$

32.(C) $I = \int_0^{100} [\tan^{-1} x]\,dx = 0 + 1 \times (100 - \tan 1)$



33.(BCD) $\int_{\alpha}^{2\alpha} \cos 2x\,dx = \frac{1}{2}\sin 2x\Big|_{\alpha}^{2\alpha} = \frac{\sin 4\alpha - \sin 2\alpha}{2} = -\sin\alpha$

$sin4\alpha = sin2\alpha - 2sin\alpha$; $\quad 4sin\alpha\ cos\alpha\, cos2\alpha = 2sin\alpha\, cos\alpha - 2sin\alpha$

$\Rightarrow \quad 2cos\alpha\, cos2\alpha = 2cos\alpha - 1 \qquad$ or $\qquad (2sin\alpha = 0 \Rightarrow \alpha = -\pi, 0)$ . . . . .(i)

$\Rightarrow \quad 4cos^3\alpha - 3cos\alpha + 1 = 0 \quad \Rightarrow \quad (cos\alpha + 1)\,(2cos\alpha - 1)^2 = 0$

$\Rightarrow \quad cos\alpha = -1$ or $\qquad cos\alpha = \frac{1}{2}$

$\alpha = -\pi$ . . . . .(ii) ; $\alpha = \frac{-\pi}{3}$ . . . . .(iii)

taking union of (i), (ii) and (iii) $\quad \alpha = -\pi, \frac{\pi}{3}, 0$

34.(D) $\int_{1/e}^{1} -\ell nx\,dx + \int_1^e \ell nx\,dx + x = |-x\ \ell nx + x|_{1/e}^{1} + |x\ \ell nx - x|_1^e = 1 - \left(\frac{1}{e} + \frac{1}{e}\right) + (e - e) - (0 - 1) = 2 - \frac{2}{e}$

35.(B) $\int_{-1}^{10} \operatorname{sgn}(\{x\})\,dx = 11\int_0^1 \operatorname{sgn}(\{x\})\,dx = 11\int_0^1 1\,dx = 11$

36.(AB) $3a(1) + 2b(1) + c = 0$; $3a(25) + 2b(5) + c = 0 \Rightarrow a = -1, b = 9, c = -15$

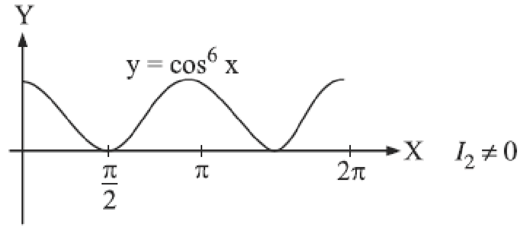
$$\left|\frac{ax^4}{4} + \frac{bx^3}{3} + \frac{cx^2}{2}\right|_{-1}^{1} = 6 \Rightarrow \frac{2b}{3} = 6 \Rightarrow b = 9$$

37.(C) $I_1 = \int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x\cos x}\,dx$ (apply $(a - x)$ property)

$$\Rightarrow I_1 = \int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \sin x\ \cos x}\,dx \quad \Rightarrow \quad 2I_1 = 0 \quad \Rightarrow \quad I_1 = 0$$

$I_2 = \int_0^{2\pi} \cos^6 x \neq 0 \qquad$ (from the graph)

$$I_3 = \int_{-\pi/2}^{\pi/2} \sin^3 x\, dx = 0 \quad [\text{as } \sin^3 x \text{ is an odd function}]$$



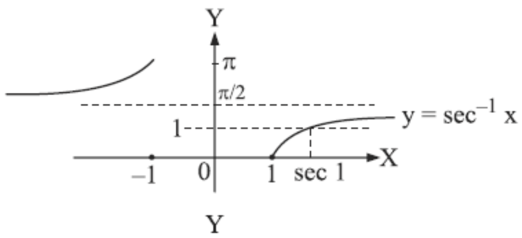
$$I_4 = \int_0^1 \ln\left(\frac{1}{x} - 1\right) dx \text{ (apply } (a-x) \text{ property)}$$

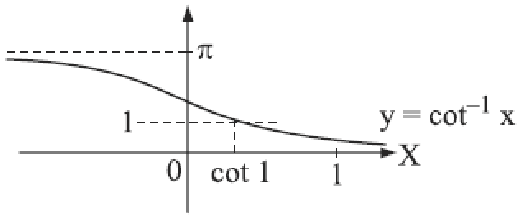
$$I_4 = \int_0^1 \ln\left(\frac{x}{1-x}\right) dx \quad \Rightarrow \quad 2I_4 = \int_0^1 \ln 1\, dx = 0$$

38.(A) $\left[\cot^{-1} x\right] = 0\ \forall x > 1$

$\left[\sec^{-1} x\right] = 0\ \forall 1 < x < \sec 1$

$$\int_1^{10\pi} [\sec^{-1} x]\, dx = \int_1^{\sec 1} 0\, dx + \int_{\sec 1}^{10\pi} 1\, dx = 10\pi - \sec 1$$





39.(C) As we have $\sin^{-1}\beta$ and $\cos^{-1}\alpha$ defined,
If $\alpha, \beta \in [0, 1]$

$\cos(\cos^{-1} x) = x\ \forall x\, [-1, 1]$; $\sin(\sin^{-1} x) = x\ \forall x \in [-1, 1]$

$$\int_{\cos\cos^{-1}\beta}^{\sin\sin^{-1}\beta} \left|\frac{\cos(\cos^{-1} x)}{\sin(\sin^{-1} x)}\right| dx = \int_\alpha^\beta 1\, dx = \beta - \alpha$$

40.(B) Put $\sin^2 x = t$; $I = \frac{1}{2}\int_0^{1/2} \frac{dt}{(1-t)+t^2} = \frac{1}{2}\int_0^{1/2} \frac{dt}{\left(t-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{\sqrt{3}}\left[\tan^{-1}\frac{t-\frac{1}{2}}{\sqrt{3}} \times 2\right]_0^{1/2} = \frac{\pi}{6\sqrt{3}}$

41.(B) $I_{n+1} = \int_0^{\pi/4} \tan^{n+1}\theta\, d\theta = \int_0^{\pi/4} \tan^{n-1}\theta \sec^2\theta\, d\theta - \int_0^{\pi/4} \tan^{n-1}\theta\, d\theta$

$$I_{n+1} + I_{n-1} = \left.\frac{\tan\theta}{n}\right|_0^{\pi/4} \quad \Rightarrow \quad n\left(I_{n+1} + I_{n-1}\right) = 1.$$

42.(B) $I = \int_{-\pi}^{\pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}}\, dx$

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx \quad \Rightarrow \quad I = \int_{-\pi}^{\pi} \frac{e^{-\sin x}}{e^{\sin x} + e^{-\sin x}}\, dx \quad \Rightarrow \quad 2I = \int_{-\pi}^{\pi} dx \quad \Rightarrow \quad I = \pi$$

43.(C) $[x^2] = 0 \quad x \in (-1, 1)$ and $I = \int_{-1}^{1} \underbrace{\log\left(\frac{2+x}{2-x}\right)}_{\text{odd function}}$

44.(A) $x = -2, -1, 1$ critical point

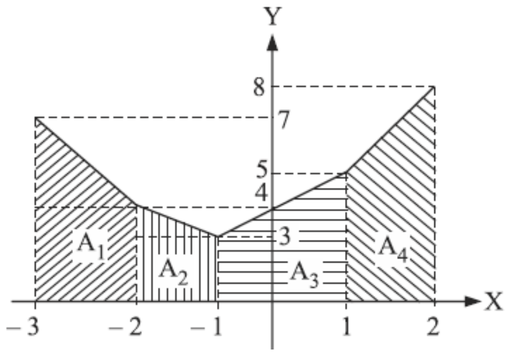
Case I : $x \le -2$ $\qquad y = -3x - 2$

Case II : $-2 \le x \le -1$ $\qquad y = 2 - x$

Case III : $-1 \le x \le 1$ $\qquad y = x + 4$

Case IV : $x \ge 1$ $\qquad y = 3x + 2$

$$A_1 + A_2 + A_3 + A_4 = 1\left(\frac{7+4}{2}\right) + 1\left(\frac{4+3}{2}\right) + 2\left(\frac{3+5}{2}\right) + 1\left(\frac{5+8}{2}\right) = \frac{11+7+16+13}{2} = \frac{47}{2}$$



45.(B) $\int_0^1 [x^2]\, dx + \int_1^{\sqrt{2}} [x^2]\, dx + \int_{\sqrt{2}}^{1.7} [x^2]\, dx = \left(\sqrt{2} - 1\right) + 2\left(1.7 - \sqrt{2}\right) = 2.4 - \sqrt{2}$.