Date Planned : __ / __ / __	Daily Tutorial Sheet - 19	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	Level – 3	Exact Duration :________

216. (i) Let $b \neq 0$ and for $j = 0, 1, 2 \ldots, n$. If S_j is the area of the region bounded by the Y-axis and the curve $xe^{ay} = \sin by, \frac{j\pi}{b} \le y \le \frac{(j+1)\pi}{b}$. Then, show that $S_0, S_1, S_2, \ldots, S_n$ are in geometric progression. Also, find their sum for $a = -1$ and $b = \pi$.

(ii) Consider a square with vertices at (1, 1), (–1, 1), (–1, –1) and (1, –1). If S is the region consisting of all points inside the square which are nearer to the origin than to any edge. Then, sketch the region S and find its area.

Paragraph for Question 217 (i)

Let $f(x) = \int x^2 \cos^2 x(2x + 6\tan x - 2x\tan^2 x)dx$ and $f(x)$ passes through the point $(\pi, 0)$

(a) If $f : R - (2n+1)\frac{\pi}{2} \to R$ then $f(x)$ be a:

(A) even function **(B)** odd function

(C) neither even nor odd **(D)** even as well as odd both

(b) The number of solution(s) of the equation $f(x) = x^3$ in $[0, 2\pi]$ be:

(A) 0 **(B)** 3 **(C)** 2 **(D)** None of these

Paragraph for Question 217 (ii)

Let $f(x)$ be a twice differentiable function defined on $(-\infty, \infty)$ such that $f(x) = f(2-x)$ and $f'\left(\frac{1}{2}\right) = f'\left(\frac{1}{4}\right) = 0$.

then:

(a) The minimum number of values where $f''(x)$ vanishes on [0, 2] is:

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5

(b) $\int_{-1}^{1} f'(1+x)x^2 e^{x^2} dx$ is equal to:

(A) 1 **(B)** π **(C)** 2 **(D)** 0

(c) $\int_0^1 f(1-t)e^{-\cos \pi t} dt - \int_1^2 f(2-t)e^{\cos \pi t} dt$ is equal to:

(A) $\int_0^2 f'(t)e^{\cos \pi t} dt$ **(B)** 1 **(C)** 2 **(D)** π

Paragraph for Question 218. (i)

Let $f(x)$ be function defined on $[0,1]$ such that $f(1)=0$ and for any $a \in (0,1]$

$\int_0^a f(x)dx - \int_a^1 f(x)dx = 2f(a) + 3a + b$ where b is constant.

(a) $b =$

(A) $\frac{3}{2e} - 3$ **(B)** $\frac{3}{2e} - \frac{3}{2}$ **(C)** $\frac{3}{2e} + 3$ **(D)** $\frac{3}{2e} + \frac{3}{2}$

(b) The length of the subtangent of the curve $y = f(x)$ at $x = 1/2$ is :

(A) $\sqrt{e} - 1$ **(B)** $\frac{\sqrt{e} - 1}{2}$ **(C)** $\sqrt{e} + 1$ **(D)** $\frac{\sqrt{e} + 1}{2}$

(c) $\int_0^1 f(x)dx =$

(A) $\frac{1}{e}$ **(B)** $\frac{1}{2e}$ **(C)** $\frac{3}{2e}$ **(D)** $\frac{2}{e}$

(ii) Let $I = \int_0^{\pi} x^6 (\pi - x)^8 dx$, then $\frac{\pi^{15}}{\left({}^{15}C_9\right) I} =$

219. (i) If $f(n) = \frac{1}{\pi} \int_0^{\pi/2} \frac{\sin^2(n\theta)d\theta}{\sin^2\theta}, n \in N$, then evaluate $\frac{f(15) + f(3)}{f(12) - f(10)}$.

(ii) if $\int_a^b |\sin x| dx = 8$ and $\int_0^{a+b} |\cos x| dx = 9$ then the value of $\frac{1}{\sqrt{2x}} \left| \int_a^b x \sin x dx \right|$ is :

(iii) If $\int_0^2 \left(3x^2 - 3x + 1\right)\cos\left(x^3 - 3x^2 + 4x - 2\right)dx = a\sin(b)$, where a and b are positive integers find the value of $(a+b)$.

220. (i) Let $f(x) = \int_0^x e^{x-y} f'(y)dy - \left(x^2 - x + 1\right)e^x$. Then number of roots of equations $f(x) = 0$ is

(ii) For a positive integer n, let $I_n = \int_{-\pi}^{\pi} \left(\frac{\pi}{2} - |x|\right)\cos nx\, dx$

Find the value of $[I_1 + I_2 + I_3 + I_4]$ where $[.]$ denotes greatest integer function.

(iii) If M be the maximum value of $72\int_0^y \sqrt{x^4 + \left(y - y^2\right)^2}\, dx$ from $y \in [0,1]$, then find $\frac{M}{6}$.

221. (i) Let a differentiable function $f(x)$ satisfies $f(x).f'(-x) = f(-x).f'(x)$ and $f(0)=1$. Find the value of $\int_{-2}^{2} \frac{dx}{1+f(x)}$.

*(ii) $T_n = \sum_{r=2n}^{3n-1} \frac{r}{r^2+n^2}, S_n = \sum_{r=2n+1}^{3n} \frac{r}{r^2+n^2}$, then $\forall n \in \{1,2,3,...\}$:

(A) $T_n > \frac{1}{2}\ln 2$ **(B)** $S_n < \frac{1}{2}\ln 2$ **(C)** $T_n < \frac{1}{2}\ln 2$ **(D)** $S_n > \frac{1}{2}\ln 2$

(iii) If the area bounded by circle $x^2+y^2=4$, the parabola $y=x^2+x+1$ and the curve $y=\left[\sin^2\frac{x}{4}+\cos\frac{x}{4}\right]$, (where [ ] denotes the greatest integer function) and x-axis is $\left(\sqrt{3}+\frac{2\pi}{3}-\frac{1}{k}\right)$, then the numerical quantity k should be :