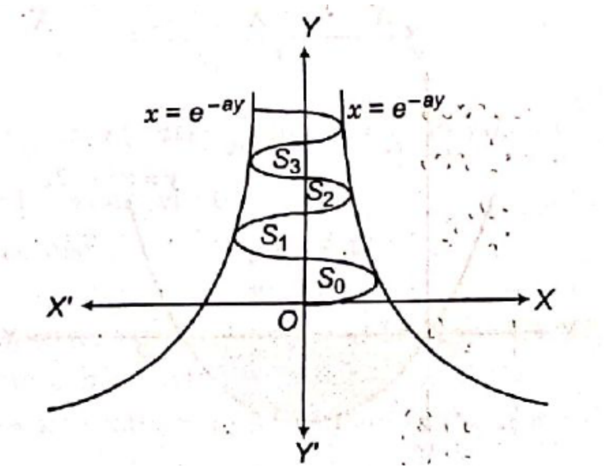
Level - 3	Daily Tutorial Sheet-19

216. **(i)** Given, $x = (\sin by)e^{-ay}$

Now, $-1 \le \sin$ by ≤ 1

$\Rightarrow \quad -e^{-ay} \le e^{-ay}$ sin by $\le e^{-ay}$

$\Rightarrow \quad -e^{-ay} \le x \le e^{-ay}$



In this case, if we take *a* and *b* positive, the values $-e^{-ay}$ and e^{-ay} becomes left bond and right bond of the curve and due to oscillating nature of sin *by*, it will oscillate between $x = e^{-ay}$ and $x = -e^{ay}$

Now, $S_j = \int_{jmb}^{(j+1)r^{th}} \sin by \cdot e^{-ay} dy$

$$\left[\begin{array}{l} \text{Since, } I = \int \sin by \cdot e^{-ay} dy \\ I = \dfrac{-e^{-ay}}{a^2+b^2}(a \sin by + b \cos by) \end{array}\right]$$

$$\therefore \quad S_j = \left|\frac{-1}{a^2+b^2}\left[e^{\frac{-a(j+1)\pi}{b}}\{a \sin (j+1)\pi + b \cos (j+1)\pi\} - e^{\frac{-aj\pi}{b}}(a \sin j\pi + b \cos j\pi)\right]\right|$$

$$S_j = \left|-\frac{1}{a^2+b^2}\left(e^{-\frac{a}{b}\pi}+1\right)\right|$$

$[\because \quad (-1)^{j+2} = (-1)^2(-1)^j = (-1)^j]$

$$= \frac{be^{-\frac{a}{b}j\pi}}{a^2+b^2}\left(a^{-\frac{a}{b}\pi}+1\right)$$

$$\therefore \qquad \frac{S_j}{S_{j-1}} = \frac{\dfrac{be^{-\frac{a}{b}j\pi}\left(e^{\frac{-a\pi}{b}}+1\right)}{a^2+b^2}}{\dfrac{be^{-\frac{a}{b}(j-1)\pi}\left(e^{\frac{-a\pi}{b}}+1\right)}{a^2+b^2}} = \frac{e^{-\frac{a}{b}j\pi}}{a^{-\frac{a}{b}(j-1)\pi}}$$

$= e^{-\frac{b}{a}\pi} =$ constant $\qquad \Rightarrow \qquad S_0, S_1, S_2, \ldots S_j$ form a GP.

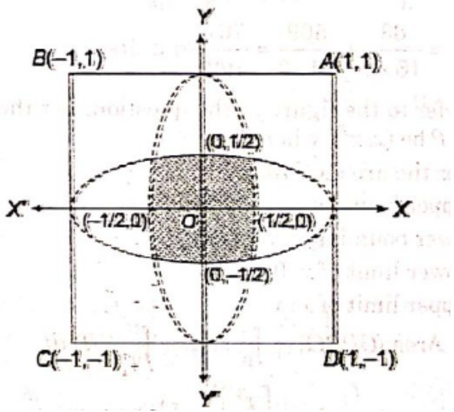
For $a = -1$ and $b = \pi$

$$S_j = \frac{\pi \cdot e^{\frac{1}{\pi}\cdot\pi j}}{(1+\pi^2)}\left(e^{\frac{1}{\pi}\cdot\pi}+1\right) = \frac{\pi \cdot e^j}{(1+\pi^2)}(1+e)$$

$$\Rightarrow \qquad \sum_{j=0}^{n} S_j = \frac{\pi\cdot(1+e)}{(1+\pi)^2}\sum_{j=0}^{n} e^j = \frac{\pi(1+e)}{(1+\pi^2)}(e^0+e^1+\ldots+e^n)$$

$$= \frac{\pi(1+e)}{(1+\pi^2)}\cdot\frac{(e^{n+1}-1)}{e-1}$$

(ii) The equation of the sides of the square are as follow:
$AB : y = 1,\ BC : x = -1,\ CD : y = -1,\ DA : x = 1$



Let the region be S and (x, y) is any point inside it.
Then, according to given conditions.

$\sqrt{x^2+y^2} < |1-x|, |1+x|, |1-y|, |1+y|$

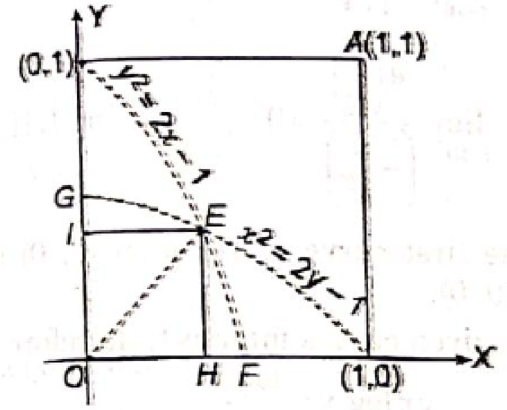
$\Rightarrow \qquad x^3+y^2 < (1-x)^2, (1+x)^2, (1-y)^2, (1+y)^2$

$\Rightarrow \qquad x^2+y^2 < x^2-2x+1, x^2+2x+1, y^2-2y+1, y^2+2y+1$

$\Rightarrow \qquad y^2 < 1-2x, y^2 < 1+2x, x^2 < 1-2y$ and $x^2 < 2y+1$

Now, in $y^2 = 1-2x$ and $y^2 = 1+2x$, the first equation represents a parabola with vertex at $\left(\frac{1}{2}, 0\right)$ and second equation represents a parabola with vertex $(-1/2, 0)$ and in $x^2 = 1-2y$ and $x^2 = 1+2y$, the first

equation represents a parabola with vertex at (0, 1/2) and second equation represents a parabola with vertex at (0, –1/2).
Therefore, the region S is lying inside the four parabolas
$y^2 = 1-2, y^3 = 1+2x, ^2 = 1+2y, x^2 = 1-2y$



Where, S is the shaded region.
Now, S is symmetrical in all four quadrants, therefore
$S = 4 \times$ Area lying in the first quadrant.

Now, $y^2 = 1-2x$ and $x^2 = 1-2y$ intersect on the line $y = x$.

The point of intersection is $E\left(\sqrt{2}-1, \sqrt{2}-1\right)$.

Area of the region EFO
= Area of ΔOEH + Area of HEFH

$$= \frac{1}{2}\left(\sqrt{2}-1\right)^2 + \int_{\sqrt{2}-1}^{1/2} \sqrt{1-2x}\, dx$$

$$= \frac{1}{2}\left(\sqrt{2}-1\right)^3 + \left[\left(1-2x\right)^{1/2} \frac{2}{3} \cdot \frac{1}{2}(-1)\right]_{\sqrt{3}-1}^{1/2} = \frac{1}{2}\left(2+1-2\sqrt{2}\right) + \frac{1}{3}\left(1+2-2\sqrt{2}\right)^{3/2}$$

$$= \frac{1}{2}\left(3-2\sqrt{2}\right) + \frac{1}{3}\left(3-2\sqrt{2}\right)^{3/2} = \frac{1}{2}\left(3-2\sqrt{2}\right) + \frac{1}{3}\left[\left(\sqrt{2}-1\right)^2\right]^{3/2}$$

$$= \frac{1}{2}\left(3-2\sqrt{2}\right) + \frac{1}{3}\left[2\left(\sqrt{2}-1-3\sqrt{2}\left(\sqrt{2}-1\right)\right]\right.$$

$$= \frac{1}{2}\left(3-2\sqrt{2}\right) + \frac{1}{3}\left[5\sqrt{2}-7\right] = \frac{1}{6}\left[9-6\sqrt{2}+10\sqrt{2}-14\right] = \frac{1}{6}\left[4\sqrt{2}-5\right] \text{ square units}$$

Similarly, area $OEGO = \frac{1}{6}\left(4\sqrt{2}-5\right)$ square units

Therefore, area of S lying in first quadrant $= \frac{2}{6}\left(4\sqrt{2}-5\right) = \frac{1}{3}\left(4\sqrt{2}-5\right)$ square units

Hence, $S = \frac{4}{3}\left(4\sqrt{2}-5\right) = \frac{1}{3}\left(16\sqrt{2}-20\right)$ square units.

Paragraph for Question 217 (i)
(i) a.(A); (b).(B)
Paragraph for Question 217 (ii)
(ii) (a).(C); (b).(D); (c).(A)
Paragraph For Question 218. (i)
(i) (a).(A); (b).(A); (c).(C)
(ii) (9)

219. **(i) (9)**
(ii) (2)
(iii) (2)
220 (i). 1; (ii) 4; (iii) 4
221. **(i) (2)**
(ii) (AB)
(iii) (6)