

Integral Calculus - 2

Date Planned : __ / __ / __	Daily Tutorial Sheet - 1	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	Level - 1	Exact Duration : _____

Note (A) : Questions having asterisk marked against them may have more than one correct answer.

(B) : Questions having  (Symbol) marked against them have a video solution.

1. Evaluate: $\int_{\log 2}^{\log 3} \frac{e^x}{1+e^x} dx$.
2. Given the function: $f(x) = \begin{cases} x^2 & ; 0 \leq x \leq 1 \\ \sqrt{x} & ; 1 \leq x \leq 2 \end{cases}$. Evaluate $\int_0^2 f(x) dx$.
3. Evaluate the integral $I = \int_0^2 |x-1| dx$.
4. Evaluate: $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx$.
5. Show that $\int_0^{\pi} x f(\sin x) dx = \frac{\pi}{2} \int_0^{\pi} f(\sin x) dx$.
6. Show that $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx = \frac{\pi^2}{2ab}$.
7. Evaluate: $\int_0^{\pi/2} \frac{x \sin x \cos x}{\cos^4 x + \sin^4 x} dx$.
8. Prove that $\int_a^b f(x) dx = (b-a) \int_0^1 f[(b-a)x+a] dx$.
9. Evaluate: $\int_{-\sqrt{2}}^{\sqrt{2}} \frac{2x^7 + 3x^6 - 10x^5 - 7x^3 - 12x^2 + x + 1}{x^2 + 2} dx$.
10. Evaluate: $\int_{-2}^2 \frac{dx}{4+x^2}$ directly as well as by the substitution $x = 1/t$. If answers do not tally, then explain why?

11. If $\int_a^b f(x)dx = I_1$, $\int_a^b g(x)dx = I_2$ then:

(A) $\int_a^b (f(x) + g(x))dx = I_1 + I_2$

(B) $\int_a^b (f(x) \cdot g(x))dx = I_1 I_2$

(C) $\int_a^b \frac{f(x)}{g(x)}dx = \frac{I_1}{I_2}$

(D) $\int_a^b \frac{dx}{f(x)} = \frac{1}{I_2}$

12. $\int_{ac}^{bc} f(x)dx$, where $c \neq 0$, is also equal to:

(A) $c \int_a^b f(cx)dx$

(B) $\frac{1}{c} \int_a^b f\left(\frac{x}{c}\right)dx$

(C) $\frac{1}{c} \int_a^b f(cx)dx$

(D) $c \int_a^b f\left(\frac{x}{c}\right)dx$

13. $\int_0^{\pi/2n} \frac{dx}{1 + (\tan nx)^n}$ is equal to $n \in N$:

(A) $\frac{n\pi}{4}$

(B) $\frac{\pi}{2n}$

(C) $\frac{\pi}{4n}$

(D) $\frac{2\pi}{n}$

14. $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1 + e^x} dx$ is equal to:

(A) 1

(B) 2

(C) -1

(D) -2

15. $\int_0^{2\pi} [|\sin x| + |\cos x|]dx$, where $[.]$ denotes the greatest integer function, is equal to:

(A) π

(B) 2π

(C) $\pi / \sqrt{2}$

(D) $\pi\sqrt{2}$