

Integral Calculus - 1

Date Planned : __ / __ / __	Daily Tutorial Sheet - 13	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	Level - 3 	Exact Duration : _____

156. If $y = \sqrt{x^2 - x + 1}$ and for $n \geq 1$, $I_n = \int x^n / y \, dx$ and $aI_3 + bI_2 + cI_1 = x^2y$, then (a, b, c) is equal to:

- (A) $\left(\frac{3}{2}, \frac{1}{2}, -1\right)$ (B) $(1, -1, 1)$ (C) $\left(3, -\frac{5}{2}, 2\right)$ (D) $\left(\frac{1}{2}, -\frac{1}{2}, 1\right)$

157. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is a function such that $f(0) = 0$, $f'(0) = 3$ and it satisfies the relation

$$f\left(\frac{x+y}{3}\right) = \frac{f(x) + f(y)}{3} \text{ for all } x, y \in \mathbb{R} \text{ then } \int f(x) \, dx \text{ equals:}$$

- (A) $3x^2 + 3$ (B) $\frac{2x^2}{3} + 3$ (C) $\frac{3}{2}x^2 + c$ (D) $3x^2 + c$

158. $\int \frac{x^2 + 1}{x\sqrt{x^2 + 2x - 1}\sqrt{1 - x^2 - x}} \, dx$ is equal to:

- (A) $2 \sin^{-1} \sqrt{x - \frac{1}{x} + 2 + c}$ (B) $2 \cos^{-1} \sqrt{x - \frac{1}{x} + 2 + c}$
 (C) $\sin^{-1} \sqrt{x - \frac{1}{x} + 2 + c}$ (D) $\cos^{-1} \sqrt{x - \frac{1}{x} + 2 + c}$

159. (i) If $\int \frac{3 \tan\left(x - \frac{\pi}{4}\right)}{\cos^2 x \sqrt{\tan^3 x + \tan^2 x + \tan x}} = K \tan^{-1}(\sqrt{\tan x + 1 + \cot x}) + C$, then the value of K is:

- (A) 2 (B) 3 (C) 6 (D) 8

(ii) If $\int \frac{1 - (\cot x)^{2010}}{\tan x + (\cot x)^{2011}} \, dx = \frac{1}{k} \log_e |(\sin x)^k + (\cos x)^k| + c$, then k is equal to:

- (A) 2010 (B) 2011 (C) 2012 (D) 2013

160. (i) $\int x^{27}(1+x+x^2)^6(6x^2+5x+4)dx =$

(A) $\frac{x^4(1+x+x^2)^7}{7} + C$

(B) $\frac{x^{28}(1+x+x^2)^7}{7} + C$

(C) $\frac{x^{28}(1+x+x^2)^7}{28} + C$

(D) None of these

(ii) If $\lim_{x \rightarrow 0} \frac{f(x)}{x^2}$ exists finitely and $\lim_{x \rightarrow 0} \left(1+x+\frac{f(x)}{x}\right)^{1/x} = e^3$, then $\int f(x) \log_e x \, dx$ is equal to:

(A) $\frac{2}{3}x^3 \left(\log_e x - \frac{1}{3}\right) + c$

(B) $\frac{x^3}{3} \left(\log_e x - \frac{1}{3}\right) + c$

(C) $\frac{2}{3}x^3 (\log_e x + 1) + c$

(D) $\frac{2}{3}x^3 (\log_e x - 1) + c$

161. $\int \frac{x(x-1) \, dx}{(x^2+1)(x+1)\sqrt{x^3+x^2+x}} = \frac{1}{2} \log \left| \frac{\sqrt{x+\frac{1}{x}+1}-1}{\sqrt{x+\frac{1}{x}+1}+1} \right| - A + c$. Then the value of A is equal to:

(A) $\cos^{-1} \sqrt{1+\frac{1}{x}}$

(B) $\tan^{-1} \sqrt{x+\frac{1}{x}+1}$

(C) $\cot^{-1} \sqrt{x+\frac{1}{x}}$

(D) $\sin^{-1} \sqrt{x+\frac{1}{x}+1}$