Date Planned : __ / __ / __	Daily Tutorial Sheet - 17	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	Level – 3	Exact Duration :________

204. (i) Let $I_n = \int_0^{\pi/2} (\sin x + \cos x)^n dx (n \geq 2)$. Then the value of $n.\ I_n - 2(n-1)I_{n-2}$ is:

(A) 5 **(B)** 9 **(C)** 2 **(D)** 7

(ii) If $\int_0^{\infty} e^{-ax} dx = \frac{1}{a}$, then $\int_0^{\infty} x^n e^{-ax} dx$ is:

(A) $\frac{(-1)^n n!}{a^{n+1}}$ **(B)** $\frac{(-1)^n (n-1)!}{a^n}$ **(C)** $\frac{n!}{a^{n+1}}$ **(D)** None of these

205. (i) Let $n \geq 1, n \in Z$. The real number $a \in (0, 1)$ that minimizes the integral $\int_0^1 |x^n - a^n| dx$ is:

(A) $\frac{1}{2}$ **(B)** 2 **(C)** 1 **(D)** $\frac{1}{3}$

(ii) $\dfrac{626\int_0^{\infty} e^{-x} \sin^{25} x\, dx}{\int_0^{\infty} e^{-x} \sin^{23} x\, dx}$ is equal to:

(A) 300 **(B)** 625 **(C)** 600 **(D)** 1200

206. If $f(x + f(y)) = f(x) + y\ \forall x, y \in R$ and $f(0) = 1$, then prove that $\int_0^2 f(2-x)dx = 2\int_0^1 f(x)dx$.

207. (i) Evaluate: $\int_0^2 \frac{dx}{(17 + 8x - 4x^2)[e^{6(1-x)} + 1]}$.

(ii) If $I = \int_1^2 \frac{dx}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$, then:

(A) $\frac{1}{3} < I < \frac{1}{\sqrt{8}}$ **(B)** $\frac{1}{4} < I < \frac{1}{3}$ **(C)** $\frac{1}{4} < I < 0$ **(D)** None of these

208. Consider the function $h(x) = \frac{g^2(x)}{2} + 3x^3 - 5$, where $g(x)$ is a continuous and differentiable function. It is given that $h(x)$ is a monotonically increasing function and $g(0) = 4$. Then which of the following is not true.

(A) $g^2(1) > 10$ **(B)** $h(5) > 3$ **(C)** $h\left(\frac{5}{2}\right) < 2$ **(D)** $g^2(-1) < 22$

Paragraph for Question 209

Let m, n be two positive real numbers and define $f(n) = \int_0^{\infty} x^{n-1} e^{-x} dx$ and $g(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx$.

It is known than $f(n)$ for $n > 0$ is finite and $g(m, n) = g(n, m)$ for $m, n > 0$.

(i) $\int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx =$

(A) $g(m, n)$ **(B)** $g(m-1, n)$ **(C)** $g(m-1, n-1)$ **(D)** $g(m, n-1)$

(ii) $\int_0^1 x^m \left(\log_e \frac{1}{x}\right)^n dx =$

(A) $\frac{f(n+1)}{(m+1)^n}$ **(B)** $\frac{f(n)}{(m+1)^{n+1}}$ **(C)** $\frac{f(n+1)}{(m+1)^{n+1}}$ **(D)** $g(m+1, n+1)$

(iii) $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx =$

(A) $g(n, m)$ **(B)** $g(m-1, n+1)$ **(C)** $g(m-1, n-1)$ **(D)** $g(m+1, n-1)$