

Date Planned : __ / __ / __	Daily Tutorial Sheet - 4	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	Level - 1	Exact Duration : _____

46. If $I_1 = \int_0^{\pi/2} \ln(\sin x) dx$; $I_2 = \int_{-\pi/4}^{\pi/4} \ln(\sin x + \cos x) dx$, then:
- (A) $I_1 = 2I_2$ (B) $I_2 = 2I_1$ (C) $I_1 = 4I_2$ (D) $I_2 = 4I_1$
47. $\int_0^1 \ln\left(\sin \frac{\pi x}{2}\right) dx$ is equal to:
- (A) $\ln 2$ (B) $-\ln 2$ (C) $\pi \ln 2$ (D) $-\pi \ln 2$
48. If $\int_{1/2}^2 \frac{1}{x} \operatorname{cosec}^{101}\left(x - \frac{1}{x}\right) dx = k$ then the value of k is:
- (A) 1 (B) $1/2$ (C) 0 (D) $1/101$
49. The value of $\int_0^{16\pi} \frac{1}{3} |\sin x| dx$ is:
- (A) $17/2$ (B) $19/2$ (C) $21/2$ (D) None of these
50. The value of $\int_0^{1000} e^{x-[x]} dx$, is ([.] denotes the greatest integer function):
- (A) $1000e$ (B) $1000(e-1)$ (C) $1001(e-1)$ (D) None of these
51. The expression $\frac{\int_0^n [x] dx}{\int_0^n \{x\} dx}$, where $[x]$ and $\{x\}$ are integral and fractional part of x and $n \in N$, is equal to:
- (A) $n+1$ (B) $\frac{1}{n}$ (C) n (D) $n-1$
52. Evaluate: (a) $\int_a^b e^x dx$; (b) $\int_a^b \sin x dx$ using limit of a sum.
53. $\lim_{n \rightarrow \infty} \frac{1}{n} \left(1 + \sqrt{\frac{n}{n+1}} + \sqrt{\frac{n}{n+2}} + \dots + \sqrt{\frac{n}{4n-3}} \right)$ is equal to:
- (A) 1 (B) 2 (C) 3 (D) None of these
54. $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=n+1}^{2n} \log_e \left(1 + \frac{r}{n} \right)$ equals:
- (A) $\log\left(\frac{27}{4e}\right)$ (B) $\log\left(\frac{27}{e^2}\right)$ (C) $\log\left(\frac{4}{e}\right)$ (D) None of these

55. The value of $\lim_{n \rightarrow \infty} \left(\frac{1}{1^3 + n^3} + \frac{2^2}{2^3 + n^3} + \dots + \frac{n^2}{n^3 + n^3} \right)$ is:
- (A) $\frac{1}{3}$ (B) $\frac{1}{3} \log(2)$ (C) $\frac{1}{2} \log(3)$ (D) $\frac{1}{3} \log(3)$
56. Evaluate: $\lim_{n \rightarrow \infty} \left[\frac{\sqrt{n}}{(3 + 4\sqrt{n})^2} + \frac{\sqrt{n}}{\sqrt{2}(3\sqrt{2} + 4\sqrt{n})^2} + \frac{\sqrt{n}}{\sqrt{3}(3\sqrt{3} + 4\sqrt{n})^2} + \dots + \frac{1}{49n} \right]$
- (A) $\frac{1}{14}$ (B) $\frac{1}{16}$ (C) $\frac{1}{18}$ (D) $\frac{1}{20}$
57. Find the limit, when $n \rightarrow \infty$ of $\frac{1}{\sqrt{(2n-1)^2}} + \frac{1}{\sqrt{(4n-2)^2}} + \frac{1}{\sqrt{(6n-3)^2}} + \dots + \frac{1}{n}$.
- (A) $\frac{\pi}{2}$ (B) π (C) $-\frac{\pi}{2}$ (D) None of these
58. The value of $\lim_{n \rightarrow \infty} \left[\frac{n}{n^2} + \frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \dots + \frac{n}{n^2 + (n-1)^2} \right]$ is:
- (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) π (D) $-\frac{\pi}{4}$
59. $\lim_{n \rightarrow \infty} \left(\frac{1}{1+n^3} + \frac{4}{8+n^3} + \dots + \frac{r^2}{r^3+n^3} + \dots + \frac{1}{2n} \right)$
- (A) $\frac{1}{2} \log 2$ (B) $\frac{1}{3} \log 2$ (C) $\frac{1}{4} \log 2$ (D) $\log 2$
60. $\lim_{n \rightarrow \infty} \left[\frac{n+1}{n^2+1^2} + \frac{n+2}{n^2+2^2} + \dots + \frac{1}{n} \right]$
- (A) $\frac{1}{2} \log 2$ (B) $-\frac{\pi}{4} + \frac{1}{2} \log 2$ (C) $\pi + \frac{1}{2} \log 2$ (D) $\frac{\pi}{4} + \frac{1}{2} \log 2$