Exercise-1

✍ **Marked questions are recommended for Revision.**

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Equation of Tangent / Normal and Common Tangents / Normals

A-1. **(i)** Find the equation of tangent to curve $y = 3x^2 + 4x + 5$ at (0, 5).

(ii) Find the equation of tangent and normal to the curve $x^2 + 3xy + y^2 = 5$ at point (1, 1) on it.

(iii) Find the equation of tangent and normal to the curve $x = \frac{2at^2}{1+t^2}$, $y = \frac{2at^3}{1+t^2}$ at the point for which $t = \frac{1}{2}$

(iv) Find the equation of tangent to the curve $y = \begin{cases} x^2 \sin 1/x & x \neq 0 \\ 0 & x = 0 \end{cases}$ at (0,0)

A-2 **(i)** Find equations of tangents drawn to the curve $y^2 - 2x^2 - 4y + 8 = 0$ from the point (1, 2).

(ii) Find the equation of all possible normals to the curve $x^2 = 4y$ drawn from the point (1,2)

A-3. **(i)** Find the point on the curve $9y^2 = x^3$ where normal to the curve has non zero x-intercept and both the x intercept and y-intercept are equal.

(ii)✍ If the tangent at (1, 1) on $y^2 = x(2 - x)^2$ meets the curve again at P, then find coordinates of P

(iii) The normal to the curve $5x^5 - 10x^3 + x + 2y + 6 = 0$ at the point P(0, –3) is tangent to the curve at some other point(s). Find those point(s)?

A-4.✍ **(i)** Find common tangent between curves $y = x^3$ and $112x^2 + y^2 = 112$

(ii) Find common normals of the curves $y = \frac{1}{x^2}$ and $x^2 + y^2 - y = 0$

A-5. **(i)** If the tangent to the curve $xy + ax + by = 0$ at (1, 1) is inclined at an angle $\tan^{-1} 2$ with positive x-axis in anticlockwise, then find a and b ?

(ii) The curve $y = ax^3 + bx^2 + 3x + 5$ touches $y = (x + 2)^2$ at (–2, 0) then $\left|\frac{a}{2}+b\right|$ is

Section (B) : Angle between curves, Orthogonal curves, Shortest/Maximum distance between two curves

B-1. Find the cosine of angle of intersection of curves $y = 2^x \ell nx$ and $y = x^{2x} - 1$ at (1, 0).

B-2. Find the angle between the curves $y = \ln x$ and $y = (\ln x)^2$ at their point of intersections.

B-3. Find the angle between the curves $y^2 = 4x + 4$ and $y^2 = 36 (9 - x)$.

B-4. Show that if the curves $ax^2 + by^2 = 1$ and $Ax^2 + By^2 = 1$ are orthogonal then $ab(A – B) = AB(a – b)$.

B-5. Find the shortest distance between line $y = x – 2$ and $y = x^2 + 3x + 2$

B-6. Find shortest distance between $y^2 = 4x$ and $(x – 6)^2 + y^2 = 1$

Section (C) : Rate of change and approximation

C-1. The length x of rectangle is decreasing at a rate of 3 cm/min and width y is increasing at a rate of 2 cm/min. When x = 10 cm and y = 6 cm, find the rate of change of (i) the perimeter, (ii) the area of rectangle.

C-2. x and y are the sides of two squares such that $y = x – x^2$. Find the rate of change of the area of the second square with respect to the first square.

C-3. A man 1.5 m tall walks away from a lamp post 4.5 m high at a rate of 4 km/hr.
(i) How fast is his shadow lengthening?
(ii) How fast is the farther end of shadow moving on the pavement?

C-4. Find the approximate change in volume V of a cube of side 5m caused by increasing its side length by 2%.

Section (D) : Monotonicity on an interval, about a point and inequalities, local maxima/minima

D-1. Show that $f(x) = \frac{x}{\sqrt{1+x}} - \ell n(1 + x)$ is an increasing function for $x > – 1$.

D-2. Find the intervals of monotonicity for the following functions.
(i) $\frac{x^4}{4} + \frac{x^3}{3} - 3x^2 + 5$ (ii) $\log_3^2 x + \log_3 x$

D-3. If g(x) is monotonically increasing and f(x) is monotonically decreasing for $x \in R$ and if (gof) (x) is defined for $x \in R$, then prove that (gof)(x) will be monotonically decreasing function. Hence prove that $(gof)(x + 1) \le (gof)(x – 1)$.

D-4. Let $f(x) = \begin{cases} x^2 ; & x \ge 0 \\ ax ; & x < 0 \end{cases}$. Find real values of 'a' such that f(x) is strictly monotonically increasing at x = 0.

D-5. Check monotonocity at following points for
(i) $f(x) = x^3 – 3x + 1$ at $x = –1, 2$
(ii) $f(x) = | x – 1 | + 2 | x – 3 | – | x + 2 |$ at $x = – 2, 0, 3, 5$
(iii) $f(x) = x^{1/3}$ at $x = 0$
(iv) $f(x) = x^2 + \frac{1}{x^2}$ at $x = 1, 2$
(v) $f(x) = \begin{cases} x^3 + 2x^2 + 5x , & x < 0 \\ 3\sin x , & x \ge 0 \end{cases}$ at $x = 0$

D-6. Prove that $\left(\dfrac{\sin\left(\frac{1}{10}\right)}{\frac{1}{10}}\right) > \left(\dfrac{\sin\left(\frac{1}{9}\right)}{\frac{1}{9}}\right)$.

D-7. Let f and g be differentiable on R and suppose f(0) = g(0) and $f'(x) \le g'(x)$ for all $x \ge 0$. Then show that $f(x) \le g(x)$ for all $x \ge 0$.

D-8. Let $f(x) = \begin{cases} 3 - x & 0 \le x < 1 \\ x^2 + \ell nb & x \ge 1 \end{cases}$. Find the set of values of b such that f(x) has a local minima at x = 1.

D-9. Find the points of local maxima/minima of following functions
(i) $f(x) = 2x^3 - 21x^2 + 36x - 20$ (ii) $f(x) = -(x - 1)^3 (x + 1)^2$
(iii) $f(x) = x\,\ell nx$

D-10. Find points of local maxima / minima of
(i) $f(x) = (2^x - 1)(2^x - 2)^2$
(ii) $f(x) = x^2e^{-x}$
(iii) $f(x) = 3\cos^4x + 10\cos^3x + 6\cos^2x - 3,\ x \in [0, \pi]$
(iv) $f(x) = 2x + 3x^{2/3}$
(v) $f(x) = \left|\dfrac{x^2 - 2}{x^2 - 1}\right|$

D-11. Draw graph of $f(x) = x|x - 2|$ and, hence find points of local maxima/minima.

Section (E) : Global maxima, Global minima, Application of Maxima and Minima

E-1. Find the absolute maximum/minimum value of following functions
(i) $f(x) = x^3$; $x \in [-2, 2]$
(ii) $f(x) = \sin x + \cos x$; $x \in [0, \pi]$
(iii) $f(x) = 4x - \dfrac{x^2}{2}$; $x \in \left[-2, \dfrac{9}{2}\right]$
(iv) $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$; $x \in [0, 3]$
(v) $f(x) = \sin x + \dfrac{1}{2}\cos 2x$; $x \in \left[0, \dfrac{\pi}{2}\right]$

E-2. Let $f(x) = x^2$; $x \in (-1, 2)$. Then show that f(x) has exactly one point of local minima but global maximum is not defined.

E-3. John has 'x' children by his first wife and Anglina has 'x + 1' children by her first husband. They both marry and have their own children. The whole family has 24 children. It is given that the children of the same parents don't fight. Then find then maximum number of fights that can take place in the family.

E-4. If the sum of the lengths of the hypotenuse and another side of a right angled triangle is given, show that the area of the triangle is a maximum when the angle between these sides is $\pi/3$.

E-5. Find the volume of the largest cylinder that can be inscribed in a sphere of radius ' r ' cm.

E-6. Show that the semi vertical angle of a right circular cone of maximum volume, of a given slant height is $\tan^{-1}\sqrt{2}$.

E-7. A running track of 440 m. is to be laid out enclosing a football field, the shape of which is a rectangle with semi circle at each end . If the area of the rectangular portion is to be maximum, find the length of its sides.

E-8. Find the area of the largest rectangle with lower base on the x-axis and upper vertices on the curve $y = 12 - x^2$.

E-9. Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible when revolved around one of its side.

E-10. The combined resistance R of two resistors R_1 & R_2 ($R_1, R_2 > 0$) is given by, $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$.

If $R_1 + R_2$ = constant. Prove that the maximum resistance R is obtained by choosing $R_1 = R_2$.

Section (F) : Rolle's Theorem, LMVT

F-1. Let f : [1, 2] → [1, 4] and g : [1, 2] → [2, 7] be two continuous bijective functions such thaf f(1) = 4 & g(2) = 7. The number of solutions of the equation f(x) = g(x) in (1, 2), is :

F-2. Verify Rolle's theorem for the function, $f(x) = \log_e\left(\frac{x^2 + ab}{x(a+b)}\right) + p$, for [a, b] where 0 < a < b.

F-3. Using Rolle's theorem prove that the equation $3x^2 + px - 1 = 0$ has at least one real root in the interval (– 1, 1).

F-4. Using Rolle's theorem show that the derivative of the function $f(x) = \begin{cases} x\sin\frac{\pi}{x} & \text{for} \quad x > 0 \\ 0 & \text{for} \quad x = 0 \end{cases}$ vanishes at an infinite set of points of the interval (0, 1).

F-5. Let f(x) be differentiable function and g(x) be twice differentiable function. Zeros of f(x), g′(x) be a, b respectively (a < b). Show that there exists at least one root of equation $f'(x)\, g'(x) + f(x)\, g''(x) = 0$ on (a, b).

F-6. If f(x) = tanx, $x \in \left[0, \frac{\pi}{5}\right]$ then show that $\frac{\pi}{5} < f\left(\frac{\pi}{5}\right) < \frac{2\pi}{5}$

F-7. If f(x) and g(x) are differentiable functions for $0 \le x \le 23$ such that f(0) = 2, g(0) = 0, f(23) = 22, g(23) = 10, then show that f′(x) = 2g′(x) for at least one x in the interval (0, 23).

F-8. If $f(x) = \begin{vmatrix} \sin^3 x & \sin^3 a & \sin^3 b \\ xe^x & ae^a & be^b \\ \frac{x}{1+x^2} & \frac{a}{1+a^2} & \frac{b}{1+b^2} \end{vmatrix}$

where $0 < a < b < 2\pi$, then show that the equation $f'(x) = 0$ has atleast one root in the interval (a, b)

F-9. A function y = f(x) is defined on [0, 6] as $f(x) = \begin{cases} -8x & ; \ 0 \le x \le 1 \\ (x-3)^3 & ; \ 1 < x < 4 \\ 2 & ; \ 4 \le x \le 6 \end{cases}$

Show that for the function y = f(x), all the three conditions of Rolle's theorem are violated on [0, 6] but still f′(x) vanishes at a point in (0, 6)

F-10. f is continuous in [a, b] and differentiable in (a, b) (where a > 0) such that $\frac{f(a)}{a} = \frac{f(b)}{b}$. Prove that there exist $x_0 \in (a, b)$ such that $f'(x_0) = \frac{f(x_0)}{x_0}$.

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Equation of Tangent / Normal and Common Tangents / Normals

A-1. Equation of normal drawn to the graph of the function defined as f(x) = $\frac{\sin x^2}{x}$, x ≠ 0 and f(0) = 0 at the origin is
(A) x + y = 0 (B) x – y = 0 (C) y = 0 (D) x = 0

A-2. Equation of the normal to the curve y = $-\sqrt{x}$ + 2 at the point (1, 1)
(A) 2x – y – 1 = 0 (B) 2x – y + 1 = 0 (C) 2x + y – 3 = 0 (D) none of these

A-3. The angle between x-axis and tangent of the curve y = (x+1) (x–3) at the point (3, 0)
(A) $\tan^{-1}\left(\frac{8}{15}\right)$ (B) $\tan^{-1}\left(\frac{15}{8}\right)$ (C) $\tan^{-1} 4$ (D) none of these

A-4. The numbers of tangent to the curve y – 2 = x^5 which are drawn from point (2,2) is / are
(A) 3 (B) 1 (C) 2 (D) 5

A-5. The equation of tangent drawn to the curve xy = 4 from point (0, 1) is
(A) $y-\frac{1}{2}=-\frac{1}{16}(x+8)$ (B) $y-\frac{1}{2}=-\frac{1}{16}(x-8)$ (C) $y+\frac{1}{2}=-\frac{1}{16}(x-8)$ (D) $y-8=-\frac{1}{16}\left(x-\frac{1}{2}\right)$

A-6. The curve y – e^{xy} + x = 0 has a vertical tangent at point
(A) (1, 1) (B) (0, 1) (C) (1, 0) (D) no point

A-7. If the tangent to the curve x = a (θ + sin θ), y = a (1 + cos θ) at θ = $\frac{\pi}{3}$ makes an angle α (0 ≤ α< π) with x-axis, then α =
(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{5\pi}{6}$

A-8. If the normal at the point (3t, 4/t) of the curve xy = 12 cuts the curve again at ($3t_1$, $4/t_1$) then find 't_1' in terms of 't'
(A) $\frac{-9}{16t^3}$ (B) $\frac{-16}{9t^3}$ (C) $\frac{9}{16t^3}$ (D) $\frac{16}{9t^3}$

A-9. The common tangent of the curves y = $x^2 + \frac{1}{x}$ and $y^2 = 4x$ is
(A) y = x + 1 (B) y = x –1 (C) y = – x + 1 (D) y = – x –1

A-10_. The area of triangle formed by tangent at (1,1) on y = x^2 + bx + c with coordinate axis is equal to 2 then the integral value of b is
(A) –3 (B) 3 (C) 2 (D) –2

Section (B) : Angle between curves, Orthogonal curves, Shortest/Maximum distance between two curves

B-1. The angle of intersection of y = a^x and y = b^x is given by
(A) $\tan\theta = \left|\frac{\log(ab)}{1-\log(ab)}\right|$ (B) $\left|\frac{\log(a/b)}{1+\log a \log b}\right|$ (C) $\left|\frac{\log(a/b)}{1-\log(a/b)}\right|$ (D) None of these

B-2. The angle between curves $x^2 + 4y^2 = 32$ and $x^2 - y^2 = 12$ is
(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

B-3. Find the angle at which two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$ intersect

(A) 0 (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

B-4. The value of a^2 if the curves $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$ and $y^3 = 16x$ cut orthogonally is

(A) 3/4 (B) 1 (C) 4/3 (D) 4

B-5. The shortest distance between curves $y^2 = 8x$ and $y^2 = 4(x-3)$ is

(A) $\sqrt{2}$ (B) $2\sqrt{2}$ (C) $3\sqrt{2}$ (D) $4\sqrt{2}$

B-6. The shortest distance between curves $\frac{x^2}{32} + \frac{y^2}{18} = 1$ and $\left(x - \frac{7}{4}\right)^2 + y^2 = 1$

(A) 15 (B) $\frac{11}{2}$ (C) $\frac{15}{4}$ (D) $\frac{11}{4}$

Section (C) : Rate of change and approximation

C-1. Water is poured into an inverted conical vessel of which the radius of the base is 2 m and height 4 m, at the rate of 77 litre/minute. The rate at which the water level is rising at the instant when the depth is 70 cm is (use $\pi = 22/7$)

(A) 10 cm/min (B) 20 cm/min (C) 40 cm/min (D) 30 cm/min

C-2 On the curve $x^3 = 12y$. The interval in which abscissa changes at a faster rate then its ordinate

(A) $(-3, 0)$ (B) $(-\infty, -2) \cup (2, \infty)$ (C) $(-2, 2)$ (D) $(-3, 3)$

C-3. A kite is 300 m high and there are 500 m of cord out. If the wind moves the kite horizontally at the rate of 5 km/hr. directly away from the person who is flying it, find the rate at which the cord is being paid?

(A) 4 (B) 8 (C) 3 (D) cannot be determined

C-4. The approximate value of tan 46° is (take $\pi = 22/7$) :

(A) 3 (B) 1.035 (C) 1.033 (D) 1.135

C-5. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases , is-

(A) $\frac{5}{6\pi}$ cm/min (B) $\frac{1}{54\pi}$ cm/min (C) $\frac{1}{18\pi}$ cm/min (D) $\frac{1}{36\pi}$ cm/min

Section (D) : Monotonicity on an interval, about a point and inequalities, local maxima/minima

D-1. The complete set of values of 'a' for which the function $f(x) = (a + 2)x^3 - 3ax^2 + 9ax - 1$ decreases for all real values of x is.

(A) $(-\infty, -3]$ (B) $(-\infty, 0]$ (C) $[-3, 0]$ (D) $[-3, \infty)$

D-2. Let $f(x) = x^3 + ax^2 + bx + 5\sin^2 x$ be an increasing function in the set of real numbers R. Then a & b satisfy the condition :

(A) $a^2 - 3b - 15 > 0$ (B) $a^2 - 3b + 15 \le 0$ (C) $a^2 + 3b - 15 < 0$ (D) $a > 0$ & $b > 0$

D-3. The function $\frac{|x-1|}{x^2}$ is monotonically decreasing at the point

(A) x = 3 (B) x = 1 (C) x = 2 (D) none of these

D-4. If $f(x) = 1 + 2x^2 + 4x^4 + 6x^6 + ... + 100x^{100}$ is a polynomial in a real variable x, then f(x) has:

(A) neither a maximum nor a minimum (B) only one maximum
(C) only one minimum (D) one maximum and one minimum

D-5_. Which of the following statement is/are true ?

(1) f(x) = sin x is increasing in interval $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

(2) f(x) = sin x is increasing in interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, \frac{5\pi}{2}\right)$

(3) f(x) = sin x is increasing at all point of the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, \frac{5\pi}{2}\right)$

(4) f(x) = sin x is increasing in intervals $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ & $\left[\frac{3\pi}{2}, \frac{5\pi}{2}\right]$

(A) all are correct (B) all are false
(C) (2) and (4) are correct (D) (1), (3) & (4) are correct

D-6_. Let f(x) = $\begin{cases} x & x \in [1,2] \\ 5-x & x \in (2,4) \\ 2 & x = 4 \\ 7-x & x \in (4,6] \end{cases}$ then which of the following statement is / are correct about f(x) ?

(A) Function is strictly increasing at point x = 2
(B) Function is strictly increasing at point x = 4
(C) Function is not increasing at point x = 2 and x = 4
(D) None of these

Section (E) : Global maxima, Global minima, Application of Maxima and Minima

E-1. The greatest, the least values of the function, $f(x) = 2 - \sqrt{1 + 2x + x^2}$, $x \in [-2, 1]$ are respectively

(A) 2, 1 (B) 2, – 1 (C) 2, 0 (D) –2, 3

E-2. Let $f(x) = (1 + b^2)x^2 + 2bx + 1$ and let m(b) be the minimum value of f(x). As b varies, the range of m(b) is

(A) [0, 1] (B) $\left(0, \frac{1}{2}\right]$ (C) $\left[\frac{1}{2}, 1\right]$ (D) (0, 1]

E-3. The radius of a right circular cylinder of greatest curved surface which can be inscribed in a given right circular cone is

(A) one third that of the cone (B) $1/\sqrt{2}$ times that of the cone
(C) 2/3 that of the cone (D) 1/2 that of the cone

E-4. The dimensions of the rectangle of maximum area that can be inscribed in the ellipse $(x/4)^2 + (y/3)^2 = 1$ are

(A) $\sqrt{8}, \sqrt{2}$ (B) 4, 3 (C) $2\sqrt{8}, 3\sqrt{2}$ (D) $\sqrt{2}$, $\sqrt{6}$

E-5. The largest area of a rectangle which has one side on the x–axis and the two vertices on the curve $y = e^{-x^2}$ is

(A) $\sqrt{2}\ e^{-1/2}$ (B) $2\ e^{-1/2}$ (C) $e^{-1/2}$ (D) none of these

E-6. Let $f(x) = \begin{cases} x^3 - x^2 + 10x - 5 & , x \le 1 \\ -2x + \log_2(b^2 - 2) & , x > 1 \end{cases}$ the set of values of b for which f(x) has greatest value at x = 1 is given by :

(A) $1 \le b \le 2$ (B) b = {1, 2}

(C) $b \in (-\infty, -1)$ (D) $\left[-\sqrt{130}, -\sqrt{2}\right) \cup \left(\sqrt{2}, \sqrt{130}\right]$

E-7. Four points A, B, C, D lie in that order on the parabola $y = ax^2 + bx + c$. The coordinates of A, B & D are known as A(– 2, 3); B(– 1, 1) and D(2, 7). The coordinates of C for which the area of the quadrilateral ABCD is greatest, is

(A) (1/2, 7/4) (B) (1/2, – 7/4) (C) (–1/2, 7/4) (D) (–1/2, –7/4)

Section (F) : Rolle's Theorem, LMVT

F-1. The function $f(x) = x^3 - 6x^2 + ax + b$ satisfy the conditions of Rolle's theorem on [1, 3]. Which of these are correct ?

(A) a =11, $b \in R$ (B) a = 11, b = – 6 (C) a = –11, b = 6 (D) a = – 11, $b \in R$

F-2. The function $f(x) = x(x + 3)e^{-x/2}$ satisfies all the conditions of Rolle's theorem on [–3, 0]. The value of c which verifies Rolle's theorem, is

(A) 0 (B) – 1 (C) – 2 (D) 3

F-3. If f(x) satisfies the requirements of Lagrange's mean value theorem on [0, 2] and if f(0) = 0 and $f'(x) \le \frac{1}{2}$ $\forall\ x \in [0, 2]$, then

(A) $|f(x)| \le 2$ (B) $f(x) \le 1$

(C) f(x) = 2x (D) f(x) = 3 for at least one x in [0, 2]

F-4_. If ab > 0 and 3a + 5b + 15 c = 0 then which of the following statement is "**INCORRECT"?**

(A) there exist exactly one root of equation $ax^4 + bx^2 + c = 0$ in (–1,0)

(B) there exist exactly one root of equation $ax^4 + bx^2 + c = 0$ in (0,1)

(C) there exist exactly two root of equation $ax^4 + bx^2 + c = 0$ in (–1,1)

(D) number of roots of equation $ax^4 + bx^2 + c = 0$ can be two in (–1,0)

F-5. Consider the function for $x \in [-2, 3]$

$f(x) = \begin{cases} -6 & ; \quad x = 1 \\ \dfrac{x^3 - 2x^2 - 5x + 6}{x - 1} & ; \quad x \ne 1 \end{cases}$. The value of c obtained by applying Rolle's theorem for which $f'(c) = 0$ is

(A) 0 (B) 1 (C) 1/2 (D) 'c' does not exist

PART - III : MATCH THE COLUMN

1. **Column – I** | **Column – II**

(A) If curves $y^2 = 4ax$ and $y = e^{-\frac{x}{2a}}$ are orthogonal then 'a' can take value — (p) 3

(B) If θ is angle between the curves $y = [|\sin x| + |\cos x|]$, ([·] denote GIF) and $x^2 + y^2 = 5$ then $\text{cosec}^2\theta$ is — (q) 1

(C) If curves $y^2 = 4a(x + a)$ and $y^2 = 4b(x + b)$ intersects each other orthogonally then $\left|\frac{a}{b}\right|$ can be equal to____ — (r) 5/4

(D) If $y = x^2 + 3x + c$ and $x = y^2 + 3y + c$ has only one common point (h, k) then $|h + k + c|$ can be equal to…… — (s) 2

2. **Column-I** | **Column-II**

(A) The number of point (s) of maxima of $f(x) = x^2 + \frac{1}{x^2}$ is — (p) 0

(B) $(\sin^{-1}x)^3 + (\cos^{-1}x)^3$ is maximum at x = — (q) 2

(C) If [a, b], (b < 1) is largest interval in which $f(x) = 3x^4 + 8x^3 - 6x^2 - 24x + 19$ is strictly increasing then $\frac{a}{b}$ is — (r) $\frac{8}{3}$

(D) If a + b = 8, a, b > 0 then minimum value of $\frac{a^3 + b^3}{48}$ is — (s) –1

3. **Column – I** | **Column – II**

(A) $f(x) = \frac{\sin x}{e^x}$, $x \in [0,\pi]$ — (p) Conditions in Rolle's theorem are satisfied.

(B) $f(x) = \text{sgn}((e^x - 1)\ \ell n x)$, $x \in \left[\frac{1}{2},\frac{3}{2}\right]$ — (q) Conditions in LMVT are satisfied.

(C) $f(x) = (x-1)^{2/5}$, $x \in [0,3]$ — (r) At least one condition in Rolle's theorem is not satisfied.

(D) $f(x) = \begin{cases} x\left(\dfrac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}\right), & x \in [-1,1] - \{0\} \\ 0, & x = 0 \end{cases}$ — (s) At least one condition in LMVT is not satisfied.

4. **Column – I** | **Column – II**

(A) A rectangle is inscribed in an equilateral triangle of side 4cm. Square of maximum area of such a rectangle is — (p) 65

(B) The volume of a rectangular closed box is 72 and the base sides are in the ratio 1 : 2. The least total surface area is — (q) 45

(C) If x and y are two positive numbers such that x + y = 60 and x^3y is maximum then value of x is — (r) 12

(D) The sides of a rectangle of greatest perimeter which is inscribed in a semicircle of radius $\sqrt{5}$ are a and b. Then $a^3 + b^3$ = — (s) 108

Exercise-2

Marked questions are recommended for Revision.

PART - I : ONLY ONE OPTION CORRECT TYPE

1. The line $\frac{x}{a}+\frac{y}{b}=1$ touches the curve $y = be^{-x/a}$ at the point

(A) (– a, 2b) (B) $\left(\frac{a}{2}, \frac{b}{2}\right)$ (C) $\left(a, \frac{b}{e}\right)$ (D) (0, b)

2. The equation of normal to the curve $x^3 + y^3 = 8xy$ at point where it is meet by the curve $y^2 = 4x$, other than origin is

(A) y = x (B) y = –x + 4 (C) y = 2x (D) y = –2x

3. The length of segment of all tangents to curve $x^{2/3} + y^{2/3} = a^{2/3}$ intercepted between coordinate axes is

(A) 2|a| (B) |a| (C) $\frac{|a|}{2}$ (D) $\frac{3|a|}{2}$

4. If tangents are drawn from the origin to the curve y = sin x, then their points of contact lie on the curve

(A) x – y = xy (B) x + y = xy (C) $x^2 - y^2 = x^2y^2$ (D) $x^2 + y^2 = x^2y^2$

5. Number of tangents drawn from the point (–1/2, 0) to the curve $y = e^{\{x\}}$. (Here { } denotes fractional part function).

(A) 2 (B) 1 (C) 3 (D) 4

6. Let f(x) = $\begin{cases} -x^2 & , \quad x < 0 \\ x^2 + 8 & , \quad x \geq 0 \end{cases}$ Equation of tangent line touching both branches of y = f(x) is

(A) y = 4x + 1 (B) y = 4x + 4 (C) y = x + 4 (D) y = x + 1

7. Minimum distance between the curves $f(x) = e^x$ & $g(x) = \ell n\, x$ is

(A) 1 (B) $\sqrt{2}$ (C) 2 (D) e

8. The point(s) on the parabola $y^2 = 4x$ which are closest to the circle, $x^2 + y^2 - 24y + 128 = 0$ is/are:

(A) (0, 0) (B) $\left(2, 2\sqrt{2}\right)$ (C) (4, 4) (D) none

9. If f(x) = $a^{\{a^{|x|}\,\mathrm{sgn}\,x\}}$; g(x) = $a^{[a^{|x|}\,\mathrm{sgn}\,x]}$ for a > 1, a ≠ 1and x ∈ R, where {} & [ ] denote the fractional part and integral part functions respectively, then which of the following statements holds good for the function h(x), where (ℓn a) h(x) = (ℓn f(x) + ℓn g(x)).

(A) 'h' is even and increasing (B) 'h' is odd and decreasing
(C) 'h' is even and decreasing (D) 'h' is odd and increasing

10. If f : [1, 10] → [1, 10] is a non-decreasing function and g : [1, 10] → [1, 10] is a non-increasing function. Let h(x) = f(g(x)) with h(1) = 1, then h(2)

(A) lies in (1, 2) (B) is more than 2 (C) is equal to 1 (D) is not defined

11. If f(x) = |ax – b| + c|x| is stricly increasing at atleast one point of non differentiability of the function where a > 0, b > 0, c > 0 then

(A) c > a (B) a > c (C) b > a + c (D) a = b

12. If g(x) is a curve which is obtained by the reflection of $f(x) = \frac{e^x - e^{-x}}{2}$ by the line y = x then
(A) g(x) has more than one tangent parallel to x-axis
(B) g(x) has more than one tangent parallel to y-axis
(C) y = –x is a tangent to g(x) at (0, 0)
(D) g(x) has no extremum

13. The set of values of p for which all the points of extremum of the function
$f(x) = x^3 - 3px^2 + 3(p^2 - 1)x + 1$ lie in the interval (– 2, 4), is:
(A) (– 3, 5) (B) (– 3, 3) (C) (– 1, 3) (D) (– 1, 4)

14. Which of the following statement is correct about e^π and π^e.
(A) $e^\pi > \pi^e$ (B) $e^\pi < \pi^e$ (C) $e^\pi = \pi^e$ (D) None of these

15^. The complete set of values of the parameter 'a' for which the point of minimum of the function
$f(x) = 1 + a^2x - x^3$ satisfies the inequality $\frac{x^2 + x + 2}{x^2 + 5x + 6} < 0$ is
(A) $(-3\sqrt{3}, -2\sqrt{3}) \cup (2\sqrt{3}, 3\sqrt{3})$ (B) $(-3\sqrt{3}, -2\sqrt{3})$
(C) $(-3\sqrt{3}, -2\sqrt{3})$ (D) $(-3\sqrt{2}, 2\sqrt{3})$

16. If $f(x) = \sin^3x + \lambda \sin^2 x$; $-\pi/2 < x < \pi/2$, then the interval in which λ should lie in order that f(x) has exactly one minima and one maxima
(A) (–3/2, 3/2) (B) (–2/3, 2/3) – {0} (C) R (D) none of these

17. In a regular triangular prism the distance from the centre of one base to one of the vertices of the other base is ℓ. The altitude of the prism for which the volume is greatest, is :
(A) $\frac{\ell}{2}$ (B) $\frac{\ell}{\sqrt{3}}$ (C) $\frac{\ell}{3}$ (D) $\frac{\ell}{4}$

18. The maximum area of the rectangle whose sides pass through the angular points of a given rectangle of sides a and b is
(A) 2 (ab) (B) $\frac{1}{2}(a + b)^2$ (C) $\frac{1}{2}(a^2 + b^2)$ (D) $\frac{a^3}{b}$

19. Let ABC is given triangle having respective sides a,b,c. D,E,F are points of the sides BC,CA,AB respectively so that AFDE is a parallelogram. The maximum area of the parallelogram is
(A) $\frac{1}{4}$bcsinA (B) $\frac{1}{2}$ bcsinA (C) bcsinA (D) $\frac{1}{8}$bcsinA

20. Square roots of 2 consecutive natural number greater than N^2 is differ by
(A) $> \frac{1}{2N}$ (B) $\geq \frac{1}{2N}$ (C) $< \frac{1}{2N}$ (D) $> \frac{1}{N}$

21. If Rolle's theorem is applicable to the function $f(x) = \frac{\ell nx}{x}$, (x > 0) over the interval [a, b] where $a \in I$, $b \in I$, then the value of $a^2 + b^2$ can be
(A) 20 (B) 25 (C) 45 (D) 10

22. If f(x) be a twice differentiable function such that $f(x) = x^2$ for x = 1, 2, 3, then
(A) $f''(x) = 2 \quad \forall x \in [1, 3]$ (B) $f''(x) = 2$ for some $x \in (1, 3)$
(C) $f''(x) = 2 \quad \forall x \in (1, 3)$ (D) $f'(x) = 2x \quad \forall x \in (1, 3)$

PART-II: NUMERICAL VALUE QUESTIONS

INSTRUCTION :

- The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto two digit.
- If the numerical value has more than two decimal places **truncate/round-off** the value to **TWO** decimal placed.

1. The number of distinct line(s) which is/are tangent at a point on curve $4x^3 = 27y^2$ and normal at other point, is :

2. The sum of the ordinates of point of contacts of the common tangent to the parabolas $y = x^2 + 4x + 8$ and $y = x^2 + 8x + 4$, is

3. Value of p for which equation $|\ell n\, x| - px = 0$ has exactly two solution is

4. A light shines from the top of a pole 50 ft. high. A ball is dropped from the same height from a point 30 ft. away from the light. If the shadow of the ball moving at the rate of 200λ ft/sec along the ground 1/2 sec. later [Assume the ball falls a distance $s = 16t^2$ ft. in 't' sec.], then |λ| is :

5. A variable ΔABC in the xy plane has its orthocentre at vertex 'B' , a fixed vertex 'A' at the origin and the third vertex 'C' restricted to lie on the parabola $y = 1 + \frac{7x^2}{36}$. The point B starts at the point (0, 1) at time t = 0 and moves upward along the y axis at a constant velocity of 2 cm/sec . If the area of the triangle increasing at the rate of 'p' cm²/sec when $t = \frac{7}{2}$ sec, then p is.

6. Function defined by $f(x) = \frac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}}$ is injective in [α – 2, ∞), then least value of α is

7. The value of $\lim_{x \to 0^+} \left(2 + \left[\frac{3x}{2\sin x + \tan}\right]\right)^{\frac{\tan^3 x - \sin^3 x}{x^5}}$ is (where [.] denotes the GIF)

[Hint: For $x \in \left(0, \frac{\pi}{2}\right)$ identify which is greater (2sinx + tanx) or (3x)}

8. If $f(x) = 2e^x - ae^{-x} + (2a - 3)x - 3$ monotonically increases for ∀ x ∈ R, then minimum value of a is

9. If the set of all values of the parameter 'a' for which the function
$f(x) = \sin 2x - 8(a + 1)\sin x + (4a^2 + 8a - 14)x$ increases for all x ∈ R and has no critical points for all x ∈ R, is $(-\infty, -m - \sqrt{n}) \cup (\sqrt{n}, \infty)$ then $(m^2 + n^2)$ is (where m, n are prime numbers) :

10. If $\ell n 2\pi < \log_2(2 + \sqrt{3}) < \ell n 3\pi$, then number of roots of the equation $4\cos(e^x) = 2^x + 2^{-x}$, is

11. For $-1 \le p \le 1$, the equation $4x^3 - 3x - p = 0$ has 'n' distinct real roots in the interval $\left[\frac{1}{2}, 1\right]$ and one of its root is $\cos(k\cos^{-1}p)$, then the value of $10n + \frac{1}{2k}$ is :

12. Least value of the function, $f(x) = 2^{x^2} - 1 + \frac{200}{2^{x^2} + 1}$ is

13. Real root of the equation $(x-1)^{2013} + (x-2)^{2013} + (x-3)^{2013} + ...+(x-2013)^{2013} = 0$ is a four digit number. Then the sum of square of the digits is :

14. The exhaustive set of values of 'a' for which the function $f(x) = \frac{a}{3}x^3 + (a+2)x^2 + (3a-10)x + 2$ possess a negative point of minimum is (p, q) then value of p is :

15. If f(x) is a polynomial of degree 6, which satisfies $\lim_{x \to 0}\left(1+\frac{f(x)}{x^3}\right)^{1/x} = e^2$ and has local maximum at x = 1 and local minimum at x = 0 and x = 2, then the value of $\left(\frac{5}{9}\right)^4 f\left(\frac{18}{5}\right)$ is :

16. Maximum value of $\left(\sqrt{-3+4x-x^2}+4\right)^2 + (x-5)^2$ (where $1 \le x \le 3$) is

17. The three sides of a trapezium are equal each being 6 cms long. Let Δ cm^2 be the maximum area of the trapezium. The value of $\frac{\sqrt{3}\,\Delta}{2}$ is :

18. A sheet of poster has its area 18 m^2. The margin at the top & bottom are 75 cms. and at the sides 50 cms. Let ℓ, n are the dimensions of the poster in meters when the area of the printed space is maximum. The value of $\frac{\ell^2+n^2+100}{7}$ is :

19. The fuel charges for running a train are proportional to the square of the speed generated in m.p.h. and costs Rs. 48/- per hour at 16 mph. What is the most economical speed if the fixed charges i.e. salaries etc. amount to Rs. 300/- per hour.

20. Let f(x) = Max. $\{x^2, (1-x)^2, 2x(1-x)\}$ where $x \in [0, 1]$ If Rolle's theorem is applicable for f(x) on largest possible interval [a, b] then the value of (4a + 6b + 11c) when $c \in (a, b)$ such that f'(c) = 0, is_____

21. For every twice differentiable function f(x) the value of $|f(x)| \le 3 \ \forall \ x \in R$ and for some α, $f(\alpha) + (f'(\alpha))^2 = 80$. Number of integral values that $(f'(x))^2$ can take between (0, 77) are equal to ___

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. If tangent to curve $2y^3 = ax^2 + x^3$ at point (a, a) cuts off intercepts α, β on co-ordinate axes, where $\alpha^2 + \beta^2 = 61$, then the value of 'a' is equal to
(A) 20 (B) 25 (C) 30 (D) – 30

2. For the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$, at point (2, – 1)
(A) length of subtangent is 7/6.
(B) slope of tangent = 6/7
(C) length of tangent = $\sqrt{(85)}/6$
(D) none of these

3. Which of the following statements is/are correct ?
(A) x + sinx is increasing function
(B) sec x is neither increasing nor decreasing function
(C) x + sinx is decreasing function
(D) sec x is an increasing function

4. If $f(x) = 2x + \cot^{-1}x + \ell n\left(\sqrt{1+x^2}-x\right)$, then f(x) :
(A) increases in [0, ∞)
(B) decreases in [0, ∞)
(C) neither increases nor decreases in [0, ∞)
(D) increases in (– ∞, ∞)

5. Let g(x) = 2f(x/2) + f(1 – x) and $f''(x) < 0$ in $0 \le x \le 1$ then g(x)

(A) decreases in $\left[0, \frac{2}{3}\right]$ (B) decreases $\left[\frac{2}{3}, 1\right]$

(C) increases in $\left[0, \frac{2}{3}\right]$ (D) increases in $\left[\frac{2}{3}, 1\right]$

6. Let $f(x) = x^{m/n}$ for $x \in R$ where m and n are integers, m even and n odd and 0 < m < n. Then

(A) f(x) decreases on (– ∞, 0] (B) f(x) increases on [0, ∞)

(C) f(x) increases on (– ∞, 0] (D) f(x) decreases on [0, ∞)

7. Let f and g be two differentiable functions defined on an interval I such that f(x) ≥ 0 and g(x) ≤ 0 for all $x \in I$ and f is strictly decreasing on I while g is strictly increasing on I then

(A) the product function fg is strictly increasing on I

(B) the product function fg is strictly decreasing on I

(C) fog(x) is monotonically increasing on I

(D) fog (x) is monotonically decreasing on I

8. Let $\phi(x) = (f(x))^3 - 3(f(x))^2 + 4f(x) + 5x + 3\sin x + 4\cos x \ \forall x \in R$, where f(x) is a differentiable function $\forall x \in R$, then

(A) ϕ is increasing whenever f is increasing (B) ϕ is increasing whenever f is decreasing

(C) ϕ is decreasing whenever f is decreasing (D) ϕ is decreasing if $f'(x) = -11$

9. If p, q, r be real, then the intervals in which, f(x) = $\begin{vmatrix} x+p^2 & pq & pr \\ pq & x+q^2 & qr \\ pr & qr & x+r^2 \end{vmatrix}$,

(A) increase is $x < -\frac{2}{3}(p^2 + q^2 + r^2)$, x > 0 (B) decrease is $(-\frac{2}{3}(p^2 + q^2 + r^2), 0)$

(C) decrease is $x < -\frac{2}{3}(p^2 + q^2 + r^2)$, x > 0 (D) increase is $(-\frac{2}{3}(p^2 + q^2 + r^2), 0)$

10. If $f(x) = \frac{x^2}{2 - 2\cos x}$; $g(x) = \frac{x^2}{6x - 6\sin x}$ where 0 < x < 1, then

(A) 'f' is increasing function (B) 'g' is decreasing function

(C) $\frac{f(x)}{g(x)}$ is increasing function (D) g(f(x)) is decreasing function

11. Which of the following statement is / are correct ?

(A) The function $y = \frac{2x^2 - 1}{x^4}$ is neither increasing nor decreasing.

(B) If f(x) is strictly increasing real function defined on R and c is a real constant, then number of Solutions of f(x) = c is always equal to one.

(C) Let f(x) = x ; $x \in (0, 1)$. f(x) does not has any point of local maxima/minima

(D) f(x) = {x} has maximum at x = 6 (here {.} denotes fractional part function).

State, in order, whether S_1, S_2, S_3, S_4 are true or false

12. Let $f(x) = \dfrac{x}{\sin x}$ & $x \in \left(0, \dfrac{\pi}{2}\right)$

Then the interval in which at least one root of equaiton lie $\dfrac{2}{x - f\left(\frac{\pi}{12}\right)} + \dfrac{3}{x - f\left(\frac{\pi}{4}\right)} + \dfrac{4}{x - f\left(\frac{5\pi}{12}\right)} = 0$

(A) $\left(f\left(\frac{\pi}{12}\right),\ f\left(\frac{\pi}{4}\right)\right)$ (B) $\left(0,\ f\left(\frac{\pi}{12}\right)\right)$ (C) $\left(f\left(\frac{5\pi}{12}\right), \infty\right)$ (D) $\left(f\left(\frac{\pi}{4}\right),\ f\left(\frac{5\pi}{12}\right)\right)$

13. Let $f(x) = (x^2 - 1)^n (x^2 + x + 1)$. f(x) has local extremum at x = 1 if

(A) n = 2 (B) n = 3 (C) n = 4 (D) n = 6

14. If $f(x) = \dfrac{x}{1 + x\tan x}$, $x \in \left(0, \dfrac{\pi}{2}\right)$, then

(A) f(x) has exactly one point of minima
(B) f(x) has exactly one point of maxima
(C) f(x) is increasing in $\left(0, \dfrac{\pi}{2}\right)$
(D) maxima occurs at x_0 where $x_0 = \cos x_0$

15. If $f(x) = a\,\ell n\,|x| + bx^2 + x$ has its extremum values at x = – 1 and x = 2, then _____

(A) a = 2 (B) b = – 1/2 (C) a = – 2 (D) b = 1/2

16. If $f(x) = \begin{cases} -\sqrt{1-x^2} & , \quad 0 \le x \le 1 \\ -x & , \quad x > 1 \end{cases}$, then

(A) Maximum of f(x) exist at x = 1
(B) Maximum of f (x) doesn't exists
(C) Minimum of $f^{-1}(x)$ exist at x = – 1
(D) Minimum of $f^{-1}(x)$ exist at x = 1

17. If $f(x) = \tan^{-1}x - (1/2)\,\ell n\,x$. Then

(A) the greatest value of f(x) on $\left[1/\sqrt{3},\ \sqrt{3}\right]$ is $\pi/6 + (1/4)\,\ell n\,3$
(B) the least value of f(x) on $\left[1/\sqrt{3},\ \sqrt{3}\right]$ is $\pi/3 - (1/4)\,\ell n\,3$
(C) f(x) decreases on $(0, \infty)$
(D) f(x) increases on $(-\infty, 0)$

18. Let $f(x) = \dfrac{40}{3x^4 + 8x^3 - 18x^2 + 60}$. Which of the following statement(s) about f(x) is (are) correct ?

(A) f(x) has local minima at x = 0.
(B) f(x) has local maxima at x = 0.
(C) Absolute maximum value of f(x) is not defined.
(D) f(x) is local maxima at x = – 3, x = 1.

19. A function $f(x) = \dfrac{x^2 - 3x + 2}{x^2 + 2x - 3}$ is -

(A) 1 is not in its domain
(B) minimum at x = – 3 and maximum at x = 1
(C) no point of maxima and minima
(D) increasing in its domain

20. Let $4x^2 + 12xy + 10y^2 - 4y + 3 = 0$.

(A) Maximum value of y is 3.
(B) Minimum value of y is 1.
(C) Maximum value of x is 3.
(D) Minimum value of x is 1.

21. For the function $f(x) = x \cot^{-1}x, x \geq 0$

(A) there is atleast one $x \in (0, 1)$ for which $\cot^{-1}x = \frac{x}{1+x^2}$

(B) for atleast one x in the interval $(0, \infty)$, $f\left(x+\frac{2}{\pi}\right) - f(x) < 1$

(C) number of solution of the equation f(x) = sec x is 1

(D) f′(x) is strictly decreasing in the interval $(0, \infty)$

22^. If f(x) = (x – 4) (x – 5) (x – 6) (x – 7) then,

(A) f''(x) = 0 has no real roots.

(B) three roots of f′(x) = 0 lie in $(4, 5) \cup (5, 6) \cup (6, 7)$.

(C) one root of the equation f′(x) = 0 is $\frac{11}{2}$

(D) graph of y = f''(x) is symmectric about $x = \frac{11}{2}$

23. Which of the following statements are true :

(A) If f(x) is differentiable function such that $f(a) \neq f(b)$ then there exist no $c \in (a, b)$ such that f′(c) = 0

(B) The function $x^{100} + \sin x - 1$ is strictly increasing in [0, 1]

(C) If a, b, c are in A.P, then at least one root of the equation $3ax^2 - 4bx + c = 0$ is positive

(D) The number of solution(s) of equation $3 \tan x + x^3 = 2$ in $(0, \pi/4)$ is 2

24. Let f(x) be a differentiable function and $f(\alpha) = f(\beta) = 0$ $(\alpha < \beta)$, then in the interval (α, β)

(A) f(x) + f '(x) = 0 has at least one root

(B) f(x) – f '(x) = 0 has at least one real root

(C) f(x) . f '(x) = 0 has at least one real root

(D) none of these

25. Which of the following inequalities are valid –

(A) $|\tan^{-1} x - \tan^{-1}y| \leq |x - y| \ \forall \ x, y \in R$

(B) $|\tan^{-1} x - \tan^{-1}y| \geq |x - y|$

(C) $|\sin x - \sin y| \leq |x - y|$

(D) $|\sin x - \sin y| \geq |x - y|$

26. For all x in [1, 2]

Let f''(x) of a non-constant function f(x) exist and satisfy $|f''(x)| \leq 2$. If f(1) = f(2), then

(A) There exist some $a \in (1, 2)$ such that f′(a) = 0

(B) f(x) is strictly increasing in (1, 2)

(C) There exist atleast one $c \in (1, 2)$ such that f′(c) > 0

(D) $|f'(x)| < 2 \ \forall \ x \in [1, 2]$

27_. For every thrice differentiable function $f : R \to [-3, 3]$ with $(f(0))^2 + (f'(0))^2 = 85$,

Which of the following statement(s) is (are) TRUE?

(A) There exist $x_0 \in (a, a + 6)$ such that $(f'(x_0))^2 \leq 1$

(B) $(f'(x))^2$ can take all real value that lies between (0, 76]

(C) There exist $\alpha \in (-6, 0)$ such that $(f(\alpha))^2 + (f'(\alpha))^2 = 10$

(D) There exists $\alpha \in (-6, 6)$ such that $|f(\alpha)| = |f''(\alpha)|$

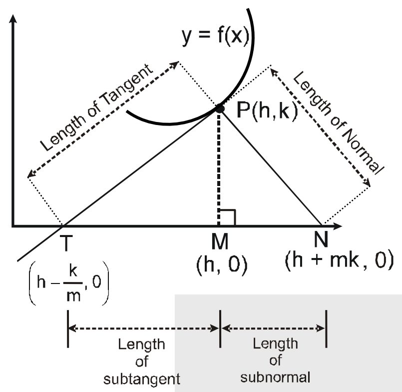
(E) There exist atleast one $c \in (-3, 3)$ such that f′(c) f'''(c) < 0.

PART - IV : COMPREHENSION

Comprehension # 1

Lengths of tangent, normal, subtangent and subnormal :

Let P (h, k) be any point on curve y = f(x). Let tangent drawn at point P meets x-axis at T & normal at point P meets x-axis at N (as shown in figure) and $m = \left.\frac{dy}{dx}\right]_{(h,k)}$ = slope of tangent.



(i) Length of Tangent = PT = $|k|\sqrt{1+\frac{1}{m^2}}$

(ii) Length of Normal = PN = $|k|\sqrt{1+m^2}$

(iii) Length of subtangent = Projection of segment PT on x-axis = TM = $\left|\frac{k}{m}\right|$

(iv) Length of subnormal = projection of line segment PN on x axis =MN = |km|

1^. Find the product of length of tangent and length of normal for the curve $y = x^3 + 3x^2 + 4x - 1$ at point x = 0.

(A) $\frac{17}{4}$ (B) $\frac{\sqrt{15}}{4}$ (C) 17 (D) $\frac{4}{\sqrt{17}}$

2. Determine 'p' such that the length of the subtangent and subnormal is equal for the curve $y = e^{px} + px$ at the point (0, 1).

(A) ±1 (B) ±2 (C) $\pm\frac{1}{2}$ (D) $\pm\frac{1}{4}$

3. Find length of subnormal to $x = \sqrt{2}\cos t$, $y = -3\sin t$ at $t = \frac{-\pi}{4}$.

(A) $\frac{2}{9}$ (B) 1 (C) $\frac{7}{2}$ (D) $\frac{9}{2}$

Comprehension # 2

Consider a function f defined by $f(x) = \sin^{-1}\sin\left(\frac{x+\sin x}{2}\right)$, $\forall x \in [0, \pi]$, which satisfies

$f(x) + f(2\pi - x) = \pi$, $\forall x \in [\pi, 2\pi]$ and $f(x) = f(4\pi - x)$ for all $x \in [2\pi, 4\pi]$, then

4. If α is the length of the largest interval on which f(x) is increasing, then α =

(A) $\frac{\pi}{2}$ (B) π (C) 2π (D) 4π

5. If f(x) is symmetric about $x = \beta$, then β =

(A) $\frac{\alpha}{2}$ (B) α (C) $\frac{\alpha}{4}$ (D) 2α

6. Maximum value of f(x) on $[0, 4\pi]$ is :

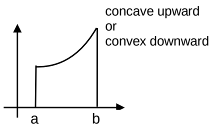
(A) $\frac{\beta}{2}$ (B) β (C) $\frac{\beta}{4}$ (D) 2β

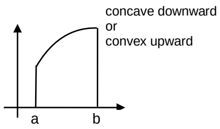
Comprehension # 3

Concavity and convexity :

If $f''(x) > 0 \ \forall \ x \in (a, b)$, then the curve $y = f(x)$ is concave up (or convex down) in (a,b) and

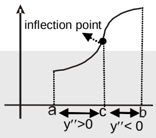
If $f''(x) < 0 \ \forall \ x \in (a, b)$ then the curve $y = f(x)$ is concave down (or convex up) in (a, b).





Inflection point :

The point where concavity of the curve changes is known as point of inflection (at inflection point $f''(x)$ is equal to 0 or undefined).



7. Number of point of inflection for $f(x) = (x - 1)^3 (x - 2)^2$, is

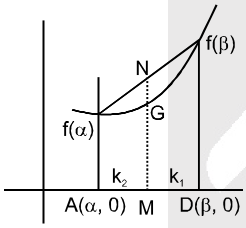
(A) 1 (B) 2 (C) 3 (D) 4

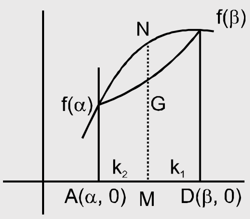
8. Exhaustive set of values of 'a' for which the function $f(x) = x^4 + ax^3 + \frac{3x^2}{2} + 1$ will be concave upward along the entire real line, is :

(A) [–1,1] (B) [–2,2] (C) [0,2] (D) [0,4]

Comprehension # 4

For a double differentiable function f(x) if $f''(x) \geq 0$ then f(x) is concave upward and if $f''(x) \leq 0$ then f(x) is concave downward





Here $M\left(\frac{k_1\alpha + k_2\beta}{k_1 + k_2}, \ 0\right)$

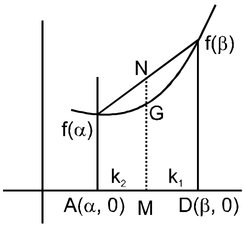
If f(x) is a concave upward in [a, b] and $\alpha, \beta \in [a, b]$ then $\frac{k_1 f(\alpha) + k_2 f(\beta)}{k_1 + k_2} \geq f\left(\frac{k_1\alpha + k_2\beta}{k_1 + k_2}\right)$,

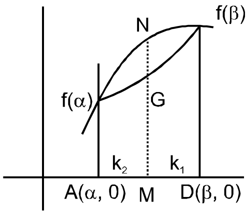
where $k_1, k_2 \in R^+$

If f(x) is a concave downward in [a, b] and $\alpha, \beta \in [a, b]$ then $\frac{k_1 f(\alpha) + k_2 f(\beta)}{k_1 + k_2} \leq f\left(\frac{k_1\alpha + k_2\beta}{k_1 + k_2}\right)$,

where $k_1, k_2 \in R^+$

then answer the following





9. Which of the following is true

(A) $\frac{\sin\alpha+\sin\beta}{2} > \sin\left(\frac{\alpha+\beta}{2}\right)$; α, β (0, π)
(B) $\frac{\sin\alpha+\sin\beta}{2} < \sin\left(\frac{\alpha+\beta}{2}\right)$; α, β ∈ (π, 2π)

(C) $\frac{\sin\alpha+\sin\beta}{2} < \sin\left(\frac{\alpha+\beta}{2}\right)$; α , β ∈(0, π)
(D) None of these

10. Which of the following is true

(A) $\frac{2^{\alpha}+2^{\beta+1}}{3} \le 2^{\frac{\alpha+2\beta}{3}}$
(B) $\frac{2\ell n\ \alpha+\ell n\beta}{3} \ge \ell n\left(\frac{2\alpha+\beta}{3}\right)$

(C) $\frac{\tan^{-1}\alpha+\tan^{-1}\beta}{2} \le \tan^{-1}\left(\frac{\alpha+\beta}{2}\right)$ a, b ∈ R^-
(D) $\frac{e^{\alpha}+2e^{\beta}}{3} \ge e^{\frac{\alpha+2\beta}{3}}$

11. Let α , β and γ are three distinct real numbers and f″ (x) < 0. Also f(x) is increasing function and let

A = $\frac{f^{-1}(\alpha)+f^{-1}(\beta)+f^{-1}(\gamma)}{3}$ and B = $f^{-1}\left(\frac{\alpha+\beta+\gamma}{3}\right)$, then order relation between A and B is ?

(A) A > B (B) A < B (C) A = B (D) none of these

Exercise-3

✍ Marked questions are recommended for Revision.

*** Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. Let f be a function defined on **R** (the set of all real numbers) such that

$f'(x) = 2010\ (x-2009)\ (x-2010)^2\ (x-2011)^3\ (x-2012)^4$, for all x ∈ **R**.

If g is a function defined on **R** with values in the interval (0, ∞) such that f(x) = ℓn (g(x)), for all x ∈ **R**, then the number of points in **R** at which g has a local maximum is **[IIT-JEE 2010, Paper-2, (3, 0)/ 79]**

2. Let f, g and h be real-valued functions defined on the interval [0, 1] by $f(x) = e^{x^2} + e^{-x^2}$, $g(x) = xe^{x^2} + e^{-x^2}$ and $h(x) = x^2e^{x^2} + e^{-x^2}$. If a, b and c denote, respectively, the absolute maximum of f, g and h on [0, 1], then **[IIT-JEE 2010, Paper-1, (3, –1)/ 84]**

(A) a = b and c ≠ b (B) a = c and a ≠ b (C) a ≠ b and c ≠ b (D) a = b = c

3. Match the statements given in **Column-I** with the intervals/union of intervals given in **Column-II** **[IIT-JEE 2011, Paper-2, (8, 0), 80]**

Column-I	**Column-II**
(A) The set $\left\{\text{Re}\left(\frac{2iz}{1-z^2}\right): z \text{ is a complex number}, \|z\|=1, z \neq \pm 1\right\}$ is	(p) $(-\infty, -1) \cup (1, \infty)$
(B) The domain of the function f(x) = $\sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is	(q) $(-\infty, 0) \cup (0, \infty)$
(C) If $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$, then the set $\left\{f(\theta) : 0 \le \theta < \frac{\pi}{2}\right\}$ is	(r) $[2, \infty)$
(D) If $f(x) = x^{3/2}(3x - 10), x \ge 0$, then f(x) is increasing in	(s) $(-\infty, -1] \cup [1, \infty)$
	(t) $(-\infty, 0] \cup [2, \infty)$

4. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is **[IIT-JEE 2011, Paper-2, (4, 0), 80]**

5. Let p(x) be a real polynomial of least degree which has a local maximum at x = 1 and a local minimum at x = 3. If p(1) = 6 p(3) = 2 , then p′(0) is **[IIT-JEE 2012, Paper-1, (4, 0), 70]**

6. Let f : IR → IR be defined as $f(x) = |x| + |x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is **[IIT-JEE 2012, Paper-1, (4, 0), 70]**

7. The number of points in $(-\infty, \infty)$, for which $x^2 - x \sin x - \cos x = 0$, is **[JEE (Advanced) 2013, Paper-1, (2, 0)/60]**

(A) 6 (B) 4 (C) 2 (D) 0

8*. A rectangular sheet of fixed perimeter with sides having their lengths in the ratio 8 : 15 is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume. The lengths of the sides of the rectangular sheet are **[JEE (Advanced) 2013, Paper-1, (4, – 1)/60]**

(A) 24 (B) 32 (C) 45 (D) 60

9. A vertical line passing through the point (h, 0) intersects the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ at the points P and Q. Let the tangents to the ellipse at P and Q meet at the point R. If $\Delta(h)$ = area of the triangle PQR, $\Delta_1 = \max\limits_{1/2 \le h \le 1} \Delta(h)$ and $\Delta_2 = \min\limits_{1/2 \le h \le 1} \Delta(h)$, then $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 =$ **[JEE (Advanced) 2013, Paper-1, (4, – 1)/60]**

10*. The function $f(x) = 2|x| + |x + 2| - ||x + 2| - 2|x||$ has a local minimum or a local maximum at x = **[JEE (Advanced) 2013, Paper-2, (3, –1)/60]**

(A) – 2 (B) $\frac{-2}{3}$ (C) 2 (D) $\frac{2}{3}$

Paragraph for Question Nos. 11 to 12

Let $f : [0, 1] \to R$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable, $f(0) = f(1) = 0$ and satisfies $f''(x) - 2f'(x) + f(x) \geq e^x$, $x \in [0, 1]$.

11. Which of the following is true for $0 < x < 1$? **[JEE (Advanced) 2013, Paper-2, (3, –1)/60]**

(A) $0 < f(x) < \infty$ (B) $-\frac{1}{2} < f(x) < \frac{1}{2}$ (C) $-\frac{1}{4} < f(x) < 1$ (D) $-\infty < f(x) < 0$

12. If the function $e^{-x} f(x)$ assumes its minimum in the interval [0, 1] at $x = \frac{1}{4}$, which of the following is true ?

[JEE (Advanced) 2013, Paper-2, (3, –1)/60]

(A) $f'(x) < f(x)$, (B) $f'(x) > f(x)$, $0 < x < \frac{1}{4}$

(C) $f'(x) < f(x)$, $0 < x < \frac{1}{4}$ (D) $f'(x) < f(x)$, $\frac{3}{4} < x < 1$

13. A line L : $y = mx + 3$ meets y - axis at E(0, 3) and the arc of the parabola $y^2 = 16x$, $0 \leq y \leq 6$ at the point $F(x_0, y_0)$. The tangent to the parabola at $F(x_0, y_0)$ intersects the y-axis at $G(0, y_1)$. The slope m of the line L is chosen such that the area of the triangle EFG has a local maximum

Match List I with List II and select the correct answer using the code given below the lists :

[JEE (Advanced) 2013, Paper-2, (3, –1)/60]

	List - I		**List - II**
P.	m =	1.	$\frac{1}{2}$
Q.	Maximum area of ΔEFG is	2.	4
R.	y_0 =	3.	2
S.	y_1 =	4.	1

Codes :

	P	Q	R	S
(A)	4	1	2	3
(B)	3	4	1	2
(C)	1	3	2	4
(D)	1	3	4	2

14*. Let $a \in R$ and let $f : R \to R$ be given by $f(x) = x^5 - 5x + a$. Then

[JEE (Advanced) 2014, Paper-1, (3, 0)/60]

(A) f(x) has three real roots if a > 4 (B) f(x) has only one real root if a > 4

(C) f(x) has three real roots if a < – 4 (D) f(x) has three real roots if – 4 < a < 4

15. The slope of the tangent to the curve $(y - x^5)^2 = x(1 + x^2)^2$ at the point (1, 3) is

[JEE (Advanced) 2014, Paper-1, (3, 0)/60]

16. A cylindrical container is to be made from certain solid material with the following constraints: It has fixed inner volume of V mm^3, has a 2 mm thick solid wall and is open at the top. The bottom of the container is solid circular disc of thickness 2 mm and is of radius equal to the outer radius of the container.

If the volume of the material used to make the container is minimum when the inner radius of the container is 10 mm, then the value of $\frac{V}{250\pi}$ is **[JEE (Advanced) 2015, P-1 (4, 0) /88]**

17*. Let f, g : [–1, 2] → R be continuous function which are twice differentiable on the interval (–1, 2). Let the values of f and g at the points –1, 0 and 2 be as given in the following table :

	x = –1	x = 0	x = 2
f(x)	3	6	0
g(x)	0	1	–1

In each of the intervals (–1, 0) and (0, 2) the function (f – 3g)" never vanishes. Then the correct statement(s) is (are) **[JEE (Advanced) 2015, P-2 (4, –2)/ 80]**

(A) f'(x) – 3g'(x) = 0 has exactly three solutions in (–1, 0) ∪ (0, 2)
(B) f'(x) – 3g'(x) = 0 has exactly one solution in (–1, 0)
(C) f'(x) – 3g'(x) = 0 has exactly one solution in (0, 2)
(D) f'(x) – 3g'(x) = 0 has exactly two solutions in (–1, 0) and exactly two solutions in (0, 2)

18. Let f : R → (0, ∞) and g : R → R be twice differentiable functions such that f" and g" are continuous functions on R. Suppose f '(2) = g(2) = 0, f"(2) ≠ 0 and g'(2) ≠ 0, If $\lim_{x\to 2} \frac{f(x)g(x)}{f'(x)g'(x)} = 1$, then

[JEE (Advanced) 2016, Paper-2, (4, –2)/62]

(A) f has a local minimum at x = 2
(B) f has a local maximum at x = 2
(C) f"(2) > f(2)
(D) f(x) – f"(x) = 0 for at least one x ∈ R

Answer Q.19, Q.20 and Q.21 by appropriately matching the information given in the three columns of the following table.

Let f(x) = x + $\log_e x$ – x$\log_e x$, x ∈ (0, ∞)

- Column1 contains information about zeros of f(x), f′(x) and f′′(x).
- Column2 contains information about the limiting behavior of f(x), f′(x) and f′′(x) at infinity.
- Column3 contains information about increasing/decreasing nature of f(x) and f′(x).

Column-1	Column-2	Column-3
(I) f(x) = 0 for some x ∈ (1, e^2)	(i) $\lim_{x\to\infty}$ f(x) = 0	(P) f is increasing in (0, 1)
(II) f′(x) = 0 for some x ∈ (1, e)	(ii) $\lim_{x\to\infty}$ f(x) = – ∞	(Q) f is decreasing in (e, e^2)
(III) f′(x) = 0 for some x ∈ (0, 1)	(iii) $\lim_{x\to\infty}$ f′(x) = – ∞	(R) f′ is increasing in (0, 1)
(IV) f′′(x) = 0 for some x ∈ (1, e)	(iv) $\lim_{x\to\infty}$ f′′(x) = 0	(S) f′ is decreasing in (e, e^2)

19. Which of the following options is the only INCORRECT combination ?

[JEE(Advanced) 2017, Paper-1,(3, –1)/61]

(A) (I) (iii) (P) (B) (II) (iv) (Q) (C) (II) (iii) (P) (D) (III) (i) (R)

20. Which of the following options is the only CORRECT combination?

[JEE(Advanced) 2017, Paper-1,(3, –1)/61]

(A) (I) (ii) (R) (B) (III) (iv) (P) (C) (II) (iii) (S) (D) (IV) (i) (S)

21. Which of the following options is the only CORRECT combination?

[JEE(Advanced) 2017, Paper-1,(3, –1)/61]

(A) (III) (iii) (R) (B) (IV) (iv) (S) (C) (II) (ii) (Q) (D) (I) (i) (P)

22. If f : R → R is a twice differentiable function such that $f''(x) > 0$ for all $x \in R$, and $f\left(\frac{1}{2}\right) = \frac{1}{2}$, f(1) = 1, then **[JEE(Advanced) 2017, Paper-2,(3, –1)/61]**

(A) $f'(1) \le 0$ (B) $f'(1) > 1$ (C) $0 < f'(1) \le \frac{1}{2}$ (D) $\frac{1}{2} < f'(1) \le 1$

23. If f : R → R is a differentiable function such that $f'(x) > 2f(x)$ for all $x \in R$, and f(0) = 1, then **[JEE(Advanced) 2017, Paper-2,(4, –2)/61]**

(A) $f(x) > e^{2x}$ in (0, ∞) (B) $f'(x) < e^{2x}$ in (0, ∞)

(C) f(x) is increasing in (0, ∞) (D) f(x) is decreasing in (0, ∞)

24. If $f(x) = \begin{vmatrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$, then **[JEE(Advanced) 2017, Paper-2,(4, –2)/61]**

(A) $f(x)$ attains its minimum at x = 0 (B) $f(x)$ attains its maximum at x = 0

(C) $f'(x) = 0$ at more than three points in (–π, π) (D) $f'(x) = 0$ at exactly three points in (–π, π)

25. For every twice differentiable function f : R → [–2, 2] with $(f(0))^2 + (f'(0))^2 = 85$, which of the following statement(s) is (are) TRUE? **[JEE(Advanced) 2018, Paper-1,(4, –2)/60]**

(A) There exist r, s ∈ R, where r < s, such that f is one-one on the open interval (r, s)

(B) There exists $x_0 \in (-4, 0)$ such that $|f'(x_0)| \le 1$

(C) $\lim_{x\to\infty} f(x) = 1$

(D) There exists $\alpha \in (-4, 4)$ such that $f(\alpha) + f''(\alpha) = 0$ and $f'(\alpha) \ne 0$

26. Let f : R → R be given by $f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0 \\ x^2 - x + 1 & 0 \le x < 1 \\ (2/3)x^3 - 4x^2 + 7x - (8/3) & 1 \le x < 3 \\ (x-2)\ell n(x-2) - x + (10/3) & x \ge 3 \end{cases}$

Then which of the following options is/are Correct ? **[JEE(Advanced) 2019, Paper-1,(4, –1)/62]**

(A) f ′ is NOT differentiable at x = 1 (B) f is increasing on (–∞,0)

(C) f is onto (D) f ′ has a local maximum at x = 1

27. Let $f(x) = \frac{\sin \pi x}{x^2}$, x > 0.

Let $x_1 < x_2 < x_3 .... < x_n < .....$ be all points of local maximum of f

and $y_1 < y_2 < y_3 < ...... < y_n < ....$ be all the points of local minimum of f

Then which of the following options is/are correct ?

(A) $|x_n - y_n| > 1$ for every n (B) $x_1 < y_1$

(C) $x_{n+1} - x_n > 2$ for every n (D) $x_n \in \left(2n, 2n + \frac{1}{2}\right)$ for every n

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Let f : **R** → **R** be defined by **[AIEEE 2010 (8, –2), 144]**

$$f(x) = \begin{cases} k - 2x, & \text{if } x \le -1 \\ 2x + 3, & \text{if } x > -1 \end{cases}$$

If f has a local minimum at x = – 1, then a possible value of k is

(1) 0 (2) $-\frac{1}{2}$ (3) –1 (4) 1

2. Let f : **R** → **R** be a continuous function defined by $f(x) = \frac{1}{e^x + 2e^{-x}}$ **[AIEEE 2010(8, –2), 144]**

Statement -1 : $f(c) = \frac{1}{3}$, for some c ∈ R.

Statement -2 : $0 < f(x) \le \frac{1}{2\sqrt{2}}$, for all x ∈ R.

(1) Statement -1 is true, Statement-2 is true ; Statement -2 is not a correct explanation for Statement -1.
(2) Statement-1 is true, Statement-2 is false.
(3) Statement -1 is false, Statement -2 is true.
(4) Statement -1 is true, Statement -2 is true; Statement-2 is a correct explanation for Statement-1.

3. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is

[AIEEE 2010 (4, –1), 144]

(1) y = 1 (2) y = 2 (3) y = 3 (4) y = 0

4. Let f be a function defined by - **[AIEEE 2011 II(4, –1), 120]**

$$f(x) = \begin{cases} \frac{\tan x}{x}, & x \ne 0 \\ 1, & x = 0 \end{cases}$$

Statement - 1 : x = 0 is point of minima of f
Statement - 2 : f '(0) = 0.
(1) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1
(3) Statement-1 is true, statement-2 is false.
(4) Statement-1 is false, statement-2 is true.

5. The shortest distance between line y – x = 1 and curve $x = y^2$ is : **[AIEEE 2011 (4, –1), 120]**

(1) $\frac{\sqrt{3}}{4}$ (2) $\frac{3\sqrt{2}}{8}$ (3) $\frac{8}{3\sqrt{2}}$ (4) $\frac{4}{\sqrt{3}}$

6. A spherical balloon is filled with 4500π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is :**[AIEEE 2012(4, –1), 120]**

(1) $\frac{9}{7}$ (2) $\frac{7}{9}$ (3) $\frac{2}{9}$ (4) $\frac{9}{2}$

7. Let a, b $\in$ R be such that the function f given by f(x) = ℓn |x| + bx^2 + ax, x $\neq$ 0 has extreme values at x = – 1 and x = 2.

Statement-1 : f has local maximum at x = – 1 and at x = 2. **[AIEEE 2012 (4, –1), 120]**

Statement-2 : a = $\frac{1}{2}$ and b = $\frac{-1}{4}$.

(1) Statement-1 is false, Statement-2 is true.
(2) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(3) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(4) Statement-1 is true, statement-2 is false.

8. The real number k for which the equation, 2x^3 + 3x + k = 0 has two distinct real roots in [0, 1] **[AIEEE - 2013, (4, –1),120]**

(1) lies between 1 and 2 (2) lies between 2 and 3
(3) lies between –1 and 0 (4) does not exist.

9. If f and g are differentiable functions in [0, 1] satisfying f(0) = 2 = g(1), g(0) = 0 and f(1) = 6, then for some c$\in$]0, 1[: **[JEE(Main) 2014, (4, – 1), 120]**

(1) f'(c) = g'(c) (2) f'(c) = 2g'(c) (3) 2f'(c) = g'(c) (4) 2f'(c) = 3g'(c)

10. If x = –1 and x = 2 are extreme points of f(x) = αlog|x| + βx^2 + x then : **[JEE(Main) 2014, (4, –1), 120]**

(1) $\alpha = 2, \beta = -\frac{1}{2}$ (2) $\alpha = 2, \beta = \frac{1}{2}$ (3) $\alpha = -6, \beta = \frac{1}{2}$ (4) $\alpha = -6, \beta = -\frac{1}{2}$

11. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then **[JEE(Main) 2016, (4, – 1), 120]**

(1) (4 – π) x = πr (2) x = 2r (3) 2x = r (4) 2x = (π + 4) r

12. Consider f(x) = $\tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$, x $\in \left(0, \frac{\pi}{2}\right)$. A normal to y = f(x) at x = $\frac{\pi}{6}$ also passes through the point : **[JEE(Main) 2016, (4, – 1), 120]**

(1) $\left(0, \frac{2\pi}{3}\right)$ (2) $\left(\frac{\pi}{6}, 0\right)$ (3) $\left(\frac{\pi}{4}, 0\right)$ (4) (0, 0)

13. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is : **[JEE(Main) 2017, (4, – 1), 120]**

(1) 12.5 (2) 10 (3) 25 (4) 30

14. The normal to the curve y(x – 2)(x – 3) = x + 6 at the point where the curve intersects the y-axis passes through the point : **[JEE(Main) 2017, (4, – 1), 120]**

(1) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ (2) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (3) $\left(\frac{1}{2}, -\frac{1}{3}\right)$ (4) $\left(\frac{1}{2}, \frac{1}{3}\right)$

15. The radius of a circle, having minimum area, which touches the curve y = 4 – x^2 and the lines, y = |x| is **[JEE(Main) 2017, (4, – 1), 120]**

(1) $2\left(\sqrt{2}+1\right)$ (2) $2\left(\sqrt{2}-1\right)$ (3) $4\left(\sqrt{2}-1\right)$ (4) $4\left(\sqrt{2}+1\right)$

16. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angels , then the value of b is : **[JEE(Main) 2018, (4, – 1), 120]**

(1) 4 (2) $\frac{9}{2}$ (3) 6 (4) $\frac{7}{2}$

17. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in R - \{-1, 0, 1\}$. If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of h(x) is : **[JEE(Main) 2018, (4, – 1), 120]**

(1) $-2\sqrt{2}$ (2) $2\sqrt{2}$ (3) 3 (4) – 3

18. Let A(4, –4) and B(9, 6) be points on the parabola, $y^2 = 4x$. Let C be chosen on the arc AOB of the parabola, where O is the origin, such that the area of ΔACB is maximum. Then, the area (in sq. units) of ΔACB, is : **[JEE(Main) 2019, Online (09-01-19),P-2 (4, – 1), 120]**

(1) $30\frac{1}{2}$ (2) $31\frac{3}{4}$ (3) $31\frac{1}{4}$ (4) 32

19. A helicopter is flying along the curve given by $y - x^{3/2} = 7$, $(x \ge 0)$. A soldier positioned at the point $\left(\frac{1}{2}, 7\right)$ wants to shoot down the helicopter when it is nearest to him. Then this nearest distance is :

[JEE(Main) 2019, Online (10-01-19),P-2 (4, – 1), 120]

(1) $\frac{1}{2}$ (2) $\frac{1}{3}\sqrt{\frac{7}{3}}$ (3) $\frac{1}{6}\sqrt{\frac{7}{3}}$ (4) $\frac{\sqrt{5}}{6}$

20. Let $f(x) = \frac{x}{\sqrt{a^2 + x^2}} - \frac{(d - x)}{\sqrt{b^2 + (d - x)^2}}$, $x \in R$, where a, b and d are non-zero real constant. Then :

[JEE(Main) 2019, Online (11-01-19),P-2 (4, – 1), 120]

(1) f is neither increasing nor decreasing function of x
(2) f is an increasing function of x
(3) f is not a continuous function of x
(4) f is a decreasing function of x

21. Let f : [0,2] $\to$ R be a twice differentiable function such that f'' (x) > 0, for all $x \in (0,2)$.
If $\phi(x) = f(x) + f(2-x)$, then ϕ is : **[JEE(Main) 2019, Online (08-04-19),P-1 (4, – 1), 120]**
(1) decreasing on (0,1) and increasing on (1,2).
(2) increasing on (0,2)
(3) increasing on (0,1) and decreasing on (1,2)
(4) decreasing on (0.2)

22. The shortest distance between the line y = x and the curve $y^2 = x - 2$ is :

[JEE(Main) 2019, Online (08-04-19),P-1 (4, – 1), 120]

(1) $\frac{7}{4\sqrt{2}}$ (2) 2 (3) $\frac{7}{8}$ (4) $\frac{11}{4\sqrt{2}}$

23. If m is the minimum value of k for which the function $f(x) = x\sqrt{kx - x^2}$ is increasing in the interval [0,3] and M is the maximum value of f in [0,3] when k = m, then the ordered pair (m,M) is equal to :

[JEE(Main) 2019, Online (12-04-19),P-1 (4, – 1), 120]

(1) $(4, 3\sqrt{2})$ (2) $(4, 3\sqrt{3})$ (3) $(3, 3\sqrt{3})$ (4) $(5, 3\sqrt{6})$

24. A Water tank has the shape of an inverted right circular cone, whose semi–vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$.

Water is poured into it at a constant rate of 5 cubic meter per minute. Then the rate (in m/ min), at which the level of water is rising at the instant when the depth of water in the tank is 10 m; is :

[JEE(Main) 2019, Online (09-04-19),P-2 (4, – 1), 120]

(1) $1/5\pi$ (2) $2/\pi$ (3) $1/15\pi$ (4) $1/10\pi$

25. Let f(x) be a polynomial of degree 5 such that x = ±1 are its critical points. If $\lim_{x\to 0}\left(2+\frac{f(x)}{x^3}\right) = 4$, then which one of the following is not true? **[JEE(Main) 2020, Online (07-01-20),P-2 (4, –1), 120]**

(1) x = 1 is a point of minima and x = –1 is a point of maxima of f.

(2) f(1) –4f(–1) = 4.

(3) x = 1 is a point of maxima and x = –1 is a point of minimum of f.

(4) f is an odd function.

26. Let $f(x) = x\cos^{-1}(-\sin|x|)$, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then which of the following is true?

(1) $f'(0) = -\frac{\pi}{2}$ **[JEE(Main) 2020, Online (08-01-20),P-1 (4, –1), 120]**

(2) f' is decreasing in $\left(-\frac{\pi}{2}, 0\right)$ and increasing in $\left(0, \frac{\pi}{2}\right)$

(3) f is not differentiable at x = 0

(4) f' is increasing in $\left(-\frac{\pi}{2}, 0\right)$ and decreasing in $\left(0, \frac{\pi}{2}\right)$

27. If c is a point at which Rolle's theorem holds for the function, $f(x) = \log_e\left(\frac{x^2+\alpha}{7x}\right)$ in the interval [3, 4], where $\alpha \in R$, then f''(c) is equal to : **[JEE(Main) 2020, Online (08-01-20),P-1 (4, –1), 120]**

(1) $\frac{\sqrt{3}}{7}$ (2) $-\frac{1}{24}$ (3) $-\frac{1}{12}$ (4) $\frac{1}{12}$

28. Let f be any function continuous on [a, b] and twice differentiable on (a, b). If for all $x \in (a, b)$, $f'(x) > 0$ and $f''(x) < 0$, then for any $c \in (a, b)$, $\frac{f(c)-f(a)}{f(b)-f(c)}$ is greater than

[JEE(Main) 2020, Online (09-01-20),P-1 (4, –1), 120]

(1) 1 (2) $\frac{b-c}{c-a}$ (3) $\frac{c-a}{b-c}$ (4) $\frac{b+a}{b-a}$

Answers

EXERCISE - 1

PART -I

Section (A) :

A-1. **(i)** $y = 4x + 5$ (ii) $y + x = 2$ (tangent) , $y = x$ (normal)

(iii) $16x + 13y = 9a$ **(iv)** $y = 0$

A-2 **(i)** $2x + y = 4$, $y = 2x$ **(ii)** $x + y = 3$

A-3. **(i)** $\left(4, \frac{8}{3}\right)$ **(ii)** (9/4, 3/8) **(iii)** (1, –1) , (–1, –5)

A-4. **(i)** $y = 12x - 16$ or $y = 12x + 16$ **(ii)** $x - 2y + 1 = 0$ or $2y + x - 1 = 0$

A-5. **(i)** $a = 1$, $b = -2$ **(ii)** **1**

Section (B) :

B-1. 1 **B-2.** 45° at (1, 0) and $\tan^{-1}\left(\frac{e}{e^2+2}\right)$ at (e, 1)

B-3. 90° **B-5.** $\frac{3}{\sqrt{2}}$ **B-6.** $\sqrt{20} - 1$

Section (C) :

C-1. (i) – 2 cm/min (ii) 2 cm^2/min **C-2.** $2x^2 - 3x + 1$ **C-3.** (i) 2 km/hr (ii) 6 km/h

C-4. 7.5 m^3

Section (D) :

D-2. (i) M.D. in $(-\infty, -3]$
M.I. in $[-3, 0]$
M.D. in $[0, 2]$
M.I. in $[2, \infty)$

(ii) M.D. in $\left(0, \frac{1}{\sqrt{3}}\right]$
M.I. in $\left[\frac{1}{\sqrt{3}}, \infty\right)$

D-4. $a \in R^+$

D-5. **Ans.** (i) Neither increasing nor decreasing at x = –1 and increasing at x = 2
(ii) at x = – 2 decreasing
at x = 0 decreasing
at x = 3 neither increasing nor decreasing
at x = 5 increasing
(iii) Strictly increasing at x = 0
(iv) Strictly increasing at x = 2, neither I nor D at x = 1
(v) Strictly increasing at x = 0

D-8. $b \in (0, e]$

D-9. (i) local max at x = 1, local min at x = 6

(ii) local max. at $x = -\frac{1}{5}$, local min. at x = – 1

(iii) local mini at $x = \frac{1}{e}$, No local maxima

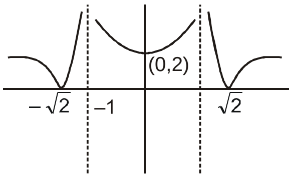
D-10. **(i)** local maxima at $x = \log_2 \frac{4}{3}$ and local minima at x = 1

(ii) local min at 0, local max at 2

(iii) local max at $x = 0, \frac{2\pi}{3}$, local min at $x = \frac{\pi}{2}, \pi$

(iv) local maxima at –1 and local minma at 0

(v) local minima at $x = \pm\sqrt{2}$, 0



(v) local minima at $x = \pm, \sqrt{2}$ 0

D-11. local max at x = 1, local min at x = 2.

Section (E) :

E-1. (i) max = 8, min. = – 8

(ii) max = $\sqrt{2}$, min = – 1

(iii) max. = 8, min. = – 10

(iv) max. = 25, min = – 39

(v) max. at x = π/6, max. value = 3/4; min. at x = 0 and π/2, min. value = 1/2

E-3. F = 191 **E-5.** $\frac{4\pi r^3}{3\sqrt{3}}$ **E-7.** 110 m, $\frac{220}{\pi}$ m **E-8.** 32 sq. units

E-9. 12cm, 6 cm

Section (F) :

F-1. 1

PART -II

Section (A) :

A-1. (A) **A-2.** (A) **A-3.** (C) **A-4.** (C) **A-5.** (B) **A-6.** (C) **A-7.** (D)
A-8. (B) **A-9.** (A) **A-10.** (A)

Section (B) :

B-1. (B) **B-2.** (D) **B-3.** (D) **B-4.** (C) **B-5.** (B) **B-6.** (D)

Section (C) :

C-1. (B) **C-2** (C) **C-3.** (A) **C-4.** (B) **C-5.** (C)

Section (D)

D-1. (A) **D-2.** (B) **D-3.** (A) **D-4.** (C) **D-5.** (D) **D-6.** (C)

Section (E) :

E-1. (C) **E-2.** (D) **E-3.** (D) **E-4.** (C) **E-5.** (A) **E-6.** (D) **E-7.** (A)

Section (F) :

F-1. (A) **F-2.** (C) **F-3.** (B) **F-4.** (D) **F-5.** (C)

PART -III

1. (A → p, q, r,s); (B → r) ; (C → p,q,r,s); (D →q)

2. (A) → (p), (B) → (s), (C) → (q), (D) → (r)

3. (A) → (p,q), (B) → (r,s), (C) → (r,s), (D) → (r,s)

4. (A) → (r), (B) → (s), (C) → (q),(D) → (p)

EXERCISE - 2

PART -I

1.	(D)	**2.**	(A)	**3.**	(B)	**4.**	(C)	**5.**	(B)	**6.**	(B)	**7.**	(B)
8.	(C)	**9.**	(D)	**10.**	(C)	**11.**	(A)	**12.**	(D)	**13.**	(C)	**14.**	(A)
15.	(A)	**16.**	(D)	**17.**	(B)	**18.**	(B)	**19.**	(A)	**20.**	(C)	**21.**	(A)
22.	(B)												

PART -II

1.	02.00	**2.**	24.00	**3.**	00.36 or 00.37	**4.**	07.50
5.	09.42 or 09.43	**6.**	02.00	**7.**	02.82 or 02.83	**8.**	00.50
9.	29.00	**10.**	04.00	**11.**	11.50	**12.**	26.28
13.	50.00	**14.**	03.33	**15.**	32.00	**16.**	36.00
17.	40.50	**18.**	19.85 or 19.86	**19.**	40.00	**20.**	10.83
21.	76.00						

PART -III

1.	(CD)	**2.**	(ABC)	**3.**	(AB)	**4.**	(AD)	**5.**	(BC)	**6.**	(AB)	**7.**	(AD)
8.	(AD)	**9.**	(AB)	**10.**	(ABC)	**11.**	(AC)	**12.**	(AD)	**13.**	(ACD)	**14.**	(BD)
15.	(AB)	**16.**	(AC)	**17.**	(ABC)	**18.**	(ACD)	**19.**	(AC)	**20.**	(AB)	**21.**	(BD)
22.	(BCD)	**23.**	(BC)	**24.**	(ABC)	**25.**	(AC)	**26.**	(ACD)	**27.**	(ABCDE)		

PART -IV

1.	(A)	**2.**	(C)	**3.**	(D)	**4.**	(C)	**5.**	(B)	**6.**	(A)	**7.**	(C)
8.	(B)	**9.**	(C)	**10.**	(D)	**11.**	(A)						

EXERCISE - 3

PART -I

1. 1 **2.** (D) **3.** (A) $\rightarrow$ (s), (B) $\rightarrow$ (t), (C) $\rightarrow$ (r),(D) $\rightarrow$ (r)

4.	2	**5.**	(9)	**6.**	(5)	**7.**	(C)	**8.**	(AC)	**9.**	**9**	**10.**	(AB)
11.	(D)	**12.**	(C)	**13.**	(A)	**14.**	(BD)	**15.**	(8)	**16.**	**4**	**17.**	(B,C)
18.	(A,D)	**19.**	(D)	**20.**	(C)	**21.**	(C)	**22.**	(B)	**23.**	(A,C)	**24.**	(BC)
25.	(ABD)	**26.**	(A,C,D)	**27.**	(ACD)								

PART -II

1.	(3)	**2.**	(4)	**3.**	(3)	**4.**	(2)	**5.**	(2)	**6.**	(3)	**7.**	(2)
8.	(4)	**9.**	(2)	**10.**	(1)	**11.**	(2)	**12.**	(1)	**13.**	(3)	**14.**	(2)
15.	(3)	**16.**	(2)	**17.**	(2)	**18.**	(3)	**19.**	(3)	**20.**	(2)	**21.**	(1)
22.	(1)	**23.**	(2)	**24.**	(1)	**25.**	(1)	**26.**	(2)	**27.**	(4)	**28.**	(3)