Level – 1	**Daily Tutorial Sheet - 2**

16.(B) $I=\int_0^{\pi} f(x)\,.\,\sin x\,dx+\int_0^{\pi} f''(x)\sin x\,dx$

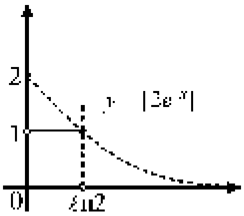
$=\left|f(x)(-\cos x)\right|_0^{\pi}-\int_0^{\pi} f'(x)(-\cos x)dx+\left|\sin x\left(f'(x)\right)\right|_0^{\pi}-\int_0^{\pi}\cos x\,.\,f'(x)dx=5=\left|f'(x)\sin x-f(x)\cos x\right|_0^{\pi}=5$

$\Rightarrow \quad f(\pi)+f(0)=5 \quad \Rightarrow \quad f(0)=3$

17.(C) $\int_0^1 x\,.\,f''(2x)dx=\left|x\,.\,\frac{f'(2x)}{2}\right|_0^1-\int_0^1 1\,.\,\frac{f'(2x)}{2}dx=\left|x\,.\,\frac{f'(2x)}{2}-\frac{f(2x)}{4}\right|_0^1=\frac{f'(2)}{2}-\frac{f(2)}{4}+\frac{f(0)}{4}=\frac{5}{2}-\frac{3}{4}+\frac{1}{4}=2$

18.(B) $\int_0^{4\pi}\frac{dx}{\cos^2 x(2+\tan^2 x)}=\int_0^{4\pi}\frac{\sec^2 x}{2+\tan^2 x}dx=8\int_0^{\frac{\pi}{2}}\frac{\sec^2 x}{2+\tan^2 x}dx=8\int_0^{\frac{\pi}{2}}\frac{dt}{2+t^2}=4\sqrt{2}\tan^{-1}\frac{t}{\sqrt{2}}\Big|_0^{\infty}=2\sqrt{2}\pi$

19.(C) $I=\int_0^{\ell n 2} 1\,.\,dx+\int_{\ell n 2}^{\infty} 0\,.\,dx=\ell n 2$



20.(A) $\int_1^4 (\{x\})^{[x]}dx=\int_1^2 (x-1)dx+\int_2^3 (x-2)^2dx+\int_3^4 (x-3)^3dx$

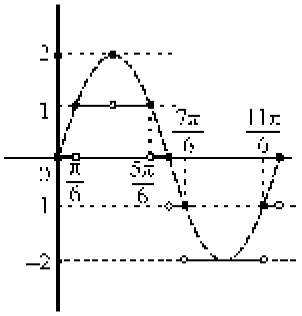
$=\int_0^1 t\,dt+\int_0^1 t^2dt+\int_0^1 t^3dx=\left[\frac{t^2}{2}+\frac{t^3}{3}+\frac{t^4}{4}\right]_0^1=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=\frac{13}{12}$

21.(A) $S=\int_0^1\left[f(x)+f(1+x)+f(2+x)+\ldots\ldots.+f(n-1+x)\right]dx$

$=\int_0^1 f(x)dx+\int_1^2 f(x)dx+\int_2^3 f(x)dx+\ldots\ldots.+\int_{n-1}^{n} f(x)dx=\int_0^n f(x)dx$

22.(A) $I=\int_0^{\pi/6} 0\,.dx+\int_{\pi/6}^{\pi/2} 1.dx+\int_{\pi/2}^{5\pi/6} 1.dx+\int_{5\pi/6}^{\pi} 0.dx+\int_{\pi}^{7\pi/6} -1dx+\int_{7\pi/6}^{11\pi/6} -2dx+\int_{11\pi/6}^{2\pi} -1dx=\frac{2\pi}{3}-\frac{2\pi}{6}-2\frac{4\pi}{6}=-\pi$

$\Rightarrow \quad |\,I\,|=\pi$



23.(C) $I = \int_0^{\pi/2}\left(\frac{e^{\sin x}.\cos x}{1+e^{\tan x}} + \frac{e^{|\sin(-x)|}\cos x}{1+e^{\tan(-x)}}\right)dx = \int_0^{\pi/2} e^{\sin x}.\cos x\, dx = \int_0^1 e^t dt = e - 1$

24.(A) $I = \int_0^2 [x^2]dx - \int_0^2 1.dx = \int_0^1 0dx + \int_1^{\sqrt{2}} 1dx + \int_{\sqrt{2}}^{\sqrt{3}} 2dx + \int_{\sqrt{3}}^{2} 3dx - 2 = 3 - \sqrt{3} - \sqrt{2}$

25.(B) $I = \int_a^b \frac{x^n}{x^n + (16-x)^n}dx = \int_a^b \frac{(a+b-x)^n}{(a+b-x)^n + (16-a-b+x)^n}dx$

$I = \int_a^b \frac{x^n}{x^n + (16-x)^n}dx = \int_a^b \frac{(16-x)^n}{(16-x)^n + x^n}dx$ [only way $I = 6\ \forall\ n \in R$ possible when $a + b = 16$]

$\Rightarrow \quad I = \frac{1}{2}\int_a^b \frac{x^n + (16-x)^n}{x^n + (16-x)^n}dx = \frac{b-a}{2} = 6$; $\quad a + b = 16$ and $b - a = 12 \Rightarrow a = 2, b = 14, x \in R$

26.(D) $af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5 \Rightarrow a^2 f(x) + abf\left(\frac{1}{x}\right) = \frac{a}{x} - 5a$

Replace x by $\frac{1}{x}$ $\Rightarrow$ $b^2 f(x) + baf\left(\frac{1}{x}\right) = bx - 5b$ $\Rightarrow$ $(a^2 - b^2)\, f(x) = \frac{a}{x} - bx - 5(a-b)$

27.(B) $f(x) = -f(a-x)$

$I = \int_0^a \frac{dx}{1+e^{f(x)}}dx = \int_0^a \frac{dx}{1+e^{f(a-x)}} = \int_0^a \frac{dx}{1+e^{-f(x)}} = \int_0^a \frac{e^{f(x)}}{1+e^{f(x)}}dx \Rightarrow I = \frac{1}{2}\int_0^a \frac{1+e^{f(x)}}{1+e^{f(x)}}dx = \frac{1}{2}(a-0) = \frac{a}{2}$

28.(B) $I = \int_{-\pi/4}^{\pi/4}\left(a|\sin x| + \frac{b\sin x}{1+\cos x} + c\right)dx = 0$

$\Rightarrow \quad \int_0^{\pi/4}\left[\left(a|\sin x| + \frac{b\sin x}{1+\cos x} + c\right)\right]dx + \int_0^{\pi/4}\left(a|\sin(-x)| + \frac{b\sin(-x)}{1+\cos(-x)} + c\right)dx = 0$

$\Rightarrow \quad \int_0^{\pi/4}(2a|\sin x| + 2c)dx = 0 \Rightarrow 2a\left(\frac{-1}{\sqrt{2}} + 1\right) + 2c\,.\,\frac{\pi}{4} = 0$

29.(D) $I = \int_1^3 |(2-x)\log_e x \mid dx = \int_1^3 |(2-x)| \log_e x \; dx \quad [\because \log_e x > 0 \;\forall\; x \in (1,3)] = \int_1^2 (2-x)\log x dx + \int_2^3 (x-2)\log x \, dx$

Use : $\int \log x \, dx = x \log x - x + c$ and $\int x \log x \, dx = \frac{x^2}{2}\log x - \frac{x^2}{4} + c$

$$\Rightarrow \qquad I = 2\left[x\log x - x\right]_1^2 - \left[\frac{x^2}{2}\log x\right]_1^2 + \left[\frac{x^2}{4}\right]_1^2 + \left[x^2\frac{\log x}{2}\right]_2^3 - \left[\frac{x^2}{4}\right]_2^3 - \left[2x \log x\right]_2^3 + 2\left[x\right]_2^3$$

$$= 4\ell n 2 - \frac{3}{2}\ell n 3 - \frac{1}{2} = \ell n \frac{16}{3\sqrt{3}} - \frac{1}{2}$$

30.(C) $I = \int_0^\pi \frac{1}{1+3^{\cos x}} dx = \int_0^\pi \frac{dx}{1+3^{\cos(\pi - x)}} = \int_0^\pi \frac{dx}{1+3^{-\cos x}} = \int_0^\pi \frac{3^{\cos x}}{3^{\cos x}+1} dx \quad \Rightarrow \quad I = \frac{1}{2}\int_0^\pi \left(\frac{1+3^{\cos x}}{1+3^{\cos x}}\right) dx = \frac{\pi}{2}$