CONTENTS

▶ **STATISTICS :**

JEE (Main) Syllabus

Statistics : Calculation of mean, median, mode of grouped and ungrouped data. Calculation of standard deviation, variance and mean deviation for grouped and ungrouped data.

Statistics

1. Measures of central tendency :

An average value or central value of a distribution is the value of variable which is representative of the entire distribution, this represenive value are called the measures of central tendency are of following type.

(A) Mathematical average
(B) Positional average
(i) Arithmetic mean or mean
(i) Median
(ii) Geometrical mean
(ii) Mode
(iii) Harmonic mean

2. Mean (Arithmetic mean)

If x_1, x_2, x_3 ,.......x_n are n values of variate x_i then their A.M. $\overline{x}$ is defined as

$$\overline{x} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n} = \frac{\sum\limits_{i=1}^{n} x_i}{n}$$

If x_1, x_2, x_3, ... x_n are values of veriate with frequencies f_1, f_2, f_3,.........f_n then their A.M. is given by

$$\overline{x} = \frac{f_1 x_1 \times f_2 x_2 + f_3 x_3 + ... f_n f_n}{f_1 + f_2 + f_3 + ... + f_n} = \frac{\sum\limits_{i=1}^{n} f_i x_i}{N} \text{ , where } N = \sum\limits_{i=1}^{n} f_i$$

(i) Properties of arithmetic mean :

(a) Sum of deviation of variate from their A.M. is always zero that is $\Sigma \left(x_i - \overline{x} \right) = 0$.

(b) Sum of square of deviation of variate from their A.M. is minimum that is

$\Sigma \left(x_i - \overline{x} \right)^2$ is minimum

(c) If $\overline{x}$ is mean of variate x_i then

A.M. of $(x_i + \lambda) = \overline{x} + \lambda$

A.M. of $\lambda_i . x_i = \lambda. \overline{x}$

A.M. of $(ax_i + b) = a\overline{x} + b$

(ii) Merits of arithmetic mean :

(a) It is rigidly defined.

(b) It is based on all the observation taken.

(c) It is calculated with reasonable ease.

(d) It is least affected by fluctuations in sampling.

(e) It is based on each observation and so it is a better representative of the data.

(f) It is relatively reliable

(g) Mathematical analysis of mean is possible.

(iii) Demerits of Arithmetic Mean :

(a) It is severely affected by the extreme values.

(b) It cannot be represented in the actual data since the mean does not coincide with any of the observed value.

(c) It cannot be computed unless all the items are known.

Example # 1 : Find mean of data 2, 4, 5, 6, 8, 17.

Solution : Mean = $\dfrac{2+4+5+6+8+17}{6} = 7$

Example # 2 : Find the mean of the following distribution :

x :	4	6	9	10	15
f :	5	10	10	7	8

Solution : Calculation of Arithmetic Mean

x_i	f_i	$f_i x_i$
4	5	20
6	10	60
9	10	90
10	7	70
15	8	120
	$N = \sum f_i = 40$	$\sum f_i x_i = 360$

∴ Mean = $\overline{X} = \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{360}{40} = 9$

Example # 3 : Find the mean wage from the following data :

Wage (in Rs) :	800	820	860	900	920	980	1000
No. of workers :	7	14	19	25	20	10	5

Solution : Let the assumed mean be A = 900 and h = 20.

Calculation of Mean

Wage (in Rs) x_i	No. of workers f_i	$d_i = x_i - A = x_i - 900$	$u_i = \dfrac{x_i - 900}{20}$	$f_i u_i$
800	7	−100	−5	−35
820	14	−80	−4	−56
860	19	−40	−2	−38
900	25	0	0	0
920	20	20	1	20
980	10	80	4	40
1000	5	100	5	25
$N = \sum f_i = 100$				$\sum f_i u_i = -44$

We have,

N = 100, $\sum f_i u_i = -44$, A = 900 and h = 20

∴ Mean = $\overline{X} = A + h\left(\dfrac{1}{N}\sum f_i u_i\right) \Rightarrow \overline{X} = 900 + 20 \times \dfrac{-44}{100} = 900 - 8.8 = 891.2$

Hence, mean wage = Rs. 891.2

3.   Geometric mean :

If $x_1, x_2, x_3, \ldots \ldots x_3$ are n positive values of variate then their geometric mean G is given by

$G = (x_1 x_2 x_3 \ldots x_n)^{1/n}$

$\Rightarrow$ $G = \text{antilog}\left[\dfrac{1}{n}\sum_{i=1}^{n}\log x_i\right]$

4. Median :

The median of a series is values of middle term of series when the values are written is ascending order or descending order. Therefore median, divide on arranged series in two equal parts

(i)   For ungrouped distribution :

If n be number of variates in a series then

$$\text{Median} = \begin{cases} \left(\dfrac{n+1}{2}\right)^{th} \text{term, (when n is odd)} \\ \text{Mean of } \left(\dfrac{n}{2}\right)^{th} \text{and } \left(\dfrac{n}{2}+2\right)^{th} \text{term (when n is even)} \end{cases}$$

(ii)   For ungrouped frequency distribution :

First we calculate cumulative frequency (sum of all frequencies). Let it be N then

$$\text{Median} \begin{cases} \left(\dfrac{N+1}{2}\right)^{th} \text{term (when n odd)} \\ \text{Mean of } \left(\dfrac{N}{2}\right) \& \left(\dfrac{N+2}{2}\right) \text{(when n is even)} \end{cases}$$

5. Merits and demerits of median :

The following are some merits and demerits of median :

(i)   Merits :

(a) It is easy to compute and understand.

(b) It is well defined an ideal average should be

(c) It can also be computed in case of frequency distribution with open ended classes.

(d) It is not affected by extreme values.

(e) It can be determined graphically.

(f) It is proper average for qualitative data where items are not measured but are scored.

(ii)   Demerits :

(a) For computing median data needs to be arranged in ascending or descending order.

(b) It is not based on all the observations of the data.

(c) It cannot be given further algebraic treatment.

(d) It is affected by fluctuations of sampling.

(e) It is not accurate when the data is not large.

(f) In some cases median is determined approximately as the mid-point of two observations whereas for mean this does not happen.

Example # 4 : Find the median of observations 4, 6, 9, 4, 2, 8, 10

Solution : Values in ascending order are 2, 4, 4, 6, 8, 9, 10

here n = 7 so $\dfrac{n+1}{2} = 4$

so median = 4th observaiton = 6

Example # 5 : Obtain the median for the following frequency distribution :

x	11	13	15	18	21	23	30	40	50
f	8	10	11	16	20	25	15	9	6

Solution :

x	f	cf
11	8	8
13	10	18
15	11	29
18	16	45
21	20	65
23	25	90
30	15	105
40	9	114
50	6	120
	N = 120	

Here, $N = 120 \Rightarrow \dfrac{N}{2} = 60$

We find that the cummulative frequency just greater than $\dfrac{N}{2}$ i.e., 60 is 65 and the value of x corresponding to 65 is 21. Therefore, Median = 21.

6. Harmonic Mean :

If $x_1, x_2, x_3, \ldots x_3$ are n non-zero values of variate then their harmonic mean H is defined as

$$H = \frac{n}{\dfrac{1}{x_i} + \dfrac{1}{x_2} + \ldots + \dfrac{1}{x_n}} = \frac{n}{\displaystyle\sum_{i=1}^{n} \dfrac{1}{x_i}}$$

7. Mode :

If a frequency distribution the mode is the value of that variate which have the maximum frequency. Mode for

(i) For ungrouped distribution :
The value of variate which has maximum frequency.

(ii) For ungrouped frequency distribution :
The value of that variate which have maximum frequency.
Relationship between mean, median and mode.

- In symmetric distribution, mean = mode = median
- In skew (moderately asymmetrical) distribution,
  median divides mean and mode internally in 1 : 2 ratio.

$$\Rightarrow \quad median = \frac{2(Mean) + (Mode)}{3}$$

8. Merits and demerits of mode :

The following are some merits and demerits of mode :

(i) Merits :
(a) It is readily comprehensible and easy to compute. In some case it can be computed merely by inspection.
(b) It is not affected by extreme values. It can be obtained even if the extreme values are not known.
(c) Mode can be determined in distributions with open classes.
(d) Mode can be located on graph also.

(ii) **Demerits :**

    (a)   It is ill-defined. It is not always possible to find a clearly defined mode. In some cases, we may come across distributions with two modes. Such distributions are called bimodal. If a distribution has more than two modes, it is said to be multimodal.

    (b)   It is not based upon all the observation.

    (c)   Mode can be calculated by various formulae as such the value may differ from one to other. Therefore, it is not rigidly defined.

    (d)   It is affected to a greater extent by fluctuations of sampling.

Example # 6 : Find mode of data 2, 4, 6, 8, 8, 12, 17, 6, 8, 9.
Solution :   8 occurs maximum number of times so mode = 8

9. Measure of dispersion :

It is measure of deviation of its value about their central values. It gives an idea of scatterdness of different values from the central values.

Types :

(i) **Range** :

Difference between greatest values & least values of variates of a distribution are called the range of distribution.

Also coefficient of range $= \dfrac{\text{difference of extreme values}}{\text{sum of extreme values}} = \dfrac{L-S}{L+S}$

where L = largest value and S = smallest value

(ii) **Mean deviation :**

Mean deviation of a distribution is, the mean of absolute value of deviation of variate from their statistical average (median, mean or mode).

If A is any statistical average then mean deviation about A is defined as

Mean deviation $= \dfrac{\sum\limits_{i=1}^{n} |x_i - A|}{n}$

Mean deviation $= \dfrac{\sum\limits_{i=1}^{n} f_i |x_i - A|}{N}$ (for frequency distribution)

Example # 7 : Calculate mean deviation about median for the following data
    3, 9, 5, 3, 12, 10, 18, 4, 7, 19, 21.

Solution :   Data in ascending order is 3, 3, 4, 5, 7, 9, 10, 12, 18, 19, 21

Median $= \dfrac{n+1}{2}$ th value = 6$^{\text{th}}$ value = 9

Mean deviation about median $= \dfrac{\sum\limits_{i=1}^{11} |x_i - \text{median}|}{11} = \dfrac{58}{11}$

Example # 8 : Find mean deviation from mean

x	5	7	9	10	12	15
f	8	6	2	2	2	6

Solution :

x	f	fx	$x - \bar{x}$	$|x - \bar{x}|$	$f|x - \bar{x}|$
5	8	40	−4	4	32
7	6	42	−2	2	12
9	2	18	0	0	0
10	2	20	1	1	2
12	2	24	3	3	6
15	6	90	6	6	36
	N = 26	$\sum fx = 234$			$\sum f|x - \bar{x}| = 88$

$\bar{x} = \dfrac{\sum fx}{\sum f} = 9$ Now, M.D. $(\bar{x}) = \dfrac{\sum f|x - \bar{x}|}{\sum f} = \dfrac{88}{26} = 3.38$

(iii) **Variance** :

It is the mean of squares of deviation of variate from their mean. It is denoted by σ^2 or var(x). The positive square root of the variance are called the standard deviation. It is denoted by σ or S.D.

so standard deviation $= +\sqrt{\text{variance}}$ formula

$$\sigma_x{}^2 = \frac{\Sigma(x_i - \overline{x})^2}{n} \qquad \Rightarrow \qquad \sigma_x{}^2 = \frac{\displaystyle\sum_{i=1}^{n} x_i^2}{n} - \left(\frac{\displaystyle\sum_{i=1}^{n} x_i}{n}\right)^2 = \frac{\displaystyle\sum_{i=1}^{n} x_i^2}{n} - \left(\overline{x}\right)^2$$

$$\sigma_d{}^2 = \frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma di}{n}\right)^2 \text{ , where } d_i = x_i - a \text{ , where a = assumed mean}$$

coefficient of S.D. $= \left(\dfrac{\sigma}{\overline{x}}\right) \Rightarrow$ coefficient of variation $= \left(\dfrac{\sigma}{\overline{x}}\right) \times 100$ (in percentage)

(a) **Properties of variance :**

• var($x_i + \lambda$) = var(x_i)   • var($\lambda.x_i$) = λ^2(var x_i)   • var($a\, x_i + b$) = a^2(var x_i)

where λ, a, b are constant.

Example # 9 : Find the mean and variance of first n natural numbers.

Solution : $\overline{x} = \dfrac{\sum x}{n} = \dfrac{1+2+3+\ .\ .\ .\ .+\ n}{n} = \dfrac{n+1}{2}$

Variance $= \dfrac{\sum x^2}{n} - (\overline{x})^2 = \dfrac{1^2 + 2^2 + 3^2 +\ .\ .\ .\ .+\ n^2}{n} - \left(\dfrac{n+1}{2}\right)^2 = \dfrac{n^2-1}{12}$

Example # 10 : Find the variance and standard deviation of the following frequency distribution :

Variable (x_i)	2	4	6	8	10	12	14	16
Frequency (f_i)	4	4	5	15	8	5	4	5

Solution :   Calculation of variance and standard deviation

Variable x_i	Frequency f_i	$f_i x_i$	$x_i = \overline{X}$ $= x_i - 9$	$(x_i - \overline{X})^2$	$f_i(x_i - \overline{X})^2$
2	4	8	−7	49	196
4	4	16	−5	25	100
6	5	30	−3	9	45
8	15	120	−1	1	15
10	8	80	1	1	8
12	5	60	3	9	45
14	4	56	5	25	100
16	5	80	7	49	245
	$N = \sum f_i = 50$	$\sum f_i x_i = 450$			$\sum f_i(x_i - \overline{X})^2 = 754$

Here N = 50, $\sum f_i x_i = 450$

$\therefore \qquad \overline{X} = \dfrac{1}{N}\ (\sum f_i x_i) = \dfrac{450}{50} = 9$

We have $\sum f_i(x_i - \overline{X})^2 = 754$

$\therefore \qquad \text{Var(X)} = \dfrac{1}{N}\left[\sum f_i(x_i - \overline{X})^2\right] = \dfrac{754}{50} = 15.08$

S.D. $= \sqrt{\text{Var(X)}} = \sqrt{15.08} = 3.88$

Example # 11 : Calculate the mean and standard deviation for the following data :

Wages upto (in Rs.)	15	30	45	60	75	90	105	120
No. of workers	12	30	65	107	157	202	222	230

Solution : We are given the cummulative frequency distribution. So first we will prepare the frequency distribution as given below :

Class Interval	Cummlative frequency	Mid–values	Frequency	$u_i = \dfrac{x_i - 67.5}{15}$	$f_i u_i$	$f_i u_i^2$
0 – 15	12	7.5	12	– 4	– 48	192
15 – 30	30	22.5	18	– 3	– 54	162
30 – 45	65	37.5	35	– 2	– 70	140
45 – 60	107	52.5	42	– 1	– 42	42
60 – 75	157	67.5	50	0	0	0
75 – 90	202	82.5	45	1	45	45
90 – 105	222	97.5	20	2	40	80
105 – 120	230	112.5	8	3	24	72
			$\sum f_i = 230$		$\sum f_i u_i = -105$	$\sum f_i u_i^2 = 733$

Here A = 67.5, h = 15, N = 230, $\sum f_i u_i = -105$ and $\sum f_i u_i^2 = 733$

∴ Mean = $A + h\left(\dfrac{1}{N}\sum f_i\ u_i\right) = 67.5 + 15\left(\dfrac{-105}{230}\right) = 67.5 - 6.85 = 60.65$

and Var(X) = $h^2\left[\dfrac{1}{N}\sum f_i\ u_i^2 - \left(\dfrac{1}{N}\sum f_i\ u_i\right)^2\right]$

⇒ Var(X) = $225\left[\dfrac{733}{230} - \left(\dfrac{-105}{230}\right)^2\right] = 225[3.18 - 0.2025] = 669.9375$

∴ S.D. = $\sqrt{\text{Var (X)}} = \sqrt{669.9375} = 25.883$

10. For NCERT and Board purpose :
(i) Arithmetic mean :
Arithmetic mean of continuous grouped data :
Take mid points of given classes as x_i and use formula as given for discrete grouped data.

Example # 12 : Find the mean of the following frequency distribution :

Class-interval :	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of workers f :	7	10	15	8	10

Solution : Calculation of Mean

Class-interval	Mid-values (x_i)	Frequency f_i	$d_i = x_i - 25$	$u_i = \dfrac{x_i - 25}{10}$	$f_i u_i$
0 – 10	5	7	–20	–2	–14
10 – 20	15	10	–10	–1	–10
20 – 30	25	15	0	0	0
30 – 40	35	8	10	1	8
40 – 50	45	10	20	2	20
		N = $\sum f_i$ = 50			$\sum f_i u_i$ = 4

We have,
A = 25, h = 10, N = 50 and $\sum f_i u_i$ = 4.

∴ Mean = $A + h\left\{\dfrac{1}{N}\sum f_i u_i\right\}$ ⇒ mean = $25 + 10 \times \dfrac{4}{50} = 25.8$

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Example # 13 : Find the mean marks of students from the following cumulative frequency distribution :

Marks	Number of students	Marks	Number of students
0 and above	80	60 and above	28
10 and above	77	70 and above	16
20 and above	72	80 and above	10
30 and above	65	90 and above	8
40 and above	55	100 and above	0
50 and above	43		

Solution : Here we have, the cumulative frequency distribution. So, first we convert it into an ordinary frequency distribution. We observe that there are 80 students getting marks greater than or equal to 0 and 77 students have secured 10 and more marks. Therefore, the number of students getting marks between 0 and 10 is 80 – 77 = 3.

Similarly, the number of students getting marks between 10 and 20 is 77 – 72 = 5 and so on. Thus, we obtain the following frequency distribution.

Marks	Number of students	Marks	Number of students
0 – 10	3	50 – 60	15
10 – 20	5	60 – 70	12
20 – 30	7	70 – 80	6
30 – 40	10	80 – 90	2
40 – 50	12	90 – 100	8

Now, we compute arithmetic mean by taking 55 as the assumed mean.
Computation of Mean

Marks	Mid-value (x_i)	Frequency (f_i)	$u_i = \dfrac{x_i - 55}{10}$	$f_i u_i$
0 – 10	5	3	–5	–15
10 – 20	15	5	–4	–20
20 – 30	25	7	–3	–21
30 – 40	35	10	–2	–20
40 – 50	45	12	–1	–12
50 – 60	55	15	0	0
60 – 70	65	12	1	12
70 – 80	75	6	2	12
80 – 90	85	2	3	6
90 – 100	95	8	4	32
Total		$\Sigma f_i = 80$		$\Sigma f_i u_i = -26$

We have,

$N = \Sigma f_i = 80$, $\Sigma f_i u_i = -26$, $A = 55$ and $h = 10$

$\therefore \quad \overline{X} = A + h \left\{ \dfrac{1}{N} \Sigma f_i u_i \right\} \quad \Rightarrow \quad \overline{X} = 55 + 10 \times \dfrac{-26}{80} = 55 - 3.25 = 51.75$ Marks

(ii) **Median :**

(a) **Median of continuous frequency distribution :**

Let the number of observation be N. Prepare the cumulative frequency table. Find the median class i.e. the class in which the observation whose cumulative frequency is equal to or just greater than $\dfrac{N}{2}$ lies.

The median value is given by the formula : Median

$(M) = \ell + \left[\dfrac{\left(\dfrac{N}{2}\right) - c}{f} \right] \times h$ where

N = total frequency = $\sum f_i$

ℓ = lower limit of median class

f = frequency of the median class

c = cumulative frequency of the class preceding the median class

h = class interval (width) of the median class

Example # 14 : Calculate the median from the following distribution :

Class	$5-10$	$10-15$	$15-20$	$20-25$	$25-30$	$30-35$	$35-40$	$40-45$
Frequency	5	6	15	10	5	4	2	2

Solution :

Class	Frequency	Cumulative Frequency
$5-10$	5	5
$10-15$	6	11
$15-20$	15	26
$20-25$	10	36
$25-30$	5	41
$30-35$	4	45
$35-40$	2	47
$40-45$	2	49
	N = 49	

We have, N = 49 $\therefore$ $\dfrac{N}{2} = \dfrac{49}{2} = 24.5$

The cummulative frequency just greater than $\dfrac{N}{2}$ is 26 and the corresponding class is 15-20.

Thus 15-20 is the median class such that $\ell = 15$, $f = 15$, F = 11 and h = 5

$\therefore$ Median $= \ell + \dfrac{\dfrac{N}{2} - F}{f} \times h$ $= 15 + \dfrac{24.5 - 11}{15} \times 5$ $= 15 + \dfrac{13.5}{3} = 19.5$

(iii) **Mode :**

(a) **Mode for continuous frequency distribution :**

First find the modal class i.e. the class which has maximum frequency. The modal class can be determined either by inspecting or with the help of grouping.

The mode is given by the formula :

Mode $= \ell + \dfrac{f_m - f_{m-1}}{2f_m - f_{m-1} - f_{m+1}} \times h$

where ℓ = lower limit of the modal class

h = width of the modal class

f_m = frequency of the modal class

f_{m-1} = frequency of the class preceding modal class

f_{m+1} = frequency of the class succeeding modal class

Example # 15 : Compute the mode for the following frequency distribution :

Size of items	0 − 4	4 − 8	8 − 12	12 − 16	16 − 20	20 − 24	24 − 28	28 − 32	32 − 36	36 − 40
Frequency	5	7	9	17	12	10	6	3	1	0

Solution : Here, the maximum frequency is 17 and the corresponding class is 12-16 So 12-16 is the modal class.

We have, ℓ = 12, h = 4, f = 17, f_1 = 9 and f_2 = 12

$\therefore$ Mode = $\ell + \dfrac{f - f_1}{2f - f_1 - f_2} \times h \Rightarrow$ Mode = $12 + \dfrac{17 - 9}{34 - 9 - 12} \times 4$

$\Rightarrow$ Mode = $12 + \dfrac{8}{13} \times 4 = 12 + \dfrac{32}{13} = 12 + 10.66 = 32.66$

(iv)   Mean deviation :

(a)   Mean deviation of continuous frequency distribution : For calculating mean deviation of a continuous frequency distribution, the procedure is same as for a discrete frequency distribution. The only difference is that here we have to obtain the midpoints of the various classes and take the deviations of these mid point from the given average A.

Example # 16 : Find the mean deviation about the median of the following frequency distribution :

Class	0 − 6	6 − 12	12 − 18	18 − 24	24 − 30
Frequency	8	10	12	9	5

Solution :   Calculation of mean deviation about the median

Class	Mid values (x_i)	Frequency (f_i)	Cumulative Frequency (c.f.)	$\lvert x_i - 14 \rvert$	$f_i \lvert x_i - 14 \rvert$
0 − 6	3	8	8	11	88
6 − 12	9	10	18	5	50
12 − 18	15	12	30	1	12
18 − 24	21	9	39	7	63
24 − 30	27	5	44	13	65
		$N = \sum f_i = 44$			$\sum f_i \lvert x_i - 14 \rvert = 278$

Here N = 44, so $\dfrac{N}{2}$ = 22 and the cumulative frequency just greater than $\dfrac{N}{2}$ is 30. Thus 12-18 is the median class.

Now   Median = $\ell + \dfrac{N/2 - F}{f} \times h$, where ℓ = 12, h = 6, f = 12, F = 18

or   Median = $12 + \dfrac{22 - 18}{12} \times 6 = 12 + \dfrac{4 \times 6}{12} = 14$

Mean deviation about median = $\dfrac{1}{N} \sum f_i \lvert x_i - 14 \rvert = \dfrac{278}{44} = 6.318$

Example # 17 : Find the mean deviation from the mean for the following data :

Classes	$10-20$	$20-30$	$30-40$	$40-50$	$50-60$	$60-70$	$70-80$
Frequencies	2	3	8	14	8	3	2

Solution : We prepare the table as follows :

Computation of mean deviation from mean

Classes	Mid–values (x_i)	frequencies f_i	$f_i\, x_i$	$\left|x_i - \overline{X}\right| = \left|x_i - 45\right|$	$f_i\left|x_i - \overline{X}\right|$
$10-20$	15	2	30	30	60
$20-30$	25	3	75	20	60
$30-40$	35	8	280	10	80
$40-50$	45	14	630	0	0
$50-60$	55	8	440	10	80
$60-70$	65	3	195	20	60
$70-80$	75	2	150	30	60
		$n = \sum f_i = 40$	$\sum f_i x_i = 1800$		$\sum f_i \left|x_i - \overline{X}\right| = 400$

We have, N = 40 and $\sum f_i\; x_i$ = 1800 $\therefore$ $\overline{X} = \dfrac{\sum f_i\; x_i}{N} = \dfrac{1800}{40} = 45$

Now $\quad \sum f_i\; \left|x_i - \overline{X}\right|$ = 400 and N = $\sum f_i = 40$

$\therefore \quad$ M.D. = $\dfrac{1}{N} \sum f_i\; \left|x_i - \overline{X}\right| = \dfrac{400}{40} = 10$

(v) **Variance and standard deviation :**

(a) **Variance of a grouped or continuous frequency distribution :** In a grouped or continuous frequency distribution, any of the formulae discussed in discrete frequency distribution can be used.

Example # 18 : Calculate the mean and standard deviation for the following distribution :

Marks	$20-30$	$30-40$	$40-50$	$50-60$	$60-70$	$70-80$	$80-90$
No. of Students	3	6	13	15	14	5	4

Solution : Calculation of Standard deviation

Class interval	Frequency f_i	Mid–values x_i	$u_i = \dfrac{x_i - 55}{10}$	$f_i u_i$	u_i^2	$f_i u_i^2$
$20-30$	3	25	-3	-9	9	27
$30-40$	6	35	-2	-12	4	24
$40-50$	13	45	-1	-13	1	13
$50-60$	15	55	0	0	0	0
$60-70$	14	65	1	14	1	14
$70-80$	5	75	2	10	4	20
$80-90$	4	85	3	12	9	36
	$N = \sum f_i = 60$			$\sum f_i u_i = 2$		$\sum f_i u_i^2 = 134$

Here N = 60, $\sum f_i u_i = 2$, $\sum f_i u_i^2 = 134$ and h = 10

$\therefore \quad$ Mean $= \overline{X} = A + h\left(\dfrac{1}{N}\sum f_i\; u_i\right) \Rightarrow \quad \overline{X} = 500 + 10\left(\dfrac{2}{60}\right) = 55.333$

$\text{Var(X)} = h^2\left[\dfrac{1}{N}\sum f_i\; u_i^2 - \left(\dfrac{1}{N}\sum f_i\; u_i\right)^2\right] = 100\left[\dfrac{134}{60} - \left(\dfrac{2}{60}\right)^2\right] = 222.9$

$\therefore \quad$ S.D. = $\sqrt{\text{Var (X)}} = \sqrt{222.9} = 14.94$

Example # 19 : Suppose that samples of polythene bags from two manufactures, A and B are tested by a prospective buyer for bursting pressure, with the following results :

Bursting presure in kg	Number of bags manufactued by manufactuer	
	A	B
5 – 10	2	9
10 – 15	9	11
15 – 20	29	18
20 – 25	54	32
25 – 30	11	27
30 – 35	5	13

Which set of the bag has the highest average bursting pressure ? Which has more uniform pressure ?

Solution: For determining the set of bags having higher average bursting pressure, we compute mean and for finding out set of bags having more uniform pressure we compute coefficient of variation.

Manufacturer A :

Computation of mean and standard deviation

Bursting pressure	Mid– values x_i	f_i	$u_i = \dfrac{x_i - 17.5}{5}$	$f_i u_i$	$f_i u_i^2$
5 – 10	7.5	2	– 2	– 4	8
10 – 15	12.5	9	– 1	– 9	9
15 – 20	17.5	29	0	0	0
20 – 25	22.5	54	1	54	54
25 – 30	27.5	11	2	22	44
30 – 35	32.5	5	3	15	45
		$N = \sum f_i = 110$	$\sum u_i = 3$	$\sum f_i u_i = 78$	$\sum f_i u_i^2 = 160$

$$\overline{X}_A = a + h \left(\frac{\sum f_i \ u_i}{N} \right)$$

$$\Rightarrow \qquad \overline{X}_A = 17.5 + 5 \times \frac{78}{110} \qquad [\because \ h = 5, \ a = 17.5]$$

$$\Rightarrow \qquad \overline{X}_A = 17.5 + 3.5 = 21 \qquad \Rightarrow \qquad \sigma_A^2 = h^2 \left[\frac{1}{N} \sum f_i \ u_i^2 - \left(\frac{1}{N} \sum f_i \ u_i \right)^2 \right]$$

$$\Rightarrow \qquad \sigma_A^2 = 25 \left[\frac{160}{110} - \left(\frac{78}{110} \right)^2 \right] \qquad \Rightarrow \qquad \sigma_A^2 = 25 \left(\frac{17600 - 6084}{110 \times 110} \right) = 23.79$$

$$\Rightarrow \qquad \sigma_A = \sqrt{23.79} \ = 4.87$$

$$\therefore \qquad \text{Coefficient of variation} = \frac{\sigma_A}{\overline{X}_A} \times 100 = \frac{4.87}{21} \times 100 = 23.19$$

Manufacurer B :

Bursting pressure	Mid-value x_i	f_i	$u_i = \dfrac{x_i - 17.5}{5}$	$f_i u_i$	$f_i u_i^2$
5 – 10	7.5	9	− 2	− 18	36
10 – 15	12.5	11	− 1	− 11	11
15 – 20	17.5	18	0	0	0
20 – 25	22.5	32	1	32	32
25 – 30	27.5	27	2	54	108
30 – 35	32.5	13	3	39	117
		$N = \sum f_i = 110$	$\sum u_i = 3$	$\sum f_i u_i = 96$	$\sum f_i u_i^2 = 304$

$$\overline{X}_B = a + h\left(\frac{\sum f_i \ u_i}{N}\right) \Rightarrow \overline{X}_B = 17.5 + 5 \times \frac{96}{110} = 17.5 + 4.36 = 21.81$$

$$\sigma_B^2 = h^2\left[\frac{1}{N}\left(\sum f_i \ u_i^2\right) - \left(\frac{1}{N}\sum f_i \ u_i\right)^2\right]$$

$$\Rightarrow \qquad \sigma_B^2 = 25\left[\frac{304}{110} - \left(\frac{96}{110}\right)^2\right] \Rightarrow \sigma_B^2 = 25\left(\frac{33440 - 9216}{110 \times 110}\right) = 50.04$$

$$\sigma_B = 7.07$$

$\therefore$ Coefficient of variation $= \dfrac{\sigma_B}{\overline{X}_B} \times 100 = \dfrac{7.07}{21.81} \times 100 = 32.41$

We observe that the average brusting pressure is higher for manufacturer B. So, bags manufactured by B have higher bursting pressure.

The coefficient of variation is less for manufacturer A. So bags manufactured by A have more uniform pressure.

Exercise-1

✎ **Marked questions are recommended for Revision**

OBJECTIVE QUESTIONS (SINGLE CHOICE CORRECT)

Section (A) : Mean, median & mode

A-1. Find the A.M. of the series 1, 2, 4, 8, 16, ..., 2^n

(1) $\dfrac{2^{n+1}-1}{n+1}$ (2) $\dfrac{2^{n+2}-1}{n}$ (3) $\dfrac{2^n-1}{n+1}$ (4) $\dfrac{2^n-1}{n}$

A-2. The average weight of 9 men is x kg. After another men joins the group, the average increases by 5%. Still another man joins and average returns to old level of x kg. Which one of the following true?

(1) the 10th & 11th men weight same

(2) the 10th man weight half as much as the 11th man

(3) the 10th man weight as much as the 11th man

(4) None of these

A-3. N observations on a variable x are $x_i = A + iB$ for i = 1, 2, 3, ..., n where A, B are real constants. The mean of the observation is

(1) $A + B\,\dfrac{(n+1)}{2}$ (2) $nA + B\,\dfrac{(n+1)}{2}$ (3) $A + Bn\,\dfrac{(n+1)}{2}$ (4) $A + B\left(\dfrac{n}{2}\right)$

A-4. Find the median of values 10, 14, 11, 9, 8, 12, 6.

(1) 8   (2) 10   (3) 9   (4) 11

A-5. If a variable takes the discrete values $\alpha + 4$, $\alpha - \dfrac{7}{2}$, $\alpha - \dfrac{5}{2}$, $\alpha - 3$, $\alpha - 2$, $\alpha + \dfrac{1}{2}$, $\alpha - \dfrac{1}{2}$, $\alpha + 5$ ($\alpha > 0$), then the median is

(1) $\alpha - \dfrac{5}{4}$ (2) $\alpha - \dfrac{1}{2}$ (3) $\alpha - 2$ (4) $\alpha + \dfrac{5}{4}$

A-6. Find the mode of data 1, 2, 5, 3, 2, 3, 0, 5, 2.

(1) 3   (2) 1   (3) 5   (4) 2

A-7. ✎ Consider the following statements related to measure of central tendency of 50 positive numbers

1. Median is not influenced by extreme values in set of numbers

2. The harmonic mean is unreliable if one or more of the numbers is non-zero

which of the above statement is/are correct

(1) 1 only   (2) 2 only   (3) both 1 and 2   (4) neither 1 nor 2

A-8. $M(x_1, x_2, x_3 .... x_n)$ defines a measure of central tendency based on n values $x_1, x_2, x_3, ...x_n$ consider the following measured of central tendency

(1) Arithmetic mean   (2) Median   (3) Geometric mean

which of the above measure satisfie/ satisfies the property

$$\dfrac{M\left(x_1,x_2,x_3............x_n\right)}{M\left(y_1,y_2,y_3............y_n\right)} = M.\left(\dfrac{x_1}{y_1},\dfrac{x_2}{y_2},\dfrac{x_3}{y_3},......,\dfrac{x_n}{y_n}\right)?$$

select the correct answer using the code below

(1) 1 only   (2) 2 only   (3) 3 only   (4) 1 and 3

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MAINSTTS-14

A-9. If values a, b, c,...... j, p occurs with frequencies $^{10}C_0$, $^{10}C_1$, $^{10}C_2$,..., $^{10}C_{10}$ then mode is

(1) a   (2) e   (3) f   (4) k

A-10. The mean of 21 observations (all different) is 40. If each observations greater than the median are increased 21, then mean of observations will become

(1) 50   (2) 50.5   (3) 30   (4) 45

A-11. The mean of series x_1, x_2, x_3, ... x_n is $\overline{x}$, then mean of the series $x_i + 2i$, i = 1, 2, 3, . . . , n will be

(1) $\overline{x} + n$ (2) $\overline{x} + n + 1$ (3) $\overline{x} + 2$ (4) $\overline{x} + 2n$

A-12. If the difference between mean and mode is 63, the difference between mean and median is

(1) 189   (2) 21   (3) 31.5   (4) 48.5

A-13. Mean of variates 1.2.3, 2.3.4, ..., n(n + 1)(n + 2)

(1) $\dfrac{n(n+1)(n+2)}{4}$ (2) $\dfrac{(n+1)(n+2)(n+3)}{2}$ (3) $\dfrac{(n+1)(n+2)(n+3)}{4}$ (4) $\dfrac{n(n+1)(n+3)}{2}$

Section (B) : Range, coefficient of range, mean deviation and coefficient of mean deviaiton.

B-1. Range of data 7, 8, 2, 1, 3, 13, 18 is

(1) 10   (2) 15   (3) 17   (4) 11

B-2. Coefficient of range 5, 2, 3, 4, 6, 8, 10 is

(1) $\dfrac{2}{3}$ (2) $\dfrac{1}{3}$ (3) $\dfrac{3}{5}$ (4) $\dfrac{1}{2}$

B-3 The scores of a batsman in ten innings are : 38, 70, 48, 34, 42, 55, 63, 46, 54, 44. Find the mean deviation about median.

(1) $\dfrac{43}{5}$ (2) $\dfrac{44}{5}$ (3) $\dfrac{41}{5}$ (4) $\dfrac{42}{5}$

B-4. The mean deviation about median of variates 13, 14, 15, ... , 99, 100 is

(1) 1936   (2) 21.5   (3) 23.5   (4) 22

B-5. The mean deviation of an ungrouped data is 50. If each observation is increased by 2%, then the new mean deviation is

(1) 50   (2) 51   (3) 49   (4) 50.5

B-6. The mean deviation of an ungrouped data is 80. If each observation is decreased by 5%, then the new mean deviation is

(1) 76   (2) 77   (3) 78   (4) 79

B-7. The mean deviation from mean of the observations a, a + d, a + 2d, ... a + 2nd is

(1) $\dfrac{n(n+1)d^2}{3}$ (2) $\dfrac{n(n+1)}{2}d^2$ (3) $a + \dfrac{n(n+1)d^2}{2}$ (4) $\dfrac{n(n+1)\,|\,d\,|}{(2n+1)}$

B-8. Mean deviation from the mean for the observations – 1, 0, 4 is

(1) $\sqrt{\dfrac{14}{3}}$ (2) $\dfrac{2}{3}$ (3) 2   (4) 3

B-9. If $\overline{X}$ is the mean of x_1, x_2, x_3 ...... x_n. Then the algebraic sum of the deviations about mean $\overline{X}$ is

(1) 0   (2) $\dfrac{\overline{X}}{n}$ (3) $n\,\overline{X}$ (4) $(n–1)\,\overline{X}$

B-10. If the algebraic sum of deviations of 20 observation from 30 is 20, then the mean of the observation is

(1) 30   (2) 30.1   (3) 29   (4) 31.

Section (C) : Variance, Standard deviation and coefficient of variation.

C-1. Variance of first 20 natural number is

(1) $\dfrac{133}{4}$ (2) $\dfrac{379}{12}$ (3) $\dfrac{133}{2}$ (4) $\dfrac{399}{4}$

C-2. The mean & variance of 7 observations are 8, 16 respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the LCM of remaining two observations is

(1) 16   (2) 24   (3) 20   (4) 14

C-3. If n = 10, $\overline{x} = \sqrt{12}$, $\sum x^2 = 1560$, then standard deviation σ is

(1) 12   (2) 13   (3) $\sqrt{166}$ (4) $\sqrt{12}$

C-4. The mean of distribution is 4 if coefficient of variation is 58%. Then standard deviation of distribution is

(1) 2.23   (2) 3.23   (3) 2.32   (4) 2.75

C-5. The sum of squares of deviations for 10 observations taken from mean 50 is 250. The co-efficient of variation is

(1) 50%   (2) 10%   (3) 40%   (4) 30%

C-6. Standard deviation is independent of

(1) change of scale & origin   (2) change of scale but not origin

(3) change of origin but not scale   (4) neither change of scale non origin

C-7 What is standard deviation of the set of observations 32, 28, 29, 30, 31?

(1) 1.6   (2) $\sqrt{2}$ (3) 2   (4) None of these

C-8. If the standard deviation of $x_1, x_2, ... x_n$ is 3.5, then the standard deviation of

$-2x_1 - 3, -2x_2 - 3, ...-2x_n - 3$ is

(1) -7 (2) -4 (3) 7   (4) 1.75

C-9. The marks of some students were listed out of a maximum 100. The standard deviation of marks was found to be 9. Subsequently the marks raised to a maximum of 150 and standard deviation of new marks was calculeted. The new standard deviation

(1) 9   (2) 13.5   (3) -13.5 (4) -9

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MAINSTTS-16

Exercise-2

✍ **Marked questions are recommended for Revision**

PART - I : OBJECTIVE QUESTIONS (SINGLE CHOICE CORRECT)

1. Find the arithmetic mean of $^{2n+1}C_0$, $^{2n+1}C_1$, ...$^{2n+1}C_n$.

(1) $\dfrac{2^{2n+2}}{n}$ 　　　　(2) $\dfrac{2^{2n}}{n+1}$ 　　　　(3) $\dfrac{2^{n+1}}{n+1}$ 　　　　(4) None of these

2. A variable takes the values of 0, 1, 2, ... , n with frequencies proportional to the binomial coefficients $^nC_0$, $^nC_1$, $^nC_2$, ... , ^nC_n, then mean of the distribution is

(1) $\dfrac{n(n+1)}{4}$ 　　(2) $\dfrac{n}{2}$ 　　(3) $\dfrac{n(n-1)}{2}$ 　　(4) $\dfrac{n(n+1)}{2}$

3. Following is the record of goals scored by team A in football session

Numbers of goals scored	0	1	2	3	4
Numbers of match is	1	9	7	5	3

for team 'B' mean number of goals scored per match was 2 goals with standard deviation 1.25 . The team which is more consistant

(1) A 　　　　　　　　　　　　(2) B
(3) A & B both are equal 　　　　(4) neither A nor B

4.✍ The mean of two samples of sizes 200 and 300 were found to be 25, 10 respectively. Their standarddeviations were 3 and 4 respectively. Find the variance of combined sample of size 500
(1) 70 　　　　(2) 60 　　　　(3) 67.2 　　　　(4) 80

5.✍ The first of the two samples has 100 items with mean 15 and S.D. 3. If the whole group has 250 items with mean 15.6 and S.D. = $\sqrt{13.44}$ then S.D. of the second group is
(1) 5 　　　　(2) 4 　　　　(3) 6 　　　　(4) 3.52

6.✍ The average marks of 10 students in a class was 60 with standard deviation 4. While the average marks of other 10 students was 40 with a standard deviation 6. If all the 20 students are taken together, their standard deviation will be

(1) 5.0 　　　　(2) 7.5 　　　　(3) 9.8 　　　　(4) 11.2

7. The mean and variance of 5 observations of an experiment are 4 and 5.2 respectively. From these observations three are 1, 2 and 6 and $\lambda = |x_1 - x_2| + 8$ where x_1 & x_2 are remaining observations. Then number of solution of equation $10 - x^2 - 2x = \lambda$ are
(1) 1 　　　　(2) 2 　　　　(3) 3 　　　　(4) 4

8.✍ The mean and variance of 10 numbers were calculated as 11.3 and 3.3 respectively. It was subsequently found that one of the a number was misread as 10 instead of 12. How does the variance change.
(1) variance decreases 　　　　　　(2) variance increases
(3) nothing can be said about variance 　　(4) variance remains unchanged.

Comprehension # 1_ (Q. No. 9 to 11)

As median divides an arranged series into two equal parts, in similar way quartile divides an arranged series in 4 equal part. For ungrouped frequency distribution formula of finding i^{th} quartile =

$$Q_i = \left\{ i \cdot \left(\frac{N+1}{4} \right) \right\}^{th} \text{ term, } i = 1, 2, 3.$$

Quartile deviation : half of difference between upper quartile & lower quartile.

$\Rightarrow$ Quartile deviation (Q.D.) = $\frac{1}{2}$ $(Q_3 - Q_1)$ $\Rightarrow$ Coefficient of quartile deviation = $\frac{Q_3 - Q_1}{Q_3 + Q_1}$.

9. If 1, 2, 3, 4, 5, 6, 7, are numbers then Q_1 & Q_3 are respectively

(1) 2, 4   (2) 2, 6   (3) 4, 6   (4) 3, 5

10. Quartile deviation of the following numbers 10, 8, 12, 11, 14, 9, 6 is

(1) 2   (2) 10   (3) 1/2   (4) 1

11. Coefficient of quartile deviation of numbers 6, 8, 9, 10, 11, 12, 14 is

(1) 1/5   (2) 1/10   (3) 2/5   (4) 1/4

**Comprehension # 2 (Q. No. 12 to 14)**

To analyse data using mean, median and mode, we need to use the most appropriate measure of central tendency. The mean is useful for predicting future results when there are no extreme values in the data set. The median may be more useful than the mean when there are extreme values in the data set as it is not affected by the extreme values. The median is most commonly quoted figure used to measure property prices as mean property price is affected by a few expensive properties that are not representative of the general property market. The mode is useful when the most common item or characteristic of a data set is required. The mode has applications in printing. It is important to print more of the most popular books.

12. For the data shown, the value of appropriate measure of central tendency is

No. of staff	1	2	4	5	3	2
Salary (In rupees)	15000	10000	7000	12000	90000	95000

(1) 95000   (2) 18350   (3) 90000   (4) 12000

13. For a normally distributed sample as shown, then most appropriate representative of data is
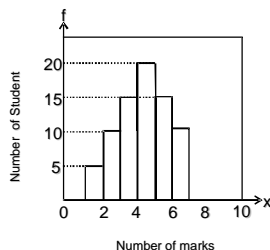
(1) Mean   (2) Mode
(3) Median   (4) any one of mean or median

14. Based upon collection of data of numbers of days it snows, rains or it is sunny in a month for three month December, January and February of last year, the weather forecast points that snow is likely to be in January. Which measure is used for this forecast ?

(1) Mean   (2) Mode   (3) Median   (4) Range

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

DIRECTIONS :

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

(1) Both the statements are true.

(2) Statement-I is true, but Statement-II is false.

(3) Statement-I is false, but Statement-II is true.

(4) Both the statements are false.

A-1. **Statement-1 :** If $\sum_{i=1}^{9}(x_i - 8) = 9$ and $\sum_{i=1}^{9}(x_i - 8)^2 = 45$ then S.D. of $x_1, x_2,......, x_9$ is 2.

Statement-2 : S.D. is independent of change of origin.

A-2. **Statement-1 :** Mean cannot be represented graphically.

Statement-2 : Mean may not coincide with anyone of the actual values.

Section (B) : MATCH THE COLUMN

B-1

	Column - I		Column - II
(1)	Better measure of central tendency for data 1, 7, 8, 9, 9 is	(p)	Mean
(2)	Which is not independent of change of scale ?	(q)	Median
(3)	Which is not dependent on change of origin ?	(r)	Mode
(4)	The value of range of data is always greater than or equal to	(s)	S.D.

B-2 Let $n \in N$.

	Column I		Column II
(1)	S.D. of 2, 4, 6, ... 2n ; $n \in N$	(p)	n
(2)	S.D. of 1, 3, 5, ..., 2n – 1 ; $n \in N$	(q)	$\dfrac{n+1}{2}$
(3)	Mean of 1,3,5, ...2n–1 ; $n \in N$	(r)	$\sqrt{\dfrac{n^2-1}{3}}$
(4)	Median of 1,3,5,.....,2n – 1	(s)	$\sqrt{\dfrac{n^2-1}{12}}$

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. If the daily earnings (in rupees) of 12 workers in a factory are 16, 11, 3, 7, 5, 28, 9, 31, 28, 43, 15 ,17, then which of the following is(are) true

(1) mean = 17.75

(2) median = 15.5

(3) mode = 28

(4) mean deviation about mean = $9.8\overline{3}$

C-2 If first sample of 25 variates has the mean 40 and standard deviation 5 and a second sample of 35 variates has the mean 45 and standard deviation 2, then which of the following is/are true

(1) mean of combined sample space = 42.917

(2) mean of combined sample space = 32.9

(3) standard deviation of combined sample space = 3.34

(4) standard deviation of combined sample space = 4.34

C-3. Which of the following is false :

(1) Algebraic sum of deviations of observation from their mean is zero

(2) If S.D. of x_i(i = 1, 2, 3, ... n) is σ then S.D. of hx_i is $h\sigma$

(3) Median is severely affected by fluctuations in extreme values

(4) Mean deviations of a given set of observations is least when taken about their median.

Exercise-3

✍ **Marked questions are recommended for Revision**

* Marked Questions may have more than one correct option.

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. The mean of the number a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the folowing gives possible values of a and b ? **[AIEEE 2008, (3, –1), 105]**

(1) a = 3, b = 4   (2) a = 0, b = 7   (3) a = 5, b = 2   (4) a = 1, b = 6

2. **Statement-I** The variance of first n even natural numbers is $\dfrac{n^2-1}{4}$ **[AIEEE 2009, (4, –1), 144]**

Statement-II The sum of first n natural numbers is $\dfrac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\dfrac{n(n+1)(2n+1)}{6}$.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

3. If the mean deviation of numbers 1, 1 + d, 1 + 2d, ...,1 + 100d from their mean is 255, then the value of d is equal to- **[AIEEE 2009, (4, –1), 144]**

(1) 10.0   (2) 20.0   (3) 10.1   (4) 20.2

4.✍ For two data sets, each of size 5, the variance are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is

[AIEEE 2010, (4, –1), 144

(1) $\dfrac{11}{2}$ (2) 6   (3) $\dfrac{13}{2}$ (4) $\dfrac{5}{2}$

| **MAINSTTS-20**

5. If the mean deviation about the median of the numbers a, 2a, ..., 50a is 50, then |a| equals :

[AIEEE 2011, (4, –1), 120]

(1) 2   (2) 3   (3) 4   (4) 5

6. A scientist is weighing each of 30 fishes. Their mean weight worked out is 30 gm and a standard deviation of 2 gm. Later, it was found that the measuring scale was misaligned and always under reported every fish weight by 2 gm. The correct mean and standard deviation (in gm) of fishes are respectively :   **[AIEEE 2011, (4, –1), 120]**

(1) 32, 2   (2) 32, 4   (3) 28, 2   (4) 28, 4

7. Let x_1, x_2, ..., x_n be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be the variance

Statement-1 : Variance of $2x_1$, $2x_2$, ..., $2x_n$ is $4\sigma^2$.   **[AIEEE 2012, (4, –1), 120]**

Statement-2 : Arithmetic mean $2x_1$, $2x_2$, ..., $2x_n$ is $4\bar{x}$.

(1) Statement-1 is false, Statement-2 is true.

(2) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.

(3) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.

(4) Statement-1 is true, statement-2 is false.

8. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given ?   **[AIEEE - 2013, (4, –1),360]**

(1) mean   (2) median   (3) mode   (4) variance

9. The variance of first 50 even natural number is   **[AIEEE - 2014, (4, –1),360]**

(1) $\dfrac{833}{4}$ (2) 833   (3) 437   (4) $\dfrac{437}{4}$

10. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is   **[JEE(Main) 2015, (4, – 1), 120]**

(1) 16.8   (2) 16.0   (3) 15.8   (4) 14.0

11. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true?

[JEE(Main) 2016, (4, – 1), 120]

(1) $3a^2 - 32a + 84 = 0$ (2) $3a^2 - 34a + 91 = 0$ (3) $3a^2 - 23a + 44 = 0$ (4) $3a^2 - 26a + 55 = 0$

12. If $\sum\limits_{i=1}^{9}(x_i - 5) = 9$ and $\sum\limits_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items x_1, x_2, ... , x_9 is :

[JEE(Main) 2018, (4, – 1), 120]

(1) 2   (2) 3   (3) 9   (4) 4

13. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. the variance (in cm^2) of the height of these six students is :

[JEE(Main) 09-01-19 P-1, (4, – 1), 120]

(1) 20   (2) 22   (3) 16   (4) 18

14. A data consists of n observations :   **[JEE(Main) 09-01-19 P-2, (4, – 1), 120]**

x_1, x_2, ..., x_n. If $\sum\limits_{i=1}^{n}(x_i + 1)^2 = 9n$ and $\sum\limits_{i=1}^{n}(x_i - 1)^2 = 5n$, then the standard deviation of this data is :

(1) 5   (2) 2   (3) $\sqrt{5}$ (4) $\sqrt{7}$

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15. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is :   **[JEE(Main) 10-01-19 P-1, (4, – 1), 120]**

(1) 4 : 9   (2) 6 : 7   (3) 5 : 8   (4) 10 : 3

16. If mean and standard deviation of 5 observations x_1, x_2, x_3, x_4, x_5, are 10 and 3, respectively, then the variance of 6 observations $x_1, x_2, ...x_5$ and –50 is equal to : **[JEE(Main) 10-01-19 P-2, (4, – 1), 120]**

(1) 586.5   (2) 507.5   (3) 582.5   (4) 509.5

17. The outcome of each of 30 items was observed; 10 items gave an outcome $\dfrac{1}{2}$ – d each, 10 items gave outcome $\dfrac{1}{2}$ each and the remaining 10 items gave outcome $\dfrac{1}{2}$ + d each. If the variance of this outcome data is $\dfrac{4}{3}$ then |d| equals :   **[JEE(Main) 11-01-19 P-1, (4, – 1), 120]**

(1) $\sqrt{2}$ (2) $\dfrac{2}{3}$ (3) 2   (4) $\dfrac{\sqrt{5}}{2}$

18. If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observations is :

[JEE(Main) 12-01-19 P-1, (4, – 1), 120]

(1) 50   (2) 51   (3) 31   (4) 30

19. The mean and the variance of five observation are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4 ; then the absolute value of the difference of the other two observations, is

[JEE(Main) 12-01-19 P-2, (4, – 1), 120]

(1) 1   (2) 3   (3) 5   (4) 7

Answers

EXERCISE - 1

PART- I

Section (A)

A-1.	(1)	**A-2.**	(4)	**A-3.**	(1)	**A-4.**	(2)	**A-5.**	(1)	**A-6.**	(4)	**A-7.**	(1)
A-8.	(3)	**A-9.**	(3)	**A-10.**	(1)	**A-11.**	(2)	**A-12.**	(2)	**A-13.**	(3)		

Section (B)

B-1.	(3)	**B-2.**	(1)	**B-3**	(1)	**B-4.**	(4)	**B-5**	(2)	**B-6.**	(1)	**B-7.**	(4)
B-8.	(3)	**B-9.**	(1)	**B-10.**	(4)								

Section (C)

C-1.	(1)	**C-2.**	(2)	**C-3.**	(1)	**C-4.**	(3)	**C-5.**	(2)	**C-6.**	(3)	**C-7**	(2)
C-8.	(3)	**C-9.**	(2)										

EXERCISE - 2

PART- I

1.	(2)	**2.**	(2)	**3.**	(1)	**4.**	(3)	**5.**	(2)	**6.**	(4)	**7.**	(1)
8.	(1)	**9.**	(2)	**10.**	(1)	**11.**	(1)	**12.**	(4)	**13.**	(1)	**14.**	(2)

PART- II

Section (A)

A-1.	**(1)**	**A-2.**	**(1)**

Section (B)

B-1   (A) → (q, r) ; (B) → (p, q, r, s) ; (C) → (s) ; (D) → (s)

B-2   A → r ; B → r ; C → p ; D → p

Section (C)

C-1.	(1,2,3,4)	**C-2**	(1,4)	**C-3.**	(2,3)

EXERCISE - 3

PART- I

1.	(1)	**2.**	(4)	**3.**	(3)	**4.**	(1)	**5.**	(3)	**6.**	(1)	**7.**	(4)
8.	(4)	**9.**	(2)	**10.**	(4)	**11.**	(1)	**12.**	(1)	**13.**	(1)	**14.**	(3)
15.	(1)	**16.**	(2)	**17.**	(1)	**18.**	(3)	**19.**	(4)				

Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

This Section is not meant for classroom discussion. It is being given to promote self-study and self testing amongst the Resonance students.

Max. Marks : 120   **Max. Time : 1 Hr.**

Important Instructions :

1. The test is of **1 hour** duration and max. marks 120.
2. The test consists **30** questions, **4 marks** each.
3. Only one choice is correct **1 mark** will be deducted for incorrect response. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instructions 3 above.

---

1. The mean weight of 150 person in a group is 60 kg. The mean weight of men in the group is 70 kg and that of women is 55kg. Find the number of men.
    (1) 50   (2) 75   (3) 100   (4) 25

2. Then mean of 11 observations is 25. If each observation is decreased by 5, the new mean will be
    (1) 25   (2) 30   (3) 20   (4) 15

3. The marks of some students were listed out of a maximum 60. The standard deviation of marks was found to be 5. Subsequently the marks raised to a maximum of 100 and variance of new marks was calculeted .The new variance

    (1) $\dfrac{25}{3}$ (2) $\dfrac{625}{3}$ (3) $\dfrac{625}{9}$ (4) $\dfrac{15}{9}$

4. Range of data 13, 14, 19, 21, 17, 14, 14, 12 is
    (1) 7   (2) 14   (3) 9   (4) 21

5. Variance of first 10 natural numbers is

    (1) $\dfrac{133}{4}$ (2) $\dfrac{33}{4}$ (3) 33   (4) $\dfrac{33}{2}$

6. Find the median of values 12, 17, 19, 8, 4, 23, 27
    (1) 27   (2) 23   (3) 17   (4) 18

7. Find the mode of the data 3, 1, 1, 2, 3, 0, –3, 4, 1 2, 3, 3, 5
    (1) 1   (2) 2   (3) 0   (4) 3

8. Coefficient of range 5, 2, 4, 3, 8, 11, is

    (1) $\dfrac{9}{13}$ (2) $\dfrac{1}{11}$ (3) $\dfrac{6}{10}$ (4) $\dfrac{6}{16}$

9. If difference between mean and mode is 3, the difference between mean and median is
    (1) 3   (2) 1   (3) 4   (4) 2

---

10 Mode of the data:

xi (variate)	frequency
$^4C_1$	$^8C_0$
$^4C_2$	$^8C_2$
$^4C_3$	$^8C_4$
$^4C_4$	$^8C_6$

(1) $^4C_2$ (2) $^8C_2$ (3) $^8C_4$ (4) $^4C_3$

11. If $\sum_{i=1}^{11}(x_i - 4) = 11$ and $\sum_{i=1}^{11}(x_i - 4)^2 = 44$ then find variance of $x_1, x_2, x_3 .....x_{11}$.

(1) 4   (2) 3   (3) 7   (4) 11

12 Mean of 1, 4, 7, 10, 13...........n terms is

(1) $(3n-1)n$ (2) $(3n-1)\dfrac{n}{2}$ (3) $\dfrac{3n-1}{2}$ (4) $(3n-1)$

13. If $var(x_i) = \lambda$ then $var(2x_i + 3)$ is

(1) $2\lambda + 3$ (2) $2\lambda^2$ (3) 4λ (4) $4\lambda + 9$

14. If S. D. of $x_1, x_2, x_3.....x_4$ is 3 then S. D of $-4x_1, -4x_2,..........-4x_n$ is

(1) −12   (2) 12   (3) −6   (4) 6

15. Consider the following statement and choose correct option
(I) variance can not be negative
(ii) S.D can not be negative
(III) Median is influenced by extreme value in set of numbers.
(1) TTT   (2) FTT   (3) FTF   (4) TTF

16. The mean and variance of 7 observation are 7 and $\dfrac{100}{7}$. If 5 of the observation are 2, 4, 7, 11, 10, find the remaining 2 observations.

(1) 3, 6   (2) 3, 12   (3) 4, 11   (4) 5, 10

17. The mean of distribution is 6, If coefficient of variation is 50%, then standard deviation of distribution is
(1) 9   (2) 3   (3) 300   (4) 4

18. The mean deviation about median of variation 53, 54, 55......100 is
(1) 11.5   (2) 12   (3) 12.5   (4) 13

19. The mean of two samples of sizes 20 and 10 were found to be 11, 8 respectively. Their variance were 4 and 34 respectively. Find the variance of combined sample of size 30.
(1) 19   (2) 19.5   (3) 18.5   (4) 16

20. If standard deviation of 1, 2, 3, 4, 5 is $\sqrt{2}$ then which of the following is correct.
(1) standard deviation of 1, 4, 9, 16, 25, is 2
(2) standard deviation of 1001, 1002, 1003, 1004, 1005 is $\sqrt{2000}$
(3) standard deviation of 1001, 1002, 1003, 1004, 1005 is $\sqrt{2}$
(4) standard deviation of 1, 8, 27, 16, 25 is $\sqrt{2}$

21. The mean and variance of 100 numbers were calculated as 11 and 2 respectively. Later it was found that one of the number was misread 5 instead of 9. How does the variance change.
(1) Variance doesn't change   (2) Variance Increases
(3) Variance decreases   (4) Can't comment

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22 The variance of first 5 even natural numbers is

(1) 6   (2) 7   (3) 8   (4) 9

23 If variance of x_1, x_2, x_3, x_4, x_5 is σ^2, find variance of $3x_1+4, 3x_2+4, 3x_3+4, 3x_4+4, 3x_5+4$

(1) $4\sigma^2 + 3$ (2) $4\sigma^2 + 9$ (3) $9\sigma^2$ (4) $4\sigma^2 - 3$

24. Find the mean deviation about median of 34, 38, 42, 55, 63, 46, 54, 44, 70, 48

(1) 8.2   (2) 8.4   (3) 8.6   (4) 8.8

25. Variance of first n natural numbers.

(1) $\dfrac{n^2-1}{24}$ (2) $\dfrac{n^2-1}{12}$ (3) $\dfrac{n^2-1}{6}$ (4) $\dfrac{n^2-1}{3}$

26. In a batch of 20 students 8 have failed. The marks of the successful candidates are 23, 27, 29, 18, 17, 19, 21, 27, 20, 24, 26, 28 the median marks are

(1) 22   (2) 18   (3) 18.5   (4) can't determine

27. The coefficient of variation of two series are 60% and 70% if their standard deviation are 21 and 14, then find ratio of their AMs

(1) $\dfrac{6}{7}$ (2) $\dfrac{2}{3}$ (3) $\dfrac{4}{7}$ (4) $\dfrac{7}{4}$

28. The mean of 2 samples of sizes 50 & 40 were found to be 63 and 54. Their variance were 81 & 36. Find the variance of combined sample of size 90

(1) 9   (2) 81   (3) 3   (4) 243

29. The mean and median of some data is 14 and 12. Later it was discovered that every data element should be increased by 2 units then new mean and median will be

(1) 16, 12   (2) 16, 14   (3) 14, 12   (4) 10, 8

30. Rohan worked for a firm as given below

No. of weeks	Days each week he worked
2 weeks	1 day each week
14 weeks	2 day each week
8 weeks	5 day each week
32 weeks	7 day each week

What is the mean number of days rohan works per week

(1) 5   (2) 6   (3) 5.5   (4) 5.25

Practice Test (JEE-Main Pattern)

OBJECTIVE RESPONSE SHEET (ORS)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

1. If a variate X is expressed as a linear function of two variates U and V in the form X = a U + b V, then mean $\overline{X}$ of X is

(1) $a\overline{U} + b\overline{V}$ (2) $\overline{U} + \overline{V}$ (3) $a\overline{U} + a\overline{U}$ (4) None of these

2. The AM of n numbers of a series is $\overline{X}$. If the sum of first (n – 1) terms is k, then the nth number is

(1) $\overline{X} - k$ (2) $n\overline{X} - k$ (3) $\overline{X} - nk$ (4) $\dfrac{\overline{X}}{3}$

3. If $\overline{X}_1$ and $\overline{X}_2$ are the means of two distributions such that $\overline{X}_1 < \overline{X}_2$ and $\overline{X}$ is the mean of the combined distribution, then

(1) $\overline{X} < \overline{X}_1$ (2) $\overline{X} > \overline{X}_2$ (3) $\overline{X} = \dfrac{\overline{X}_1 + \overline{X}_2}{2}$ (4) $\overline{X}_1 < \overline{X} < \overline{X}_2$

4. Coefficient of variation of two distribution are 50% and 60% and their arithmetic means are 30 and 25 respectively. Difference of their standard deviations is

(1) 0   (2) 1   (3) 1.5   (4) 2.5

APSP Answers

PART- I

1.	(1)	**2.**	(3)	**3..**	(3)	**4.**	(3)	**5.**	(2)	**6.**	(3)	**7.**	(4)
8.	(1)	**9.**	(2)	**10**	(4)	**11.**	(2)	**12**	(3)	**13.**	(3)	**14.**	(2)
15.	(4)	**16.**	(2)	**17.**	(2)	**18.**	(2)	**19.**	(4)	**20.**	(3)	**21.**	(3)
22	(3)	**23**	(3)	**24.**	(3)	**25.**	(2)	**26.**	(3)	**27.**	(4)	**28.**	(2)
29.	(2)	**30.**	(4)										

PART- II

1.	(1)	**2.**	(2)	**3.**	(4)	**4.**	(1)

Exercise-1

✎ **Marked questions are recommended for Revision**
✎ **चिन्हित प्रश्न दोहराने योग्य प्रश्न है।**

OBJECTIVE QUESTIONS (SINGLE CHOICE CORRECT)
वस्तुनिष्ठ प्रश्न (OBJECTIVE QUESTIONS)(SINGLE CHOICE CORRECT)

Section (A) : Mean, median & mode
खण्ड (A) : माध्य, माध्यिका और बहुलक

A-1. Find the A.M. of the series 1, 2, 4, 8, 16, ..., 2^n
श्रेणी 1, 2, 4, 8, 16, ..., 2^n का समान्तर माध्य ज्ञात कीजिए।

(1*) $\dfrac{2^{n+1}-1}{n+1}$ (2) $\dfrac{2^{n+2}-1}{n}$ (3) $\dfrac{2^n-1}{n+1}$ (4) $\dfrac{2^n-1}{n}$

Sol. $\dfrac{1+2+4.....2^n}{n+1}$ = $.\ \dfrac{1}{n+1}$

A-2. The average weight of 9 men is x kg. After another men joins the group, the average increases by 5%. Still another man joins and average returns to old level of x kg. Which one of the following true?

(SCRA-2013)
(1) the 10th & 11th men weight same
(2) the 10th man weight half as much as the 11th man
(3) the 10th man weight as much as the 11th man
(4*) None of these

9 व्यक्तियों का औसत वजन x कि.ग्रा में है। समूह में कुछ दूसरे व्यक्तियों को जोड़ने के बाद औसत 5% बढ़ जाता है। फिर भी दूसरे व्यक्तियों को जोड़ने पर भी औसत पुराने स्तर x कि.ग्रा पर ही रहता है। निम्न में से कौनसा सत्य है ?
(SCRA-2013)
(1) 10वें और 11वें व्यक्ति का वजन समान है।
(2) 10वें व्यक्ति का वजन 11वें व्यक्ति के वजन से आधा है।
(3) 10वें व्यक्ति का वजन 11वें व्यक्ति के वजन से अधिक है।
(4*) इनमें से कोई नहीं

Sol. $\dfrac{w_1 + w_2 + ... + w_9}{9} = x$ ...(1)

$\dfrac{(w_1 + w_2 + ... + w_9) + w_{10}}{10} = x + \dfrac{1}{20}\ x$ ....(2)

From equation समीकरण (1) & व (2) से

$w_{10} = \dfrac{3}{2}\ x$

Nowअब $\dfrac{(w_1 + w_2 + ... + w_9) + w_{10} + w_{11}}{11} = x$ ....(3)

From equation समीकरण (1), (2) and व (3) से

$w_{11} = \dfrac{x}{2}$

A-3. N observations on a variable x are $x_i = A + iB$ for i = 1, 2, 3, ..., n where A, B are real constants. The mean of the observation is

चर x पर N प्रेक्षण $x_i = A + iB$, i = 1, 2, 3, ..., n के लिए जहाँ A, B वास्तविक अचर है, है। प्रेक्षणों का माध्य है

(1*) $A + B\dfrac{(n+1)}{2}$　　(2) $nA + B\dfrac{(n+1)}{2}$　　(3) $A + Bn\dfrac{(n+1)}{2}$　　(4) $A + B\left(\dfrac{n}{2}\right)$

Sol.　Mean माध्य $= \dfrac{\sum\limits_{i=1}^{n} A + iB}{n} = \dfrac{A.n + B\dfrac{(n(n+1))}{2}}{n}$

Mean माध्य $= A + B\dfrac{(n+1)}{2}$

A-4.　Find the median of values 10, 14, 11, 9, 8, 12, 6. **RFT-8**
10, 14, 11, 9, 8, 12, 6 की माध्यिका ज्ञात कीजिए।
(1) 8　　　　　(2*) 10　　　　　(3) 9　　　　　(4) 11
Sol.　6, 8, 9, 10, 11, 12, 14
median (माध्यिका) = 10

A-5.　If a variable takes the discrete values $\alpha + 4$, $\alpha - \dfrac{7}{2}$, $\alpha - \dfrac{5}{2}$, $\alpha - 3$, $\alpha - 2$, $\alpha + \dfrac{1}{2}$, $\alpha - \dfrac{1}{2}$, $\alpha + 5$ ($\alpha > 0$), then the median is**RFT-8**

यदि एक चर के भिन्न मान $\alpha + 4$, $\alpha - \dfrac{7}{2}$, $\alpha - \dfrac{5}{2}$, $\alpha - 3$, $\alpha - 2$, $\alpha + \dfrac{1}{2}$, $\alpha - \dfrac{1}{2}$, $\alpha + 5$ ($\alpha > 0$), है, तब माध्यिका है

(1*) $\alpha - \dfrac{5}{4}$　　　(2) $\alpha - \dfrac{1}{2}$　　　(3) $\alpha - 2$　　　(4) $\alpha + \dfrac{5}{4}$

Sol.　$\alpha - \dfrac{7}{2}$, $\alpha - 3$, $\alpha - \dfrac{5}{2}$, $\alpha - 2$, $\alpha - \dfrac{1}{2}$, $\alpha + \dfrac{1}{2}$, $\alpha + 4$, $\alpha + 5$ ($\alpha > 0$)

median माध्यिका $= \dfrac{\alpha - 2 + \alpha - \dfrac{1}{2}}{2} = \alpha - \dfrac{5}{4}$

A-6.　Find the mode of data 1, 2, 5, 3, 2, 3, 0, 5, 2.　**RFT-8**　　**[Revision Plannar_15]**
आँकड़ो 1, 2, 5, 3, 2, 3, 0, 5, 2 के लिये बहुलक है–
(1) 3　　　　　(2) 1　　　　　(3) 5　　　　　(4*) 2
Sol.　Number 2 repeats maximum number of times
Hindi　संख्या 2 की पुनरावृत्ति सबसे अधिक बार है।

A-7. Consider the following statements related to measure of central tendency of 50 positive numbers

　　　　(SCRA-2014)
1. Median is not influenced by extreme values in set of numbers
2. The harmonic mean is unreliable if one or more of the numbers is non-zero
which of the above statement is/are correct
(1*) 1 only　　　　(B) 2 only　　　　(3) both 1 and 2　　　(4) neither 1 nor 2
माना निम्न कथन 50 धनात्मक संख्याओं के केन्द्रीय प्रवृतियों के माप से सम्बन्धित है　　　**(SCRA-2014)**
1. संख्याओं के समुच्चय में माध्यिका चरम मानों से प्रभावित नहीं होती है।
2. यदि एक या अधिक संख्याऐं अशून्य है तब हरात्मक माध्य के बारे में कुछ नहीं कह सकते।
उपरोक्त कथन में से कौनसा सही है
(1*) केवल 1　　　　(B) केवल 2　　　　(3) 1 और 2 दोनों　　　(4) ना तो 1 और ना ही 2

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MAINSTTS-2

Sol. Obvious

A-8. $M(x_1, x_2, x_3 .... x_n)$ defines a measure of central tendency based on n values $x_1, x_2, x_3, ...x_n$ consider the following measured of central tendency **(SCRA-2010)**
(1) Arithmetic mean
(2) Median
(3) Geometric mean
which of the above measure satisfie/ satisfies the property

$$\frac{M(x_1, x_2, x_3............x_n)}{M(y_1, y_2, y_3............y_n)} = M. \left(\frac{x_1}{y_1}, \frac{x_2}{y_2}, \frac{x_3}{y_3}, ..., \frac{x_n}{y_n} \right)?$$

select the correct answer using the code below
(1) 1 only   (2) 2 only   (3*) 3 only   (4) 1 and 3

माना $M(x_1, x_2, x_3 .... x_n)$, क्रमशः $x_1, x_2, x_3, ...x_n$, n मानों के आधार पर केन्द्रीय प्रवृत्तियों के माप को परिभाषित करता है माना कि निम्न में से किसका केन्द्रीय प्रवृति का माप किया जाता है–
       (SCRA-2010)
(1) समान्तर माध्य
(2) माध्यिका
(3) गुणोत्तर माध्य
निम्न में से किस गुणधर्म को सन्तुष्ट करता है

$$\frac{M(x_1, x_2, x_3............x_n)}{M(y_1, y_2, y_3............y_n)} = M. \left(\frac{x_1}{y_1}, \frac{x_2}{y_2}, \frac{x_3}{y_3}, ..., \frac{x_n}{y_n} \right)?$$

नीचे दिये गये कोड के आधार पर सही उत्तर का चयन कीजिए
(1) केवल 1   (2) केवल 2   (3*) केवल 3   (4) 1 और 3

Sol. Geometric mean गुणोत्तर माध्य

$$\frac{M(x_1, x_2, x_3.....x_n)}{M(y_1, y_2, y_3.....y_n)} \equiv \frac{(x_1.x_2.x_3......x_n)^{1/n}}{(y_1.y_2.y_3......y_n)^{1/n}} = \left(\frac{x_1}{y_1}.\frac{x_2}{y_2}.....\frac{x_n}{y_n} \right)^{\frac{1}{n}} = M\left(\frac{x_1}{y_1}, \frac{x_2}{y_2}, \frac{x_3}{y_3}, ..., \frac{x_n}{y_n} \right)$$

$\Rightarrow$ While in case of 1 & 2 it is not true always
       जबकि 1 ओर 2 स्थिति में ये सदैव सत्य नहीं है।

A-9. If values a, b, c,...... j, p occurs with frequencies $^{10}C_0$, $^{10}C_1$, $^{10}C_2$,..., $^{10}C_{10}$ then mode is
यदि a, b, c,...... j, p की आवृत्तियाँ क्रमशः $^{10}C_0$, $^{10}C_1$, $^{10}C_2$,..., $^{10}C_{10}$ हैं तो बहुलक है– **[Revision Plannar_15]**
(1) a   (2) e   (3*) f   (4) k

Sol. Frequency of f = $^{10}C_5$ which has maximum value
f की बारम्बारता = $^{10}C_5$ जिसका मान अधिकतम है।

A-10. The mean of 21 observations (all different) is 40. If each observations greater than the median are increased 21, then mean of observations will become **RFT-8**   **[Revision Plannar_15]**
21 प्रेक्षणों (सभी विभिन्न) का माध्य 40 है। यदि प्रत्येक प्रेक्षण जो माध्यिका से बडे है उनको 21 से बढाया जाता है, तब नये प्रेक्षणों का माध्य होगा।
(1*) 50   (2) 50.5   (3) 30   (4) 45

Sol. Mean of 21 observation $\bar{x}$ = 40, so
Sum of numbers = 21 × 40 = 840
$\Rightarrow$ As numbers greater then median increased by 21, so 10 observations will increase by 21.
Now sum of all observations = 840 + 10×(21) = 1050
$\Rightarrow$ So now new mean is = $\frac{1050}{21}$ = 50

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MAINSTTS-4

Hindi 21 प्रेक्षणों का माध्य $\overline{x}$ = 40, इसलिए

संख्याओं का योगफल = 21 × 40 = 840

⇒ माध्यिका से बड़ी संख्याओं को 21 से बढ़ाया जाता है इसलिए 10 प्रेक्षण 21 से बढेगें

अब सभी प्रेक्षणों का योगफल = 840 + 10×(21) = 1050

⇒ इसलिए अब नया माध्य = $\dfrac{1050}{21}$ = 50 है।

A-11. The mean of series $x_1, x_2, x_3, \ldots x_n$ is $\overline{x}$, then mean of the series $x_i + 2i$, i = 1, 2, 3, . . . , n will be

श्रेणी $x_1, x_2, x_3, \ldots x_n$ का माध्य $\overline{x}$ है तब श्रेणी $x_i + 2i$, i = 1, 2, 3, . . . , n का माध्य होगा– **RFT-8**

(1) $\overline{x} + n$ (2*) $\overline{x} + n + 1$ (3) $\overline{x} + 2$ (4) $\overline{x} + 2n$

Sol. $x_1 + x_2 \ldots\ldots x_n = n\,\overline{x}$

so अतः

$$\sum_{i=1}^{n}(x_i + 2i) = \sum x_i + 2\sum i = n\,\overline{x} + \frac{2n\ (n+1)}{2}$$ so Mean अतः माध्य = $\overline{x} + (n+1)$

A-12. If the difference between mean and mode is 63, the difference between mean and median is **RFT-8**

यदि माध्य एवं बहुलक के मध्य अन्तर 63 हो, तो माध्य एवं माध्यिका के मध्य अन्तर होगा–

(1) 189 (2*) 21 (3) 31.5 (4) 48.5

Sol. Mean – Mode = 63

As mode = 3 median – 2 mean

⇒ mean – 63 = 3 median – 2 mean

⇒ mean – median = 21

माध्य – बहुलक = 63

चूंकि बहुलक = 3 माध्यिका – 2 माध्य

⇒ माध्य – 63 = 3 माध्यिका – 2 माध्य

⇒ माध्य – माध्यिका = 21

A-13. Mean of variates 1.2.3, 2.3.4, ..., n(n + 1)(n + 2) **RFT-8** **[Revision Plannar_15]**

चरों 1.2.3, 2.3.4, ..., n(n + 1)(n + 2) का माध्य है–

(1) $\dfrac{n(n+1)(n+2)}{4}$ (2) $\dfrac{(n+1)(n+2)(n+3)}{2}$ (3*) $\dfrac{(n+1)(n+2)(n+3)}{4}$ (4) $\dfrac{n(n+1)(n+3)}{2}$

Sol. Mean of variate 1.2.3, 2.3.4, 3.4.5, ..., n.(n + 1).(n + 2)

चरों 1.2.3, 2.3.4, 3.4.5, ..., n.(n + 1).(n + 2) का माध्य

For sum of series श्रेणी के योगफल के लिए

$$T_r = \frac{r(r+1)(r+2)}{4}\,[(r+3)-(r-1)] \quad\Rightarrow\quad S = \sum_{r=1}^{n}T_r = \frac{n(n+1)(n+2)(n+3)}{4}$$

Mean माध्य = $\dfrac{s}{n} = \dfrac{(n+1)(n+2)(n+3)}{4}$

Section (B) : Range, coefficient of range, mean deviation and coefficient of mean deviaiton.

B-1. Range of data 7, 8, 2, 1, 3, 13, 18 is

आंकड़ों 7, 8, 2, 1, 3, 13, 18 का परिसर है–

(1) 10 (2) 15 (3*) 17 (4) 11

Sol. Range of data = difference of extreme values = 18-1 = 17

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MAINSTTS-5

आंकड़ों का परिसर = सिरों के मानों का अन्तर = 18-1 = 17

B-2. Coefficient of range 5, 2, 3, 4, 6, 8, 10 is
5, 2, 3, 4, 6, 8, 10 के परिसर का गुणांक है–

(1*) $\dfrac{2}{3}$ (2) $\dfrac{1}{3}$ (3) $\dfrac{3}{5}$ (4) $\dfrac{1}{2}$

Sol. Coefficient of range = $\dfrac{\text{difference of extreme values}}{\text{sum of extreme values}} = \dfrac{10-2}{10+2} = \dfrac{2}{3}$

परिसर का गुणांक = $\dfrac{\text{सिरों के मानों का अन्तर}}{\text{सिरों के मानों का योगफल}} = \dfrac{10-2}{10+2} = \dfrac{2}{3}$

B-3 The scores of a batsman in ten innings are : 38, 70, 48, 34, 42, 55, 63, 46, 54, 44. Find the mean deviation about median.
एक बल्लेबाज का 10 पारियों में स्कोर है : 38, 70, 48, 34, 42, 55, 63, 46, 54, 44. माध्यिका से माध्य विचलन ज्ञात कीजिए। **RFT-8**

(1*) $\dfrac{43}{5}$ (2) $\dfrac{44}{5}$ (3) $\dfrac{41}{5}$ (4) $\dfrac{42}{5}$

Sol. 34, 38, 42, 44, 46, 48, 54, 55, 63, 70

median (माध्यिका) = $\dfrac{46+48}{2}$ = 47

$\sum |x_i - M|$ = 13 + 9 + 5 + 3 + 1 + 1 + 7 + 8 + 16 + 23 = 86

so mean diviation about median = $\dfrac{86}{10}$ = 8.6

Hindi. 34, 38, 42, 44, 46, 48, 54, 55, 63, 70

माध्यिका = $\dfrac{46+48}{2}$ = 47

$\sum |x_i - M|$ = 13 + 9 + 5 + 3 + 1 + 1 + 7 + 8 + 16 + 23 = 86

माध्यिका से माध्य विचलन = $\dfrac{86}{10}$ = 8.6

B-4. The mean deviation about median of variates 13, 14, 15, ... , 99, 100 is **RFT-8**
चरों 13, 14, 15, ... , 99, 100 का माध्यिका के सापेक्ष माध्य विचलन होगा–
(1) 1936 (2) 21.5 (3) 23.5 (4*) 22

Sol. n = 88

Median = $\dfrac{44^{th}\,\text{value} + 45^{th}\,\text{value}}{2} = \dfrac{56+57}{2} = 56.5$

माध्यिका = $\dfrac{44^{th}\,\text{value} + 45^{th}\,\text{value}}{2} = \dfrac{56+57}{2} = 56.5$

M.D.(median) = माध्यिका के सापेक्ष माध्य विचलन =

$\dfrac{\sum\limits_{i=1}^{88}|x_i - 56.5|}{88} = \dfrac{43.5 + 42.5 + ... + 0.5 + 0.5 + ... + 43.5}{88}$

$$= \frac{1+3+5+\ .\ .\ .\ +85+87}{88} = 22$$

B-5_ The mean deviation of an ungrouped data is 50. If each observation is increased by 2%, then the new mean deviation is **RFT-8**

अवर्गीकृत समूह के आकड़ों का माध्य विचलन 50 है। यदि प्रत्येक प्रेक्षण को 2% से बढ़ाया जाता है। तब नया माध्य विलयन होगा

(1) 50   (2*) 51   (3) 49   (4) 50.5

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MAINSTTS-7

Sol. If $x_1 ..... x_n$ are the observations then the new observations are $(1.02) x_1, (1.02)x_n$

यदि $x_1 ..... x_n$ प्रेक्षण है तब नये प्रेक्षण $(1.02) x_1, (1.02)x_n$ है।

Therefore the new mean इसलिए नया माध्य $= (1.02) \ \bar{x}$

New mean deviation नया माध्य विचलन $= \dfrac{1}{n}\sum |(1.02)x_i - (1.02)\bar{x}|$

$$= (1.02) \ \dfrac{1}{n}\sum |x_i - \bar{x}|$$
$$= (1.02) \times 50 = 51$$

B-6_. The mean deviation of an ungrouped data is 80. If each observation is decreased by 5%, then the new mean deviation is

अवर्गीकृत समूह के आकड़ों का माध्य विचलन 80 है। यदि प्रत्येक प्रेक्षण को 5% से घटया जाता है। तब नया माध्य विचलन है।

(1*) 76   (2) 77   (3) 78   (4) 79

Sol. If $x_1 ..... x_n$ are the observations then the new observations are $(0.95) x_1, (0.95) x_n$

यदि $x_1 ..... x_n$ प्रेक्षण है तब नये प्रेक्षण $(0.95) x_1, (0.95) x_n$ है।

Therefore the new mean इसलिए नया माध्य $= (0.95) \ \bar{x}$

New mean deviation नया माध्य विचलन $= \dfrac{1}{n}\sum |(0.95)x_i - (0.95)\bar{x}|$

$$= (0.95) \ \dfrac{1}{n}\sum |x_i - \bar{x}|$$
$$= (0.95) \times 80 = 76$$

B-7._ The mean deviation from mean of the observations a, a + d, a + 2d, ... a + 2nd is

प्रेक्षणों a, a + d, a + 2d, ... a + 2nd का माध्य से लिया गया माध्य विचलन है –

(1) $\dfrac{n(n+1)d^2}{3}$ (2) $\dfrac{n(n+1)}{2}d^2$

(3) $a + \dfrac{n(n+1)d^2}{2}$ (4*) $\dfrac{n(n+1)|d|}{(2n+1)}$

Sol. $\bar{x} = \dfrac{1}{2n+1}$ [a + (a + d) + ... + (a + 2nd)]

$= \dfrac{1}{2n+1}$ [(2n + 1) a + d (1 + 2 + ... + 2n)]

$= a + d \ \dfrac{2n}{2} \ \dfrac{(1+2n)}{2n+1} = a + nd$

M.D. from mean माध्य से माध्य विचलन $= \dfrac{1}{2n+1}$ 2|d| (1 + 2 + ... + n) $= \dfrac{n(n+1)|d|}{(2n+1)}$

B-8._ Mean deviation from the mean for the observations – 1, 0, 4 is **RFT-8**

प्रेक्षणों – 1, 0, 4 के लिए माध्य से माध्य विचलन है –

(1) $\sqrt{\dfrac{14}{3}}$ (2) $\dfrac{2}{3}$ (3*) 2   (4) 3

Sol. Mean माध्य $= \dfrac{-1+0+4}{3} = 1$

Hence M.D. (about mean) अतः माध्य से लिया गया माध्य विचलन

$$\frac{|-1-1|+|0-1|+|4-1|}{3} = 2$$

B-9_. If $\overline{X}$ is the mean of $x_1, x_2, x_3 ...... x_n$. Then the algebraic sum of the deviations about mean $\overline{X}$ is **RFT-8**

यदि $\overline{X}$ आंकड़ों $x_1, x_2, x_3 ...... x_n$. का माध्य है तब माध्य $\overline{X}$ के सापेक्ष विचलनों का बीजीय योग है –

(1*) 0   (2) $\dfrac{\overline{X}}{n}$ (3) $n\,\overline{X}$ (4) $(n–1)\,\overline{X}$

Sol. See properties of AM

समान्तरमाध्य के गुणधर्म से

B-10_. If the algebraic sum of deviations of 20 observation from 30 is 20, then the mean of the observation is

यदि 30 से 20 प्रेक्षणों के विचलनों का बीजीय योग 20 है। तब प्रेक्षण का माध्य है –

(1) 30   (2) 30.1   (3) 29   (4*) 31.

Sol. $\displaystyle\sum_{i=1}^{20}(x_i - 30) = 20 \Rightarrow \sum_{i=1}^{20} x_i - 20 \times 30 = 20$

$\displaystyle\sum_{i=1}^{20} x_i = 600 + 20 = 620$

Mean माध्य $= \dfrac{620}{20} = 31$ Ans.

Section (C) : Variance, Standard deviation and coefficient of variation.

खण्ड प्रसरण, मानक विचलन और चरिता गुणांक

C-1. Variance of first 20 natural number is

प्रथम 20 प्राकृत संख्या का प्रसरण है–

(1*) $\dfrac{133}{4}$ (2) $\dfrac{379}{12}$ (3) $\dfrac{133}{2}$ (4) $\dfrac{399}{4}$

Sol. $\overline{x} = \dfrac{\displaystyle\sum_{i=1}^{20} n}{20} = \dfrac{21}{2}$, $\dfrac{\displaystyle\sum_{i=1}^{20} n^2}{20} = \dfrac{287}{2}$ $\Rightarrow$ $\sigma^2 = \dfrac{\displaystyle\sum_{i=1}^{20} n^2}{20} - \left(\dfrac{\displaystyle\sum_{i=1}^{20} n}{20}\right)^2 = \dfrac{287}{2} - \left(\dfrac{21}{2}\right)^2 = \dfrac{133}{4}$

C-2. The mean & variance of 7 observations are 8, 16 respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the LCM of remaining two observations is   **[Revision Plannar_15]**

7 प्रेक्षणों का माध्य और प्रसरण क्रमशः 8, 16 है। यदि 5 प्रेक्षण 2, 4, 10, 12, 14 हो, तो शेष दो प्रेक्षणों का ल.स.प. (LCM) है–

(1) 16   (2*) 24   (3) 20   (4) 14

Sol. Let two observations are x and y   (माना दो प्रेक्षण x और y है)

then (तब) $\dfrac{x + y + 2 + 4 + 10 + 12 + 14}{7} = 8$

$x + y + 42 = 56 \Rightarrow x + y = 14$ ...(A)

and (और) $\dfrac{x^2 + y^2 + 4 + 16 + 100 + 144 + 196}{7} - \dfrac{(x + y + 42)^2}{49} = 16$

$\Rightarrow \dfrac{x^2 + y^2 + 460}{7} = 16 + 64 = 80$

$\Rightarrow x^2 + y^2 = 560 - 460 = 100$ ...(B)

$\therefore$ on solving (A) & (B) we get x = 6, y = 8 ((A) और (B) को हल करने पर x = 6, y = 8)

C-3. If n = 10, $\bar{x} = \sqrt{12}$, $\sum x^2 = 1560$, then standard deviation σ is   **[Revision Plannar_15]**

यदि n = 10, $\bar{x} = \sqrt{12}$, $\sum x^2 = 1560$ तब मानक विचलन σ है –

(1*) 12   (2) 13   (3) $\sqrt{166}$ (4) $\sqrt{12}$

Sol. $\sigma^2 = \dfrac{\sum x^2}{n} - (\bar{x})^2$

$\sigma^2 = \dfrac{1560}{10} - (\sqrt{12})^2 = 144 \quad \Rightarrow \quad$ S.D. $= \sqrt{\sigma^2} = 12$

C-4. The mean of distribution is 4 if coefficient of variation is 58%. Then standard deviation of distribution is

वितरण का माध्य 4 है। यदि चरिता गुणांक 58% है तब वितरण का मानक विचलन है–

(1) 2.23   (2) 3.23   (3*) 2.32   (4) 2.75

Sol. Coefficient of variation प्रसरण का गुणांक $= 0.58 = \dfrac{\sigma}{\bar{x}}$

σ(S.D.) = .58 × 4 = 2.32

C-5. The sum of squares of deviations for 10 observations taken from mean 50 is 250. The co-efficient of variation is

यदि 10 प्रेक्षणों का उनके माध्य 50 से विचलनों के वर्गों का योग 250 है तब चरिता गुणांक होगा–

(1) 50%   (2*) 10%   (3) 40%   (4) 30%

Sol. $\displaystyle\sum_{i=1}^{10}(x_i - 50)^2 = 250$

$\sigma = \sqrt{\dfrac{\displaystyle\sum_{i=1}^{10}(x_i - 50)^2}{10}} = 5$

coeff. of variation $= \dfrac{\sigma}{\bar{x}} \times 100 = 10\%$

Hindi. $\displaystyle\sum_{i=1}^{10}(x_i - 50)^2 = 250$

$\sigma = \sqrt{\dfrac{\displaystyle\sum_{i=1}^{10}(x_i - 50)^2}{10}} = 5$

चरिता गुणांक $= \dfrac{\sigma}{\bar{x}} \times 100 = 10\%$

C-6. Standard deviation is independent of   **(SCRA-2013)**

(1) change of scale & origin   (2) change of scale but not origin

(3*) change of origin but not scale   (4) neither change of scale non origin

मानक विचलन किससे स्वतंत्र है   **(SCRA-2013)**

(1) मूल बिन्दु और पैमाने के परिवर्तन से   (2) पैमाने के परिवर्तन से परन्तु मूल बिन्दु से नहीं

(3*) मूल बिन्दु के परिवर्तन से परन्तु पैमाने से नहीं   (4) ना तो मूल बिन्दु और ना ही पैमाने के परिवर्तन से

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MAINSTTS-10

Sol. Obvious

C-7 What is standard deviation of the set of observations 32, 28, 29, 30, 31?

प्रेक्षणों 32, 28, 29, 30, 31 के समुच्चय का मानक विचलन है ?

(1) 1.6　　　　　(2*) $\sqrt{2}$　　　　　(3) 2　　　　　(4) None of these

Sol. 28, 29, 30, 31, 32

Mean माध्य = 30

$\Rightarrow$　　variance प्रसरण $= \sigma^2 = \dfrac{(30-28)^2 + (30-29)^2 + (30-30)^2 + (30-31)^2 + (30-32)^2}{5}$

$\sigma^2 = 2$　　　$\Rightarrow$　　S.D. $= + \sqrt{\sigma^2} = \sqrt{2}$

C-8._ If the standard deviation of x_1, x_2, ... x_n is 3.5, then the standard deviation of $-2x_1 - 3$, $-2x_2 - 3$, ...$-2x_n - 3$ is

यदि x_1, x_2, ... x_n का मानक विचलन 3.5, है तब $-2x_1 - 3$, $-2x_2 - 3$, ...$-2x_n - 3$ का मानक विचलन है –

(1) – 7　　　　　(2) – 4　　　　　(3*) 7　　　　　(4) 1.75

Sol. We known that if हम जानते है कि यदि $d_1 = \dfrac{x_i + 3/2}{-1/2}$ so इसलिए $h = -\dfrac{1}{2}$ .Thus अतः $\sigma_d = \dfrac{1}{|h|} \sigma_x = 2 \times 3.5 = 7$

C-9._ The marks of some students were listed out of a maximum 100. The standard deviation of marks was found to be 9. Subsequently the marks raised to a maximum of 150 and standard deviation of new marks was calculeted. The new standard deviation

कुछ विद्यार्थियों के अंक, अधिकतम अंक 60 में से दिए गए है। अंको का मानक विचलन 5 पाया गया है। अंको को अधिकतम 100 तक बढ़ाया जाता है तथा नये अंक का प्रसरण ज्ञात किया जाता है। तब नया प्रसरण है –

(1) 9　　　　　(2*) 13.5　　　　　(3) –13.5　　　　　(4) –9

Sol. We known that $y_i = \dfrac{3}{2} x_i$ so $h = \dfrac{3}{2}$ Thus $\sigma_y = |h| \sigma_x = 1.5 \times 9 = 13.5$

हम जानते है कि $y_i = \dfrac{3}{2} x_i$ इसलिए $h = \dfrac{3}{2}$ इसलिए $\sigma_y = |h| \sigma_x = 1.5 \times 9 = 13.5$

Exercise-2

✍ **Marked questions are recommended for Revision**

✍ **चिन्हित प्रश्न दोहराने योग्य प्रश्न है।**

PART - I : OBJECTIVE QUESTIONS (SINGLE CHOICE CORRECT)

भाग - I : वस्तुनिष्ठ प्रश्न (OBJECTIVE QUESTIONS) (SINGLE CHOICE CORRECT)

1. Find the arithmetic mean of $^{2n+1}C_0$, $^{2n+1}C_1$, ...$^{2n+1}C_n$.

$^{2n+1}C_0$, $^{2n+1}C_1$, ...$^{2n+1}C_n$ का समांतर माध्य ज्ञात कीजिए।

(1) $\dfrac{2^{2n+2}}{n}$　　　　　(2*) $\dfrac{2^{2n}}{n+1}$

(3) $\dfrac{2^{n+1}}{n+1}$　　　　　(4) None of these इनमें से कोई नहीं।

Sol. $\dfrac{^{2n+1}C_0 +...... {}^{2n+1}C_n}{n+1} = \dfrac{2^{2n}}{n+1}$

2. A variable takes the values of 0, 1, 2, ... , n with frequencies proportional to the binomial coefficients
${}^nC_0, {}^nC_1, {}^nC_2, ... , {}^nC_n$, then mean of the distribution is $\qquad$ **[Revision Plannar_15]**

एक चर 0, 1, 2, ... , n तक बारम्बारताऐं लेता है जिनके समानुपाती द्विपद गुणांक ${}^nC_0, {}^nC_1, {}^nC_2, ... , {}^nC_n$ है तब वितरण का माध्य है

(1) $\dfrac{n(n+1)}{4}$ (2*) $\dfrac{n}{2}$ (3) $\dfrac{n(n-1)}{2}$ (4) $\dfrac{n(n+1)}{2}$

Sol. $\bar{x} = \dfrac{0 \cdot {}^nC_0 + 1 \cdot {}^nC_1 + 2 {}^nC_2 +....... + n {}^nC_n}{{}^nC_0 + {}^nC_1 + {}^nC_2 +.....+ {}^nC_n} = \dfrac{n.2^{n-1}}{2^n} = \dfrac{n}{2}$

3. Following is the record of goals scored by team A in football session
Numbers of goals scored $\qquad$ 0 $\qquad$ 1 $\qquad$ 2 $\qquad$ 3 $\qquad$ 4
Numbers of match is $\qquad$ 1 $\qquad$ 9 $\qquad$ 7 $\qquad$ 5 $\qquad$ 3
for team 'B' mean number of goals scored per match was 2 goals with standard deviation 1.25 . The team which is more consistant

फुटबाल में टीम A के द्वारा अर्जित किये गये गोलों को दर्शाया गया है।
प्राप्त किये गये गोलों की संख्या $\qquad$ 0 $\qquad$ 1 $\qquad$ 2 $\qquad$ 3 $\qquad$ 4
खेलों की संख्या $\qquad$ 1 $\qquad$ 9 $\qquad$ 7 $\qquad$ 5 $\qquad$ 3

टीम 'B' के लिए प्रत्येक खेल में प्राप्त किये गये गोलों का माध्य 2 गोल है तथा मानक विचलन 1.25 तब कौनसी टीम अधिक उपयोगी है

(1*) A $\qquad$ (2) B $\qquad$ (3) A तथा B दोनों समान है (4) ना ही A और ना ही B

Sol.

x_i	f_i	$f_i x_i$	$f_i x_i^2$
0	1	0	0
1	9	9	9
2	7	14	28
3	5	15	45
4	3	12	48
	$\sum f_i = 25$	$\sum f_i x_i = 50$	$\sum f_i x_i^2 = 130$

$\sigma^2 = \left[\dfrac{\sum f_i x_i^2}{\sum f_i} - \left(\dfrac{\sum f_i x_i}{\sum f_i} \right)^2 \right]$

$= \left[\dfrac{130}{25} - \left(\dfrac{50}{25} \right)^2 \right] = 1.2$

so variance of A = 1.2 < 1.25 = variance of B
so more consistent team = A

x_i	f_i	$f_i x_i$	$f_i x_i^2$
0	1	0	0
1	9	9	9
2	7	14	28
3	5	15	45
4	3	12	48
	$\sum f_i = 25$	$\sum f_i x_i = 50$	$\sum f_i x_i^2 = 130$

Hindi.

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$$\sigma^2 = \left[\frac{\sum f_i \ x_i^2}{\sum f_i} - \left(\frac{\sum f_i \ x_i}{\sum f_i} \right)^2 \right]$$

$$= \left[\frac{130}{25} - \left(\frac{50}{25} \right)^2 \right] = 1.2$$

चरिता A = 1.2 < 1.25 = B की चरिता

इसलिए स्थायी टीम = A

4. The mean of two samples of sizes 200 and 300 were found to be 25, 10 respectively. Their standarddeviations were 3 and 4 respectively. Find the variance of combined sample of size 500

200 एवं 300 आकार वाले दो सेम्पलों के माध्य क्रमशः 25 एवं 10 है। उनके मानक विचलन क्रमशः 3 एवं 4 है। 500 आकार वाले संयुक्त सेम्पल का प्रसरण का मान ज्ञात कीजिए। **[Revision Plannar_15]**

(1) 70   (2) 60   (3*) 67.2   (4) 80

Sol. $\dfrac{\Sigma x_i}{200} = 25, \qquad \dfrac{\Sigma y_i}{300} = 10$

$\Rightarrow \qquad \Sigma x_i = 5000, \qquad \Sigma y_i = 3000$

$\qquad\qquad \sigma_x = 3 \quad$ and $\quad \sigma_y = 4$

$\Rightarrow \qquad \dfrac{\Sigma x_i^2}{200} - (25)^2 = 9$ and $\dfrac{\Sigma y_i^2}{300} - (10)^2 = 16$

$\Rightarrow \qquad \Sigma x_i^2 = 126800$ and $\Sigma y_i^2 = 34800$

$\therefore \qquad \sigma = \dfrac{\Sigma z_i^2}{n} - \left(\dfrac{\Sigma z_i}{n} \right)^2 = \dfrac{\Sigma(x_i^2 + y_i^2)}{500} - \left(\dfrac{\Sigma x_i + \Sigma y_i}{500} \right)^2 = \dfrac{161600}{500} - \left(\dfrac{8000}{500} \right)^2 = 67.2$

Hindi $\dfrac{\Sigma x_i}{200} = 25,$ $\dfrac{\Sigma y_i}{300} = 10$

$\Rightarrow$ $\Sigma x_i = 5000,$ $\Sigma y_i = 3000$

$\sigma_x = 3$ और $\sigma_y = 4$

$\Rightarrow$ $\dfrac{\Sigma x_i^2}{200} - (25)^2 = 9$ और $\dfrac{\Sigma y_i^2}{300} - (10)^2 = 16$

$\Rightarrow$ $\Sigma x_i^2 = 126800$ और $\Sigma y_i^2 = 34800$

$\therefore$ $\sigma = \dfrac{\Sigma z_i^2}{n} - \left(\dfrac{\Sigma z_i}{n}\right)^2 = \dfrac{\Sigma(x_i^2 + y_i^2)}{500} - \left(\dfrac{\Sigma x_i + \Sigma y_i}{500}\right)^2 = \dfrac{161600}{500} - \left(\dfrac{8000}{500}\right)^2 = 67.2$

5. The first of the two samples has 100 items with mean 15 and S.D. 3. If the whole group has 250 items with mean 15.6 and S.D. $= \sqrt{13.44}$ then S.D. of the second group is

दो प्रतिदर्शों में से पहले में 100 ऑकड़े है जिनका माध्य एवं मानक विचलन 3 हैं। यदि पूरे समूह में 250 ऑकड़े है जिनका माध्य 15.6 एवं मानक विचलन $\sqrt{13.44}$ है, तो दूसरे प्रतिदर्श का मानक विचलन होगा–

(1) 5 (2*) 4 (3) 6 (4) 3.52

Sol. $\sigma_x = 3 \Rightarrow \dfrac{\sum x_i^2}{100} - (\bar{x})^2 = 9 \Rightarrow \sum x_i^2 = 23400$

$\sum z_i = 250 \times 15.6 = 3900$

$\therefore \sum y_i = \sum z_i - \sum x_i = 3900 - 1500 = 2400$

$\sigma_z^2 = 13.44 \Rightarrow \dfrac{\sum x_i^2 + \sum y_i^2}{250} - (15.6)^2 = 13.44 \Rightarrow \sum y_i^2 = 40800$

$\Rightarrow \sigma_y = \sqrt{\dfrac{\sum y_i^2}{150} - \left(\dfrac{\sum y_i}{150}\right)^2} = \sqrt{\dfrac{40800}{150} - \left(\dfrac{2400}{150}\right)^2} = 4$

6. The average marks of 10 students in a class was 60 with standard deviation 4. While the average marks of other 10 students was 40 with a standard deviation 6. If all the 20 students are taken together, their standard deviation will be

(SCRA-2015)

कक्षा में 10 विद्यार्थियों का औसत अंक 60 तथा मानक विचलन 4 है। जबकि अन्य 10 विद्यार्थियों का औसत अंक 40 अंक और मानक विचलन 6 है। यदि सभी 20 विद्यार्थियों को एक साथ लिया जाता है, तो उनका मानक विचलन होगा –

(SCRA-2015)

(1) 5.0 (2) 7.5 (3) 9.8 (4*) 11.2

Sol. $(\bar{x}) = 60,$ $(\bar{y}) = 40$

$(\sigma_x^2) = 16,$ $(\sigma_y^2) = 36$

$\sigma_x^2 = \dfrac{\Sigma x_1^2}{10} -, (\bar{x})^2$ $\qquad \sigma_y^2 = \dfrac{\Sigma y^2}{10} - (\bar{y})^2$

$\Sigma x_1^2 = 160 + (60)^2 - 10$ $\qquad \Sigma y_1^2 = 360 + (40)^2.10$

$\sigma^2\text{(overall)} = \dfrac{\Sigma x_1^2 + \Sigma y_1^2}{20} - \left(\dfrac{10\bar{x} + 10\bar{y}}{20}\right)^2 = \dfrac{520 + 52000}{20} - (50)^2$

$\sigma^2 = 2626 - 2500 = 126$

$\text{S.D.} = +\sqrt{\sigma^2} = 11.2$

7. The mean and variance of 5 observations of an experiment are 4 and 5.2 respectively. From these observations three are 1, 2 and 6 and $\lambda = |x_1 - x_2| + 8$ where x_1 & x_2 are remaining observations. Then number of solution of equation $10 - x^2 - 2x = \lambda$ are **[Revision Plannar_15]**

एक प्रयोग के 5 प्रेक्षणों का माध्य और प्रसरण क्रमशः 4 ओर 5.2 है। इन प्रेक्षणों में से तीन 1, 2 ओर 6 तथा $\lambda = |x_1 - x_2|$ + 8 जहाँ x_1 और x_2 शेष प्रेक्षण है तब समीकरण $10 - x^2 - 2x = \lambda$ के हलों की संख्या है–

(1*) 1 (2) 2 (3) 3 (4) 4

Sol. Mean $(\bar{x})$ = 4, variance = 5.2

a_1, a_2, a_3 = 1, 2, 3.

Let x_1, x_2 are remaining values

Mean $(\bar{x}) = \dfrac{a_1 + a_2 + a_3 + x_1 + x_2}{5}$ $\Rightarrow$ $x_1 + x_2 = 11$ ....(1)

variance $\sigma^2 = 5.2 = \dfrac{a_1^2 + a_2^2 + a_3^2 + x_1^2 + x_2^2}{5} - (\bar{x})^2$ $\Rightarrow$ $x_1^2 + x_2^2 = 65$ ....(2)

$\Rightarrow$ $|x_1 - x_2| = 3$

$\Rightarrow$ So $\lambda = 11 \Rightarrow 10 - x^2 - 2x = \lambda$ $\Rightarrow$ $(x + 1)^2 = 0$ one solution

Hindi माध्य $(\bar{x})$ = 4, प्रसरण = 5.2

a_1, a_2, a_3 = 1, 2, 3.

माना x_1, x_2 शेष मान है

माध्य $(\bar{x}) = \dfrac{a_1 + a_2 + a_3 + x_1 + x_2}{5}$ $\Rightarrow$ $x_1 + x_2 = 11$ ....(1)

प्रसरण $\sigma^2 = 5.2 = \dfrac{a_1^2 + a_2^2 + a_3^2 + x_1^2 + x_2^2}{5} - (\bar{x})^2$ $\Rightarrow$ $x_1^2 + x_2^2 = 65$ ....(2)

$\Rightarrow$ $|x_1 - x_2| = 3$

$\Rightarrow$ इसलिए $\lambda = 11 \Rightarrow 10 - x^2 - 2x = \lambda \Rightarrow$ $(x + 1)^2 = 0$ एक हल

8. The mean and variance of 10 numbers were calculated as 11.3 and 3.3 respectively. It was subsequently found that one of the a number was misread as 10 instead of 12. How does the variance change.

(1*) variance decreases (2) variance increases
(3) nothing can be said about variance (4) variance remains unchanged.

10 संख्याओं का माध्य और प्रसरण क्रमशः 11.3 और 3.3 से दिया है। बाद में ये पाया गया है कि गलती से एक संख्या को

12 के स्थान पर 10 पढ़ लिया गया है, तब प्रसरण किस तरह बदलता है **(SCRA-2011)**

(1*) प्रसरण घटता है। (2) प्रसरण बढ़ता है।
(3) प्रसरण के बारे में कुछ नहीं कहा जा सकता है। (4) प्रसरण अपरिवर्तित रहता है।

Sol. Let x_n misread value $(x_n) = 10$ $(x_n)_{actual} = 12$

माना x_n जो गलत पढ़ा गया है $(x_n) = 10$ $(x_n)_{actual} = 12$

$\sigma^2 = 3.3$ $\bar{x} = 11.3$ $\Rightarrow$ $\sum\limits_{i=1}^{n-1} x_i = 113 - 10 = 103 = 10.(\bar{x}) - 10$

$\sigma^2 = \dfrac{\sum\limits_{i=1}^{n-1} x_i^2 + x_n^2}{10} - (\bar{x})^2$

$$\sum_{i=1}^{n-1} x_i^2 = -67 + 10\left(\overline{x}\right)^2 \qquad ...(1)$$

$$\Rightarrow \qquad (\sigma^2)_{actual} = \dfrac{\displaystyle\sum_{i=1}^{n} x_1^2 + \left(x_n\right)^2_{actual}}{10} - \left(\overline{x}\right)_{actual}$$

$$\Rightarrow \qquad (\sigma^2)_{actual} = \dfrac{\displaystyle\sum_{i=1}^{n} x_1^2 + \left(x_n\right)^2_{actual}}{10} - \left(\overline{x}\right)_{actual}$$

$$= \dfrac{-67 + 10\left(\overline{x}\right)^2 + 144}{10} - \left(\dfrac{10\left(\overline{x}\right) - 10 + 12}{10}\right)$$

$$= (\sigma^2_{actual}) = 3.14$$

Comprehension # 1_ (Q. No. 9 to 11) **[Revision Plannar_15]**
अनुच्छेद # 1_ (प्र० सं० 9 से11)

As median divides an arranged series into two equal parts, in similar way quartile divides an arranged series in 4 equal part. For ungrouped frequency distribution formula of finding i^{th} quartile =

$Q_i = \left\{ i.\left(\dfrac{N+1}{4}\right)\right\}^{th}$ term, i = 1, 2, 3.

Quartile deviation : half of difference between upper quartile & lower quartile.

$\Rightarrow$ Quartile deviation (Q.D.) = $\dfrac{1}{2}$ $(Q_3 - Q_1)$ $\Rightarrow$ Coefficient of quartile deviation = $\dfrac{Q_3 - Q_1}{Q_3 + Q_1}$.

चूंकि माध्यिका व्यवस्थित की गई श्रेणी को दो बराबर भागों में विभाजित करती है इसी प्रकार चतुर्थक व्यवस्थित की गई श्रेणी को 4 चार बराबर भागों में विभाजित करता है। असमान वितरण असमान समूह आवृत्ति वितरण के लिए iवाँ चतुर्थक का   सूत्र

$= Q_i = \left\{ i.\left(\dfrac{N+1}{4}\right)\right\}^{th}$ पद, i = 1, 2, 3

चतुर्थक विचलन : उच्च चतुर्थक ओर निम्न चतुर्थक के मध्य अन्तर का आधा

$\Rightarrow$ चतुर्थक विचलन (Q.D.) = $\dfrac{1}{2}$ $(Q_3 - Q_1)$ $\Rightarrow$ चतुर्थक विचलन का गुणांक = $\dfrac{Q_3 - Q_1}{Q_3 + Q_1}$

9. If 1, 2, 3, 4, 5, 6, 7, are numbers then Q_1 & Q_3 are respectively
यदि 1, 2, 3, 4, 5, 6, 7 संख्याऐं है तब Q_1 और Q_3 क्रमशः है–
(1) 2, 4   (2*) 2, 6   (3) 4, 6   (4) 3, 5

Sol. 1, 2, 3, 4, 5, 6, 7,

$Q_1 = \left(1.\left(\dfrac{7+1}{4}\right)\right)^{nd} = 2^{nd}$ term पद = 2

$Q_3 = \left(3.\left(\dfrac{7+1}{4}\right)\right)^{th} = 6^{th}$ term पद = 6

10. Quartile deviation of the following numbers 10, 8, 12, 11, 14, 9, 6 is
निम्न संख्याओं 10, 8, 12, 11, 14, 9, 6 का चतुर्थक विचलन है–
(1*) 2   (2) 10   (3) 1/2   (4) 1

Sol. 6, 8, 9, 10, 11, 12, 14

Q_1 = 2nd term पद = 8

Q_3 = 6th term पद = 12

Quartile deviation चतुर्थक विचलन = $\dfrac{1}{2}$ $(Q_3 - Q_1)$ = $\dfrac{1}{2}$ $(12 - 8)$ = 2

11. Coefficient of quartile deviation of numbers 6, 8, 9, 10, 11, 12, 14 is
संख्याओं 6, 8, 9, 10, 11, 12, 14 के चतुर्थक विचलन का गुणांक है –
(1*) 1/5   (2) 1/10   (3) 2/5   (4) 1/4

Sol. Coefficient of quartile deviation is चतुर्थक विचलन का गुणांक = $\dfrac{Q_3 - Q_1}{Q_3 + Q_1}$

6, 8, 9, 10, 11, 12, 14

Q_1 = 8 $\Rightarrow$ coefficient गुणांक = $\dfrac{12 - 8}{12 + 8}$ = $\dfrac{4}{20}$ = $\dfrac{1}{5}$

Q_3 = 12

Comprehension # 2_ (Q. No. 12 to 14)
अनुच्छेद # 2_ (प्र० सं० 12 से 14)

To analyse data using mean, median and mode, we need to use the most appropriate measure of central tendency. The mean is useful for predicting future results when there are no extreme values in the data set. The median may be more useful than the mean when there are extreme values in the data set as it is not affected by the extreme values. The median is most commonly quoted figure used to measure property prices as mean property price is affected by a few expensive properties that are not representative of the general property market. The mode is useful when the most common item or characteristic of a data set is required. The mode has applications in printing. It is important to print more of the most popular books.

माध्य, माध्यिका एवं बहुलक का उपयोग कर आँकड़ो का विश्लेषण करने के लिये हमें केन्द्रीय प्रवृत्ति की सर्वाधिक उपयुक्त माप का उपयोग करना चाहिये। माध्य, भविष्य के परिणामों की भविष्यवाणी करने के लिये तब उपयोगी रहता है जब आँकड़ो के समुच्चय में सीमान्त मान नहीं दिये गये हो। माध्यिका तब अधिक उपयोगी रहती है जब आँकड़ो के समुच्चय में सीमान्त मान दिये गये हो। क्योंकि माध्यिका सीमान्त मानों से प्रभावित नहीं होती है। सम्पत्ति की कीमतों के मापन हेतु सर्वाधिक व्यापक रूप से माध्यिका का ही उपयोग किया जाता है। क्योंकि सम्पति कीमतों का माध्य, कुछ अत्यधिक मंहगी सम्पति से ही प्रभावित होता है जो कि सामान्य सम्पति बाजार का प्रतिनिधित्व नहीं करती। बहुलक तब अधिक उपयोगी होती है जब आँकड़ो के समुच्चय में सर्वाधिक बार प्रयुक्त आँकड़ों या गुण की आवश्यकता हो। बहुलक का अनुप्रयोग छपाई में होता है। सर्वाधिक प्रसिद्ध किताबों की अधिक छपाई महत्वपूर्ण है

12. For the data shown, the value of appropriate measure of central tendency is
दिये गये आँकड़ों के लिये केन्द्रीय प्रवृत्ति की उपर्युक्त माप का मान है

No. of staff	1	2	4	5	3	2
Salary (In rupees)	15000	10000	7000	12000	90000	95000

(1) 95000   (2) 18350   (3) 90000   (4*) 12000

Sol. As data has outliners at 90,000 and 95,000 we should use median in place of mode

& Median = $\dfrac{10\text{th value} + 11\text{th value}}{2}$ = 12000

दिये गये आँकडों में 90,000 तथा 95,000 सामान्य प्रारूप से भिन्नता रखते है। अतः बहुलक के स्थान पर माध्यिका का उपयोग करते है

एवं माध्यिका $= \dfrac{10\ \text{वाँ मान} + 11\ \text{वाँ मान}}{2} = 12000$

13. For a normally distributed sample as shown, then most appropriate representative of data is

Number of marks
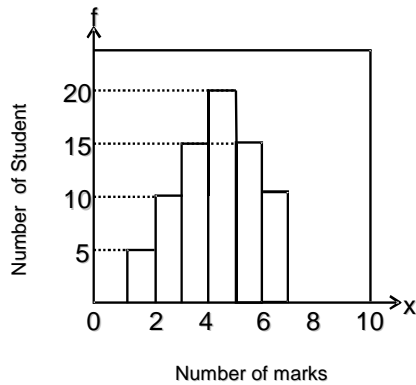
(1*) Mean   (2) Mode
(3) Median   (4) any one of mean or median

दर्शाये गये सामान्य रूप से बंटित प्रतिदर्श के लिये, आँकड़ो को प्रदर्शित करने का सबसे सही तरीका होगा

प्राप्तकों की संख्या
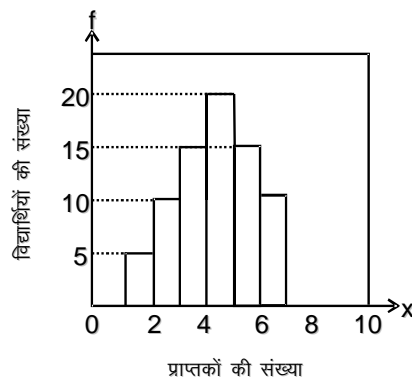
(1*) माध्य   (2) बहुलक
(3) माध्यिका   (4) माध्य या माध्यिका में से कोई भी एक

Sol. For a normally distributed data, we many use either mean or median. However, mean is preferred as it include all the values in the data set for its calculation and any change in any of the scores will affect the value of the mean which is not the case with median or mode.

आँकडो के सामान्य बंटन के लिये, हम माध्य या माध्यिका का उपयोग कर सकते है। फिर भी माध्य को प्रमुखता दी जाती है क्योंकि इससे गणना करने में आँकडो के समुच्चय के सभी मानों को शामिल किया जाता है तथा किसी भी मान में परिवर्तन, माध्य को प्रभावित करता है जो कि माध्यिका या बहुलक के साथ नहीं होता।

14. Based upon collection of data of numbers of days it snows, rains or it is sunny in a month for three month December, January and February of last year, the weather forecast points that snow is likely to be in January. Which measure is used for this forecast ?
(1) Mean   (2*) Mode   (3) Median   (4) Range

पिछले वर्ष के तीन महीनों दिसम्बर, जनवरी एवं फरवरी में प्रत्येक महीने में कितने दिन बर्फबारी होगी कितने दिन सूर्य दिखेगा एवं कितने दिन वर्षा होगी, के लिये इकठ्ठे किये गयें आँकड़ों के आधार पर जनवरी माह में बर्फबारी होने के मौसम के बारे में भविष्यवाणी की जाती है। इस भविष्यवाणी हेतु कौनसा मापन ठीक होगा ?
(1) माध्य   (2*) बहुलक   (3) माध्यिका   (4) परिसर

Sol. Obviously स्पष्टतया

PART - II : MISCELLANEOUS QUESTIONS

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MAINSTTS-19

भाग-II : विविध प्रकार के प्रश्न (MISCELLANEOUS QUESTIONS)

Section (A) : ASSERTION/REASONING
Section (A) : कथन/कारण (ASSERTION/REASONING)

DIRECTIONS :

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.
(1) Both the statements are true.
(2) Statement-I is true, but Statement-II is false.
(3) Statement-I is false, but Statement-II is true.
(4) Both the statements are false.

निर्देशः प्रत्येक प्रश्न के 4 विकल्प है। (1), (2), (3) तथा (4) है, जिनमें से सिर्फ एक सही है।
(1) दोनो कथन सत्य है।
(2) कथन-I सत्य है, परन्तु कथन-II असत्य है।
(3) कथन-I असत्य है, परन्तु कथन-II सत्य है।
(4) दोनो कथन असत्य है।

A-1. **STATEMENT-1 :** If $\sum_{i=1}^{9}(x_i - 8) = 9$ and $\sum_{i=1}^{9}(x_i - 8)^2 = 45$ then S.D. of $x_1, x_2, ..., x_9$ is 2.

कथन-1 : यदि $\sum_{i=1}^{9}(x_i - 8) = 9$ तथा $\sum_{i=1}^{9}(x_i - 8)^2 = 45$ तब $x_1, x_2, ..., x_9$ का मानक विचलन 2 है –

STATEMENT-2 : S.D. is independent of change of origin.
कथन-2 : मानक विचलन मूल बिन्दु परिवर्तन से स्वतंत्र है। **[Revision Plannar_15]**

Ans. (1)

Sol. $\text{S.D.}(x_i) = \text{S.D.}(x_i - 8) = \sqrt{\dfrac{\sum(x_i - 8)^2}{n} - \left(\dfrac{\sum(x_i - 8)}{n}\right)^2} = \sqrt{\dfrac{45}{9} - 1} = 2$

A-2_. **STATEMENT-1 :** Mean cannot be represented graphically.
कथन-1 : माध्य को आरेख द्वारा प्रदर्ीत नहीं कर सकते।
STATEMENT-2 : Mean may not coincide with anyone of the actual values.
कथन-2 : माध्य, किसी भी वास्तविक मान से संपाती नहीं हो सकता।

Ans. **(1)**
Sol. Obviously स्पष्टतया

Section (B) : MATCH THE COLUMN
Section (B) : कॉलम को सुमेलित कीजिए (MATCH THE COLUMN)

Match the column **[Revision Plannar_15]**

B-1

	Column - I		Column - II
(A)	Better measure of central tendency for data 1, 7, 8, 9, 9 is	(p)	Mean
(B)	Which is not independent of change of scale ?	(q)	Median
(C)	Which is not dependent on change of origin ?	(r)	Mode
(D)	The value of range of data is always greater than or equal to	(s)	S.D.

सुमेलित कीजिये

स्तम्भ- I		स्तम्भ- II

(A) आँकड़ों 1, 7, 8, 9, 9 की केन्द्रीय प्रवृत्ति मापन हेतु उपर्युक्त माप है   (p) माध्य

(B) स्केल (scale) परिवर्तन से जो अप्रभावित नही रहता है   (q) माध्यिका

(C) मूल बिन्दु परिवर्तन से जो स्वतंत्र रहता है   (r) बहुलक

(D) आँकड़ो के परिसर का मान जिसके बराबर या अधिक होता है   (s) मानक विचलन

(1) (A) → (r) ; (B) → (p, q, s) ; (C) → (s) ; (D) → (s)
(2*) (A) → (q, r) ; (B) → (p, q, r, s) ; (C) → (s) ; (D) → (s)
(3) (A) → (q) ; (B) → (p, q, r, s) ; (C) → (s) ; (D) → (r)
(4) (A) → (q, r) ; (B) → (q, r, s) ; (C) → (p) ; (D) → (s)

Ans. (A) → (q, r) ; (B) → (p, q, r, s) ; (C) → (s) ; (D) → (s)

Sol.
(A) Due to low value 1, mean is not preferred
(B) Mean, Median, Mode and S.D. are dependent on change of scale.
(C) S.D. is independent of change of origin.
(D) Range is always greater than or equal to S.D.

Hindi.
(A) छोटे मान 1 के कारण माध्य प्राथमिकता पर नहीं होगा।
(B) माध्य, माध्यिका, बहुलक एवं मानक विचलन चारो ही स्केल परिवर्तन पर निर्भर है।
(C) मानक विचलन मूल बिन्दु परिवर्तन से स्वतंत्र होता है।
(D) परिसर सदैव मानक विचलन के बराबर या उससे अधिक होता है।

B-2_ Let $n \in N$.

	Column I		Column II
(A)	S.D. of 2, 4, 6, ... 2n ; $n \in N$	(p)	n
(B)	S.D. of 1, 3, 5, ..., 2n – 1 ; $n \in N$	(q)	$\dfrac{n+1}{2}$
(C)	Mean of 1,3,5, ...2n–1 ; $n \in N$	(r)	$\sqrt{\dfrac{n^2-1}{3}}$
(D)	Median of 1,3,5,.....,2n – 1	(s)	$\sqrt{\dfrac{n^2-1}{12}}$

माना $n \in N$.

	स्तम्भ I		स्तम्भ II
(A)	2, 4, 6, ... 2n ; $n \in N$ का मानक विचलन	(p)	n
(B)	1, 3, 5, ..., 2n – 1 ; $n \in N$ का मानक विचलन	(q)	$\dfrac{n+1}{2}$
(C)	1,3,5, ...2n–1 ; $n \in N$ का मानक विचलन	(r)	$\sqrt{\dfrac{n^2-1}{3}}$
(D)	1,3,5,.....,2n – 1 की माध्यिका	(s)	$\sqrt{\dfrac{n^2-1}{12}}$

Sol. A → r ; B → r ; C → p ; D → p
(A) = 2 {S.D. of 2, 4, 6, ..., 2n

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MAINSTTS-21

= 2 {S.D. of 1,2,3, ...n}

$= \sqrt{\dfrac{n^2 - 1}{12}} = \sqrt{\dfrac{n^2 - 1}{3}}$

(B) S.D. of 1,3,5, ..., 2n – 1

= S.D. of 2,4,6, ..., 2n का मानक विचलन

$= \sqrt{\dfrac{n^2 - 1}{3}}$ [from part (A)]

(C) Mean of 1,3,5, ... 2n – 1 का माध्य

$= \dfrac{1 + 3 + 5.... + (2n - 1)}{n}$

$= \dfrac{n^2}{n} = n$

(D) When n is odd, then जब n विषम है तब

median माध्यिका $= \dfrac{n+1}{2}$ th odd natural number विषम प्रकृत संख्या

$= 2\left(\dfrac{n+1}{2}\right) - 1 = n$

When n is even, then जब n सम है तब

median माध्यिका $= \left(\dfrac{n}{2} + 1\right)$th odd number $\Big)$ विषम प्रकृत संख्या

$= \dfrac{1}{2}\left\{2\left(\dfrac{n}{2}\right) - 1 + 2\left(\dfrac{n}{2} + 2\right) - 1\right\}$

$= \dfrac{1}{2}$ (n – 1 + n + 1) = n

Note kth odd number = 2k – 1   नोट : k की विषम संख्या = 2k – 1

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT
Section (C) : एक या एक से अधिक सही विकल्प प्रकार (ONE OR MORE THAN ONE OPTIONS CORRECT)

C-1._ If the daily earnings (in rupees) of 12 workers in a factory are 16, 11, 3, 7, 5, 28, 9, 31, 28, 43, 15 ,17, then which of the following is(are) true

(1*) mean = 17.75   (2*) median = 15.5

(3*) mode = 28   (4*) mean deviation about mean = $9.8\overline{3}$

यदि एक फैक्ट्री में 12 कर्मचारियों का दैनिक वेतन 16, 11, 3, 7, 5, 28, 9, 31, 28, 43, 15 ,17, है तब निम्न में से कौनसा कथन सत्य है।

(1) माध्य = 17.75   (2) माध्यिका = 15.5

(3) बहुलक = 28   (4) माध्य के सापेक्ष माध्य विचलन = $9.8\overline{3}$

Sol. Mean $= \dfrac{\text{Sum of the observation}}{\text{number of observation}}$

For median , arrange the items in increasing order as 3, 5,7, 9, 11, 15, 16, 17, 28, 28, 31, 43.

Here , number of items is 12 (even number)

Median $= \dfrac{1}{2}\left(\left(\dfrac{12}{2}\right)\text{th item} + \left(\dfrac{12}{2} + 1\right)\text{th item}\right) = \dfrac{1}{2}$ (6th item + 7 th item)

$$= \frac{1}{2} \ (15+16) = 15.5$$

Hindi : माध्य $\dfrac{\text{प्रेक्षणों का योग}}{\text{प्रेक्षणों की संख्या}}$

माध्यिका के लिए बढ़ते क्रम में 3, 5,7, 9, 11, 15, 16, 17, 28, 28, 31, 43.

माध्यिका $= \dfrac{1}{2} \left(\left(\dfrac{12}{2}\right)\text{th item} + \left(\dfrac{12}{2}+1\right)\text{th item} \right) = \dfrac{1}{2}$ (6th item + 7 th item) = (15+16) = 15.5

C-2_ If first sample of 25 variates has the mean 40 and standard deviation 5 and a second sample of 35 variates has the mean 45 and standard deviation 2, then which of the following is/are true

(1*) mean of combined sample space = 42.917

(2) mean of combined sample space = 32.9

(3) standard deviation of combined sample space = 3.34

(4*) standard deviation of combined sample space = 4.34

यदि 25 चरों के प्रथम नमूनों का माध्य 40 है तथा मानक विचलन 5 है तथा 35 चरों के दूसरे नमूनों के माध्य 45 तथा मानक विचलन 2 है तब निम्न में से कौनसा सत्य है ?

(1*) संयुक्त नमूनों का माध्य = 42.917

(2) संयुक्त नमूनों का माध्य = 32.9

(3) संयुक्त नमूनों का मानक विचलन = 3.34 है।

(4*) संयुक्त नमूनों का मानक विचलन = 4.34 है।

Sol. Here यहाँ , $n_1 = 25$, $n_2 = 35$.

$\bar{x}_1 = 40, \bar{x}_2 = 45,$

$\sigma_1 = 5$ and $\sigma_2 = 2$

Let माना $\bar{x}$ be the mean and σ , the standard deviation of the two samples taken together, then माना $\bar{x}$ माध्य है तथा, σ दो नमूनों को एक साथ मान विचलन है।

$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} = \frac{25 \times 40 + 35 \times 45}{25 + 35} = \frac{2575}{60} = 42.917$$

Also तथा, $\sigma^2 = \dfrac{1}{n_1 + n_2}\left[n_1 \sigma_1^2 + n_2 \sigma_2^2 + \dfrac{n_1 n_2}{n_1 + n_2}(\bar{x}_1 - \bar{x}_2)^2 \right]$

$$= \frac{1}{25 + 35}\left[25 \times (5)^2 + 35 \times (2)^2 + \frac{25 \times 35}{25 + 35}(40 - 45)^2 \right] = \frac{1}{60}[625 + 140 + 364.58]$$

$$= \frac{1129.58}{60} 18.83 \ \text{nearly लगभग}$$

$\Rightarrow \sigma = 4.34$ nearly लगभग

C-3. Which of the following is false :

(1) Algebraic sum of deviations of observation from their mean is zero

(2*) If S.D. of x_i (i = 1, 2, 3, ... n) is σ then S.D. of hx_i is $h\sigma$

(3*) Median is severely affected by fluctuations in extreme values

(4) Mean deviations of a given set of observations is least when taken about their median.

निम्नलिखित में से कौनसा असत्य है

(1) प्रक्षणों का उनके माध्य से विचलनों का बीजीय योग शून्य होता है।

(2*) यदि x_i (i = 1, 2, 3, ... n) का मानक विचलन σ है, तो hx_i का मानक विचलन $h\sigma$ होगा।

(3*) माध्यिका अन्तमानों में होने वाले परिवर्तनों से अत्यधिक प्रभावित होती हे।

(4) दिये गये प्रक्षणों के समुच्चय का माध्य विचलन न्यूनतम होगा यदि यह उनकी माध्यिका के सापेक्ष लिये गये हो।

Sol. (1) $\sum d_i = \sum (x_i - \overline{x}) = n\overline{x} - n\overline{x} = 0$

(2) If S.D. of x_i (i = 1, 2, 3, ... n) is σ then S.D. of hx_i is $|h|\sigma$

(3) यदि x_i (i = 1, 2, 3, ... n) का मानक विचलन σ है, तो hx_i का मानक विचलन $|h|\sigma$ होगा।

Exercise-3

Marked questions are recommended for Revision
* चिन्हित प्रश्न एक से अधिक सही विकल्प वाले प्रश्न है -

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)
भाग - I : JEE (MAIN) / AIEEE (पिछले वर्षों) के प्रश्न

1. The mean of the number a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the folowing gives possible values of a and b ? **RFT-8**   **[AIEEE 2008 (3,–1),225]**

संख्याओं a, b, 8, 5, 10 का माध्य 6 है तथा चरिता 6.80 है तब निम्न में से a तथा b के मान है ?

                                                        [AIEEE 2008 (3,–1),225]

(1*) a = 3, b = 4   (2) a = 0, b = 7   (3) a = 5, b = 2   (4) a = 1, b = 6

Sol. $\dfrac{a+b+8+5+10}{5} = 6 \Rightarrow a + b = 7$ ...(1)

$\dfrac{(a-6)^2 + (b-6)^2 + 2^2 + 1^2 + 4^2}{5} = 6.80 \Rightarrow (a-6)^2 + (b-6)^2 = 13$

solve (हल करने पर) a = 3, b = 4

2. **Statement-I** The variance of first n even natural numbers is $\dfrac{n^2-1}{4}$ **[AIEEE 2006 (4,–1),225]**

Statement-II The sum of first n natural numbers is $\dfrac{n(n+1)}{2}$ and the sum of squares of first n natural

numbers is $\dfrac{n(n+1)(2n+1)}{6}$.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4*) Statement-1 is False, Statement-2 is True

कथन–1 : प्रथम n सम प्राकृत संख्याओं की चरिता $\dfrac{n^2-1}{4}$ है।   **[AIEEE 2009]**

कथन–2 : प्रथम n प्राकृत संख्याओं का योगफल $\dfrac{n(n+1)}{2}$ है तथा उनके वर्गों का योगफल $\dfrac{n(n+1)(2n+1)}{6}$ है।

(1) कथन–1 सत्य है, कथन–2 सत्य है ; कथन–2, कथन–1 का सही स्पष्टीकरण है।
(2) कथन–1 सत्य है, कथन–2 सत्य है ; कथन–2, कथन–1 का सही स्पष्टीकरण नहीं है।
(3) कथन–1 सत्य है, कथन–2 असत्य है।
(4*) कथन–1 असत्य है, कथन–2 सत्य है।

Sol. **Statement-1 :** $\dfrac{\sum n^2}{n} - \dfrac{\sum n}{n} = \dfrac{(n+1)\ (2n+1)}{6} - \dfrac{n+1}{2}$

$= \left(\dfrac{n+1}{6}\right) (2n + 1 - 3)$

Statement-2 : Obvious

Hindi. कथन -1 : $\dfrac{\sum n^2}{n} - \dfrac{\sum n}{n} = \dfrac{(n+1)\ (2n+1)}{6} - \dfrac{n+1}{2} = \left(\dfrac{n+1}{6}\right) (2n + 1 - 3)$

कथन-2 : स्पष्टतया

3. If the mean deviation of numbers 1, 1 + d, 1 + 2d, ...,1 + 100d from their mean is 255, then the value of d is equal to- **[AIEEE 2009]**

यदि संख्याओं 1, 1 + d, 1 + 2d, ...,1 + 100d का उनके माध्य से माध्य विचलन 255 है तब d मान बराबर है– **[AIEEE 2009]**

(1) 10.0   (2) 20.0   (3*) 10.1   (4) 20.2

Sol. $\overline{x} = \dfrac{1+(1+d)+(1+2d)+(1+100d)}{101} = 1 + 50d$

Mean deviation (माध्य विचलन) $= \dfrac{\sum\limits_{i=0}^{100}|x_i - \overline{x}|}{101} = \sum \dfrac{|(1+id)-(1+50d)|}{101}$

$= \sum\limits_{i=0}^{100} \dfrac{|(i-50)\ d|}{101} = 225 \Rightarrow \dfrac{(50+49+1+0+1+50)d}{101} = 225$

$\Rightarrow \dfrac{50\times 51}{101}.d = 225 \Rightarrow d = 10.1$

4. For two data sets, each of size 5, the variance are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is **[AIEEE 2010]**

प्रत्येक 5 के आकार वाले, आँकड़ों के दो समुच्चयों का प्रसरण 4 व 5 और उनके संगत माध्य क्रमशः 2 व 4 है। संयुक्त आँकड़ो के समुच्चय का प्रसरण है– **[AIEEE 2010]**

(1*)   (2) 6   (3)   (4)

Sol. $\sigma_x^2 = 4 \Rightarrow \dfrac{\sum x_i^2}{n} - \left(\dfrac{\sum x_i}{n}\right)^2 = 4$

$\Rightarrow \dfrac{\sum x_i^2}{5} - (2)^2 = 4 \Rightarrow \sum x_i^2 = 40$

similarly (इसी प्रकार) $\sum y_i^2 = 105$

$\therefore \ \sigma^2 = \dfrac{\sum x_i^2 + \sum y_i^2}{10} - \left(\dfrac{\sum x_i + \sum y_i}{10}\right)^2 = \dfrac{145}{10} - \left(\dfrac{10+20}{10}\right)^2 = 5.5$

5. If the mean deviation about the median of the numbers a, 2a, ..., 50a is 50, then |a| equals : **[AIEEE 2011]**

संख्याओं a, 2a, ...,50a का माध्य के सापेक्ष माध्य विचलन 50 है, तो |a| का मान है : **[AIEEE 2011]**

(1) 2   (2) 3   (3*) 4   (4) 5

Sol. Median = 25.5 a

Mean deviation about median = 50

माध्यिका = 25.5 a

माध्यिका के सापेक्ष माध्य विचलन = 50

$\Rightarrow \dfrac{\sum |x_i - 25.5a|}{50} = 50$

$\Rightarrow 24.5\,a + 23.5a + ... + 0.5a + 0.5a + ... + 24.5a = 2500$

$\Rightarrow a + 3a + 5a + ... + 49a = 2500$

$$\Rightarrow \frac{25}{2} \ (50a) = 2500 \ \Rightarrow \ a = 4$$

6. A scientist is weighing each of 30 fishes. Their mean weight worked out is 30 gm and a standard deviation of 2 gm. Later, it was found that the measuring scale was misaligned and always under reported every fish weight by 2 gm. The correct mean and standard deviation (in gm) of fishes are respectively : **RFT-8**

[AIEEE 2011]

एक वैज्ञानिक 30 मछलियों में से प्रत्येक को तोलता है। उनके भार का माध्य 30 ग्राम है तथा मानक विचलन 2 ग्राम है। बाद में यह पाया गया कि तोलने की मशीन, ठीक न होने के कारण प्रत्येक मछली का भार 2 ग्राम कम बताती है, तो मछलियों का सही माध्य तथा मानक विचलन (ग्राम में) है :

[AIEEE 2011]

(1*) 32, 2   (2) 32, 4   (3) 28, 2   (4) 28, 4

Sol. Correct mean = observed mean + 2

सही माध्य = अवलोकित माध्य + 2

30 + 2 = 32

Correct S.D. = observed S.D. = 2

सही मानक विचलन = अवलोकित मानक विचलन = 2

7. Let x_1 , x_2 , ..., x_n be n observations, and let $\overline{x}$ be their arithmetic mean and σ^2 be the variance

Statement-1 : Variance of $2x_1$, $2x_2$, ..., $2x_n$ is $4\sigma^2$.   **[AIEEE-2012]**

Statement-2 : Arithmetic mean $2x_1$, $2x_2$, ..., $2x_n$ is $4\overline{x}$.

(1) Statement-1 is false, Statement-2 is true.

(2) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.

(3) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.

(4*) Statement-1 is true, statement-2 is false.

माना n प्रेक्षण x_1 , x_2 , ..., x_n हैं तथा उनका समान्तर माध्य $\overline{x}$ तथा प्रसरण σ^2 है।

कथन -1 : $2x_1$, $2x_2$, ..., $2x_n$ का प्रसरण $4\sigma^2$ है।

कथन -2 : $2x_1$, $2x_2$, ..., $2x_n$ का समान्तर माध्य $4\overline{x}$ है।

(1) कथन-1 असत्य है, कथन-2 सत्य है।

(2) कथन-1 सत्य है, कथन-2 सत्य है; कथन-2 कथन-1 की सही व्याख्या है।

(3) कथन-1 सत्य है, कथन -2 सत्य है, कथन-2, कथन -1 की सही व्याख्या नहीं है।

(4*) कथन -1 सत्य है, कथन-2 असत्य है।

Sol. A.M. of $2x_1$, $2x_2$ ..... $2x_n$ is $\dfrac{2x_1 + 2x_2 + ... + 2x_n}{n}$

$$= 2\left(\frac{x_1 + x_2 + ... + x_n}{n}\right) = 2\overline{x}$$

So statement-2 is false

variance $(2x_i) = 2^2$ variance $(x_i) = 4\sigma^2$

so statement-1 is true.

Hindi $2x_1$, $2x_2$ ..... $2x_n$ का समान्तर माध्य $\dfrac{2x_1 + 2x_2 + ... + 2x_n}{n}$ है।

$$= 2\left(\frac{x_1 + x_2 + ... + x_n}{n}\right) = 2\overline{x}$$

अतः कथन-2 असत्य है।

प्रसरण $(2x_i) = 2^2$ प्रसरण $(x_i) = 4\sigma^2$

अतः कथन -1 सत्य है।

8. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given ? **RFT-8** **[AIEEE - 2013, (4, –¼),360]**

(1) mean (2) median (3) mode (4*) variance

एक कक्षा के सभी विद्यार्थियों ने गणित में कम अंक प्राप्त किए। अध्यापक ने प्रत्येक विद्यार्थी को 10 रियायती अंक देने का निर्णय लिया। रियायती अंक देने के बाद भी निम्न में से कौनसा से सांख्यिकी माप नहीं बदलेगा ? **[AIEEE - 2013, (4, –¼),360]**

(1) माध्य (2) माध्यक (3) बहुलक (4) प्रसरण

Sol. (4)

If initially all marks were x_i then $\sigma_1^2 = \dfrac{\sum(x_i - \bar{x})^2}{N}$

Now each is increased by 10

$$\sigma_2^2 = \dfrac{\sum\left[(x_i + 10) - (\bar{x} + 10)\right]^2}{N} = \sigma_1^2$$

So variance will not change whereas mean, median and mode will increase by 10.

Hindi. (4)

यदि प्रारम्भिक अंक x_i है तब $\sigma_1^2 = \dfrac{\sum(x_i - \bar{x})^2}{N}$

अब सभी को 10 से बढ़ाने पर

$$\sigma_2^2 = \dfrac{\sum\left[(x_i + 10) - (\bar{x} + 10)\right]^2}{N} = \sigma_1^2$$

अतः प्रसरण नहीं बदलेगा जबकि माध्य, माध्यिका और बहुलक 10 से बढ़ जायेगा।

9. The variance of first 50 even natural number is**RFT-8**

प्रथम 50 सम प्राकृत संख्या का प्रसरण है – **[AIEEE 2014]**

(1) $\dfrac{833}{4}$ (2*) 833 (3) 437 (4) $\dfrac{437}{4}$

Sol. First 50 even natural numbers प्रथम 50 सम प्राकृत संख्याऐं है

Mean माध्य $= \dfrac{\sum\limits_{i=1}^{50} 2n}{50} + = \dfrac{2+4+6+.....+100}{50} = 2\left(\dfrac{50\times 51}{2\times 50}\right) = 51$

variance प्रसरण $= \dfrac{\sum\limits_{i=1}^{50}(2n)^2}{50} - = 42$ $\quad = 4\dfrac{2^2+4^2+......+100^2}{50} - 2601 = 833$

10. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is **[JEE(Main) 2015, (4, – ¼), 120]**

(1) 16.8 (2) 16.0 (3) 15.8 (4*) 14.0

16 प्रेक्षणों वाले आंकड़ों का माध्य 16 है। यदि एक प्रेक्षण जिसका मान 16 है, को हटा कर, 3 नये प्रेक्षण जिनके मान 3, 4 और 5 है, आंकड़ों में मिला दिये जाते है, तो नये आंकड़ों का माध्य है– **[JEE(Main) 2015, (4, – ¼), 120]**

(1) 16.8   (2) 16.0   (3) 15.8   (4) 14.0

Sol. $\dfrac{x_1 + x_2 .... x_{16}}{16} = 16$

If यदि $x_1 = 16$

$\dfrac{x_1 + x_2 .... x_{10} - 16 + 3 + 4 + 5}{18}$

$= \dfrac{16 \times 10 - 16 + 12}{18} = \dfrac{240 + 12}{18} = \dfrac{252}{18} = 14$

11. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true?

[JEE(Main) 2016, (4, – 1), 120]

यदि संख्याओं 2, 3, a तथा 11 का मानक विचलन 3.5 है, तो निम्न में से कौन–सा सत्य है ?

(1) $3a^2 - 32a + 84 = 0$ (2) $3a^2 - 34a + 91 = 0$ (3) $3a^2 - 23a + 44 = 0$ (4) $3a^2 - 26a + 55 = 0$

Ans. **(1)**
Sol. Standard deviation of numbers 2, 3, a and 11 is 3.5
संख्याओं 2, 3, a तथा 11 का मानक विचलन 3.5 है।

$\therefore \quad (3.5)^2 = \dfrac{\sum x_i^2}{4} - (\overline{x})^2$

$\Rightarrow \quad (3.5)^2 = \dfrac{4 + 9 + a^2 + 121}{4} - \left(\dfrac{2 + 3 + a + 11}{4}\right)^2$

on solving, we get (हल करने पर प्राप्त होता है)
$3a^2 - 32a + 84 = 0$

12. If $\sum\limits_{i=1}^{9}(x_i - 5) = 9$ and $\sum\limits_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, ..., x_9$ is :

[JEE(Main) 2018, (4, – 1), 120]

यदि $\sum\limits_{i=1}^{9}(x_i - 5) = 9$ तथा $\sum\limits_{i=1}^{9}(x_i - 5)^2 = 45$ है, तो नौ प्रेक्षणों $x_1, x_2, ..., x_9$ का मानक विचलन है –

(1) 2   (2) 3   (3) 9   (4) 4

Sol. **(1)**
SD is independent of shifting of origin मूल बिन्दु को स्थानान्तरित करने पर मानक विचलन स्वतंत्र रहता है –

So S.D. $= + \sqrt{\text{var}(x_i - 5)} = \sqrt{\dfrac{1}{9}(45) - \dfrac{9}{9}} = 2$

13. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. the variance (in cm^2) of the height of these six students is :

[JEE(Main) 09-01-19 P-1, (4, – 1), 120]

एक कक्षा के 5 विद्यार्थियों की ऊँचाइयों का माध्य 150 से.मी. तथा प्रसरण 18 वर्ग से.मी. है। 156 से.मी. ऊँचाई वाला एक नए विद्यार्थी उनसे आ मिला। इन छः विद्यार्थियों की ऊँचाइयों का प्रसरण है–

(1) 20　　　　　(2) 22　　　　　(3) 16　　　　　(4) 18

Ans. **(1)**

Sol. Let 5 students are x_1, x_2, x_3, x_4, x_5

Given $x = \dfrac{\Sigma x_i}{5} = 150$　$\Rightarrow$　$\sum_{i=1}^{5} x_i = 750$　.....(1)

$\dfrac{\Sigma x_i^2}{5} - (\bar{x})^2 = 18 \Rightarrow$　$\dfrac{\Sigma x_i^2}{5} - (150)^2 = 18$

$\Rightarrow$　$\Sigma x_i^2 = (22500 + 18) \times 5 \Rightarrow$　$\sum_{i=1}^{5} x_i^2 = 112590$　...... (2)

Height of new student = 156 (Let x_6)

now $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 750 + 156$

$\bar{x}_{new} = \dfrac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = \dfrac{906}{6} = 151$　........(3)

Variance (new) $= \dfrac{\Sigma x_i^2}{6} - (\bar{x})^2$

$= \dfrac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2}{6} - (151)^2$

from equation (2) and (3)

var (new) $= \dfrac{112590 + (156)^2}{6} - (151)^2 = 22\,821 - 22801 = 20$

Hindi. माना 5 विद्यार्थि x_1, x_2, x_3, x_4, x_5 है।

दिया है $x = \dfrac{\Sigma x_i}{5} = 150$　$\Rightarrow$　$\sum_{i=1}^{5} x_i = 750$　.....(1)

$\dfrac{\Sigma x_i^2}{5} - (\bar{x})^2 = 18 \Rightarrow$　$\dfrac{\Sigma x_i^2}{5} - (150)^2 = 18$

$\Rightarrow$　$\Sigma x_i^2 = (22500 + 18) \times 5 \Rightarrow$　$\sum_{i=1}^{5} x_i^2 = 112590$　...... (2)

नये विद्यार्थी की ऊँचाई = 156 (माना x_6)

अब $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 750 + 156$

$\bar{x}_{new} = \dfrac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = \dfrac{906}{6} = 151$　........(3)

प्रसरण (new) $= \dfrac{\Sigma x_i^2}{6} - (\bar{x})^2$

$= \dfrac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2}{6} - (151)^2$

समीकरण (2) तथा (3) से

प्रसरण (new) = $\dfrac{112590 + (156)^2}{6} - (151)^2$ = 22 821 – 22801 = 20

14. A data consists of n observations : **[JEE(Main) 09-01-19 P-2, (4, – 1), 120]**

$x_1, x_2, ..., x_n$. If $\sum\limits_{i=1}^{n}(x_i + 1)^2 = 9n$ and $\sum\limits_{i=1}^{n}(x_i - 1)^2 = 5n$, then the standard deviation of this data is :

आँकड़ों के एक समूह में n प्रेक्षण $x_1, x_2, ..., x_n$ हैं। यदि $\sum\limits_{i=1}^{n}(x_i + 1)^2 = 9n$ तथा $\sum\limits_{i=1}^{n}(x_i - 1)^2 = 5n$ है, तो इन आँकड़ों का मानक विचलन है–

(1) 5 　　　　(2) 2 　　　　(3) $\sqrt{5}$ 　　　　(4) $\sqrt{7}$

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Ans. **(3)**

Sol. $\sum\limits_{i=1}^{n}(x_i + 1)^2 = 9n \quad \Rightarrow \quad \sum x_i^2 + 2\sum x_i + n = 9n$

$\sum x_i^2 + 2\sum x_i = 8n$ 　　　　............(i)

$\sum\limits_{i=1}^{n}(x_i - 1)^2 = 5n \quad \Rightarrow \quad \sum x_i^2 - 2\sum x_i + n = 5n$

$\Rightarrow \quad \sum x_i^2 - 2\sum x_i = 4n$ 　　　　............(ii)

from (i) and (ii) 　　　(समीकरण (i) व (ii) से)

$\sum x_i^2 = 6n$ and $\sum x_i = n$

hence standard deviation 　　　(अतः मानक विचलन)

= $\sqrt{\dfrac{\sum x_i^2}{n} - \left(\dfrac{\sum x_i}{n}\right)^2} = \sqrt{\dfrac{6n}{n} - \left(\dfrac{n}{n}\right)^2} = \sqrt{5}$

15. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is : **[JEE(Main) 10-01-19 P-1, (4, – 1), 120]**
पाँच प्रेक्षणों का माध्य 5 है तथा उनका प्रसरण 9.20 है। यदि इन दिए गए पाँच प्रेक्षणों में से तीन 1, 3 तथा 8 हैं, तो अन्य दो प्रेक्षणों का एक अनुपात है:

(1) 4 : 9 　　　(2) 6 : 7 　　　(3) 5 : 8 　　　(4) 10 : 3

Ans. **(1)**

Sol. $\mu = \dfrac{1+3+8+x+y}{5}$

$25 = 12 + x + y \Rightarrow x + y = 13$ 　　　.... (1)

$\sigma^2 = \dfrac{\sum(x_i - \mu)^2}{N}$

$9.2 = \dfrac{1+9+64+x^2+y^2}{5} - 25$

$34.2 \times 5 = 74 + x^2 + y^2$

$171 = 74 + x^2 + y^2$

$97 = x^2 + y^2$ ...(2)

$(x + y)^2 = x^2 + y^2 + 2xy$

$169 - 97 = 2xy \Rightarrow xy = 36$

$T = 4,9$

So ratio is अतः अनुपात $\dfrac{4}{9}$ or $\dfrac{9}{4}$

16. If mean and standard deviation of 5 observations $x_1, x_2, x_3, x_4, x_5,$ are 10 and 3, respectively, then the variance of 6 observations $x_1, x_2, ...x_5$ and –50 is equal to : **[JEE(Main) 10-01-19 P-2, (4, – 1), 120]**

यदि पाँच प्रेक्षणों x_1, x_2, x_3, x_4, x_5 का माध्य तथा मानक विचलन क्रमशः 10 तथा 3 है, तो 6 प्रेक्षणों $x_1, x_2, ...x_5$ तथा –50 का प्रसरण है :

(1) 586.5 (2) 507.5 (3) 582.5 (4) 509.5

Ans. **(2)**

Sol. $\dfrac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 10 \Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 50$ ...(1)

$\dfrac{\sum x_1^2}{5} - (\bar{x})^2 = 9$

$\dfrac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}{5} - 100 = 9$

$x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = 545$ ...(2)

$\bar{x}_{new} = \dfrac{x_1 + x_2 + x_3 + x_4 + x_5 - 50}{6} = 0$

variance new नया प्रसरण $= \dfrac{\sum\limits_{i-1}^{6} x_1^2}{6} - (\bar{x}_{new})^2$

$= \dfrac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + 2500}{6} - 0$

$= \dfrac{545 + 2500}{6} = \dfrac{3045}{6} = 507.5$

17. The outcome of each of 30 items was observed; 10 items gave an outcome $\dfrac{1}{2} - d$ each, 10 items gave outcome $\dfrac{1}{2}$ each and the remaining 10 items gave outcome $\dfrac{1}{2} + d$ each. If the variance of this outcome data is $\dfrac{4}{3}$ then |d| equals : **[JEE(Main) 11-01-19 P-1, (4, – 1), 120]**

30 आइटम (items) का परिणाम देखा गया; इनमें से 10 आइटम में प्रत्येक ने परिणाम $\dfrac{1}{2} - d$ दिया, 10 आइटम में प्रत्येक ने परिणाम $\dfrac{1}{2}$ दिया तथा बाकी 10 आइटम में प्रत्येक ने परिणाम $\dfrac{1}{2} + d$ दिया। यदि इन आँकड़ों का प्रसरण $\dfrac{4}{3}$ है, तो |d| बराबर है -

(1) $\sqrt{2}$ (2) $\dfrac{2}{3}$ (3) 2 (4) $\dfrac{\sqrt{5}}{2}$

Ans. **(1)**

Sol. $\sigma^2 = \dfrac{\Sigma x^2}{N} - \mu^2$

$$= \dfrac{10\left(\dfrac{1}{2}+d\right)^2 + 10 \times \dfrac{1}{4} + 10\left(\dfrac{1}{2}-d\right)^2}{30} - \left(\dfrac{10\left(\dfrac{1}{2}+d\right) + 10x\dfrac{1}{2} + 10\left(\dfrac{1}{2}-d\right)}{30}\right)^2$$

$$= \dfrac{20\left(\dfrac{1}{4}+d^2\right)+\dfrac{5}{2}}{30} - \dfrac{1}{9}\left(\dfrac{3}{2}\right)^2 = \dfrac{5+20d^2+\dfrac{5}{2}}{30} - \dfrac{1}{4} = \dfrac{\dfrac{3}{2}+4d^2}{6} - \dfrac{1}{4} = \dfrac{3+8d^2}{12} - \dfrac{1}{4} = \dfrac{2d^2}{3} = \dfrac{4}{3}$$

$\Rightarrow |d| = \sqrt{2}$

18. If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observations is :

यदि 50 प्रेक्षणों के 30 से विचलनों (deviations) का योग 50 है, तो इन प्रेक्षणों का माध्य है–

[JEE(Main) 12-01-19 P-1, (4, – 1), 120]

(1) 50　　　　　(2) 51　　　　　(3) 31　　　　　(4) 30

Ans. **Bonus or (3)**

Sol. It should be bonus as deviations means $|x_i – 30|$ and not $x_i – 30$. But if we take algebraic sum then the solution is

यह प्रश्न बोनस होना चाहिए क्योंकि विचलनों का अर्थ $|x_i – 30|$ है जबकि $x_i – 30$ नहीं है। परन्तु यदि हम बीजिय योग लेते है तब हल है।

Given दिया गया $\displaystyle\sum_{i=1}^{50}(x_i - 30) = 50$

$\Rightarrow \displaystyle\sum x_i = 30(50) + 50 \Rightarrow \dfrac{\displaystyle\sum x_i}{50} = 31$

19. The mean and the variance of five observation are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4 ; then the absolute value of the difference of the other two observations, is

[JEE(Main) 12-01-19 P-2, (4, – 1), 120]

पांच प्रेक्षणों का माध्य तथा प्रसरण क्रमशः 4 तथा 5.20 है। यदि इन प्रेक्षणों में से तीन 3, 4 तथा 4 है, तो अन्य दो प्रेक्षणों के अन्तर का निरपेक्ष (absolute) मान है–

(1) 1　　　　　(2) 3　　　　　(3) 5　　　　　(4) 7

Ans. **(4)**

Sol. $4 = \dfrac{3+4+4+x_1+x_2}{5} \Rightarrow x_1 + x_2 = 9$

$5.2 = \dfrac{\Sigma x_i^2}{5} - \mu^2 \Rightarrow 5.2 = \dfrac{9+16+16+x_1^2+x_2^2}{5} - 16$

$\Rightarrow x_1^2 + x_2^2 = 65 \Rightarrow 81 - 2x_1 x_2 = 65 \Rightarrow x_1 x_2 = 8$

$(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1 x_2 = 81 - 32 = 49$

$\Rightarrow |x_1 - x_2| = 7$

Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

JEE(Main) Pattern Practice paper (30 SCQ, 1 hr, 120 Marks).

This Section is not meant for classroom discussion. It is being given to promote self-study and self testing amongst the Resonance students.

Max. Marks : 120   **Max. Time : 1 Hr.**

Important Instructions :

1. The test is of **1 hour** duration and max. marks 120.
2. The test consists **30** questions, **4 marks** each.
3. Only one choice is correct **1 mark** will be deducted for incorrect response. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instructions 3 above.

Max. Marks : 120   **Max. Time : 1 Hr.**

महत्त्वपूर्ण निर्देश :

1. परीक्षा की अवधि 1 घंटे है, तथा अधिकतम अंक **120** है।
2. इस परीक्षा पुस्तिका में **30** प्रश्न है। प्रत्येक **4** अंक का है।
3. केवल एक विकल्प सही है। प्रत्येक प्रश्न के गलत उत्तर के लिये **1** अंक लिया जायेगा। यदि उत्तर पुस्तिका में किसी प्रश्न का उत्तर नहीं दिया गया हो तो कुल प्राप्तांक से कोई कटौती नहीं कि जायेगी।
4. प्रत्येक प्रश्न का केवल एक ही सही उत्तर है। एक से अधिक उत्तर देने पर उसे गलत उत्तर माना जायेगा और उपरोक्त निर्देश **3** के अनुसार अंक काट लिये जायेंगे।

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1. The mean weight of 150 person in a group is 60 kg. The mean weight of men in the group is 70 kg and that of women is 55kg. Find the number of men.

   समूह में 150 व्यक्तियों का भारित माध्य 60 kg है। समूह में पुरूषों का माध्य भार 70 kg है, तथा औरतों का माध्य भार 55kg है, तब पुरूषों की संख्या है–

   (1*) 50   (2) 75   (3) 100   (4) 25

   Sol. Let the number of men be x

   माना पुरूषों की संख्या x है

   $$\frac{70x + 55(150 - x)}{150} = 60 \quad x = 50$$

2. Then mean of 11 observations is 25. If each observation is decreased by 5, the new mean will be

   11 प्रेक्षणों का माध्य 25 है। यदि प्रत्येक प्रेक्षण को 5 से घटाया जाता हैं तब नया माध्य होगा–

   (1) 25   (2) 30   (3*) 20   (4) 15

   Sol. $\bar{x}_{new} = \bar{x}_{old} - 5$

3.._ The marks of some students were listed out of a maximum 60. The standard deviation of marks was found to be 5. Subsequently the marks raised to a maximum of 100 and variance of new marks was calculeted .The new variance

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MAINSTTS-34

कुछ विद्यार्थियों के अंक, अधिकतम अंक 60 में से दिए गए है। अंको का मानक विचलन 5 पाया गया है। अंको को अधिकतम 100 तक बढ़ाया जाता है तथा नये अंक का प्रसरण ज्ञात किया जाता है। तब नया प्रसरण है –

(1) $\dfrac{25}{3}$ (2) $\dfrac{625}{3}$ (3*) $\dfrac{625}{9}$ (4) $\dfrac{15}{9}$

Sol. We known that $y_i = \dfrac{100}{60} x_i = \dfrac{5}{3} x_i$ so $h = \dfrac{5}{3}$ Thus $\sigma_y = |h|\, \sigma_x = \dfrac{5}{3} \times 5 = \dfrac{25}{3}$

so new variance $= \left(\dfrac{25}{3}\right)^2 = \dfrac{625}{9}$

हम जानते है कि $y_i = \dfrac{100}{60} x_i = \dfrac{5}{3} x_i$ इसलिए $h = \dfrac{5}{3}$ अतः $\sigma_y = |h|\, \sigma_x = \dfrac{5}{3} \times 5 = \dfrac{25}{3}$

इसलिए नया प्रसरण $= \left(\dfrac{25}{3}\right)^2 = \dfrac{625}{9}$

4. Range of data 13, 14, 19, 21, 17, 14, 14, 12 is
आंकड़ो 13, 14, 19, 21, 17, 14, 14, 12 का परिसर है–
(1) 7   (2) 14   (3*) 9   (4) 21

Sol. Range परिसर = 21 – 12 = 9

5. Variance of first 10 natural numbers is
प्रथम 10 प्राकृत संख्याओं का प्रसरण हैं–

(1) $\dfrac{133}{4}$ (2*) $\dfrac{33}{4}$ (3) 33   (4) $\dfrac{33}{2}$

Sol. variance प्रसरण $(x_i) = \dfrac{\sum x_i^2}{n} - \left(\dfrac{\sum x_i}{n}\right)^2$

$1^2 + 2^2 + 3^2 \ldots\ldots 10^2 = \dfrac{n(n+1)(2n+1)}{6} = \dfrac{10 \times 11 \times 21}{6} = 385$

$1 + 2 + 3 \ldots\ldots 10 = \dfrac{n(n+1)}{2} = \dfrac{(10)(11)}{2} = 55$

$\text{var}(xi) = \dfrac{385}{10} - \left(\dfrac{55}{10}\right)^2 = \dfrac{825}{100} = \dfrac{33}{4}$

6. Find the median of values 12, 17, 19, 8, 4, 23, 27
12, 17, 19, 8, 4, 23, 27 की माध्यिका है–
(1) 27   (2) 23   (3*) 17   (4) 18

Sol. 4, 8, 12, 17, 19, 23, 27, $\Rightarrow$ Median माध्यिका = 17

7. Find the mode of the data 3, 1, 1, 2, 3, 0, –3, 4, 1 2, 3, 3, 5
3, 1, 1, 2, 3, 0, –3, 4, 1 2, 3, 3, 5 का बहुलक है–
(1) 1   (2) 2   (3) 0   (4*) 3

Sol. Most frequent data = 3
अधिक बार आकड़ा = 3

8. Coefficient of range 5, 2, 4, 3, 8, 11, is
5, 2, 4, 3, 8, 11, के परिसर का गुणांक है–

(1*) $\dfrac{9}{13}$ (2) $\dfrac{1}{11}$ (3) $\dfrac{6}{10}$ (4) $\dfrac{6}{16}$

Sol. $\dfrac{L-S}{L+S} = \dfrac{11-2}{11+2} = \dfrac{9}{13}$

9. If difference between mean and mode is 3, the difference between mean and median is
   यदि माध्य और बहुलक का अंतर 3 है, तब माध्य और माध्यिका के मध्य अन्तर है–
   (1) 3 (2*) 1 (3) 4 (4) 2

Sol. Mode + 2 mean = 3 median
   (mean –3) + 2 mean = 3 median
   3(mean – median) = 3

Hindi. Mode + 2 माध्य = 3 माध्यिका
   (mean –3) + 2 माध्य = 3 माध्यिका
   3 (माध्य – माध्यिका) = 3

10 Mode of the data:
   आकड़ो का बहुलक

xi (variate) चर	frequency बारम्बारता
$^4C_1$	$^8C_0$
$^4C_2$	$^8C_2$
$^4C_3$	$^8C_4$
$^4C_4$	$^8C_6$

(1) $^4C_2$ (2) $^8C_2$ (3) $^8C_4$ (4*) $^4C_3$

Sol. Most frequent data
   सबसे अधिक आंकड़ा

11. If $\displaystyle\sum_{i=1}^{11}(x_i - 4) = 11$ and $\displaystyle\sum_{i=1}^{11}(x_i - 4)^2 = 44$ then find variance of $x_1, x_2, x_3 ..... x_{11}$.

   यदि $\displaystyle\sum_{i=1}^{11}(x_i - 4) = 11$ और $\displaystyle\sum_{i=1}^{11}(x_i - 4)^2 = 44$ तब $x_1, x_2, x_3 ..... x_{11}$. का प्रसरण है–

   (1) 4 (2*) 3 (3) 7 (4) 11

Sol. variance $(x_i - 4) = \text{var}(x_i) = \dfrac{44}{11} - \left(\dfrac{11}{11}\right)^2 = 3$

   प्रसरण $(x_i - 4) = \text{var}(x_i) = \dfrac{44}{11} - \left(\dfrac{11}{11}\right)^2 = 3$

12 Mean of 1, 4, 7, 10, 13...........n terms is
   1, 4, 7, 10, 13...........n का माध्य है–

   (1) (3n–1)n (2) (3n–1) $\dfrac{n}{2}$ (3*) $\dfrac{3n-1}{2}$ (4) (3n–1)

Sol. $S_n = \dfrac{n}{2}[2 + (n-1)3]$

   $\bar{x} = \dfrac{S_n}{n} = \dfrac{1}{2}[3n-1]$

13. If var(x_i) = λ then var(2x_i + 3) is

यदि var(x_i) = λ हो तो var($2x_i$ + 3) है–

(1) 2λ + 3   (2) 2λ²   (3*) 4λ   (4) 4λ + 9

Sol. variance प्रसरण (ax_i + b) = a^2 var(x_i)

14. If S. D. of $x_1, x_2, x_3.....x_4$ is 3 then S. D of $-4x_1, -4x_2..........-4x_n$ is

यदि $x_1, x_2, x_3.....x_4$ का मानक विचलन 3 है, तब $-4x_1, -4x_2..........-4x_n$ का मानक विचलन है–

(1) –12   (2*) 12   (3) –6   (4) 6

Sol. variance प्रसरण (ax_i) = a^2var(x_i)

S.D. (ax_i) = |a| $\sqrt{\text{var}(x_i)}$

15. Consider the following statement and choose correct option

माना कि निम्न कथन और सही विकल्प को चुनिए–

(I) variance can not be negative

प्रसरण ऋणात्मक नही हो सकता है।

(ii) S.D can not be negative

मानक विचलन ऋणात्मक नहीं हो सकता है।

(III) Median is influenced by extreme value in set of numbers.

संख्याओं के समुच्चय में माध्यिका, चरम मानों से परिवर्तित होती है।

(1) TTT   (2) FTT   (3) FTF   (4*) TTF

Sol. Variance प्रसरण (x_i) = $\dfrac{\sum\limits_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

16. The mean and variance of 7 observation are 7 and $\dfrac{100}{7}$. If 5 of the observation are 2, 4, 7, 11, 10, find the remaining 2 observations.

7 प्रेक्षणों का माध्य और प्रसरण क्रमशः 7 और $\dfrac{100}{7}$ है, यदि प्रेक्षण के 5 चर 2, 4, 7, 11, 10, है तब शेष 2 प्रेक्षणों को ज्ञात कीजिए।

(1) 3, 6   (2*) 3, 12   (3) 4, 11   (4) 5, 10

Sol. $\dfrac{\sum x_i}{7}$ = 7

2+ 4 + 7 + 11 + 10 + a + b = 49   ⇒   a + b = 15

$\dfrac{\sum(x_i - \bar{x})^2}{7} = \dfrac{100}{7}$

⇒   25 + 9 + 0 + 16 + 99 + $(7 - a)^2$ + $(7 - b)^2$ = 100

⇒ $(7 - a)^2$ + $(7 - b)^2$ = 41

⇒   a = 3

         b = 12

17. The mean of distribution is 6, If coefficient of variation is 50%, then standard deviation of distribution is

वितरण का माध्य 6 है, यदि चरिता का गुणांक 50% है, तब वितरण का मानक विचलन है–

(1) 9   (2*) 3   (3) 300   (4) 4

Sol. $\dfrac{\sigma}{\bar{x}} \times 100$ = coefficient of variation   (प्रसरण गुणांक) ⇒ σ = 3

18. The mean deviation about median of variation 53, 54, 55......100 is

चरों 53, 54, 55......100 की माध्यिका के सापेक्ष माध्य विचलन है–

(1) 11.5   (2*) 12   (3) 12.5   (4) 13

Sol. Median माध्यिका $= \dfrac{24^{th}\,term + 25^{th}\,term}{2} = \dfrac{76 + 77}{2} = 76.5$

mean deviation माध्य विचलन $= \dfrac{(23.5 + 22.5 ...... 0.5 + 0.5 + ... 23.5)}{48} = 12$

19. The mean of two samples of sizes 20 and 10 were found to be 11, 8 respectively. Their variance were 4 and 34 respectively. Find the variance of combined sample of size 30.

आकार 20 और 10 के दो नमूनों का माध्य क्रमशः 11 और 8 पाये जाते है। यदि प्रसरण क्रमशः 4 और 34 है तब आकार 30 के संयुक्त नमूने का प्रसरण है–

(1) 19   (2) 19.5   (3) 18.5   (4*) 16

Sol. $\sum x_i = 20 \times 11 = 220$ $\Rightarrow$ $\sum y_i = 10 \times 8 = 80$

variance प्रसरण $(x_i) = \dfrac{\sum x_i^2}{20} - (121)$ $\Rightarrow$ $2500 = \sum x_i^2$

variance प्रसरण $(y_i) = \dfrac{\sum y_i^2}{10} - 64$ $\Rightarrow$ $980 = \sum y_i^2$

$\sum x_i^2 + \sum y_i^2 = \left(x_i^2 + y_i^2\right) = 3480$ $\Rightarrow \sum x_i + \sum y_i = \dfrac{\sum x_i^2 + y_i^2}{30} - \left(\dfrac{\sum x_i + y_i}{30}\right)^2 = 116 - 100 = 16$

20. If standard deviation of 1, 2, 3, 4, 5 is $\sqrt{2}$ then which of the following is correct.

यदि 1, 2, 3, 4, 5 का मानक विचलन $\sqrt{2}$ है। तब निम्न में से कौनसा सही है–

(1) standard deviation of 1, 4, 9, 16, 25, is 2

1, 4, 9, 16, 25, का मानक विचलन 2 है।

(2) standard deviation of 1001, 1002, 1003, 1004, 1005 is $\sqrt{2000}$

1001, 1002, 1003, 1004, 1005 का मानक विचलन $\sqrt{2000}$ है।

(3*) standard deviation of 1001, 1002, 1003, 1004, 1005 is $\sqrt{2}$

1001, 1002, 1003, 1004, 1005 का मानक विचलन $\sqrt{2}$ है।

(4) standard deviation of 1, 8, 27, 16, 25 is $\sqrt{2}$

1, 8, 27, 16, 25 का मानक विचलन $\sqrt{2}$ है।

Sol. Standard deviation is independent of change of origin bu not scale.

मानक विचलन, मूल बिन्दू के परिवर्तन से स्वतंत्र है परन्तु पैमाने से नहीं–

21. The mean and variance of 100 numbers were calculated as 11 and 2 respectively. Later it was found that one of the number was misread 5 instead of 9. How does the variance change.

100 संख्याओं का माध्य और प्रसरण क्रमशः 11 व 2 है, बाद में पाया गया है कि संख्याओं को गलती से 9 की जगह 5 पढ़ा गया। तब प्रसरण कैसे बदलता है –

(1) Variance doesn't change   (2) Variance Increases

(3*) Variance decreases   (4) Can't comment

(1) प्रसरण परिवर्तित नही होता।   (2) प्रसरण बढ़ता है।

(3*) प्रसरण घटता है।   (4) टिप्पणी नही कर सकते।

Sol. Actual data is more close to mean, therefore less variance

सही आंकड़ा, माध्य से सबसे नजदीक है इसलिए प्रसरण कम है।

22 The variance of first 5 even natural numbers is

प्रथम 5 सम प्राकृत संख्याओं का प्रसरण है–

(1) 6   (2) 7   (3*) 8   (4) 9

Sol. Variations) प्रसरण $= \dfrac{2^2 + 4^2 + 6^2 + 8^2 + 10^2}{5} - \left(\dfrac{2 + 4 + 6 + 8 + 10}{5}\right)^2 = 44 - 36 = 8$

23 If variance of x_1, x_2, x_3, x_4, x_5 is σ^2, find variance of $3x_1 + 4$, $3x_2 + 4$, $3x_3 + 4, 3x_4 + 4, 3x_5 + 4$

यदि x_1, x_2, x_3, x_4, x_5 का प्रसरण σ^2 है तब $3x_1 + 4$, $3x_2 + 4$, $3x_3 + 4, 3x_4 + 4, 3x_5 + 4$ का प्रसरण है–

(1) $4\sigma^2 + 3$ (2) $4\sigma^2 + 9$ (3*) $9\sigma^2$ (4) $4\sigma^2 - 3$

Sol. $\text{Var}(3x_i + 4) = 9\sigma^2$

24. Find the mean deviation about median of 34, 38, 42, 55, 63, 46, 54, 44, 70, 48

34, 38, 42, 55, 63, 46, 54, 44, 70, 48 माध्यिका के सापेक्ष माध्य विचलन है–

(1) 8.2   (2) 8.4   (3*) 8.6   (4) 8.8

Sol. Arranging the data in ascending order 34, 38, 42, 44, 46, 48, 54, 55, 63, 70

आंकड़ो का बढ़ते क्रम में 34, 38, 42, 44, 46, 48, 54, 55, 63, 70

median माध्यिका $= \dfrac{46 + 48}{2} = 47$

$\sum |xi - 47| = 9 + 23 + 1 + 13 + 5 + 8 + 16 + 1 + 7 + 3 = 86.$

Mean deviation मानक विचलन $= \dfrac{\sum |xi - 47|}{10} = 8.6$

25. Variance of first n natural numbers.

प्रथम n प्राकृत संख्याओं का प्रसरण है–

(1) $\dfrac{n^2 - 1}{24}$ (2*) $\dfrac{n^2 - 1}{12}$ (3) $\dfrac{n^2 - 1}{6}$ (4) $\dfrac{n^2 - 1}{3}$

Sol. $\bar{x} = \dfrac{n(n+1)}{2n} = \dfrac{n+1}{2} \Rightarrow \sigma^2 = \left(\dfrac{\sum\limits_{i=1}^{n} x_i^2}{n}\right) - \left(\dfrac{\sum ni}{n}\right)^2 = \dfrac{n(n+1)(2n+1)}{6n} - \left(\dfrac{(n+1)}{2}\right)^2 = \dfrac{n^2 - 1}{12}$

26. In a batch of 20 students 8 have failed. The marks of the successful candidates are 23, 27, 29, 18, 17,
19,   21, 27, 20, 24, 26, 28 the median marks are

20 विद्यार्थियों के बेच में 8 फेल होते है, यदि सफलता पूर्वक विद्यार्थियों के अंक 23, 27, 29, 18, 17, 19,1,27, 20, 24, 26, 28 है, तब माध्यिका अंक है–

(1) 22   (2) 18

(3*) 18.5

(4) can't determine ज्ञात नहीं कर सकते

Sol. Arrange marks in ascending order

बढ़ते क्रम में अंक

(...............), 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29

8 boys failed लड़के फेल होते है

median माध्यिका $= \dfrac{18 + 19}{2} = 18.5$

27. The coefficient of variation of two series are 60% and 70% if their standard deviation are 21 and 14, then find ratio of their AMs

दो श्रेणियों का चरिता गुणांक 60% और 70% है। यदि उनके मानक विचलन 21 तथा 14 है, तब उनके माध्यों का अनुपात ज्ञात कीजिए–

(1) $\dfrac{6}{7}$ (2) $\dfrac{2}{3}$ (3) $\dfrac{4}{7}$ (4*) $\dfrac{7}{4}$

Sol. $\dfrac{\sigma_1}{\bar{x}_1} = 0.6$; $\dfrac{\sigma_2}{\bar{x}_2} = 0.7$

$\dfrac{\sigma_1}{\sigma_2} = \dfrac{0.6 \times \bar{x}_1}{0.7 \times \bar{x}_2} \Rightarrow \dfrac{21}{14} \times \dfrac{0.7}{0.6} = \dfrac{\bar{x}_1}{\bar{x}_2} \Rightarrow \dfrac{7}{4} = \dfrac{\bar{x}_1}{\bar{x}_2}$

28. The mean of 2 samples of sizes 50 & 40 were found to be 63 and 54. Their variance were 81 & 36. Find the variance of combined sample of size 90

आकार 50 और 40 के 2 नमूनों का माध्य 63 और 54 पाया गया है, इसका प्रसरण 81 और 36 है। आकार 90 के संयुक्त नमूने का प्रसरण ज्ञात कीजिए।

(1) 9 (2*) 81 (3) 3 (4) 243

Sol. $\sum x_i = 63 \times 50 = 3150$; $\sum y_i = 40 \times 54 = 2160$

$var(x_i) = 81 = \dfrac{\sum x_i^2}{50} - (63)^2$, $var(y_i) = 36 = \dfrac{\sum y_i^2}{40} - (54)^2$

$\sum x_i^2 = 202500$

$\sum y_i^2 = 118080$

combined variance संयुक्त प्रसरण $= \dfrac{\left(\sum x_i^2 + \sum y_i^2\right)}{90} - \left(\dfrac{\sum x_i + \sum y_i}{90}\right)^2$

$= \dfrac{320580}{90} - (59)^2$

$= 3562 - 3481$

$= 81$

29. The mean and median of some data is 14 and 12 . Later it was discovered that every data element should be increased by 2 units then new mean and median will be

कुछ आकड़ों का माध्य और माध्यिका 14 और 12 है। बाद में यह पाया गया है कि प्रत्येक अवयव को 2 से बढ़ाया जाता है तब नया माध्य और माध्यिका होगी।

(1) 16, 12 (2*) 16, 14 (3) 14, 12 (4) 10, 8

Sol. Mean and median both will be increased by 2

माध्य और माध्यिका, 2 से बढ़ायी जाती है।

30. Rohan worked for a firm as given below

रोहन एक कम्पनी के लिए निम्न प्रकार कार्य करता है–

No. of weeks	Days each week he worked
2 weeks	1 day each week
14 weeks	2 day each week
8 weeks	5 day each week
32 weeks	7 day each week

सप्ताह की संख्या	सप्ताह के प्रत्येक दिन के कार्य
2 सप्ताह	1 प्रत्येक सप्ताह में एक दिन
14 सप्ताह	2 प्रत्येक सप्ताह में एक दिन
8 सप्ताह	5 प्रत्येक सप्ताह में एक दिन
32 सप्ताह	7 प्रत्येक सप्ताह में एक दिन

What is the mean number of days rohan works per week

प्रत्येक सप्ताह पर रोहन के कार्य के दिनों की संख्या का माध्य है–

(1) 5 (2) 6 (3) 5.5 (4*) 5.25

Sol. $\dfrac{(2\times1)+(14\times2)+(8\times5)+(32\times7)}{2+14+8+32} = \dfrac{294}{56} = 5.25$

Practice Test (JEE-Main Pattern)
OBJECTIVE RESPONSE SHEET (ORS)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

Practice Questions: 20-50 depending on chapter length.

1._ If a variate X is expressed as a linear function of two variates U and V in the form X = a U + b V, then mean $\bar{X}$ of X is

यदि चर X, दो चर U और V के रेखीय फलन है जिसका रूप X = a U + b V तब X का माध्य $\bar{X}$ है।

(1*) $a\bar{U} + b\bar{V}$ (2) $\bar{U}+\bar{V}$ (3) $a\bar{U} + a\bar{U}$ (4) None of these

Sol. We have हम जानते है : $\Sigma X = a \Sigma U + b\Sigma V$. Therefore इसलिए,

$$\bar{X} = \frac{1}{n}\sum X = a.\frac{1}{n}\sum U + b.\frac{1}{n}\sum V = a\bar{U} + b\bar{V}$$

2._ The AM of n numbers of a series is $\bar{X}$. If the sum of first (n – 1) terms is k, then the nth number is

श्रेणी की n संख्याओं का समान्तर माध्य $\bar{X}$ है। यदि प्रथम (n – 1) पदों का योगफल k है तब nवीं संख्या है –

(1) $\bar{X} - k$ (2*) $n\bar{X} - k$ (3) $\bar{X} - nk$ (4) $\dfrac{\bar{X}}{3}$

Sol. Let the n-numbers be $x_1, x_2 , ..., x_n$ then,

माना n-संख्याऐं $x_1, x_2 , ..., x_n$ तब

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\Rightarrow \bar{X} = \frac{x_1 + x_2 + ...x_{n-1} + x_n}{n}$$

$$\Rightarrow \overline{X} = \frac{k + x_n}{n} \qquad\qquad [\because x_1 + x_2 + ... x_{n-1} = k]$$

$$\Rightarrow x_n = n - \overline{X} k$$

3._ If $\overline{X}_1$ and $\overline{X}_2$ are the means of two distributions such that $\overline{X}_1 < \overline{X}_2$ and $\overline{X}$ is the mean of the combined distribution, then

यदि $\overline{X}_1$ और $\overline{X}_2$ दो वितरणों के माध्य इस प्रकार है कि $\overline{X}_1 < \overline{X}_2$ और $\overline{X}$ संयुक्त वितरण का माध्य है

(1) $\overline{X} < \overline{X}_1$ 　　　　(2) $\overline{X} > \overline{X}_2$ 　　　　(3) $\overline{X} = \dfrac{\overline{X}_1 + \overline{X}_2}{2}$ 　　　　(4*) $\overline{X}_1 < \overline{X} < \overline{X}_2$

Sol. Let n_1 and n_2 be the number of observations in two groups having means $\overline{X}_1$ and $\overline{X}_2$ respectively. Then

माना n_1 और n_2 दो समूहों में प्रेक्षणों की संख्या है जिनके माध्य $\overline{X}_1$ और $\overline{X}_2$ क्रमशः है तब

$$\overline{X} = \frac{n_1 \overline{X} + n_2 \overline{X}_2}{n_1 + n_2}$$

Now अब, $\overline{X} - \overline{X}_1 \qquad = \dfrac{n_1 \overline{X}_1 + n_2 \overline{X}_2}{n_1 + n_2} - \overline{X}$

$$= \frac{n_2 (\overline{X}_2 - \overline{X}_1)}{n_1 + n_2} > 0 \qquad\qquad [\because \overline{X}_2 - \overline{X}_1 \,]$$

$$\Rightarrow \overline{X}_1 - \overline{X}_2 \qquad\qquad\qquad\qquad .... (i)$$

And और, $\overline{X} - \overline{X}_2 = \dfrac{n(\overline{X}_1 - \overline{X}_2)}{n_1 + n_2} < 0 \qquad\qquad [\because \overline{X}_2 > \overline{X}_1 \,]$

$$\Rightarrow \overline{X} < \overline{X}_2 \qquad\qquad\qquad\qquad .... (ii)$$

From (i) and और (ii) से $\overline{X}_1 < \overline{X} < \overline{X}_2$.

4. Coefficient of variation of two distribution are 50% and 60% and their arithmetic means are 30 and 25 respectively. Difference of their standard deviations is

दो वितरणों के चरिता के गुणांक 50% तथा 60% है तथा उनके समान्तर माध्य क्रमशः 30 और 25 है। तब उनके मानक विचलों का अन्तर है।

(a*) 0 　　　　(B) 1 　　　　(C) 1.5 　　　　(D) 2.5

Sol. माना Let σ_1 and और σ_2 be the standard deviations of the two data, then दो आकड़ों का मानक विचलन है।

$$\frac{50}{100} = \frac{\sigma_1}{30} \text{ and और } \frac{60}{100} = \frac{\sigma_2}{25}$$

$$\Rightarrow \sigma_1 = 15 \text{ and और } \sigma_2 = 15$$

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