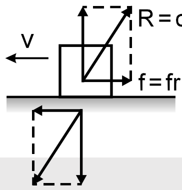
FRICTION

1. FRICTION

When two bodies are kept in contact, electromagnetic forces act between the charged particles (molecules) at the surfaces of the bodies. Thus, each body exerts a contact force on the other. The magnitudes of the contact forces acting on the two bodies are equal but their directions are opposite and therefore the contact forces obey Newton's third law.



The direction of the contact force acting on a particular body is not necessarily perpendicular to the contact surface. We can resolve this contact force into two components, one perpendicular to the contact surface and the other parallel to it (figure. The perpendicular component is called the normal contact force or normal force (generally written as N) and the parallel component is called friction (generally written as f).

Therefore if R is contact force then R = $\sqrt{f^2 + N^2}$

2. REASONS FOR FRICTION

(i) Inter-locking of extended parts of one object into the extended parts of the other object.

(ii) Bonding between the molecules of the two surfaces or objects in contact.

3. FRICTION FORCE IS OF TWO TYPES.

a. Kinetic b. Static

(a) Kinetic Friction Force

Kinetic friction exists between two contact surfaces only when there is **relative motion** between the two contact surfaces. It stops acting when relative motion between two surfaces ceases.

DIRECTION OF KINECTIC FRICTION ON AN OBJECT

It is opposite to the relative velocity of the object with respect to the other object in contact considered.

Note that its direction is not opposite to the force applied it is opposite to the relative motion of the body considered which is in contact with the other surface.

MAGNITUDE OF KINETIC FRICTION

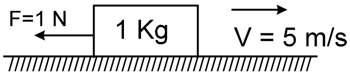
The magnitude of the kinetic friction is proportional to the normal force acting between the two bodies. We can write

$$f_k = \mu_k N$$

where N is the normal force. **The proportionality constant μ_k is called the coefficient of kinetic friction and its value depends on the nature of the two surfaces in contact.**

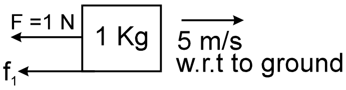
Solved Examples

Example 1. Find the direction of kinetic friction force

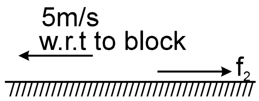


(a) on the block, exerted by the ground. (b) on the ground, exerted by the block.

Solution : (a)



(b)



where f_1 and f_2 are the friction forces on the block and ground respectively.

Example 2. In above example correct relation between magnitude of f_1 and f_2 is

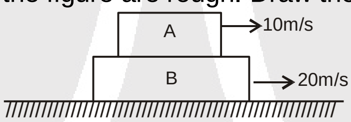
(A) $f_1 > f_2$ (B) $f_2 > f_1$

(C) $f_1 = f_2$ (D) not possible to decide due to insufficient data.

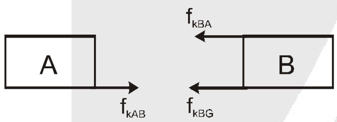
Solution : By Newton's third law the above friction forces are action-reaction pair and equal but opposite to each other in direction. Hence (C).

Also note that the direction of kinetic friction has nothing to do with applied force F.

Example 3. All surfaces as shown in the figure are rough. Draw the friction force on A & B

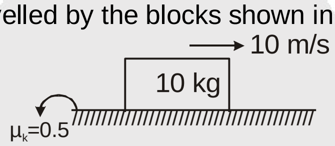


Solution :



Kinetic friction acts in such a way so as to reduce relative motion.

Example 4. Find out the distance travelled by the blocks shown in the figure before it stops.



Solution :

$N - 10\,g = 0$

$N = 100\ N$

$f_x = \mu_k N$

$\mu = \mu_s = \mu_k$ when not mentioned

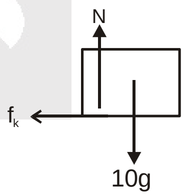
$f_x = 0.5 \times 100 = 50\ N$

$F_x = ma$

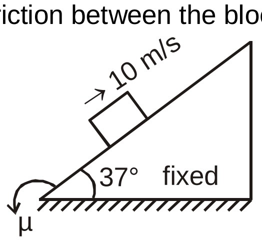
$50 = 10\,a \Rightarrow a = 5$

$\therefore\ v^2 = u^2 + 2as$

$0^2 = 10^2 + 2\,(-5)\,(S)$ $\quad\therefore\quad S = 10\ m$



Example 5. Find out the distance travelled by the block on incline before it stops. Initial velocity of the block is 10 m/s and coefficient of friction between the block and incline is $\mu = 0.5$.



Solution : $N = mg \cos 37°$

$\therefore\ mg \sin 37° + \mu N = ma$

$a = 10\ m/s^2$ down the incline

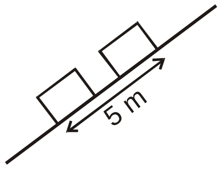
Now $v^2 = u^2 + 2as$

$0 = 10^2 + 2(-10)\ S$

$\therefore\ S = 5\ m$

Example 6. Find the time taken in the above example by the block to reach the initial position.

Solution : $a = g \sin 37° - \mu g \cos 37°$

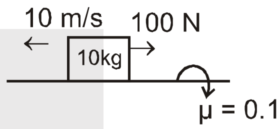


$\therefore\ a = 2\ m/s^2$ down the incline

$\therefore\ S = ut + \frac{1}{2}at^2 \Rightarrow S = \frac{1}{2} \times 2 \times t^2$

$\therefore\ t = \sqrt{5}$ sec.

Example 7. A block is given a velocity of 10 m/s and a force of 100 N in addition to friction force is also acting on the block. Find the retardation of the block?



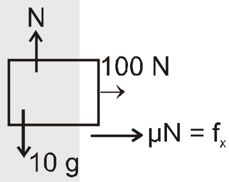
Solution : As there is relative motion

$\therefore$ Kinetic friction will act to reduce this relative motion.

$f_k = \mu N = 0.1 \times 10 \times 10 = 10\ N$

$100 + 10 = 10a$

$a = \frac{110}{10} = 11\ m/s^2$



Example 8. A body is projected up along a rough inclined plane from the bottom with some velocity. It travels up the incline and then returns back. If the time of ascent is t_a and time of descent is t_d, then

(A) $t_a = t_d$ (B) $t_a > t_d$ (C) $t_a < t_d$ (D) data insufficient

Solution : Let velocity of projection be V and velocity of the block when it returns back = V'

then $V > V'$ (since some K.E. is lost to friction)

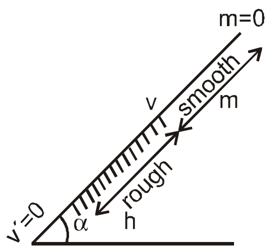
Hence average velocity during ascent > average velocity during descent

$\Rightarrow\quad t_a < t_d$

Example 9. The upper portion of an inclined plane of inclination α is smooth and the lower portion is rough. A particle slides down from rest from the top and just comes to rest at the foot. If the ratio of the smooth length to rough length is m : n, the coefficient of friction is :

(A) $\left[\frac{m+n}{n}\right]\tan\alpha$ (B) $\left(\frac{m+n}{n}\right)\cot\alpha$ (C) $\left(\frac{m-n}{n}\right)\cot\alpha$ (D) $\frac{1}{2}$

Solution : On smooth surface $a_1 = g \sin \alpha$



$\therefore\ v^2 = u^2 + 2a_1s_1 = 0 + 2\ g \sin \alpha\ .m$

On rough surface

$a_2 = g \sin \alpha - \mu g \cos \alpha$

$\therefore\ v'^2 = v^2 + 2a_2s_2$

$O = 2mg \sin \alpha + 2g (\sin \alpha - \mu \cos \alpha)n$

$\Rightarrow\quad \mu = \left[\frac{m+n}{n}\right]\tan \alpha$

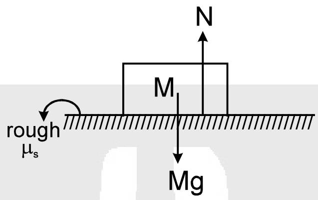
(b) STATIC FRICTION

It exists between the two surfaces when there is tendency of relative motion but no relative motion along the two contact surface.

For example consider a bed inside a room ; when we gently push the bed with a finger, the bed does not move. This means that the bed has a tendency to move in the direction of applied force but does not move as there exists static friction force acting in the opposite direction of the applied force.

Solved Example

Example 10. What is value of static friction force on the block?



Solution : In horizontal direction as acceleration is zero.

Therefore $\Sigma F = 0$.

$\therefore\ f = 0$

Direction of static friction force :

The static friction force on an object is opposite to its impending motion relative to the surface.

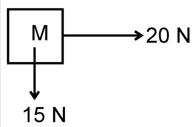
Following steps should be followed in determining the direction of static friction force on an object.

(i) Draw the free body diagram with respect to the other object on which it is kept.
(ii) Include pseudo force also if contact surface is accelerating.
(iii) Decide the resultant force and the component parallel to the surface of this resultant force.
(iv) The direction of static friction is opposite to the above component of resultant force.

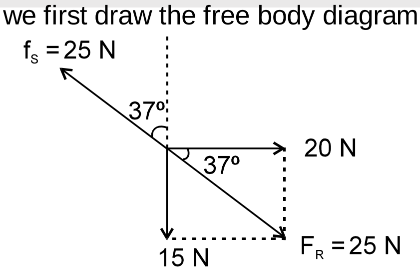
Note : Here once again the static friction is involved when there is no relative motion between two surfaces.

Solved Example

Example 11. In the following figure an object of mass M is kept on a rough table as seen from above. Forces are applied on it as shown. Find the direction of static friction if the object does not move.



Solution : In the above problem we first draw the free body diagram of find the resultant force.



As the object doe not move this is not a case of kinetic friction. The direction of static friction is opposite to the direction of the resultant force F_R as shown in figure by f_s. Its magnitude is equal to 25 N.

4. MAGNITUDE OF KINETIC AND STATIC FRICTION

Kinetic friction :

The magnitude of the kinetic friction is proportional to the normal force acting between the two bodies. We can write

$$f_k = \mu_k N$$

where N is the normal force. **The proportionality constant μ_k is called the coefficient of kinetic friction and its value depends on the nature of the two surfaces in contact.** If the surfaces are smooth μ_k will be small, if the surfaces are rough μ_k will be large. It also depends on the materials of the two bodies in contact.

Static friction :

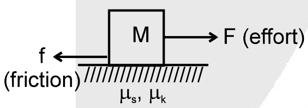
The magnitude of static friction is equal and opposite to the external force exerted, till the object at which force is exerted is at rest. **This means it is a variable and self adjusting force. However it has a maximum value called limiting friction.**

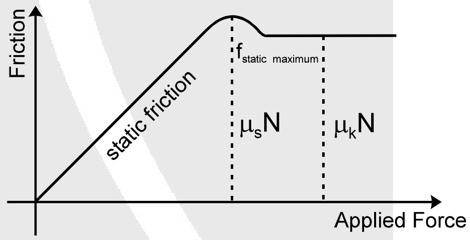
$$f_{max} = \mu_s N$$

The actual force of static friction may be smaller than $\mu_s N$ and its value depends on other forces acting on the body. The magnitude of frictional force is equal to that required to keep the body at relative rest.

$$0 \le f_s \le f_{smax}$$

Here μ_s and μ_k are proportionality constants. μ_s is called coefficient of static friction and μ_k is called coefficient of kinetic friction. They are dimensionless quantities independent of shape and area of contact. It is a property of the two contact surfaces. $\mu_s > \mu_k$ for a given pair of surfaces. If not mentioned then $\mu_s = \mu_k$ can be taken. Value of μ can be from 0 to ∞.



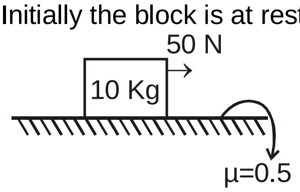


Following table gives a rough estimate of the values of coefficient of static friction between certain pairs of materials. The actual value depends on the degree of smoothness and other environmental factors. For example, wood may be prepared at various degrees of smoothness and the friction coefficient will vary.

Material	μ_s	Material	μ_s
Steel and steel	0.58	Copper and copper	1.60
Steel and brass	0.35	Teflon and teflon	0.04
Glass and glass	1.00	Rubber tyre on dry concrete road	1.0
Wood and wood	0.35		
Wood and metal	0.40	Rubber tyre on wet concrete road	0.7

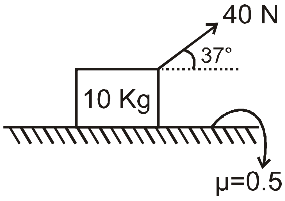
Solved Example

Example 12. Find acceleration of block. Initially the block is at rest.



Solution : zero

Example 13. Find out acceleration of the block. Initially the block is at rest.



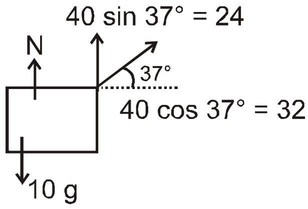
Solution : $N + 24 - 100 = 0$ for vertical direction

$\therefore$ N = 76 N

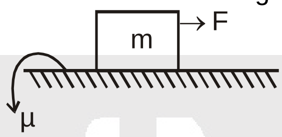
Now $0 \le f_s \le \mu_s N \Rightarrow 0 \le f_s \le 76 \times 0.5$

$0 \le f_s \le 38$ N $\therefore$ 32 < 38 Hence f = 32

$\therefore$ Acceleration of block is zero.



Example 14. Find out acceleration of the block for different ranges of F.

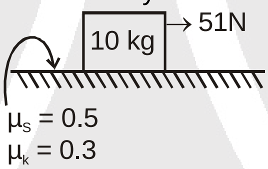


Solution : $0 \le f \le \mu_S N \Rightarrow 0 \le f \le \mu_S mg$

$a = 0$ if $F \le \mu_S mg$

$a = \dfrac{F - \mu Mg}{M}$ if $F > \mu Mg$

Example 15. Find out acceleration of the block. Initially the block is at rest.



Solution : $0 \le f_s \le \mu_S N$

$0 \le f_s \le 50$

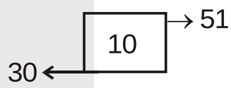
Now 51 > 50

$\therefore$ Block will move but if the block starts moving then kinetic friction is involved.

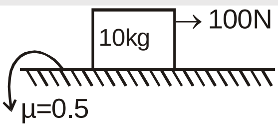
$K_F = \mu_k N = 0.3 \times 100 = 30$ N

$\therefore$ $51 - 30 = 10$ a

$\therefore$ a = 2.1 m/s^2



Example 16. Find out the minimum force that must be applied on the block vertically downwards so that the block doesn't move.



Solution : $100 - f_s = 0$

$\therefore$ $f_s = 100$ .....(1)

$F + 10\,g = N \Rightarrow N = 100 + F$ .....(2)

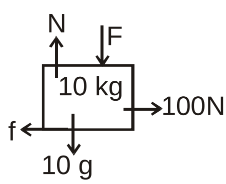
Now $0 \le f_S \le \mu N$

$100 \le 0.5$ N

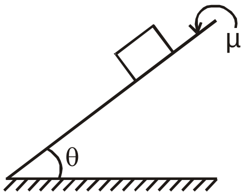
$100 \le 0.5$ [100 + F]

$200 \le 100 + F$; $F \ge 100$ N

$\therefore$ Minimum F = 100 N



Example 17. The angle of inclination is slowly increased. Find out the angle at which the block starts moving.



Solution : $0 \leq f \leq \mu_S N$

mg sinθ > f_{smax}

mg sin θ > μN

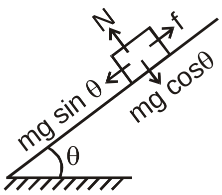
mg sinθ > μ mg cos θ

$\therefore$ tan θ > μ

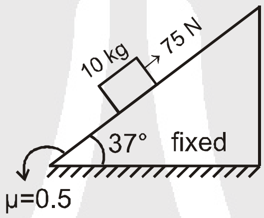
$\theta = \tan^{-1} \mu$

for tan $\theta \leq \mu$ no sliding on inclined plane.

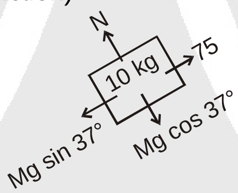
This method is used for finding out the value of μ practically.



Example 18. Find out the acceleration of the block. If the block is initially at rest.

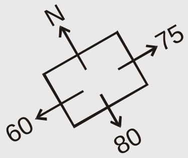


Solution : (FBD of the block excluding friction)



N = 10 g cos 37° = 80 N

Now $0 \leq f_s \leq \mu N$; $0 \leq f_s \leq 0.5 \times 80$ $\therefore$ $f_s \leq 40$ N



We will put value of f in the last i.e. in the direction opposite to resultant of other forces. f acts down the incline and its value is of = 75 – 60 = 15 N

So acceleration is zero

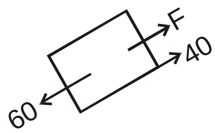
Example 19. In the above problem how much force should be added to 75 N force so that block starts to move up the incline.

Solution : $\therefore$ 60 + 40 = 75 + f_{extra} $\therefore$ f_s = 25 N

Example 20. In the above problem what is the minimum force by which 75 N force should be replaced with so that the block does not move.

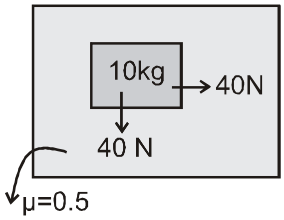
Solution : In this case the block has a tendency to move downwards.

Hence friction acts upwards.



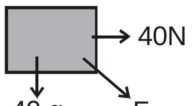
$\therefore$ F + 40 = 60 $\therefore$ F = 20 N

Example 21. Top view of a block on a table is shown ($g = 10\ m/s^2$).



Find out the acceleration of the block.

Solution :

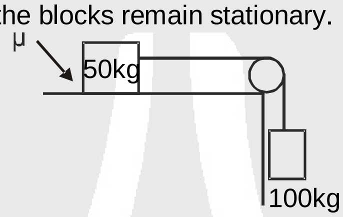


$F_R = 40\sqrt{2} \cong 56N$

Now $f_s \le \mu N$ $\quad \therefore f_s \le 50$; $\quad F_R > f_{smax}$

Hence the block will move. $a = \dfrac{40\sqrt{2} - 50}{10} = \left(4\sqrt{2} - 5\right) m/s^2$

Example 22. Find minimum μ so that the blocks remain stationary.



Solution : T = 100 g = 1000N

$\therefore$ f = 1000 to keep the block stationary

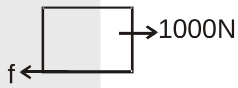
Now f_{max} = 1000N

μN = 1000

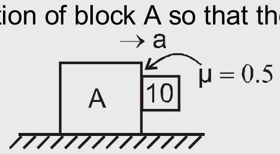
μ = 2

Can μ be greater than 1 ?

Yes $0 < \mu \le \infty$



Example 23. Find out minimum acceleration of block A so that the 10 kg block doesn't fall.



Solution : Applying NL in horizontal direction

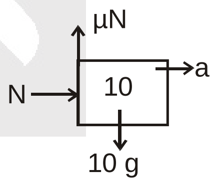
N = 10 a ......(1)

Applying NL in vertical direction

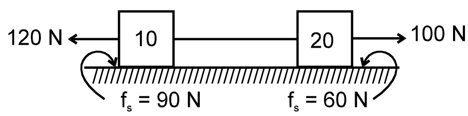
10 g = μ N ......(2)

10 g = μ 10 a from (1) & (2)

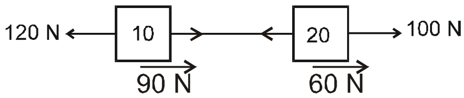
$\therefore\ a = \dfrac{g}{\mu} = 20\ m/s^2$



Example 24. Find the tension in the string in situation as shown in the figure below. Forces 120 N and 100 N start acting when the system is at rest and the maximum value of static friction on 10 kg is 90 N and that on 20 kg is 60N?

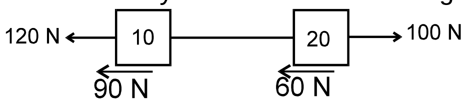


Solution : (i) Let us assume that system moves towards left then as it is clear from FBD, net force in horizontal direction is towards right. Therefore the assumption is not valid.



Above assumption is not possible as net force on system comes towards right. Hence system is not moving towards left.

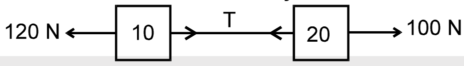
(ii) Similarly let us assume that system moves towards right.



Above assumption is also not possible as net force on the system is towards left in this situation.

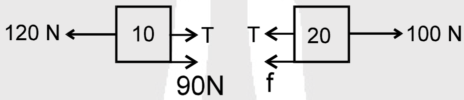
Hence assumption is again not valid.

Therefore it can be concluded that the system is stationary.



$f_{max} = 90N$ $\quad$ $f_{max} = 60N$

Assuming that the 10 kg block reaches limiting friction first then using FBD's.



$120 = T + 90 \qquad \Rightarrow \qquad T = 30$ N

Also $T + f = 100$

$\therefore \quad 30 + f = 100$

$f = 70$ N which is not possible as the limiting value is 60 N for this surface of block.

$\therefore$ Our assumption is wrong and now taking the 20 kg surface to be limiting we have

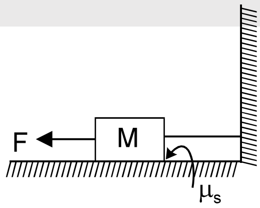
$T + 60 = 100$ N $\qquad \Rightarrow \qquad T = 40$ N

Also $f + T = 120$ N $\quad \Rightarrow \qquad f = 80$ N

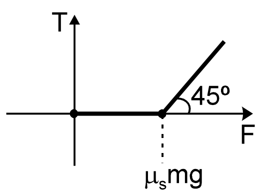
This is acceptable as static friction at this surface should be less than 90 N.

Hence the tension in the string is T = 40 N.

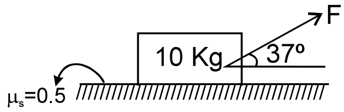
Example 25. In the following figure force F is gradually increased from zero. Draw the graph between applied force F and tension T in the string. The coefficient of static friction between the block and the ground is μ_s. {Initially string is horizontal & has zero tension.}



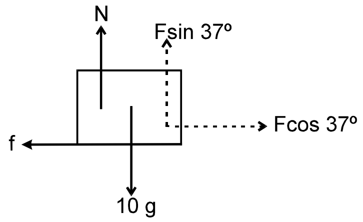
Solution : As the external force F is gradually increased from zero it is compensated by the friction and the string bears no tension. When limiting friction is achieved by increasing force F to a value till $\mu_s mg$, the further increase in F is transferred to the string.



Example 26. Force F is gradually increased from zero. Determine whether the block will first slide or lift up?



Solution : There are minimum magnitude of forces required both in horizontal and vertical direction either to slide on lift up the block. The block will first slide on lift up will depend upon which minimum magnitude of force is lesser.



For vertical direction to start lifting up

$F \sin 37^o + N - Mg \geq 0.$

N becomes zero just lifting condition.

$F_{lift} \geq \dfrac{10g}{3/5}$ $\quad\quad\therefore\quad F_{lift} \geq \dfrac{500}{3}$ N

For horizontal direction to start sliding $F \cos 37 \geq \mu_s N$

$F \cos 37^o > 0.5\,[10g - F \sin 37^o]$ $\;(\because\;\; N = 10\,g - F \sin 37^o)$

Hence$F_{slide} > \dfrac{50}{\cos 37^o + 0.5 \sin 37^o}$

$F_{slide} > \dfrac{500}{11}$ N

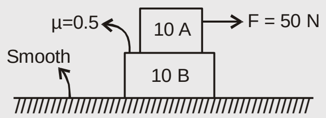
$F_{lift} > \dfrac{500}{3}$ N. $\quad\Rightarrow\;\; F_{slide} < F_{lift}$

Therefore the block will begin to slide before lifting.

TWO BLOCK PROBLEMS

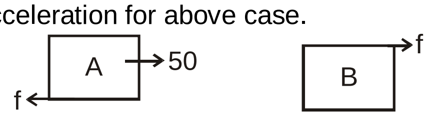
Solved Example

Example 27. Find the acceleration of the two blocks. The system is initially at rest and the friction coefficient are as shown in the figure?



Solution : **Method of solving**

Step 1 : Make force diagram.
Step 2 : Show static friction force by f because value of friction is not known.
Step 3 : Calculate separately for two cases.
Case 1 : Move together
Step 4 : Calculate acceleration.
Step 5 : Check value of friction for above case.
Step 6 : If required friction is less than available it means they will move together else move separately.
Step 7: (a) above acceleration will be common acceleration for both
Case 2 : Move separately
Step 7(b) If they move separately then kinetic friction is involved. Whose value is μN.
Step 8 : Calculate acceleration for above case.



$f_{max} = \mu N$ $\quad\quad\therefore\; f \leq 50$ N (available friction)

Move together

(i) $a = \dfrac{50}{10+10} = 2.5$ m/s^2

(ii) Check friction for B : f = 10 × 2.5 = 25

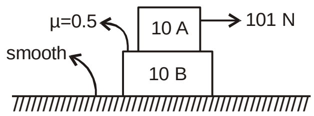
25 N is required which is less than available friction hence they will move together.

and $a_A = a_B = 2.5$ m/s^2

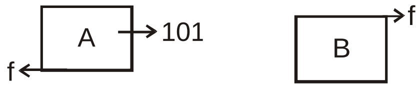
Move separately

No need to calculate

Example 28. Find the acceleration of the two blocks. The system is initially at rest and the friction coefficient are as shown in the figure?

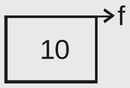


Solution : $f_{max} = 50$ N $\quad \therefore f \le 50$ N



(i) If they move together $a = \dfrac{101}{20} = 5.05$ m/s^2

(ii) Check friction on B



f = 10 × 5.05 = 50.5 (required)

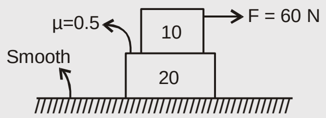
50.5 > 50 (therefore required > available). Hence they will not move together.

(iii) Hence they move separately so kinetic friction is involved.

$\therefore$ for $a_A = \dfrac{101-50}{10} = 5.1$ m/s^2 $\Rightarrow$ $a_B = \dfrac{50}{10} = 5$ m/s^2

Also $a_A > a_B$ as force is applied on A.

Example 29. Find the acceleration of the two blocks. The system is initially at rest and the friction coefficient are as shown in the figure?



Solution :

Move Together

$a = \dfrac{60}{30} = 2$ m/s^2

Check friction on 20 kg.

f = 20 x 2

f = 40 (which is required)

40 < 50 (therefore required < available)

$\therefore$ will move together.

Move Separately

No need to calculate.

Example 30. In above example find maximum F for which two blocks will move together.

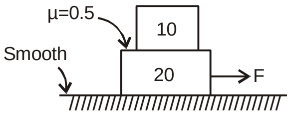
Solution : Observing the critical situation where friction becomes limiting.

$\therefore F - f_{max} = 10\,a$ ......(1)

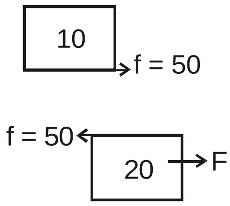
$f_{max} = 20\,a$ ......(2)

$\therefore F = 75$ N

Example 31. Initially the system is at rest. find out minimum value of F for which sliding starts between the two blocks.



Solution : At just sliding condition limiting friction is acting.



F – 50 = 20 a ....(1)

f = 10 a ....(2)

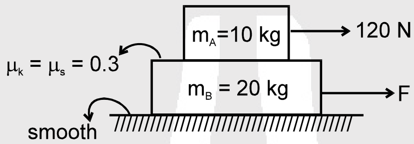
50 = 10 a

$\therefore$ a = 5 m/s^2

hence F = 50 + 20 × 5 = 150 N

$\therefore$ F_{min} = 150 N

Example 32. In the figure given below force F applied horizontally on lower block, is gradually increased from zero. Discuss the direction and nature of friction force and the accelerations of the block for different values of F (Take g = 10 m/s^2).



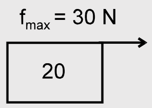
Solution : In the above situation we see that the maximum possible value of friction between the blocks is $\mu_s m_A g$ = 0.3 × 10 × 10 = 30 N.

Case (i) : When F = O.

Considering that there is no slipping between the blocks the acceleration of system will be

$$a = \frac{120}{20+10} = 4 \text{ m/s}^2$$

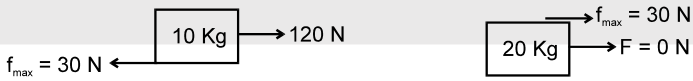
But the maximum acceleration of B can be obtained by the following force diagram.



$a_B = \dfrac{30}{20}$ = 1.5 m/s^2 ($\because$ only friction force by block A is responsible for producing acceleration in block B)

Because 4 > 1.5 m/s^2 we can conclude that the blocks do not move together.

Now drawing the F.B.D. of each block, for finding out individual accelerations.

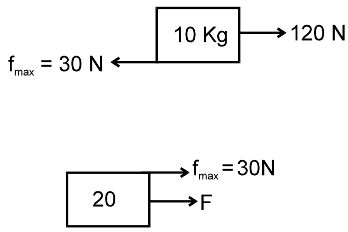


$$a_A = \frac{120-30}{10} = 9 \text{ m/s}^2 \text{ towards right}$$

$$a_B = \frac{30}{20} = 1.5 \text{ m/s}^2 \text{ towards right.}$$

Case (ii) F is increased from zero till the two blocks just start moving together.

As the two blocks move together the friction is static in nature and its value is limiting. FBD in this case will be

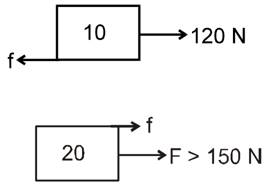


$$a_A = \frac{120-30}{10} = 9 \text{ m/s}^2 \Rightarrow a_B = \frac{F+30}{20} = a_A \Rightarrow \frac{F+30}{20} = 9$$

$\therefore$ F = 150 N

Hence when 0 < F < 150 N the blocks do not move together and the friction is kinetic. As F increases acceleration of block B increases from 1.5 m/s^2.
At F = 150 N limiting static friction start acting and the two blocks start moving together.

Case (iii) When F is increased above 150 N.
In this scenario the static friction adjusts itself so as to keep the blocks moving together. The value of static friction starts reducing but the direction still remains same. This happens continuously till the value of friction becomes zero. In this case the FBD is as follows



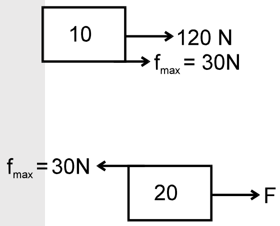
$a_A = a_B = \frac{120 - f}{10} = \frac{F + f}{20}$

$\therefore$ when friction force f gets reduced to zero the above accelerations become

$a_A = \frac{120}{10} = 12 \text{ m/s}^2 \Rightarrow a_B = \frac{F}{20} = a_A = 12 \text{ m/s}^2 \qquad \therefore \quad F = 240 \text{ N}$

Hence when 150 $\leq$ F $\leq$ 240 N the static friction force continuously decreases from maximum to zero at F = 240 N. The accelerations of the blocks increase from 9 m/s^2 to 12 m/s^2 during the change of force F.

Case (iv) When F is increased again from 240 N the direction of friction force on the block reverses but it is still static. F can be increased till this reversed static friction reaches its limiting value. FBD at this juncture will be
The blocks move together therefore.

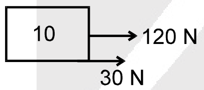


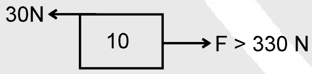
$a_A = \frac{120 + 30}{10} = 15 \text{ m/s}^2$

$\Rightarrow \quad a_B = \frac{F - 30}{20} = a_A = 15 \text{ m/s}^2 \quad \therefore \quad \frac{F - 30}{20} = 15 \text{ m/s}^2$

Hence F = 330 N.

Case (v) When F is increased beyond 330 N. In this case the limiting friction is achieved and slipping takes place between the blocks (kinetic friction is involved).





$\therefore \quad a_A = 15 \text{ m/s}^2$ which is constant

$a_B = \frac{F - 30}{20} \text{ m/s}^2$ where F > 330 N.

Exercise-1

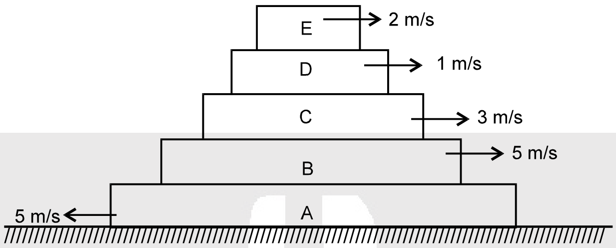
Marked Questions can be used as Revision Questions.

PART - I : SUBJECTIVE QUESTIONS

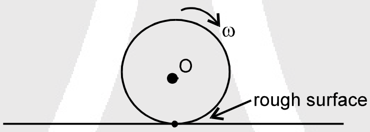
Section (A) : Kinetic Friction

A-1. Suppose you are running fast in a field. When you suddenly find a snake in front of you, you stop quickly. Which force is responsible for your deacceleration ?

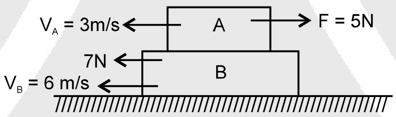
A-2.# In the given diagram find the direction of friction forces on each block and on the ground (Assume all surfaces are rough and all velocities are with respect to ground).



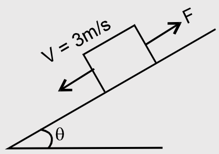
A-3.# The wheel shown in the figure is fixed at 'O' and is in contact with a rough surface as shown. The wheel rotates with an angular velocity ω. What is the direction and nature of friction force on the wheel and on the ground.



A-4.# In the following figure, find the direction of friction on the blocks and ground .



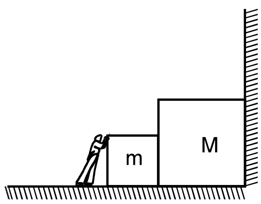
A-5.# In the following figure, find the direction and nature of friction on the block.



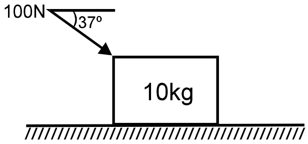
A-6. A block is shot with an initial velocity $5 ms^{-1}$ on a rough horizontal plane. Find the distance covered by the block till it comes to rest. The coefficient of kinetic friction between the block and plane is 0.1.

Section (B) : Static Friction

B-1.# The person applies F force horizontally on the smaller block as shown in figure. The coefficient of static friction is μ between the blocks and the surface. Find the force exerted by the vertical wall on mass M. What is the value of action-reaction forces between m and M?



B-2.# In the figure shown calculate the angle of friction. The block is just about to slide. Take g = 10 m/s^2.

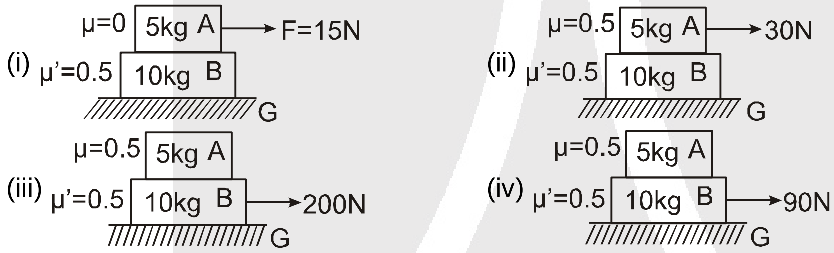


B-3. What is the minimum value of force (in following two cases) required to pull a block of mass m on a horizontal surface having coefficient of friction μ? Also find the angle this force makes with the horizontal.
(a) If force is parallel to horizontal surface
(b) If force is in any direction (Also find the angle this force makes with the horizontal.)

Section (C) : Miscellaneous Questions

C-1. A body of mass 5 kg is kept on a rough horizontal surface. It is found that the body does not slide if a horizontal force less than 30 N is applied to it. Also it is found that it takes 5 seconds to slide throughout the first 10 m if a horizontal force of 30 N is applied and the body is gently pushed to start the motion. Taking g= 10 m/s^2, calculate the coefficients of static and kinetic friction between the block and the surface.

C-2.# In the given figures find the accelerations and the friction forces involved :

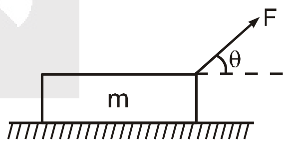


PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Kinetic Friction

A-1. Starting from rest a body slides down a 45º inclined plane in twice the time it takes to slide down the same distance in the absence of friction. The co-efficient of friction between the body and the inclined plane is:
(A) 0.75 (B) 0.33 (C) 0.25 (D) 0.80

A-2.# A wooden block of mass m resting on a rough horizontal table (coefficient of friction = μ) is pulled by a force F as shown in figure. The acceleration of the block moving horizontally is :



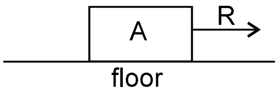
(A) $\frac{F \cos \theta}{m}$ (B) $\frac{\mu F \sin \theta}{M}$

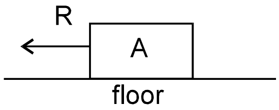
(C) $\frac{F}{m}(\cos \theta + \mu \sin \theta) - \mu g$ (D) none of these

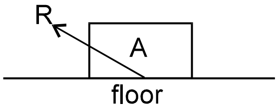
Section (B) : Static Friction

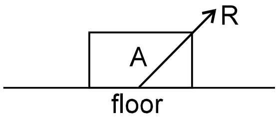
B-1. If the normal force is doubled, the co-efficient of friction is :
(A) halved (B) doubled (C) tripled (D) not changed

B-2.# A box 'A' is lying on the horizontal floor of the compartment of a train running along horizontal rails from left to right. At time 't', it decelerates. Then the resultant contact force R by the floor on the box is given best by :

(A) 

(B) 

(C) 

(D) 

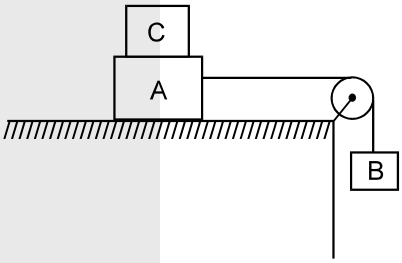
B-3. A block of mass 1 kg lies on a horizontal surface in a truck. The coefficient of static friction between the block and the surface is 0.6 If the acceleration of the truck is 5 m/s^2, the frictional force acting on the block is :

(A) 5 N (B) 6 N (C) 10 N (D) 15 N

B-4. A block of mass 2 kg rests on a rough inclined plane making an angle of 30^0 with the horizontal. The coefficient of static friction between the block and the plane is 0.7. The frictional force on the block is :

(A) 9.8 N (B) $0.7 \times 9.8\sqrt{3}$ N (C) 9.8×7 N (D) 0.8×9.8 N

B-5. Two masses A and B of 10 kg and 5 kg respectively are connected with a string passing over a frictionless pulley fixed at the corner of a table as shown. The coefficient of static friction of A with table is 0.2. The minimum mass of C that may be placed on A to prevent it from moving is



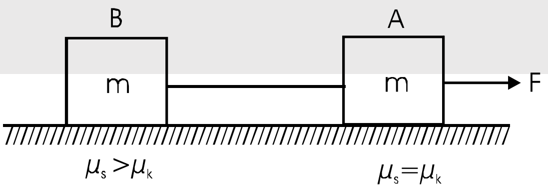
(A) 15 kg (B) 10 kg
(C) 5 kg (D) 12 kg

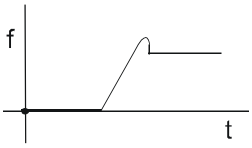
Section (C) : Miscellaneous Questions

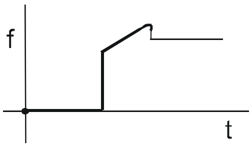
C-1. A 60 kg body is pushed horizontally with just enough force to start it moving across a floor and the same force continues to act afterwards. The coefficient of static friction and sliding friction are 0.5 and 0.4 respectively. The acceleration of the body is :

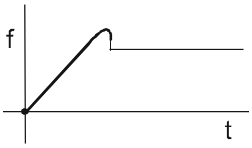
(A) 6 m/s^2 (B) 4.9 m/s^2 (C) 3.92 m/s^2 (D) 1 m/s^2

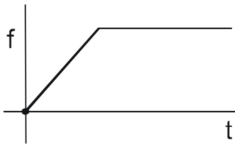
C-2.# A force F = t is applied to block A as shown in figure. The force is applied at t = 0 seconds when the system was at rest and string is just straight without tension. Which of the following graphs gives the friction force between B and horizontal surface as a function of time 't'.



(A) 

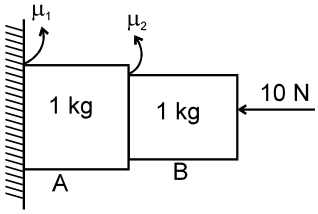
(B) 

(C) 

(D) 

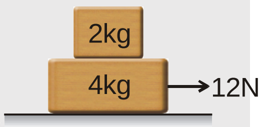
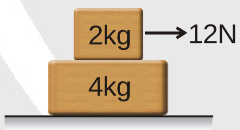
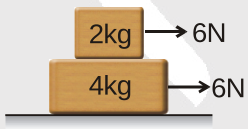
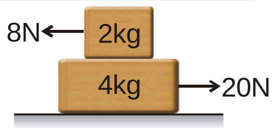
PART - III : MATCH THE COLUMN

1.# In the given figure find the accelerations of blocks A and B for the following cases (g = 10 m/s^2)



Column – I	Column – II
(A) $\mu_1 = 0$ and $\mu_2 = 0.1$	(p) $a_A = a_B = 9.5$ m/s^2
(B) $\mu_2 = 0$ and $\mu_1 = 0.1$	(q) $a_A = 9$ m/s^2, $a_B = 10$ m/s^2
(C) $\mu_1 = 0.1$ and $\mu_2 = 1.0$	(r) $a_A = a_B = g = 10$ m/s^2
(D) $\mu_1 = 1.0$ and $\mu_2 = 0.1$	(s) $a_A = 1$, $a_B = 9$ m/s^2

2.# Column II gives certain situations involving two blocks of mass 2 kg and 4 kg. The 4 kg block lies on a smooth horizontal table. There is sufficient friction between both the blocks and there is no relative motion between the blocks in all situation. Horizontal forces act on one or both blocks as shown. Column I gives certain statement related to figures given in column II. Match the statements in column I with the figure in column II.

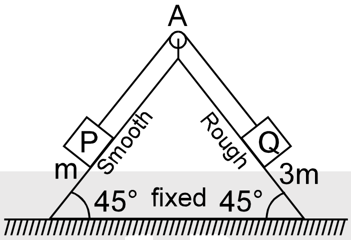
Column I	Column II
(A) Magnitude of frictional force is maximum.	(p) 
(B) Magnitude of friction force is least.	(q) 
(C) Friction force on 2 kg block is towards right.	(r) 
(D) Friction force on 2 kg block is towards left.	(s) 

Exercise-2

✍ Marked Questions may have for Revision Questions.

PART - I : ONLY ONE OPTION CORRECT TYPE

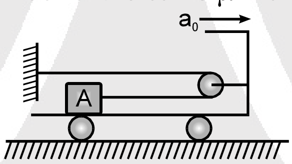
1.✍ A fixed wedge with both surface inclined at 45° to the horizontal as shown in the figure. A particle P of mass m is held on the smooth plane by a light string which passes over a smooth pulley A and attached to a particle Q of mass 3m which rests on the rough plane. The system is released from rest. Given that the acceleration of each particle is of magnitude $\frac{g}{5\sqrt{2}}$ then



the tension in the string is :

(A) mg (B) $\frac{6mg}{5\sqrt{2}}$ (C) $\frac{mg}{2}$ (D) $\frac{mg}{4}$

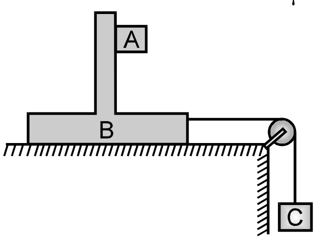
2.#✍ Starting from rest, A flat car is given a constant acceleration $a_0 = 2\ m/s^2$. A cable is connected to a crate A of mass 50 kg as shown. Neglect the friction between floor and car wheels and mass of pulley. The cofficient of friction between crate & floor of the car is $\mu = 0.3$. The tension in cable is -



(A) 700 N (B) 350 N (C) 175 N (D) 0

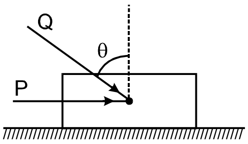
3. A uniform rope lies on a table with some portion hanging. The rope begins to slide when the length of hanging part is 25 % of entire length. The co-efficient of friction between rope and table is:
(A) 0.33 (B) 0.25 (C) 0.5 (D) 0.2

4#. In the arrangement shown mass of the block B and A are 2 m and, 8 m respectively. Surface between B and floor is smooth. The block B is connected to block C by means of a pulley. If the whole system is released then the minimum value of mass of the block C so that the block A remains stationary with respect to B is : (Co-efficient of friction between A and B is μ and pulley is ideal)



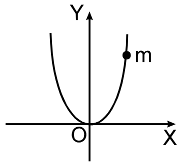
(A) $\frac{m}{\mu}$ (B) $\frac{2m}{\mu+1}$ (C) $\frac{10\ m}{1-\mu}$ (D) $\frac{10m}{\mu-1}$

5.# A block of mass m lying on a rough horizontal plane is acted upon by a horizontal force P and another force Q inclined at an angle θ to the vertical. The minimum value of coefficient of friction between the block and the surface for which the block will remain in equilibrium is:



(A) $\frac{P+Q\sin\theta}{mg+Q\cos\theta}$ (B) $\frac{P\cos\theta+Q}{mg-Q\sin\theta}$ (C) $\frac{P+Q\cos\theta}{mg+Q\sin\theta}$ (D) $\frac{P\sin\theta-Q}{mg-Q\cos\theta}$

6. A bead of mass m is located on a parabolic wire (equation $x^2 = ay$) with its axis vertical and vertex directed downward as in figure. If the coefficient of friction is μ, the highest distance above the x-axis at which the particle will be in equilibrium is



(A) μa (B) $\mu^2 a$

(C) $\frac{1}{4}\mu^2 a$ (D) $\frac{1}{2}\mu a$

7. A 1.5 kg box is initially at rest on a horizontal surface when at t = 0 a horizontal force $\vec{F} = (1.8t)\hat{i}N$ (with t in seconds) is applied to the box. The acceleration of the box as a function of time t is given by : (g = 10m/s^2)

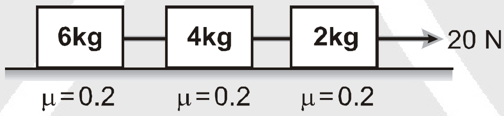
$\vec{a} = 0$ for $0 \le t \le 2.85$

$\vec{a} = (1.2t - 2.4)\hat{i}$ m/s^2 for $t > 2.85$

The coefficient of kinetic friction between the box and the surface is :

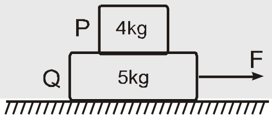
(A) 0.12 (B) 0.24 (C) 0.36 (D) 0.48

8.# In the arrangement shown tension in the string connecting 4kg and 6kg masses is



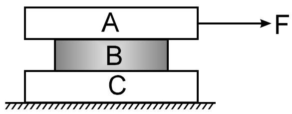
(A) 8N (B) 12N (C) 6N (D) 4N

9.# In the given figure the coefficient of friction between 4kg and 5 kg blocks is 0.2 and between 5 kg block and ground is 0.1. Choose the correct statements



(A) Minimum force needed to cause system to move is 17 N
(B) When force is 4N static friction at all surfaces is 4N to keep system at rest
(C) Maximum acceleration of 4kg block is 2m/s^2
(D) Slipping between 4kg and 5 kg blocks starts when F is > 17N

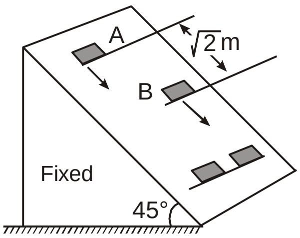
10.# Given m_A = 30 kg, m_B = 10 kg, m_C = 20 kg. Between A and B friction coefficient μ_1 = 0.3, between B and C friction coefficient μ_2 = 0.2 and between C and ground μ_3 = 0.1. The least horizontal force F to start the motion of any part of the system of three blocks resting upon one another as shown in figure is (g = 10 m/s^2)



(A) 60 N (B) 90 N (C) 80 N (D) 150 N

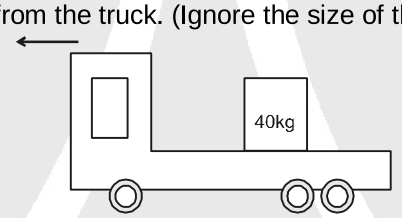
PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. Two blocks A and B of equal masses are sliding down along straight parallel lines on an inclined plane of 45°. Their coefficients of kinetic friction are μ_A = 0.2 and μ_B = 0.3 respectively. At t = 0, both the blocks are at rest and block A is $\sqrt{2}$ meter behind block B. The time (in second) from the initial position where the front faces of the blocks come in line on the inclined plane as shown in figure. (Use g = 10 ms^{-2}.) **[JEE 2004 (Scr.) 3/84]**

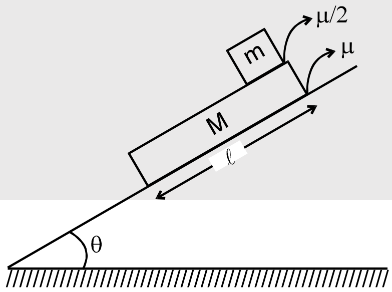


2. A block of mass 2 kg is pushed against a rough vertical wall with a force of 30 N, coefficient of static friction being 0.5. Another horizontal force of 15 N is applied on the block in a direction parallel to the wall. What is the acceleration of block (in m/s^2) ?

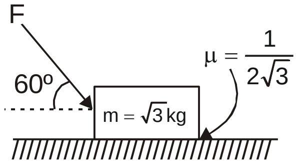
3.# The rear side of a truck is open and a box of 40 kg mass is placed 5 m away from the open end as shown in figure. The coefficient of friction between the box and the surface below it is 0.15. On a straight road, the truck starts from rest and accelerates with 2 ms^{-2}. Find the distance (in m) travelled by the truck by the time box falls from the truck. (Ignore the size of the box).



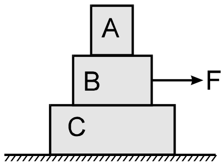
4.# In the given situation it is known that when released the blocks slide. Find the time (in second) when the small block will fall off from the larger block. (The size of m is very –very small then M, see figure). If m = 1 kg, M = 4 kg, ℓ = 4 m, θ = 37°, μ = 0.4.



5.# What is the maximum value of the force F (in newton) such that the block shown in the arrangement, does not move : **[JEE 2003 (Screening); 3/90]**



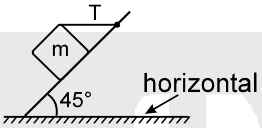
6.# In the figure shown, the coefficient of static friction between C and ground is 0.5, coefficient of static friction between A and B is 0.25, coefficient of static friction between B and C is zero. Find the minimum value of force 'F' (in newton), to cause sliding between A and B. Masses of A, B and C are respectively 2 kg, 4 kg and 5 kg.



7. A small body was launched up an inclined plane set at an angle α = 15° against the horizontal. The coefficient of friction is k, if the time of the ascent of the body is η = 2.0 times less than the time of its descent. Find value of 100 k

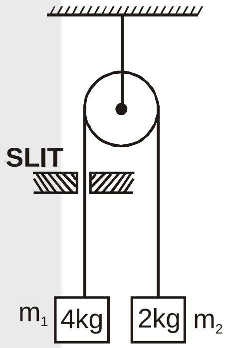
PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. # A block of mass 15 kg is resting on a rough inclined plane as shown in figure. The block is tied up by a horizontal string which has a tension of 50 N. The coefficient of friction between the surfaces of contact may be (g = 10 m/s^2)



(A) 1/2 (B) 2/3 (C) 3/4 (D) 1/4

2.# Two masses m_1 = 4 kg and m_2 = 2kg are connected with an inextensible, massless string that passes over a frictionless pulley and through a slit, as shown. The string is vertical on both sides and the string on the left is acted upon by a constant friction force 10 N by the slit as it moves. (Use g = 10 m/s^2)



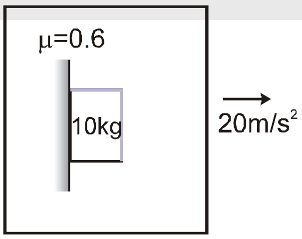
(A) Acceleration of mass m_1 is $\frac{5}{3}$ m/s^2, downwards.

(B) Tension in the string is same throughout.

(C) Force exerted by the string on mass m_2 is $\frac{70}{3}$ N.

(D) If positions of both the masses are interchanged, then 2kg mass moves up with an acceleration $\frac{10}{3}$ m/s^2.

3. # Car is accelerating with acceleration = 20 m/s^2. A box of mass m = 10 kg that is placed inside the car, it is put in contact with the vertical wall of car as shown. The friction coefficient between the box and the wall is μ = 0.6.



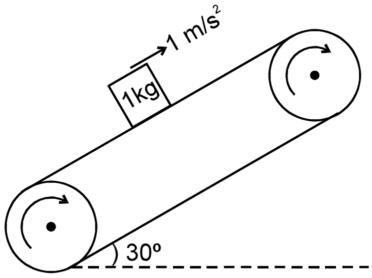
(A) The acceleration of the box will be 20 m/sec^2

(B) The friction force acting on the box will be 100 N

(C) The contact force between the vertical wall and the box will be $100\sqrt{5}$ N

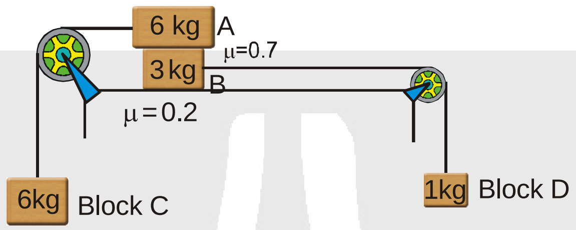
(D) The net contact force between the vertical wall and the box is only of electromagnetic in nature.

4.# A block of mass 1 kg is stationary with respect to a conveyer belt that is accelerating with 1 m/s^2 upwards at an angle of 30º as shown in figure. Which of the following is/are correct?



(A) Force of friction on block is 6 N upwards along the inclined plane.
(B) Force of friction on block is 1.5 N upwards along the inclined plane.
(C) Contact force between the block & belt is 10.5 N.
(D) Contact force between the block & belt is $5\sqrt{3}$ N.

5.# An arrangement of the masses and pulleys is shown in the figure. Strings connecting masses A and B with pulleys are horizontal and all pulleys and strings are light. Friction coefficient between the surface and the block B is 0.2 and between blocks A and B is 0.7. The system is released from rest.
(use g = 10 m/s^2)

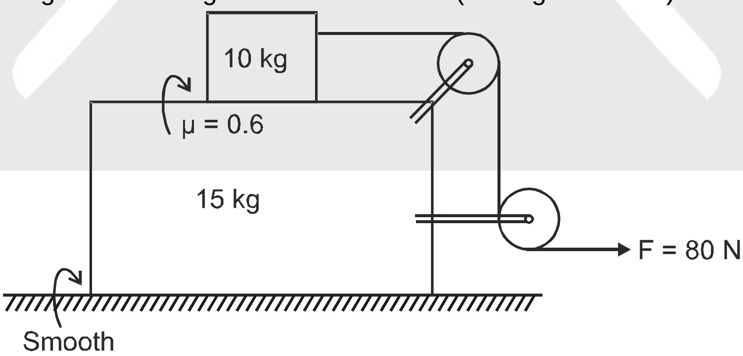


(A) The magnitude of acceleration of the system is 2 m/s^2 and there is no slipping between block A and block B.
(B) The magnitude of friction force between block A and block B is 42 N.
(C) Acceleration of block C is 1 m/s^2 downwards.
(D) Tension in the string connecting block B and block D is 12 N.

PART - IV : COMPREHENSION

Comprehension - 1

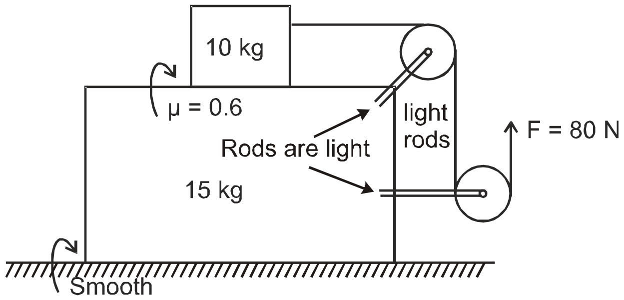
A block of mass 15 kg is placed over a frictionless horizontal surface. Another block of mass 10 kg is placed over it, that is connected with a light string passing over two pulleys fastened to the 15 kg block. A force F = 80 N is applied horizontally to the free end of the string. Friction coefficient between two blocks is 0.6. The portion of the string between 10 kg block and the upper pulley is horizontal as shown in figure Pulley string & connecting rods are massless. (Take g = 10 m/s^2)



1. The magnitude of acceleration of the 10 kg block is :
(A) 3.2 m/s^2 (B) 2.0 m/s^2 (C) 1.6 m/s^2 (D) 0.8 m/s^2

2. If applied force F = 120 N, then magnitude of acceleration of 15 kg block will be :
(A) 8 m/s^2 (B) 4 m/s^2 (C) 3.2 m/s^2 (D) 4.8 m/s^2

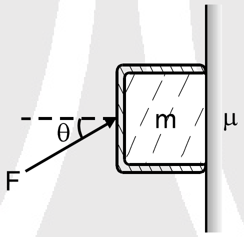
3. Continuing with the situation, if the force F = 80 N is directed vertically as shown, the acceleration of the 10 kg block will be :



(A) 2 m/s^2, towards right (B) 2 m/s^2, towards left
(C) 6 m/s^2, towards left (D) 16/5 m/s^2, towards right

Comprehension # 2

Impending state of motion is a critical border line between static and dynamic states of a body. A block of mass m is supported on a rough vertical wall by applying a force F as shown in figure. Coefficient of static friction between block and wall is μ_S. The block under the influence of F $\sin\theta$ may have a tendency to move upward or it may be assumed that F $\sin\theta$ just prevents downward fall of the block. Read the above passage carefully and answer the following questions.



4. The minimum value of force F required to keep the block stationary is :

(A) $\frac{mg}{\mu\cos\theta}$ (B) $\frac{mg}{\sin\theta+\mu\cos\theta}$ (C) $\frac{mg}{\sin\theta-\mu\cos\theta}$ (D) $\frac{mg}{\mu\tan\theta}$

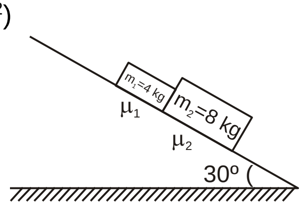
5. The value of F for which friction force between the block and the wall is zero.

(A) mg (B) $\frac{mg}{\sin\theta}$ (C) $\frac{mg}{\cos\theta}$ (D) $\frac{mg}{\tan\theta}$

6. If F is the force applied on the block as shown and F_{min} is the minimum value of force required to keep the block stationary. Then choose the correct alternative.
(A) If F < F_{min} ; the block slides downward
(B) If F = F_{min} ; the block slides upward
(C) In each case (for any value of F) the friction force f < mg
(D) All the above

Comprehension # 3

In the figure shown below the friction between the 4 kg block and the incline as μ_1 and between 8 kg and incline is μ_2. (Take g = 10 m/s^2)



7. If μ_1 = 0.2 and μ_2 = 0.3 then find acceleration of m_1 and m_2 ?
(A) $a_1 = a_2 = 2.7$ m/s^2 (B) $a_1 = 3.2$ m/s^2, $a_2 = 2.4$ m/s^2
(C) $a_1 = a_2 = 3.2$ m/s^2 (D) $a_1 = 2.4$ m/s^2, $a_2 = 3.2$ m/s^2

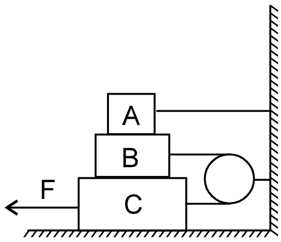
8. If $\mu_1 = 0.3$ and $\mu_2 = 0.2$ then find acceleration of m_1 and m_2 ?

(A) $a_1 = a_2 = 2.7$ m/s^2 (B) $a_1 = 3.2$ m/s^2, $a_2 = 2.4$ m/s^2

(C) $a_1 = a_2 = 3.2$ m/s^2 (D) $a_1 = 2.4$ m/s^2, $a_2 = 3.2$ m/s^2

Comprehension # 4

M_A = 3 kg, M_B = 4 kg and M_C = 8 kg. Friction cofficient between any two surfaces is 0.25. Pulley is frictionless and string is massless. Block, A is connected to the wall through a horizontal massless rigid rod as shown in figure. (g=10m/s^2)



9. Find the value of F to keep C moving with constant speed

(A) 60 N (B) 40 N (C) 80 N (D) 100 N

10. If F is 200 N then find acceleration of B

(A) 5 m/s^2 (B) 10 m/s^2 (C) 4 m/s^2 (D) zero

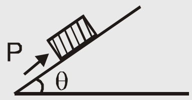
Exercise-3

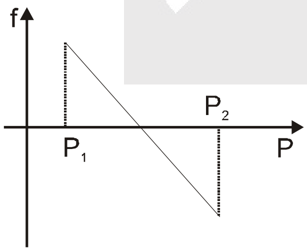
✍ **Marked Questions may have for Revision Questions.**

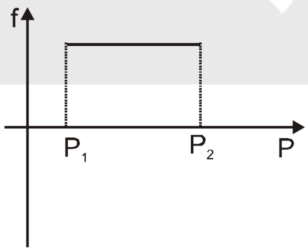
*** Marked Questions may have more than one correct option.**

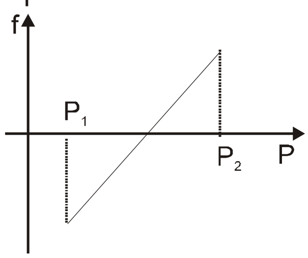
PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

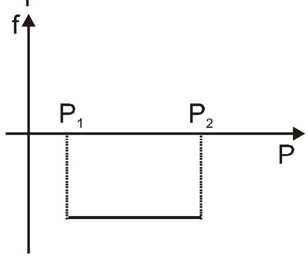
1.#✍ A block of mass m is on inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like : **[JEE 2010, 3/163, –1]**



(A)


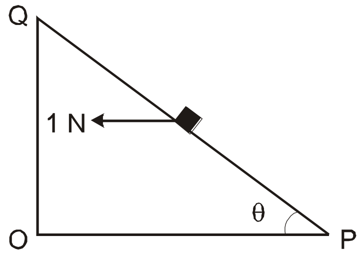
(B)


(C)


(D)


2. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define k = 10 μ, then value of k is **[JEE 2011, 4/160]**

3.*# A small block of mass 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its center of mass as shown in the figure. The block remains stationary if (take g = 10 m/s^2) **[IIT-JEE-2012, Paper-1; 4/70]**



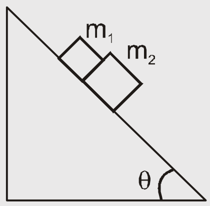
(A) θ = 45°

(B) θ > 45° and a frictional force acts on the block towards P.

(C) θ > 45° and a frictional force acts on the block towards Q.

(D) θ < 45° and a frictional force acts on the block towards Q.

4.# A block of mass m_1 = 1 kg and another mass m_2 = 2 kg , are placed together (see figure) on an inclined plane with angle of inclination θ. Various values of θ are given in List I. The coefficient of friction between the block m_1 and the plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to μ = 0.3. In List II expression for the friction on block m_2 given. Match the correct expression of the friction in List II with the angles given in List I, and choose the correct option. The acceleration due to gravity is denoted by g. **[JEE (Advanced) 2014; 3/60, –1]**
[**Useful information** : tan(5.5°) ≈ 0.1 ; tan (11.5°) ≈ 0.2 ; tan(16.5º ≈ 0.3)]



List-I			**List-II**
P.	θ = 5°	**1.**	$m_2 g \sin\theta$
Q.	θ = 10°	**2.**	$(m_1 + m_2)g \sin\theta$
R.	θ = 15°	**3.**	$\mu m_2 g \cos\theta$
S.	θ = 20°	**4.**	$\mu(m_1 + m_2)g \cos\theta$

Code :

(A) P-1, Q-1, R-1, S-3 (B) P-2, Q-2, R-2, S-3 (C) P-2, Q-2, R-2, S-4 (D) P-2, Q-2, R-3, S-3

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

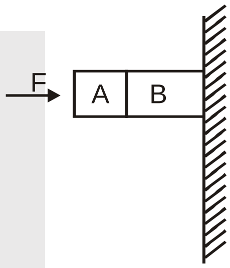
1. The minimum force required to start pushing a body up a rough (friction coefficient μ) inclined plane is F_1 while the minimum force needed to prevent it from sliding down is F_2. If the inclined plane makes an angle θ from the horizontal such that $\tan\theta = 2\mu$ then the ratio F_1/F_2 is :
[AIEEE 2011, 11 May; 4/120, –1]

(1) 1 (2) 2 (3) 3 (4) 4

2. A block of mass m is placed on a surface with a vertical cross section given by $y = \frac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is :
[JEE (Main) 2014 ; 4/120, –1]

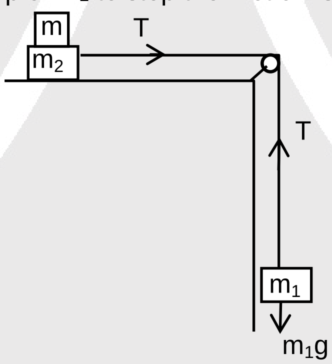
(1) $\frac{1}{6}$ m (2) $\frac{2}{3}$ m (3) $\frac{1}{3}$ m (4) $\frac{1}{2}$ m

3. Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is (Assume system in equilibrium):
[JEE (Main) 2015; 4/120, –1]



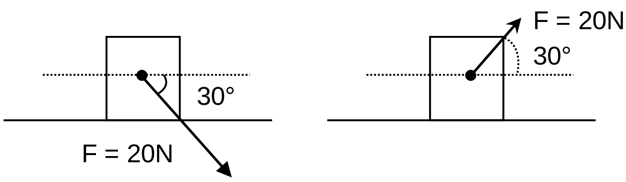
(1) 100N (2) 80N
(3) 120N (4) 150N

4. Two masses m_1 = 5kg and m_2 = 10kg connected by an inextensible string over a frictionless pulley are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is : **[JEE (Main) 2018; 4/120, –1]**



(1) 43.3 kg (2) 10.3 kg (3) 18.3 kg (4) 27.3 kg

5. A block of mass 5kg is (i) pushed in case (A) and (ii) pulled in case (B), by a force F = 20N, making an angle of 30° with the horizontal, as shown in the figures. The coefficient of friction between the block and floor is μ = 0.2. The difference between the accelerations of the block, in case (B) and case (A) will be : (g = 10ms^{-2}) **[JEE (Main) 2019 April; 4/120, –1]**



(A) (B)

(1) 0.8 ms^{-2} (2) 0 ms^{-2} (3) 3.2 ms^{-2} (4) 0.4 ms^{-2}

Answers

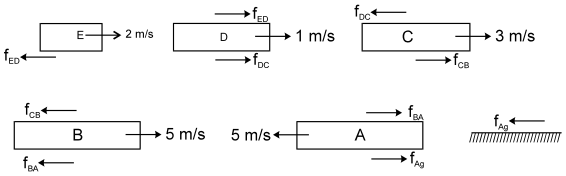
Exercise–1

PART - I

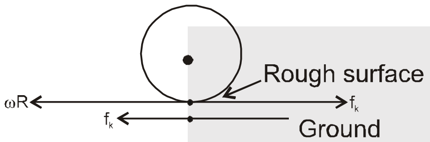
Section (A)

A-1. Frictional force, which is a type of electromagnetic force.

A-2.

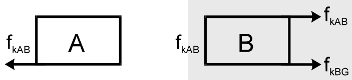


A-3.



Kinetic friction is involved.

A-4.



A-5. Up the incline, kinetic friction.

A-6. 12.5 m

Section (B)

B-1. action-reaction force between M and vertical wall

$N = 0$ for $F\mu \le (M+m)g$

$N = F - \mu(M+m)g$ for $F > \mu(M+m)g$

Action-reaction force between m and M

$N = F - \mu mg$ for $F > \mu mg$

and $N = 0$ for $F < \mu mg$

B-2. $\theta = \tan^{-1}\dfrac{1}{2}$

B-3. (a) μmg (b) $\dfrac{\mu mg}{\sqrt{1+\mu^2}}$, $\tan^{-1}\mu$.

Section (C)

C-1. $\mu_s = 0.60$, $\mu_k = 0.52$

C-2. (i) $a_A = 3\ m/s^2$, $a_B = 0$, $f_{AB} = 0$, $f_{BG} = 0$

(ii) $a_A = 1\ m/s^2$, $a_B = 0$, $f_{AB} = 25N$, $f_{BG} = 25N$

(iii) $a_A = 5\ m/s^2$; $a_B = 10\ m/s^2$; $f_{AB} = 25N$; $f_{BG} = 75N$

(iv) $a_A = 1 m/s^2$; $a_B = 1 m/s^2$; $f_{AB} = 5N$; $f_{BG} = 75N$

PART - II

Section (A)

A-1.	(A)	**A-2.**	(C)

Section (B)

B-1.	(D)	**B-2.**	(C)	**B-3.**	(A)
B-4.	(A)	**B-5.**	(A)		

Section (C)

C-1.	(D)	**C-2.**	(A)

PART - III

1. (A) r, (B) q, (C) p , (D) s

2. (A) s (B) r (C) p, s (D) q, r

Exercise–2

PART - I

1.	(B)	**2.**	(B)	**3.**	(A)
4.	(D)	**5.**	(A)	**6.**	(C)
7.	(B)	**8.**	(A)	**9.**	(C)
10.	(A)				

PART - II

1.	2	**2.**	5	**3.**	20
4.	2	**5.**	20	**6.**	15
7.	16				

PART - III

1. (A) (B) (C) **2.** (A) (C)

3. (A) (B) (C) (D) **4.** (A) (C) **5.** (A) (D)

PART - IV

1.	(A)	**2.**	(B)	**3.**	(A)
4.	(B)	**5.**	(B)	**6.**	(A)
7.	(A)	**8.**	(D)	**9.**	(C)
10.	(B)				

Exercise–3

PART - I

1.	(A)	**2.**	k = 5	**3.**	(A) (C)
4.	(D)				

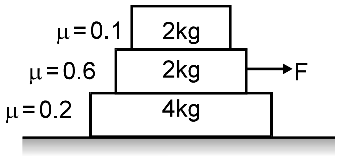
PART - II

1.	(3)	**2.**	(1)	**3.**	(3)
4.	(4)	**5.**	(1)		

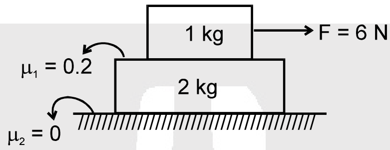
High Level Problems (HLP)

SUBJECTIVE QUESTIONS

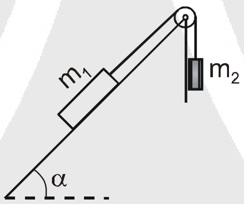
1.# In the situation shown in figure, for what value of minmum horizontal force F (in Newton), sliding between middle and lower block will start ? (Take g = 10 m/s^2)



2.# In the situation shown find the accelerations of the blocks. Also find the accelerations if the force is shifted from the upper block to the lower block.

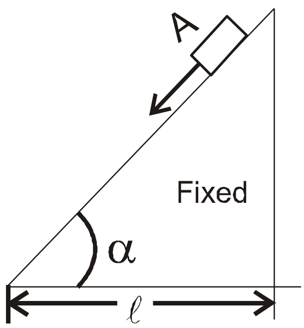


3.# The inclined plane of Fig. forms an angle α = 30° with the horizontal. The mass ratio $m_2/m_1 = \eta = 2/3$. The coefficient of friction between the body m_1 and the inclined plane is equal to k = 0.10. The masses of the pulley and the threads are negligible. Find the magnitude and the direction of acceleration of the body m_2 when the system of masses starts moving.

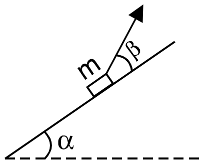


4. A small mass slides down an inclined plane of inclination θ with the horizontal. The co-efficient of friction is $\mu = \mu_0 x$ where x is the distance through which the mass slides down and μ_0, a constant. Then find
(a) Maximum speed of particle
(b) How much distance it will cover to get that maximum speed

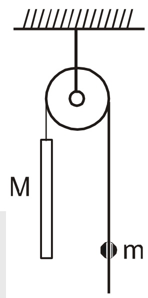
5.# A small body A starts sliding down from the top of a wedge (Fig.) whose base is equal to ℓ = 2.10 m. The coefficient of friction between the body and the wedge-surface is k = 0.140. At what value of the angle will the time of sliding be the least ? What will it be equal to ?



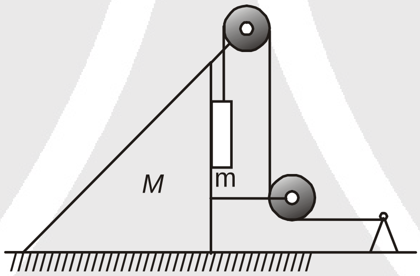
6.# A bar of mass m is pulled by means of a thread up an inclined plane forming an angle α with the horizontal (fig.). The coefficient of friction is equal to k. Find the angle β which the thread must form with the inclined plane for the tension of the thread to be minimum. What is it equal to ?



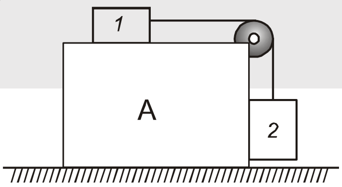
7.# In the arrangement shown in Fig. the mass of the rod M exceeds the mass m of the ball. The ball has an opening permitting it to slide along the thread with some friction. The mass of the pulley, mass of the string and the friction in its axle are negligible. At the initial moment the ball was located opposite the lower end of the rod. When set free, both bodies began moving with constant accelerations. Find the friction force between the ball and the thread if t seconds after the beginning of motion the ball got opposite the upper end of the rod. The rod length equals ℓ.



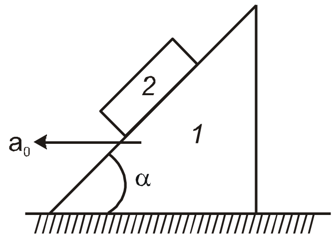
8.# In the arrangement shown in figure the masses of the wedge M and the body m are known. The appreciable friction exists only between the wedge and the body m, the friction coefficient being equal to k. The masses of the pulley and the thread are negligible. Find the acceleration of the body m relative to the horizontal surface on which the wedge Slides.



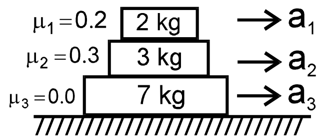
9.# What is the minimum acceleration with which bar A (figure) should be shifted horizontally to keep bodies 1 and 2 stationary relative to the bar? The masses of the bodies are equal and the coefficient of friction between the bar and the bodies is equal to k. The masses of the pulley and the threads are negligible, the friction in the pulley is absent.



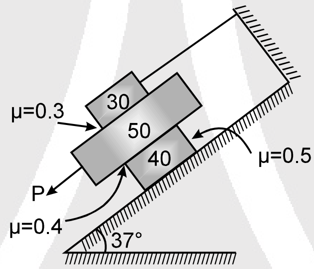
10.# Prism 1 with bar 2 of mass m placed on it gets a horizontal acceleration a_0 directed to the left (figure). At what maximum value of this acceleration will the bar be still stationary relative to the prism, if the coefficient of friction between them $k < \cot \alpha$?



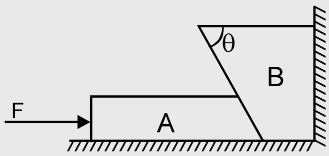
11. Find the accelerations a_1, a_2, a_3 of the three blocks shown in figure. If a horizontal force of 10N is applied on (i) 2 kg block, (ii) 3 kg block, (iii) 7 kg block. (Take g = 10 m/s^2)



12.# The three flat blocks as shown in the figure are positioned on the 37º incline and a force P parallel to the inclined plane is applied to the middle block. The upper block is prevented from moving by a wire which attaches it to the fixed support. The masses of three blocks in kg and coefficient of static friction for each of the three pairs of contact surfaces are shown in the figure. Determine the maximum value which force P may have before slipping take place anywhere. (g = 10 m/s^2)

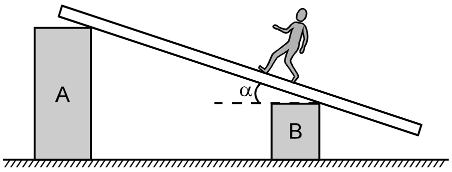


13.# In the figure shown, the coefficient of static friction between block B and the wall is 2/3 and the coefficient of kinetic friction between B and the wall is 1/3. Other contacts are smooth. Find the minimum force 'F' required to lift B, up. Now if the force applied on A is slightly increased than the calculated value of minimum force, then find the acceleration of B. Mass of A is 2m and the mass of B is m. Take tan θ = 3/4.

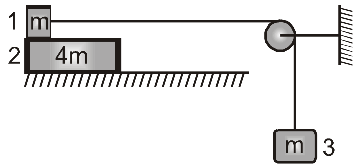


14. A plank of mass m_1 with a bar of mass m_2 placed on it lies on a smooth horizontal plane. A horizontal force growing with time t as F = kt (k is constant) is applied to the bar. Find how the accelerations of the plank a_1 and of the bar a_2 depend on t, if the coefficient of friction between the plank and the bar is equal to μ. Draw the approximate plots of these dependences.

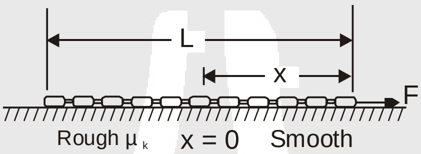
15.# A plank is held at an angle α to the horizontal (Fig.) on two fixed supports A and B. The plank can slide against the supports (without friction) because of its weight Mg. With what acceleration and in what direction, a man of mass m should move so that the plank does not move.


A
B
α

16.# In figure block 1 has one fourth mass and one fourth length of block 2 (mass 4m and length ℓ). No friction exists between block 2 and surface on which it rests. Coefficient of friction is μ_k between 1 & 2. The distance block 2 moves when only half of block 1 is still on block 2 is $\frac{n\mu_k \ell}{8(2-3\mu_k)}$. Then find value of n.



17.# A heavy chain with mass per unit length 'ρ' is pulled by the constant force F along a horizontal surface consisting of a smooth section and a rough section. The chain is initially at rest on the rough surface with x = 0. If the coefficient of kinetic friction between the chain and the rough surface is μ_k, then what is the velocity v (in m/s) of the chain when x = L, if the force F is greater than $\mu_k \rho gL$ in order to initiate the motion. If F = 21N, μ = 0.5, L = 1 m, ρ = 2 kg/m



HLP Answers

1. 30 N **2.** Upper block 4 m/s^2, lower block 1 m/s^2 ; Both blocks 2 m/s^2

3. $a_2 = g(\eta - \sin\alpha - k\cos\alpha) / (\eta + 1) = 0.05\ g$. **4.** **(a)** $v_{max} = \sqrt{\frac{g\sin\theta\tan\theta}{\mu_0}}$ **(b)** $x = \frac{\tan\theta}{\mu_0}$

5. $\tan 2\alpha = (-1/k)$, $\alpha = 49^o$, t_{min} = 1.0s **6.** $\tan\beta = k$; $T_{min} = mg(\sin\alpha + k\cos\alpha) / \sqrt{1+k^2}$.

7. $F_{fr} = 2mM\ell / (M-m)t^2$ **8.** $a = g/\sqrt{2}\ (2 + k + M/m)$ **9.** $w_{min} = g(1-k)/(1+k)$

10. $w_{max} = g(1 + k\cot\alpha) / (\cot\alpha - k)$

11. (i) a_1 = 3 m/s^2, $a_2 = a_3$ = 0.4 m/s^2 , (ii) $a_1 = a_2 = a_3 = \frac{5}{6}$ m/s^2 , (iii) same as (b)

12. P = 12 N **13.** (i) $F_{min} = \frac{3}{2}mg$ (ii) $b = \frac{3g}{22}$

14. When $t \le t_0$, the accelerations $a_1 = a_2 = kt/(m_1 + m_2)$; when $t \ge t_0$
$a_1 = \mu g m_2 / m_1$, $a_2 = (kt - \mu m_2 g)/m_2$. Here $t_0 = \frac{\mu g(m_1+m_2)}{k} \times \frac{m_2}{m_1}$

15. $g\sin\alpha\left(1+\frac{M}{m}\right)$ down the incline **16.** 7 **17.** 4

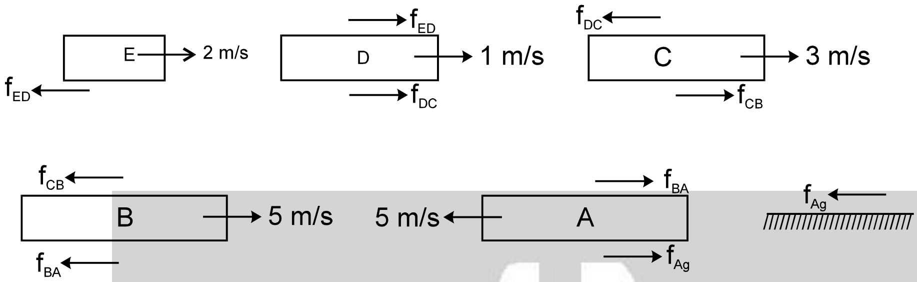
SOLUTIONS OF FRICTION

EXERCISE-1

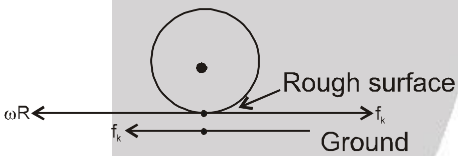
PART-I

A-1. Force along and opposite to the direction of motion is friction.

A-2.



A-3.



Kinetic friction is involved.

A-4. Direction of friction is such that it opposes the relative velocity between the contact surface.

A-5. Friction is kinetic because there is relative motion. Direction of friction is such that it opposes the relative velocity.

A-6. $a = -\mu mg/m = -\mu g = -1\ m/s^2$

$V_f^2 - V_i^2 = 2as, \qquad \therefore (V_f = 0, V_i = 5\ m/s)$

$\Rightarrow \qquad s = \dfrac{25}{2\times 1} = 12.5m.$

B-1. action-reaction force between M and vertical wall

$N = 0$ for $F\mu \le (M+m)g$

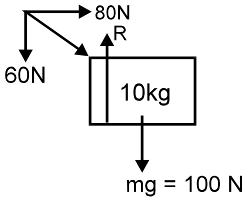
$N = F - \mu(M+m)g$ for $F > \mu(M+m)g$

Action-reaction force between m and M

$N = F - \mu mg$ for $F > \mu mg$

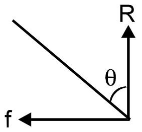
and $N = 0$ for $F < \mu mg$

B-2.



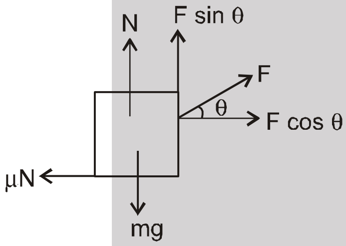
R = mg + 60 = 160 N

f = 80 N ($\because$ No sliding)



Angle of friction $\theta = \tan^{-1}\frac{f}{R} = \tan^{-1}\frac{80}{160} \Rightarrow \theta = \tan^{-1}\frac{1}{2}$

B-3.



$N = mg - F\sin\theta$

$F\cos\theta = \mu N = \mu\,[mg - F\sin\theta]$

$F = \dfrac{\mu\ mg}{\cos\theta + \mu\sin\theta}$

F is minimum when $\cos\theta + \mu\sin\theta$ is max

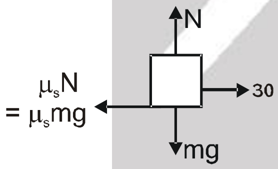
$\Rightarrow \dfrac{d}{d\theta}(\cos\theta + \mu\sin\theta) = 0 \quad \Rightarrow -\sin\theta + \mu\cos\theta = 0 \qquad \Rightarrow \mu = \tan\theta$

or $\quad \theta = \tan^{-1}\mu$

also $\cos\theta + \mu\sin\theta = \sqrt{1+\mu^2}$ for $\theta = \tan^{-1}\mu$

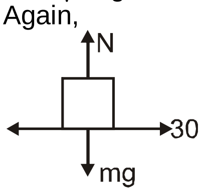
thus $F_{min} = \dfrac{\mu mg}{\sqrt{1+\mu^2}}$

C-1.



$30 = \mu_s mg \quad \Rightarrow \quad 30 = \mu_s \times 5 \times 10 \quad \Rightarrow \quad \mu_s = 0.6.$

Again,

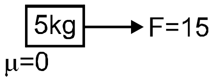


$\mu_k N$

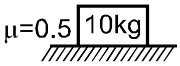
$\mu_k mg$

$S = \frac{1}{2}at^2$

$\Rightarrow \quad a = \dfrac{2S}{t^2} = \dfrac{2\times 10}{25} = 0.8. \quad ; \qquad 30 - \mu_k mg = m \times 0.8 \quad \Rightarrow \quad \mu_k = \dfrac{30 - m\times 0.8}{mg} = 0.52.$

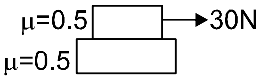
C-2.(i) 

$a_A = \frac{F}{m} = \frac{15}{5} = 3$



$a_B = \frac{0}{10} = 0$

$f_{AB} = 0$, $f_{BG} = 0$.

(ii) 

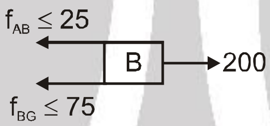
$f_{BG} \le 75$

Since f_{AB} can't be greater than f_{BG} therefore acceleration of B will be zero.

and $\quad a_A = \frac{30-25}{5} = 1\text{m/sec}^2$

$f_{AB} = 25$ N, $f_{BG} = 25$ N.

(iii) 



$f_{AB} \le 25 \quad \Rightarrow \quad a_A \le \frac{25}{5} \quad$ or $\quad a_A \le 5$

Let there is no sliding between A and B then common acceleration of A and B.

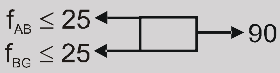
$= \frac{200-75}{15} = 8.33$

Since $a_A \le 5 \quad \Rightarrow$ Hence, there will be sliding between A and B in that case.

$a_A = 5\text{ m/sec}^2$, $a_B = \frac{200-100}{10} = 10\text{ m/sec}^2$

$f_{AB} = 25$ N, $f_{BG} = 75$ N.

(iv) 



$a_A \le 5$

Let A and B move together then common acceleration.

$= \frac{90-75}{15} = 1\text{m/sec}^2$

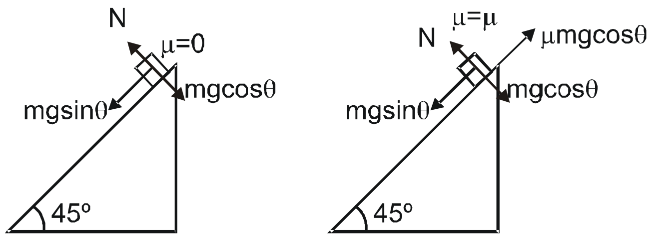
As common acceleration is less than a_A hence A and B will move together

$\therefore \ a_A = 1\text{m/sec}^2$, $a_B = 1\text{m/sec}^2$

$f_{AB} = m_A \times 1 = 5\text{N}$, $\qquad f_{BG} = 75$ N.

PART-II

A-1.



Let acceleration in I[st] case is a_1 and that in second case is a_2

Now , $\frac{1}{2}a_1t^2 = \frac{1}{2}a_2(2t)^2 \quad \Rightarrow \quad a_2 = \frac{a_1}{4}$ ....(i)

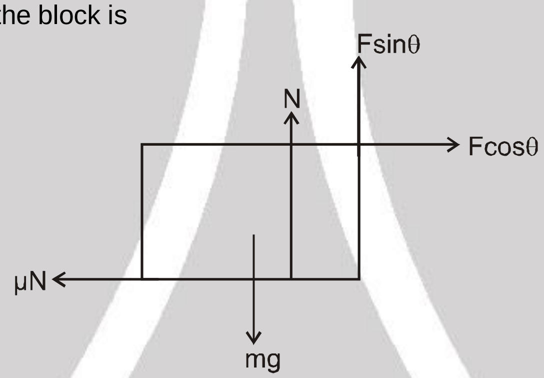
Clearly $a_1 = \frac{mg\sin\theta}{m} = g\sin\theta$ ....(ii)

and $a_2 = \frac{mg\sin\theta - \mu mg\cos\theta}{m} = g\sin\theta - \mu g\cos\theta$ ....(iii)

From (i), (ii) and (iii),

we get $\mu = 0.75$.

A-2. The normal reaction on the block is



$N = mg - F\sin\theta$

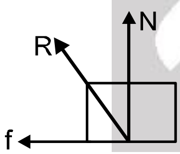
∴ Net force on block is

$F\cos\theta - \mu N = F\cos\theta - \mu mg + \mu F\sin\theta$

or acceleration of the block is

$a = \frac{F(\cos\theta + \mu\sin\theta) - \mu mg}{m} = \frac{F}{m}(\cos\theta + \mu\sin\theta) - \mu g$

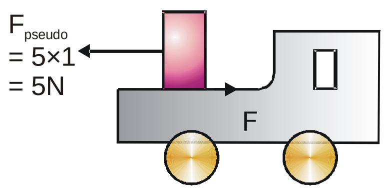
B-1. μ does not depend on normal reaction.



B-2. Acceleration of train will be from right to left.

⇒ Pseudo force will act on the box from left to right therefore friction will act from right to left.

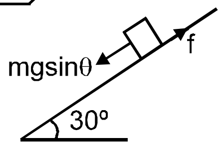
B-3. Solving from the frame of truck



(w.r.t. truck)

$f \le \mu mg = 6 \quad \Rightarrow \quad f = 5N.$

B-4.



since $\mu > \tan\theta$

the block will not slide therefore $f = mg\sin\theta$

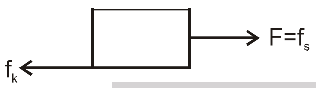
$= 2 \times 9.8 \times \frac{1}{2} = 9.8$ N.

B-5. Apply Newton's law for system along the string

$m_B\, g = \mu(m_A + m_C) \times g$

$\Rightarrow \quad m_C = \frac{m_B}{\mu} - m_A = \frac{5}{0.2} - 10 = 15$ kg

C-1.



$a = \frac{f_s - f_k}{m} = \frac{(\mu_s - \mu_k)mg}{m} = (\mu_S - \mu_k)\,g$

$= (0.5 - 0.4)10 = 1$ m/sec^2

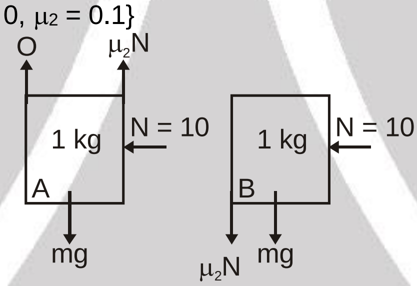
C-2. When F is less than $\mu_s mg$ then tension in the string is zero.

When $\mu_s mg \le F < \mu_s 2mg$ then friction on block B is static.

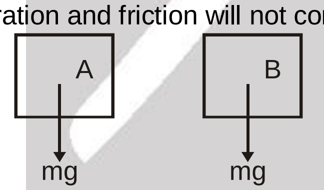
If F is further increase friction on block B is kinetic.

PART – III

1. (i) FBD in (case (i)) $\{\mu_1 = 0, \mu_2 = 0.1\}$

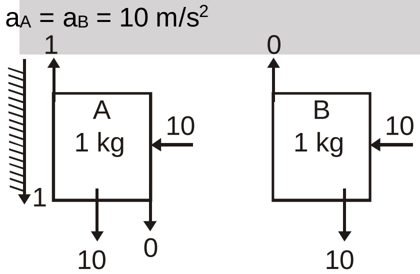


While friction's work is to oppose the relative motion and here if friction comes then relative motion will start and without friction there is no relative motion so both the block move together with same acceleration and friction will not come.



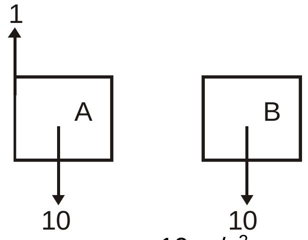
$\Rightarrow \quad a_A = a_B = 10$ m/s^2

(ii)



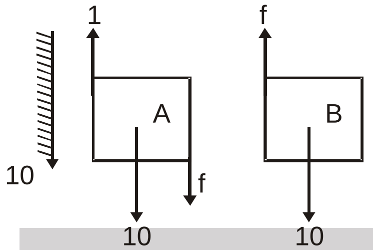
Friction between wall and the block opposes relative motion. Since wall is stationary so friction will try to stop block A also and maximum friction will act between wall and block while there is no friction between block.

Note : Friction between wall and block will oppose relative motion between wall and block only.

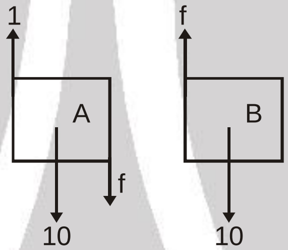


$a_A = 9\ m/s^2$; $a_B = 10\ m/s^2$

(iii)

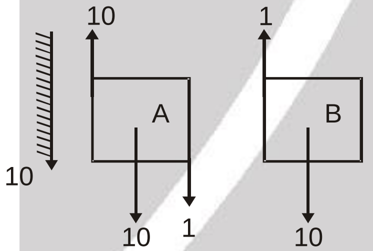


Maximum friction between wall and block will be 1N, but maximum friction available between block A and B is 10 N but if this will be there then relative motion will increase while friction is to oppose relative motion. So friction will come less than 10N so friction will be static.



by system (20 – 1) = 2 × a $\Rightarrow$ $a = \frac{19}{2} = 9.5\ m/s^2$

(iv)



$a_A = \frac{11-10}{1} = 1\ m/s^2$

$a_B = \frac{10-1}{1} = 9\ m/s^2$

2. The acceleration of two block system for all cases is $a = 2\ m/s^2$

In option (p) the net force on 2 kg block is frictional force

$\therefore$ Frictional force on 2 kg block is f = 2 × 2 = 4N towards right

In option (q) the net force on 4 kg block is frictional force

$\therefore$ Frictional force on 4 kg block is f = 4 × 2 = 8N towards right

In option (r) the net force on 2 kg block is 2 × 2 = 4N

$\therefore$ Friction force f on 2 kg block is towards left.

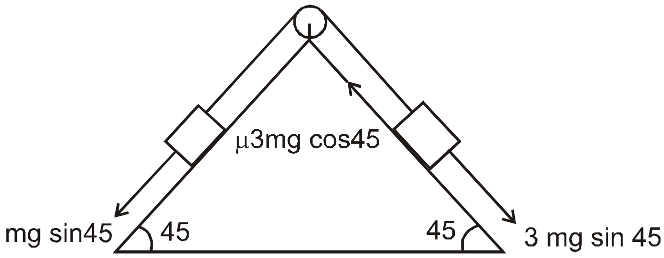
$\therefore$ 6 – f = 2 × 2 or f = 2N

In option (s) the net force on 2 kg block is ma = 2 × 2 = 4N towards right.

$\therefore$ Friction force on 2 kg block is 12N towards right.

EXERCISE-2
PART-I

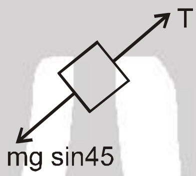
1.



Applying Newton's law for the system of m and 3m along the length of the string, we get
3mg sin45 – μ3mg cos 45 – mg sin45 = (3m + m)a

$\Rightarrow \quad \mu = \frac{2}{5}$ as $\quad a = \frac{g}{5\sqrt{2}}$

now making the F.B.D. of m we get

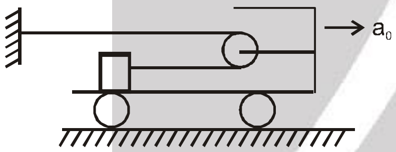


T – mg sin 45 = m a

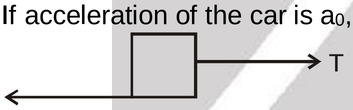
$\Rightarrow \quad T = \frac{mg}{5\sqrt{2}} + \frac{mg}{\sqrt{2}}$

$T = \frac{6\ mg}{5\sqrt{2}}$

2.

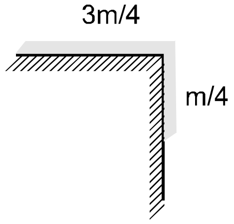


If acceleration of the car is a_0, acceleration of the block $2a_0 = 2 \times 2 = 4$ m/s^2 ($\rightarrow$)



F = μN = 0.3 × 50 × 10 = 150
T – F = ma
$\Rightarrow$ T – 150 = 50 × 4
$\Rightarrow$ T = 350 N.

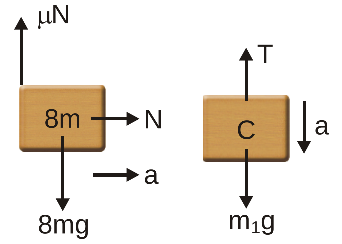
3. Apply system equation



$\frac{m}{4} g = \frac{3m}{4} g \times \mu$

$\Rightarrow \quad \mu = 1/3 = 0.33$

4. FBD of A



If the acceleration of 'C' is a

For block 'A' $\quad N = 8\,ma$ ....(1)

$\quad 8\text{ mg} - \mu N = 0$ ....(2)

and acceleration a can be written by the equation of system (A + B + C)

$m_1 g = (10\text{ m} + m_1)\, a$ ....(3)

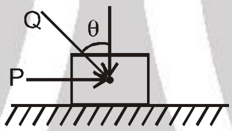
$$8\text{ mg} = \mu 8m\left(\frac{m_1 g}{10m + m_1}\right)$$

$$10\text{ m} + m_1 = \mu\, m_1$$

$$10\text{ m} = (\mu - 1)\, m_1 \qquad \Rightarrow m_1 = \frac{10m}{\mu - 1}$$ **Ans.**

5. $N = mg + Q\cos\theta$

$P + Q\sin\theta \le \mu N$



$$\Rightarrow \quad \mu \ge \frac{P + Q\sin\theta}{N}, \qquad \mu \ge \frac{P + Q\sin\theta}{mg + Q\cos\theta}$$

6. For the sliding not to occur when

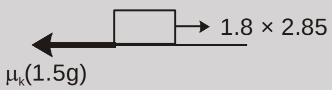
$\tan\theta \le \mu$

$$\tan\theta = \frac{dy}{dx} = \frac{2x}{a} = \frac{2\sqrt{ya}}{a} = 2\sqrt{\frac{y}{a}}$$

$$\therefore \quad 2\sqrt{\frac{y}{a}} \le \mu \qquad \text{or} \qquad y \le \frac{a\mu^2}{4}$$

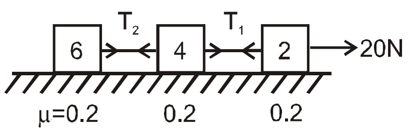
7._ $1.8\,t - \mu_k\, 15 = 1.5\,(1.2\,t - 2.4)$

for t = 2.85 sec.



$\mu_k = 0.24$

8.



(i) Let the blocks do not move

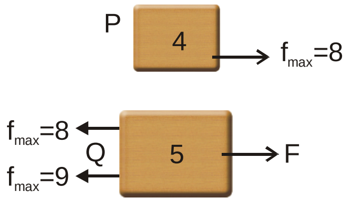
then $T_1 = 20 - 4$

$T_2 = T_1 - 8 = 20 - 4 - 8 = 8$ N

Since T_2 < max possible friction force for 6 kg block

hence it will be at rest and this assumption is right. Therefore tension in the string connecting 4kg and 6 kg block = 8N

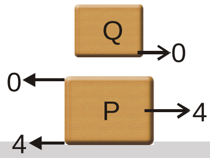
9. So block 'Q' is moving due to force while block 'P' due to friction.
Friction direction on both P + Q blocks as shown.



First block 'Q' will move and P will move with 'Q' so by FBD taking 'P' and 'Q' as system

$F - 9 = 0 \qquad \Rightarrow F = 9$ N

When applied force is 4 N then FBD



4 kg block is moving due to friction and maximum friction force is 8 N.
So acceleration = 8/4 = 2 m/s^2 = a_{max}.
Slipping will start when Q has +ve acceleration equal to maximum acceleration of P i.e. 2 m/s^2.
$F - 17 = 5 \times 2 \quad \Rightarrow F = 27$ N.

10. **(A)** Limiting friction between A & B = 90 N
Limiting friction between B & C = 80 N
Limiting friction between C & ground = 60 N
Since limiting friction is least between C and ground, slipping will occur at first between C and ground. This will occur when F = 60 N.

PART-II

1. $a_A = g\,[\sin 45 - \mu_A \cos 45] = \dfrac{8}{\sqrt{2}}$, $a_B = g\,[\sin 45 - \mu_B \cos 45] = \dfrac{7}{\sqrt{2}}$

$a_{AB} = a_A - a_B = g\,(\mu_B - \mu_A)\cos 45 = \dfrac{1}{\sqrt{2}}$, $s_{AB} = \sqrt{2}$

Now $s_{AB} = \dfrac{1}{2}\, a_{AB}\, t^2 \quad \Rightarrow \quad \sqrt{2} = \dfrac{1}{2} \times \dfrac{1}{\sqrt{2}}\, t^2 \quad \Rightarrow \quad t = 2$ sec.

2. $F = \sqrt{20^2 + 15^2} = 25$
$f_r = 0.5 \times 30 = 15$
$a = \dfrac{25-15}{2} = 5$ m/s^2.

3. $a_{block} = \dfrac{\mu mg}{m} = \mu g = 0.15 \times 10 = 1.5$
$a_T = 2$
$S_T - S_b = 5$

$\Rightarrow \quad \dfrac{1}{2}\, a_T\, t^2 - \dfrac{1}{2}\, a_B\, t^2 = 5$

$\Rightarrow \quad \dfrac{1}{2}\, t^2\, [2 - 1.5] = 5$

$\Rightarrow \quad t^2 = 20$

$S_T = \dfrac{1}{2}\, a_T t^2$

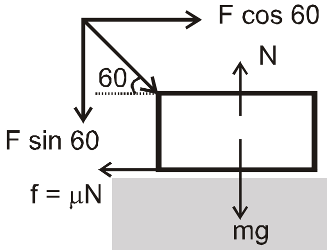
$= \dfrac{1}{2} \times 2 \times 20 = 20$ m.

4. $a_m = (mg \sin\theta - \frac{\mu}{2} mg \cos\theta) / m = g \sin\theta - \frac{\mu}{2} g \cos\theta$

$$a_M = \frac{Mg\sin\theta + \frac{\mu}{2}mg\cos\theta - \mu(M+m)\ \ g\cos\theta}{M}$$

$$S_{mM} = \frac{1}{2} a_{mM}t^2 \quad \Rightarrow \quad t = \sqrt{\frac{2s_{mM}}{a_{mM}}} = \sqrt{\frac{2\ell}{a_m - a_M}} = \sqrt{\frac{4\ell M}{\mu g\cos\theta(M+m)}} = 2 \text{ sec}$$

5.



$N = mg + F \sin 60 = \sqrt{3} \times 10 + \frac{F\sqrt{3}}{2}$ ......(i)

$F \cos 60 \ = \mu N$ ......(ii)

$\Rightarrow \quad \frac{F}{2} = \frac{1}{2\sqrt{3}} \times (10\sqrt{3} + \frac{F\sqrt{3}}{2})$

$\Rightarrow \quad \frac{F}{2} = 5 + \frac{F}{4} \quad \Rightarrow \quad \frac{F}{4} = 5 \quad \Rightarrow \quad F = 20$ N

6. The F.B.D. of A and B are

For sliding to start between A and B, the frictional $f = \mu N = \frac{1}{4} \times 2 \times 10 = 5$ N $= f_{max}$

Applying Newton's second law to system of A + B

$F = (m_A + m_B)\, a = 6a$ ....(1)

Applying Newton's second law to A

$f = m_A\, a \Rightarrow a_{max} = \frac{f_{max}}{m_A} = \frac{5}{2} = 2.5$ m/s^2 ....(2)

from (1) and (2) $F_{min} = (m_A + m_B)\ 2.5$ m/s^2 $= 6 \times 2.5 = 15$ N

Ans. $F_{min} = 15$ N

7. This problem can be solved in two steps :

Step I When the body is moving up on the inclined plane (shown in fig. A)

Here $N = mg \cos\alpha$

The acceleration of the body is

$$a_1 = -\left(\frac{mg\sin\alpha + \mu N}{m}\right) = -(g \sin\alpha + \mu g \cos\alpha)$$

Let body is projected with speed v_0 along the inclined plane (along the x-axis). After time t, body reaches at point P ($v = 0$) (as shown in fig.B).

Let OP = s

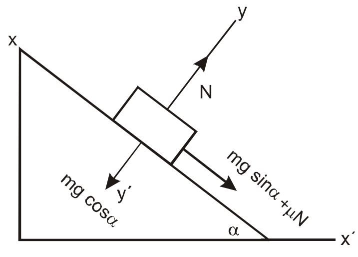


Fig. A

$$s = \left(\frac{\nu_0 + \nu}{2}\right) t_1 = \frac{\nu_0}{2}\, t_1$$

$a_1 = \dfrac{0 - \nu_0}{t_1}$ $\quad\quad \therefore\ \nu_0 = -\, a_1 t_1$

$\therefore \quad s = -\,\dfrac{a_1 t_1^2}{2}$

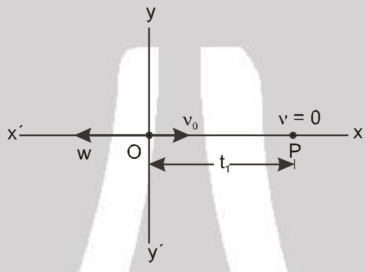


Fig. B

$\therefore \quad t_1 = \sqrt{-\dfrac{2s}{a_1}}$

$= \sqrt{\dfrac{2s}{g\sin\alpha + \mu g\cos\alpha}}$ $\quad\quad$ ....(1)

Step II : When the body starts to return from point P, the acceleration is

$a_2 = \dfrac{mg\sin\alpha - \mu mg\cos\alpha}{m}$ (In fig.C)

$= g \sin \alpha - \mu g \cos \alpha$

$\therefore \quad s = \dfrac{1}{2}\, a_2 t_2^2$

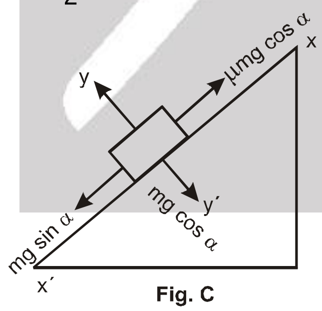


Fig. C

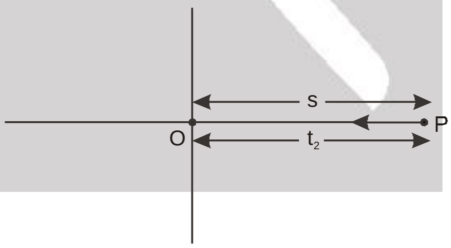


Fig. D

$\therefore \quad t_2 = \sqrt{\dfrac{2s}{a_2}} = \sqrt{\dfrac{2s}{g\sin\alpha - \mu g\cos\alpha}}$ $\quad\quad$ ....(ii)

According to the problem,

$t_2 = \eta t_1$,

Putting the value of t_1 and t_2 from equations (i) and (ii), we get

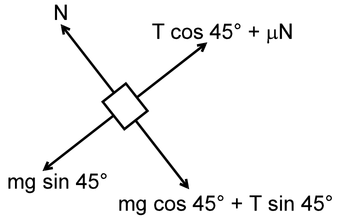
$\mu = \left[\dfrac{(\eta^2 - 1)}{(\eta^2 + 1)}\right] \tan \alpha = 0.16 = k$

than 100 k = 16 Ans.

PART-III

1. The free body diagram of the block is

N is the normal reaction exerted by inclined plane on the block.



Applying Newton's second law to the block along and normal to the incline.

$mg \sin 45° = T \cos 45° + \mu N$ .... (1)

$N = mg \cos 45° + T \sin 45°$ .... (2)

On solving we get

$\mu = 1/2$

so any value of μ greater than 0.5 is answer

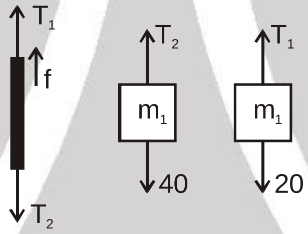
2. Applying NLM on the part that moves through slit.

$T_2 - f - T_1 = 0$

For 4 kg mass $40 - T_2 = 4a$

For 2 kg mass $T_1 - 20 = 2a$

On solving $10 = 6a$

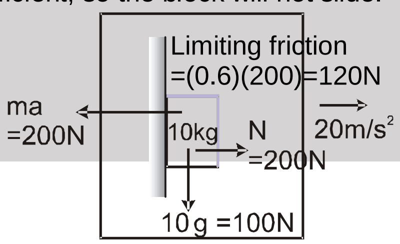


$a = \frac{5}{3}$ m/s^2

Force exerted on 2kg mass by string = $T_1 = \frac{70}{3}$ N.

Tension in the string will not be same throughout, due to the friction force exerted by the slit.

3. The breaking force is insufficient, so the block will not slide.



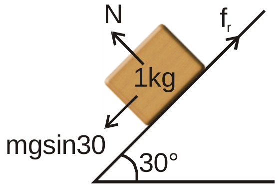
So friction force = 100 N

and acceleration will be 20 m/sec^2 only

Net contact force on the block = $\sqrt{(200)^2 + (100)^2}$ = $100\sqrt{5}$ N

All mechanical interactions are electromagnetic at microscopic level.

4. Block is moving upwards due to friction



f_r – mg sin 30 = ma

$\Rightarrow$ f_r – 1 × 10 × 1/2 = 1 × 1

$\Rightarrow$ f_r = 6 N

Contact force is the resultant of N and f_r = $\sqrt{N^2 + f_r^2}$ = $\sqrt{(mg\cos 30)^2 + (6)^2}$ = 10.5 N

5. Suppose blocks A and B move together. Applying NLM on C, A + B, and D

60 – T = 6a

T – 18 – T' = 9a

T' – 10 = 1a

On solving a = 2 m/s^2

To check slipping between A and B, we have to find friction force in this case. If it is less than limiting static friction, then there will be no slipping between A and B.

Applying NLM on A.

T – f = 6(2)

as T = 48 N

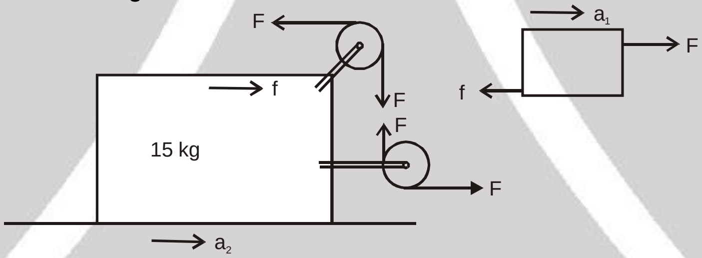
f = 36 N

and f_s = 42 N hence A and B move together.

and T' = 12 N.

PART-IV

1. First, let us check upto what value of F, both blocks move together. Till friction becomes limiting, they will be moving together. Using the FBDs



10 kg block will not slip over the 15 kg block till acceleration of 15 kg block becomes maximum as it is created only by friction force exerted by 10 kg block on it

$a_1 > a_{2(max)}$

$\frac{F-f}{10} = \frac{f}{15}$ for limiting condition as f maximum is 60 N.

F = 100 N.

Therefore for F = 80 N, both will move together.

Their combined acceleration, by applying NLM using both as system F = 25a

a = $\frac{80}{25}$ = 3.2 m/s^2

2. If F = 120 N, then there will be slipping, so using FBDs of both (friction will be 60 N)

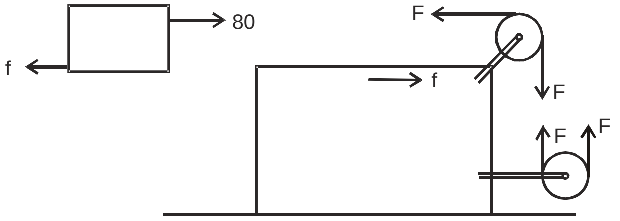
For 10 kg block

120 – 60 = 10 a $\Rightarrow$ a = 6 m/s^2

For 15 kg block

60 = 15a $\Rightarrow$ a = 4 m/s^2

3. In case 80 N force is applied vertically, then



For 10 kg block 80 – 60 = 10a

$a = 2\ m/s^2$

For 15 kg block in horizontal direction.

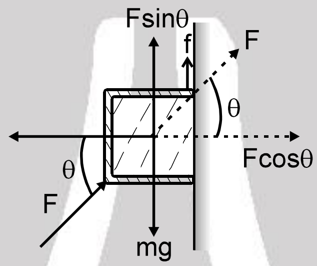
$F - f = 15a$

$a = 4/3\ m/s^2$, towards left.

4. $F \sin\theta + f = mg$

and $F\cos\theta = N$

for minimum ; $f = \mu N = \mu F\cos\theta$



$\therefore\ F_{min.} = \dfrac{mg}{\sin\theta + \mu\cos\theta}$

5. As $f = 0$ $\therefore\ F \sin\theta = mg$

$F = \dfrac{mg}{\sin\theta}$

6. If $F < F_{min}$; the block slides downward

7 to 8. (a) $\mu_2 > \mu_1$ therefore both will more together

and $a = \dfrac{(m_1 + m_2)g\sin\theta - \mu_1 m_1 g\cos\theta - \mu_2 m_2 g\cos\theta}{m_1 + m_2}$

$= g \sin\theta - \dfrac{g(\mu_1 m_1 + \mu_2 m_2)}{m_1 + m_2}\cos\theta$

$5 - \dfrac{4}{\sqrt{3}}$ (= 2.7 m/s^2) for both

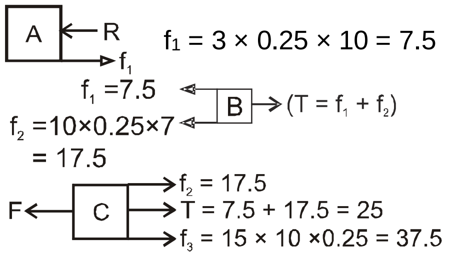
(b) $\mu_2 < \mu_1$ therefore ` $a_2 > a_1$

$a_2 = g \sin\theta - \mu_2 g \cos\theta$

$= 10 \times \dfrac{1}{2} - 0.2\times 10 \times \dfrac{\sqrt{3}}{2} = 3.2\ m/sec^2$

$a_1 = g \sin\theta - \mu_1 g \cos\theta = 5 - 1.5\sqrt{3} = 2.4\ m/sec^2$

9 to 10.

(1) 

$F = 17.5 + 25 + 37.5 = 80$ N

(2) If $F = 200$ then $a_B = a_C$

$\Rightarrow$ $T - f_1 - f_2 = m_B a$ ....(1)

$F - T - f_2 - f_3 = m_C a$ ....(2)

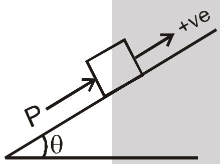
from equation (1) and (2)

$F - f_1 - 2f_2 - f_3 = (m_B + m_C)a$

$= a = \dfrac{F - f_1 - 2f_2 - f_3}{m_B + m_C} = \dfrac{200 - 7.5 - 35 - 37.5}{12}$ 10 m/sec^2

EXERCISE-3
PART-I

1.

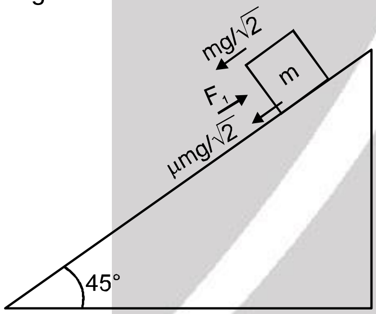


$\tan\theta > \mu$

$P_1 = mg\sin\theta - \mu mg\cos\theta$

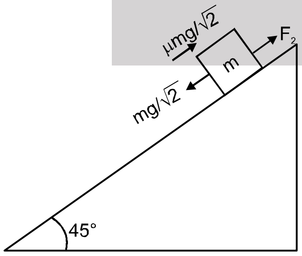
$P_2 = mg\sin\theta + \mu mg\cos\theta$

Initially block has tendency to slide down and as $\tan\theta > \mu$, maximum friction $\mu mg\cos\theta$ will act in positive direction. When magnitude P is increased from P_1 to P_2, friction reverse its direction from positive to negative and becomes maximum i.e. $\mu mg\cos\theta$ in opposite direction.



2.

$F_1 = \dfrac{mg}{\sqrt{2}} + \dfrac{\mu mg}{\sqrt{2}}$

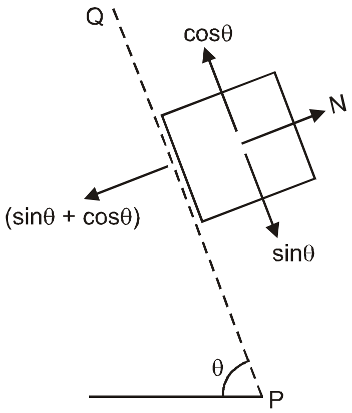


$F_2 = \dfrac{mg}{\sqrt{2}} - \dfrac{\mu mg}{\sqrt{2}}$

$F_1 = 3F_2$; $1 + \mu = 3 - 3\mu$

$4\mu = 2$; $\mu = 1/2$; $k = 10\mu$; $k = 5$ **Ans.**

3.



f = 0, If $\sin\theta = \cos\theta$ $\Rightarrow$ $\theta = 45°$

f towards Q, $\sin\theta > \cos\theta$ $\Rightarrow$ $\theta > 45°$

f towards P, $\sin\theta < \cos\theta$ $\Rightarrow$ $\theta < 45°$

4. Block will not slip if $(m_1 + m_2)\, g \sin\theta \le \mu\, m_2\, g \cos\theta$

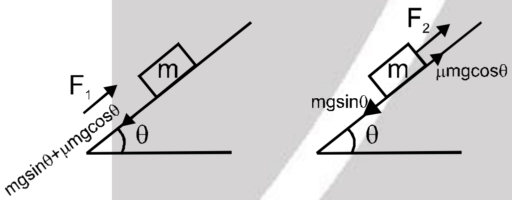
$$3 \sin\theta \le \left(\frac{3}{10}\right)(2) \cos\theta$$

$\tan\theta \le 1/5$ $\Rightarrow$ $\boxed{\theta \le 11.5°}$

(P) $\theta = 5°$	friction is static	$f = (m_1 + m_2)g \sin\theta$
(Q) $\theta = 10°$	friction is static	$f = (m_1 + m_2)g \sin\theta$
(R) $\theta = 15°$	friction is kinetic	$f = \mu m_2 g \cos\theta$
(S) $\theta = 20°$	friction is kinetic	$f = \mu m_2 g \cos\theta$

PART-II

1.

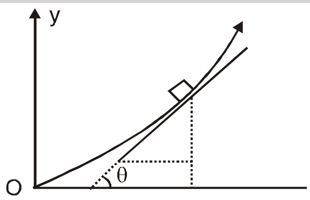


$F_1 = mg \sin\theta + \mu mg \cos\theta$; $F_2 = mg \sin\theta - \mu mg \cos\theta$

$$\frac{F_1}{F_2} = \frac{\sin\theta + \mu\cos\theta}{\sin\theta - \mu\cos\theta}$$

$$\frac{\tan\theta + \mu}{\tan\theta - \mu} = \frac{2\mu + \mu}{2\mu - \mu} = \frac{3\mu}{\mu} = 3.$$

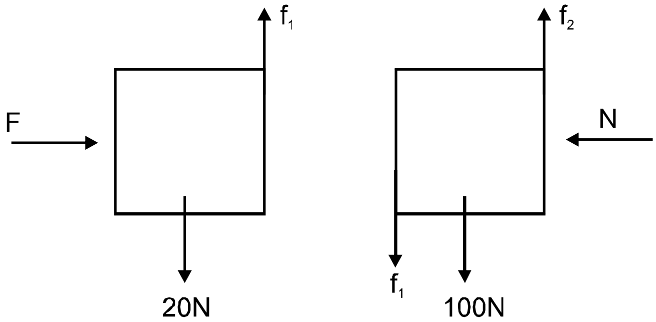
2. $dy/dx = \tan\theta = \mu$ in limiting case



$$\frac{dy}{dx} = \frac{3x^2}{6} = \frac{1}{2} \qquad \Rightarrow \qquad x = \pm 1$$

So $\quad y = 1/6$

3.



Assuming both the blocks are stationary

$N = F$

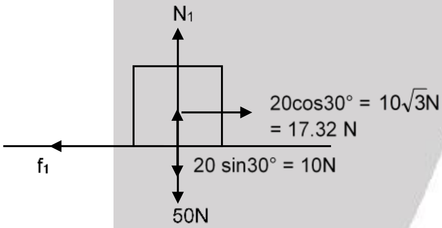
$f_1 = 20N$

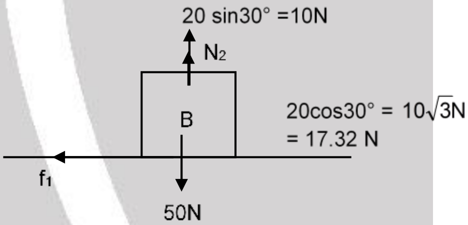
$f_2 = 100+20 = 120N$

4. $\mu(m + m_2) = m_1$

$m + m_2 = \frac{m_1}{\mu} \Rightarrow m = \frac{m_1}{\mu} - m_2$

$m = \frac{5}{0.15} - 10 = 23.33kg$





5.

$N_1 = 50 + 10 = 60N$

$F_{Lim} = 0.2 \times 60 = 12N$

$F_{applied} > 12N$

$\therefore \quad f_1 = 12N$

$\therefore \quad a_1 = \frac{17.32 - 12}{5}$

$= \frac{5.32}{5}$

$N_2 = 50 - 10 = 40N$

$F_{Lim} = 0.2 \times 40 = 8N$

$F_{applied} > 8N$

$\therefore f_1 = 8N$

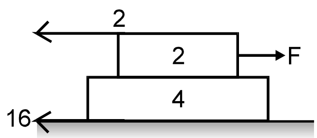
$a_2 = \frac{17.32 - 8}{5}$

$\therefore \quad a_1 - a_2 = -4/5$

$\therefore \quad |a_1 - a_2| = 4/5 = 0.8 \text{ m/s}^2$

HIGH LEVEL PROBLEMS (HLP)

1. for uppermost block $a_{max.} = 1 \text{ m/s}^2$

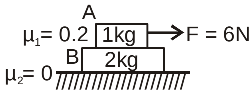


for lowermost block $a_{max} = \frac{(24-16)}{4} = 2 \text{ m/s}^2$

Hence sliding between middle and lower block will start only after sliding between middle and upper block has already started.

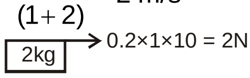
for middle block $F - 18 = 6 \times 2 \Rightarrow F = 30 \text{ N}$

2. **Case - I** : For the lower block



$a_{max} = \frac{2}{2} = 1 m/s^2$

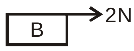
and common possible acceleration = $\frac{6}{(1+2)} = 2 \ m/s^2$



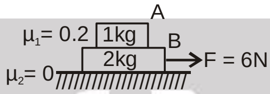
Hence, blocks move with different accelerations.

$a_A = \frac{6-2}{1} = 4 \ m/s^2$

$a_B = \frac{2}{2} = 1 \ m/s^2$



Case II : When the force is acting on the lower block maximum possible acceleration of A



$= \frac{2}{1} = 2 \ m/s^2$

and common acceleration of the two blocks $= \frac{6}{(1+2)} = 2 \ m/s^2$

Hence, both blocks move with common acceleration of $a_A = a_B = 2 \ m/s^2$

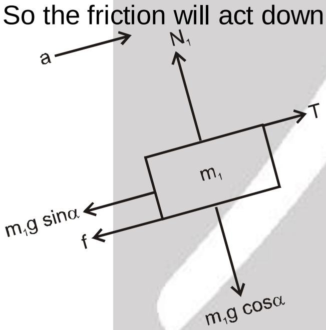
3. For checking the direction of friction, let us assume there is no friction. Then net force acting on the system along the string in vertically downward direction is given by

$m_2g - m_1g \sin \alpha = (m_1 + m_2) a$

$\eta m_1g - m_1g/2 = (m_1 + \eta m_1)a$

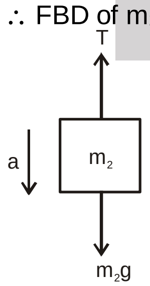
$a = \frac{\eta - \frac{1}{2}}{\eta + 1} g \quad \Rightarrow \quad a > 0.$

So the friction will act down the incline FBD of m_1 gives : –



$T - f - m_1g \sin \alpha = m_1a. \Rightarrow T - k \ m_1g \cos \alpha - m_1g \ sing \ \alpha = m_1a$ ——(i)

$\therefore$ FBD of m_2



$m_2g - T = m_2a$ ——(ii)

from (i) and (ii)

$a = \frac{g(\eta - \sin\alpha - k\cos\alpha)}{(\eta + 1)}$

Putting $\eta = 2/3$, $\alpha = 30^o$ and k = 0.1 ; a = 0.05 g (downward for m_2)

4. Acceleration of mass at distance x

$$a = g(\sin\theta - \mu_0 x \cos\theta)$$

Speed is maximum, when a = 0

$g(\sin\theta - \mu_0 x \cos\theta) = 0$

$x = \dfrac{\tan\theta}{\mu_0}$

for maximum speed

$a = g(\sin\theta - \mu_0 x \cos\theta)$

$\dfrac{vdv}{dx} = g(\sin\theta - \mu_0 x \cos\theta)$

$vdv = (g\sin\theta - \mu_0 gx \cos\theta)dx$

Integrating both sides

$\int_0^{v_{max}} vdv = \int_0^{\tan\theta/\mu_0} (g\sin\theta - \mu_0 gx\cos\theta)\ dx$

$\dfrac{v_{max}^2}{2} = \left[gx\sin\theta - \dfrac{\mu_0 gx^2\cos\theta}{2}\right]_0^{\frac{\tan\theta}{\mu_0}}$

After Applying limits

$v_{max} = \sqrt{\dfrac{g\sin\theta\tan\theta}{\mu_0}}$

5. The acceleration of block is

$$w = \frac{mg\sin\alpha - kmg\cos\alpha}{m}$$

or $\quad w = g \sin \alpha - kg \cos \alpha$

Let $\quad AB = s \therefore \cos\alpha = \dfrac{\ell}{s}$

$\therefore \quad s = \ell \sec \alpha$

$\therefore$ According to kinematics equation

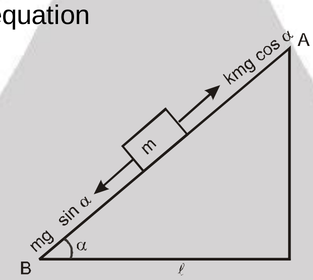


Fig. A

$s = \dfrac{1}{2} wt^2$

$\therefore \quad t = \sqrt{\dfrac{2s}{w}} = \sqrt{\dfrac{2\ell\sec\alpha}{g\sin\alpha - kg\cos\alpha}}$

or $\quad t^2 = \dfrac{2\ell\sec\alpha}{g(\sin\alpha - k\cos\alpha)} = \dfrac{2\ell}{g(\sin\alpha\cos\alpha - k\cos^2\alpha)}$

For being t_0 minimum

$\sin\alpha\cos\alpha - k\cos^2\alpha$ is maximum

$\dfrac{d}{d\alpha}(\sin\alpha\cos\alpha - k\cos^2\alpha) = 0$

$\cos^2\alpha - \sin^2\alpha + 2k\cos\alpha\sin\alpha = 0$

$\cos 2\alpha + k \sin 2\alpha = 0$

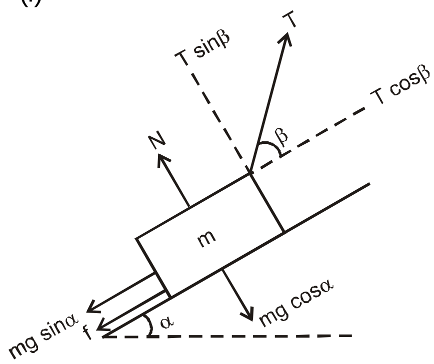
$\tan 2\alpha = \dfrac{-1}{k} \quad \Rightarrow \quad \alpha = \dfrac{1}{2}\tan^{-1}\left(\dfrac{-1}{k}\right)$

After putting the values, $\alpha = 49^o$

6. For limiting friction

$N + T \sin\beta = mg \cos\alpha$

$\Rightarrow N = mg \cos\alpha - T \sin\beta$ ——(i)



and, $T\cos\beta = mg \sin\alpha + f$

$= mg \sin\alpha + k\,(mg \cos\alpha - T \sin\beta)$ (from (i))

$\Rightarrow T(\cos\beta + k \sin\beta) = mg \sin\alpha + k\, mg \cos\alpha$

$$\Rightarrow T = \frac{mg\ (\sin\alpha + k\cos\alpha)}{(\cos\beta + k\sin\beta)}$$

for minimum T , $\cos\beta + k \sin\beta$ should be maximum

$$\frac{dy}{d\beta} = -\sin\beta + k\cos\beta = 0$$

$\Rightarrow$ $\tan\beta = k$

$$\therefore \quad T_{min} = \frac{mg(\sin\alpha + k\cos\alpha)}{\left(\frac{1}{\sqrt{1+k^2}} + \frac{k^2}{\sqrt{1+k^2}}\right)} = \frac{mg(\sin\alpha + k\cos\alpha)}{\sqrt{1+k^2}}$$

7. **Step -I :** Draw force diagram separately : In fig B, P is a point on the string

From fig. A,

$mg - f = mw_1$

$$\therefore \quad w_1 = \frac{mg - f}{m} \qquad .....(i)$$

From fig.B,,

$T = f$ .....(ii)

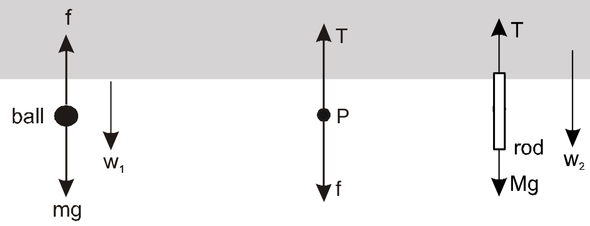


Fig. A Fig.B Fig.C

From fig.C

$Mg - T = Mw_2$

$Mg - f = Mw_2$

$$\therefore \quad w_2 = \frac{Mg - f}{M} \qquad ......(iii)$$

Step II : Apply kinematic relation :

$$s_{rel} = u_{rel}\, t \;\; \frac{1}{2}\, w_{rel}\, t^2 \qquad \text{(Shown in fig.D)}$$

Here $s_{rel} = \ell$, $u_{rel} = 0$

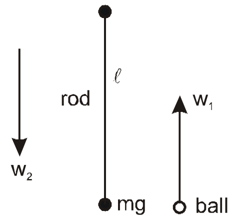


Fig. D

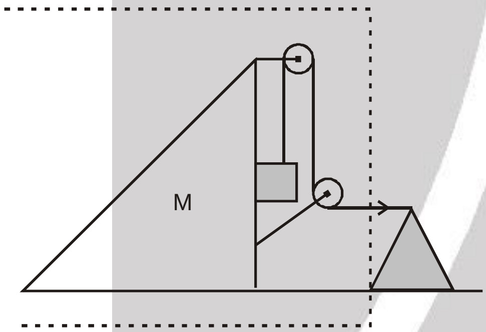
$w_{rel} = w_2 - w_1$

$\therefore \quad \ell = \frac{1}{2}(w_2 - w_1)\, t^2 \qquad \therefore \quad t = \sqrt{\frac{2\ell}{(w_2 - w_1)}}$

Putting the value of w_1 and w_2,

$$f = \frac{2\ell Mm}{(M-m)t^2}$$

8.



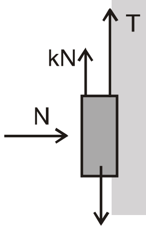
Considering the system as marked in the diagram

$\therefore \quad T = (M + m)a.$

a is the common acceleration of the two masses in horizontal direction.

Now taking the body m as system.

F.B.D. for the body will be



$\therefore \quad N = ma$ (m has acceleration a in horizontal direction)

Also $mg - kN - T = ma$ (m has acceleration a downward w.r.t. wedge because of the constraint)

or $a = \frac{mg}{M + 2m + Km}$; $\vec{a}_{bg} = \vec{a}_{bw} + \vec{a}_{wg}$

$$|\vec{a}_{bg}| = \sqrt{a^2 + a^2} = \sqrt{2}a = \frac{\sqrt{2}mg}{M + 2m + Km} = \frac{\sqrt{2}g}{2 + K + M/m}$$

9. If acceleration in bar is zero, then the body (1) will slip on bar rightward and the body (2) moves downward. To prevent slipping, net force on each body should be zero in the frame of bar (non-inertial reference frame)

In fig. A, $T = mw + kmg$ ....(i)

In fig. B, $T + kN = mg$ ....(ii)

$N = mw$ ....(iii)

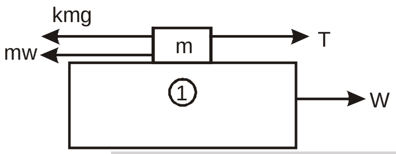


Fig. A

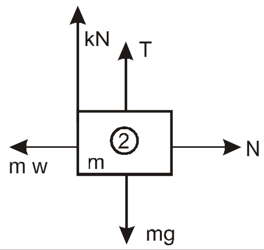


Fig. B

From (ii) and (iii), we get

$$T + kmw = mg$$

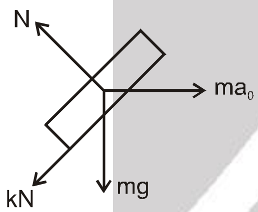
$$T = mg - kmw \quad ....(iv)$$

From (i) and (iv), we get

$$w = \frac{g(1-k)}{(1+k)} \quad ....(v)$$

Since, relative acceleration of body with respect to bar (w_{rel}) is zero : So, the value of w in eqn.(v) is minimum value of w.

10.



(a_0 is acceleration of the wedge leftward)

As a_0 increases the component of ma_0 up the up the incline increases and friction attains its max value.

Writing the force equation along the incline and perpendicular to the incline.

$$ma_0 \cos\alpha - mg \sin\alpha - kN = 0 \quad ....(i)$$

$$mg \cos\alpha + ma_0 \sin\alpha = N \quad ....(ii)$$

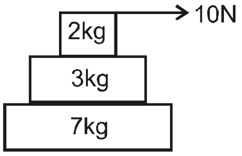
From equation (i) and (ii),

$$a_0 \cos\alpha - g \sin\alpha = kg \cos\alpha + K\, a_0 \sin\alpha$$

$$a_0 (\cos\alpha - k \sin\alpha) = g\{k \cos\alpha + \sin\alpha\}$$

$$a_0 = \frac{g\{k\cos\alpha + \sin\alpha\}}{\{\cos\alpha - k\sin\alpha\}}$$

11. (i) Assuming there is no slipping anywhere and the common acceleration of the three blocks be a



Writing the force equation F = ma

10 = 12a or a = 5/6.

Now $f_{max.}$ between 2 kg & 3kg is

$\mu_1 N = 0.2 \times 20 = 4N$.

For 2 kg block F.B.D. will be:

$F - f = ma = 2 \times 5/6$

$10 - f = 10/6$

$10 - 10/6 = f$

$50/6 = f > f_{max}$

$\therefore$ There will be slipping between 2 kg and 3 kg block. Now considering the slipping the new equation would be

$F - f_{max} = ma_1$

$10 - 4 = 2a_1$ or $a_1 = 3\ ms^{-2}$

Now lets take 3 kg & 7 kg as system and writing the force equation.

$f_{max} = 10\ a_0$.

or $4 = 10\ a_0$ $\Rightarrow$ $a_0 = 0.4\ ms^{-2}$

To check the required friction between 3 kg and 7 kg block.

F on 7 kg block $F = 7 \times 0.4 = 2.8$ N

f_{max} between 7 kg and 3 kg = $0.3\ N_2 = 0.3 \times 50 = 15$ N

$2.8 < f_{max}$

hence there is no slipping between the two blocks

(ii) Now if force is applied on 3 kg block

Assuming there is no slipping anywhere and the common acceleration of the three blocks is

$\frac{5}{6}\ ms^{-2}$

Now, if all system is going with common acceleration $\frac{5}{6} ms^{-2}$

for a = $\frac{5}{6} ms^{-2}$ required friction force between 2 kg and 3 kg block = $m_1 a = 2 \times \frac{5}{6} = \frac{5}{3}$ N < f_{max}

so there is no slipping

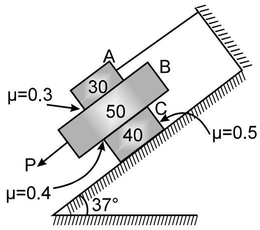
Same for 7 kg block, required friction is = $m_3 a = 7 \times \frac{5}{6} = \frac{35}{6}$ N< f_{max}

so there is no slipping

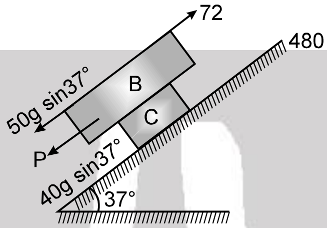
$\therefore a_1 = a_2 = a_3 = \frac{5}{6} ms^{-2}$

(iii) Same as (b

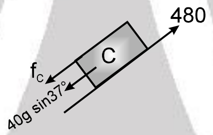
12. $f_{ABmax} = 0.3 \times 30g \cos 37°$



$f_{ABmax} = 0.3 \times 30g \cos 37° = 72$ N
$f_{BCmax} = 0.4 \times 80g \cos 37° = 256$ N
$f_{Cmax} = 0.5 \times 120g \cos 37° = 480$ N
when block 'B' is pulled two cases are possible
(1) B & C both moves together and there is just slipping at A & B contact and at C and wedge contact
(2) A & C remains stationary only 'B' tends to move downwards.
Taking case (1) Let B & C moves together and there is just slipping between A & B and between wedge and C

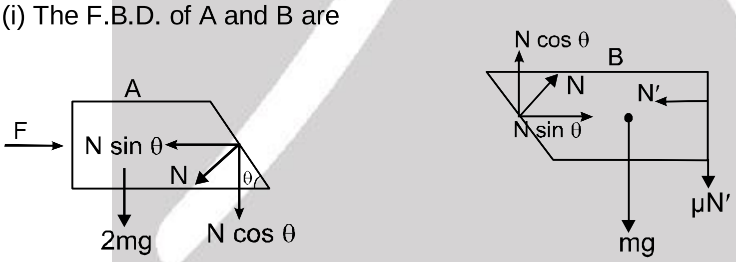


$\Rightarrow \quad P + 40\,g \sin 37° + 50\,g \sin 37° - 72 - 480 = 0$
$\Rightarrow \quad P = 12$ N
Checking friction between B & C
$f_{BC} + 40\,g \sin 37° - 480 = 0$



$f_{BC} = 240$
Which is within limit so case is correct.
So max. force for which there will be no slipping $P_{max} = 12$ N the at this force both B & C tends to move together.

13. (i) The F.B.D. of A and B are



For A to be in equilibrium ; $F = N \sin\theta$ ....(1)
For B to just lift off ; $N \cos\theta = mg + \mu_s N'$ ....(2)
For horizontal equilibrium of B ; $N' = N \sin\theta$ ....(3)
From (2) and (3)

$N(\cos\theta - \mu_s \sin\theta) = mg$ or $N = \left(\frac{4}{5} - \frac{2}{3} \times \frac{3}{5}\right) mg$ or $N = \frac{5}{2}\,mg$ ....(4)

From equation (1) $F = N \times 3/5 \Rightarrow \therefore F = 3/2\ mg$
(ii) The acceleration of the block A be a and B be b

$F - N \sin\theta = 2ma$ ....(1)
$N \cos\theta - mg - \mu_k N' = mb$ ....(2)
$N' = N \sin\theta$ ....(3)

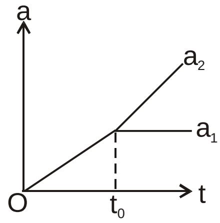
From constraint =

$a \sin\theta = b \cos\theta$ ....(4)

Solving (1), (2) , (3) and (4) we get $\Rightarrow \quad b = \frac{3g}{22}$

14. When $t \le t_0$, the accelerations $a_1 = a_2 = kt / (m_1 + m_2)$; when $t \ge t_0$

$a_1 = \mu g m_2 / m_1$, $a_2 = (kt - \mu m_2 g) / m_2$. Here $t_0 = \dfrac{\mu g(m_1 + m_2)}{k} \times \dfrac{m_2}{m_1}$

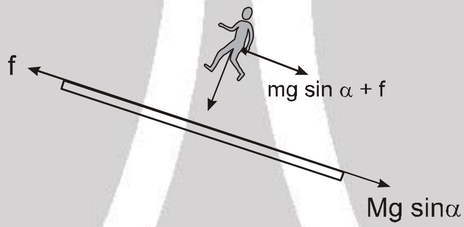


When $t \le t_0$, there is no slipping occurs and the accelerations $a_1 = a_2 = kt / (m_1 + m_2)$;

when $t \ge t_0$ maximum friction exert between plank and rod

$a_1 = \mu g m_2 / m_1$, $a_2 = (kt - \mu m_2 g) / m_2$. Here $t_0 = \dfrac{\mu g(m_1 + m_2)}{k} \times \dfrac{m_2}{m_1}$

15. F.B.D. of man and plank are



For plank be at rest, applying Newton's second law to plank along the incline

$Mg \sin \alpha = f$ ....(1)

and applying Newton's second law to man along the incline.

$mg \sin \alpha + f = ma$ ....(2)

$a = g \sin \alpha \left(1 + \dfrac{M}{m}\right)$ down the incline

Alternate Solution : :

If the friction force is taken up the incline on man, then application of Newton's second law to man and plank along incline yields.

$f + Mg \sin \alpha = 0$ ....(1)

$mg \sin \alpha - f = ma$ ....(2)

Solving (1) and (2)

$a = g \sin \alpha \left(1 + \dfrac{M}{m}\right)$ down the incline

Alternate Solution :

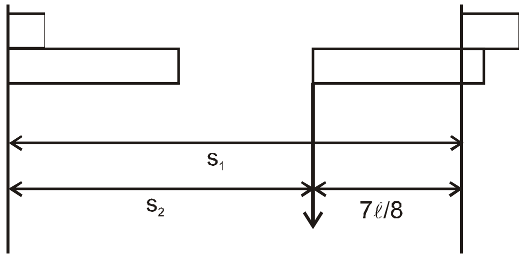
Application of Newton's seconds law to system of man + plank along the incline yields

$mg \sin \alpha + Mg \sin \alpha = ma$

$a = g \sin \alpha \left(1 + \dfrac{M}{m}\right)$ down the incline

16. $a_1 = \dfrac{mg - \mu_k mg}{2m} = \dfrac{g}{2}\ (1 - \mu_k)$

$a_2 = \dfrac{\mu_k mg}{4m} = \dfrac{\mu_k g}{4}$



$s_1 = \dfrac{1}{2} a_1 t^2$

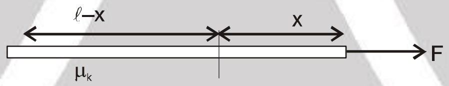
$s_2 = \dfrac{1}{2} a_2 t^2$

$s_1 - s_2 = \dfrac{7\ell}{8} \quad \Rightarrow \quad \dfrac{1}{2}\dfrac{g}{2}(1 - \mu_k)t^2 - \dfrac{1}{2}\dfrac{g}{4}\ \mu_k\ t^2 = \dfrac{7\ell}{8}$

$\Rightarrow \quad t^2 = \dfrac{7\ell}{2g(1-\mu_k) - g\mu_k} = \dfrac{7\ell}{g(2-3\mu_k)}$

$s_2 = \dfrac{1}{2}\ a_2\ t^2 = \dfrac{1}{2} \times \dfrac{\mu_k g}{4} \times \dfrac{7\ell}{g(2-3\mu_k)} = \dfrac{7\ell\mu_k}{8(2-3\mu_k)}$

17. Considering the forces on the chain for the given situation we have



$F - \mu_k\ (\ell - x)\rho g = \ell\rho a$

$\dfrac{F}{\rho\ell} - \dfrac{\mu_k(\ell - x)g}{\ell} = \dfrac{dv}{dx}.v.$

$\int_0^\ell \dfrac{F}{\rho\ell}dx - \int_0^\ell \dfrac{\mu_k(\ell - x)}{\ell} gdx = \int_0^v dvv$

$\dfrac{F}{\rho\ \ \ell} x\Big|_0^\ell - g\mu_k\left(x - \dfrac{x^2}{2\ell}\right)\Big|_0^\ell = \dfrac{v^2}{2}\Big|_0^v$

$\dfrac{F}{\rho} - g\mu_k\dfrac{\ell}{2} = \dfrac{v^2}{2}$

$\sqrt{\dfrac{2F}{\rho} - \mu_k g\ell} = v = 4$ m/s