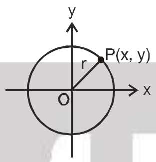
CIRCLE

Four circles to the kissing come, The smaller are the benter. The bend is just the inverse of The distance from the centre. Through their intrigue left Euclid dumb There's now no need for rule of thumb. Since zero bend's a dead straight line And concave bends have minus sign, The sum of squares of all four bends Is half the square of their sum. .... Soddy, Frederick

A circle is a locus of a point in a plane whose distance from a fixed point (called centre) is always constant (called radius).

Equation of a circle in various forms :

(a) The circle with centre as origin & radius 'r' has the equation; $x^2 + y^2 = r^2$.



(b) The circle with centre (h, k) & radius 'r' has the equation; $(x - h)^2 + (y - k)^2 = r^2$.

(c) The general equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$
with centre as (–g, –f) & radius = $\sqrt{g^2 + f^2 - c}$.

This can be obtained from the equation $(x - h)^2 + (y - k)^2 = r^2$

$\Rightarrow$ $x^2 + y^2 - 2hx - 2ky + h^2 + k^2 - r^2 = 0$

Take $-h = g, -k = f$ and $h^2 + k^2 - r^2 = c$

Condition to define circle :-

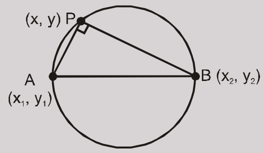
$g^2 + f^2 - c > 0$ $\Rightarrow$ real circle.

$g^2 + f^2 - c = 0$ $\Rightarrow$ point circle.

$g^2 + f^2 - c < 0$ $\Rightarrow$ imaginary circle, with real centre, that is (– g, – f)

Note : That every second degree equation in x & y, in which coefficient of x^2 is equal to coefficient of y^2 & the coefficient of xy is zero, always represents a circle.

(d) The equation of circle with (x_1, y_1) & (x_2, y_2) as extremeties of its diameter is:
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.



This is obtained by the fact that angle in a semicircle is a right angle.

$\therefore$ (Slope of PA) (Slope of PB) = – 1

$\Rightarrow \dfrac{y - y_1}{x - x_1} \cdot \dfrac{y - y_2}{x - x_2} = -1 \Rightarrow (x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Note that this will be the circle of least radius passing through (x_1, y_1) & (x_2, y_2).

Example # 1 Find the equation of the circle whose centre is (0, 3) and radius is 3.

Solution. The equation of the circle is $(x - 0)^2 + (y - 3)^2 = 3^2$

$\Rightarrow$ $x^2 + y^2 - 6y = 0$

Example # 2 Find the equation of the circle which passes through(1, –1) and two of its dimeter are $x + 2y - 5 = 0$ and $x - y + 1 = 0$

Solution. Let P be the point of intersection of the lines

$x + 2y - 5 = 0$ ....(i)

and $x - y + 1 = 0$ ....(ii)

Solving (i) and (ii), we get x = 1, y = 2. So, coordinates of centre are (1, 2). since circle passes through (1,–1) so

radius = $\sqrt{(1-1)^2+(2+1)^2}$ $\Rightarrow$ radius = 3

Hence the equation of the required circle is $(x - 1)^2 + (y + 2)^2 = 9$

Example # 3 If the equation $ax^2 + (b - 3)xy + 3y^2 + 6ax + 2by - 3 = 0$ represents the equation of a circle then find a, b

Solution. $ax^2 + (b - 3)xy + 3y^2 + 6ax + 2by - 3 = 0$

above equation will represent a circle if

coefficient of x^2 = coefficient of y^2

a = 3

coefficient of xy = 0

b = 3

Example # 4 Find the equation of a circle whose diametric end points are (x_1, y_1) and (x_2, y_2) where x_1, x_2 are the roots of $x^2 - ax + b = 0$ and y_1, y_2 are the roots of $y^2 - by + a = 0$.

Solution. We know that the equation of the circle described on the line segment joining (x_1, y_1) and (x_2, y_2) as a diameter is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.

$x^2 + y^2 - (x_1 + x_2)x - (y_1 + y_2)y + x_1x_2 + y_1y_2 = 0$

Here, $x_1 + x_2 = a$, $x_1x_2 = b$

$y_1 + y_2 = b$, $y_1y_2 = a$

So, the equation of the required circle is

$x^2 + y^2 - ax - by + a + b = 0$

Self practice problems :

(1) Find the equation of the circle passing through the point of intersection of the lines $x + 3y = 0$and $2x - 7y = 0$ and whose centre is the point of intersection of the lines $x + y + 1 = 0$ and $x - 2y + 4 = 0$.

(2) Find the equation of the circle whose centre is (1, 2) and which passes through the point (4, 6)

(3) Find the equation of a circle whose radius is 6 and the centre is at the origin.

Answers :

(1) $x^2 + y^2 + 4x - 2y = 0$ (2) $x^2 + y^2 - 2x - 4y - 20 = 0$ (3) $x^2 + y^2 = 36$.

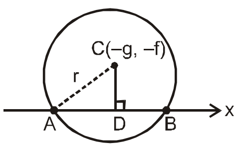
Intercepts made by a circle on the axes:

The intercepts made by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ on the co–ordinate axes are $2\sqrt{g^2-c}$ (on x-axis) & $2\sqrt{f^2-c}$ (on y-axis) respectively.

If $g^2 > c$ $\Rightarrow$ circle cuts the x axis at two distinct points.

$g^2 = c$ $\Rightarrow$ circle touches the x–axis.

$g^2 < c$ $\Rightarrow$ circle lies completely above or below the x–axis.



$AB = 2AD = 2\sqrt{r^2 - CD^2} = 2\sqrt{r^2 - f^2} = 2\sqrt{g^2 + f^2 - c - f^2} = 2\sqrt{g^2 - c}$

Example # 5 Find the locus of the centre of the circle whose x and y intercepts are a and b respectively.

Solution. Equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

x intercept = a

$2\sqrt{g^2 - c} = a \qquad g^2 - c = \frac{a^2}{4}$ ...... (i)

y intercept = b

$2\sqrt{f^2 - c} = b \qquad f^2 - c = \frac{b^2}{4}$ ...... (ii)

subtracting equation (i) and (ii)

$g^2 - f^2 = \frac{a^2 - b^2}{4}$

hence locus of centre is $x^2 - y^2 = \frac{a^2 - b^2}{4}$

Self practice problems :

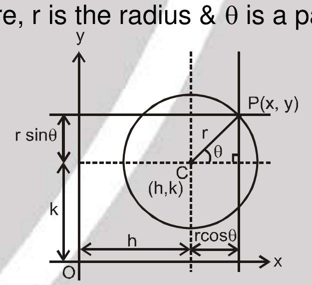
(4) Find the equation of a circle which touches the positive axis of y at a distance 3 from the origin and intercepts a distance 6 on the axis of x.

(5) Find the equation of a circle which touches positive y-axis at a distance of 2 units from the origin and cuts an intercept of 3 units with the positive direction of x-axis.

Answers : (4) $x^2 + y^2 \pm 6\sqrt{2}\ x - 6y + 9 = 0$ (5) $x^2 + y^2 - 5x - 4y + 4 = 0$

Parametric equations of a circle:

The parametric equations of $(x - h)^2 + (y - k)^2 = r^2$ are: $x = h + r\cos\theta$; $y = k + r\sin\theta$; $-\pi < \theta \le \pi$ where (h, k) is the centre, r is the radius & θ is a parameter.



Example # 6 Find the parametric equations of the circle $x^2 + y^2 + 4x + 6y + 9 = 0$

Solution. We have : $x^2 + y^2 + 4x + 6y + 9 = 0$

$\Rightarrow \quad (x + 2)^2 + (y + 3)^2 = 2^2$

So, the parametric equations of this circle are

$x = -2 + 2\cos\theta$, $y = -3 + 2\sin\theta$.

Example # 7 Find the equation of the following curve in cartesian form

$x + y = \cos\theta$, $x - y = \sin\theta$ where θ is the parameter.

Solution. We have : $x + y = \cos\theta$ .... (i)

$x - y = \sin\theta$ .... (ii)

$(i)^2 + (ii)^2$

$\Rightarrow \quad (x + y)^2 + (x - y)^2 = 1$

$x^2 + y^2 = \frac{1}{2}$

Clearly, it is a circle with centre at (0, 0) and radius $\frac{1}{\sqrt{2}}$.

Self practice problems :

(6) Find the parametric equations of circle $x^2 + y^2 - 6x + 4y - 12 = 0$

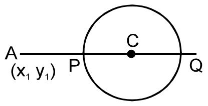
(7) Find the cartesian equations of the curve $x = 1 + \sqrt{2}\cos\theta$, $y = 2 - \sqrt{2}\sin\theta$

Answers : (6) $x = 3 + 5\cos\theta$, $y = -2 + 5\sin\theta$ (7) $(x - 1)^2 + (y - 2)^2 = 2$

Position of a point with respect to a circle:

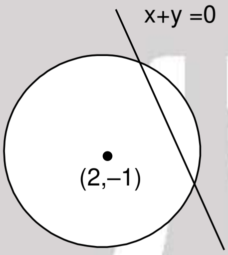
The point (x_1, y_1) is inside, on or outside the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$.
according as $S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c <, = \text{ or } > 0$.

Note : The greatest & the least distance of a point A (lies outside the circle) from a circle with centre C & radius r is AC + r & AC − r respectively.



Example # 8 Check whether the point (1, 2) lies in smaller or larger region made by circle $x^2 + y^2 - 4x + 2y - 11 = 0$ and the line $x + y = 0$

Solution : We have $x^2 + y^2 - 4x + 2y - 11 = 0$ or $S = 0$,



where $S = x^2 + y^2 - 4x + 2y - 11$.
For the point (1, 2), we have $S_1 = 1^2 + 2^2 - 4 \times 1 + 2 \times 2 - 11 < 0$
Hence, the point (1, 2) lies inside the circle
Points (1,2) and (2, –1) lie on same side of the line $x + y = 0$
Hence the point (1,2) lies in the larger region.

Self practice problem :

(8) How are the points (0, 1) (3, 1) and (1, 3) situated with respect to the circle $x^2 + y^2 - 2x - 4y + 3 = 0$?

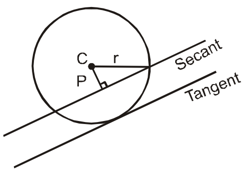
Answer : (8) (0, 1) lies on the circle ; (3, 1) lies outside the circle ; (1, 3) lies inside the circle.

Line and a circle:

Let L = 0 be a line & S = 0 be a circle. If r is the radius of the circle & p is the length of the perpendicular from the centre on the line, then:

(i) $p > r \Leftrightarrow$ the line does not meet the circle i. e. passes out side the circle.
(ii) $p = r \Leftrightarrow$ the line touches the circle. (It is tangent to the circle)
(iii) $p < r \Leftrightarrow$ the line is a secant of the circle.
(iv) $p = 0 \Rightarrow$ the line is a diameter of the circle.

Also, if $y = mx + c$ is line and $x^2 + y^2 = a^2$ is circle then
(i) $c^2 < a^2 (1 + m^2) \Leftrightarrow$ the line is a secant of the circle.
(ii) $c^2 = a^2 (1 + m^2) \Leftrightarrow$ the line touches the circle. (It is tangent to the circle)
(iii) $c^2 > a^2 (1 + m^2) \Leftrightarrow$ the line does not meet the circle i. e. passes out side the circle.



These conditions can also be obtained by solving $y = mx + c$ with $x^2 + y^2 = a^2$ and making the discriminant of the quadratic greater than zero for secant, equal to zero for tangent and less the zero for the last case.

Example # 9 For what value of λ, does the line $x + y = \lambda$ touch the circle $x^2 + y^2 - 2x - 2y = 0$

Solution. We have : $x + y = \lambda$ ......(i) and $x^2 + y^2 - 2x - 2y = 0$....... (ii)

If the line (i) touches the circle (ii), then

length of the $\perp$ from the centre (1, 1) = radius of circle (ii)

$$\Rightarrow \left|\frac{1+1-\lambda}{\sqrt{1^2+1^2}}\right| = \sqrt{2} \quad \Rightarrow \quad |2-\lambda| = 2 \quad \Rightarrow \quad \lambda = 0 \text{ or } 4$$

Hence, the line (i) touches the circle (ii) for $\lambda = 0$ or 4

Self practice problem :

(9) Find the range of values of m for which the line $y = mx + 2$ cuts the circle $x^2 + y^2 = 1$ at distinct points

Answers : (9) $m \in (-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

Slope form of tangent :

$y = mx + c$ is always a tangent to the circle $x^2 + y^2 = a^2$ if $c^2 = a^2(1 + m^2)$. Hence, of tangent is $y = mx \pm a\sqrt{1+m^2}$ and the point of contact is $\left(-\frac{a^2m}{c}, \frac{a^2}{c}\right)$.

Point form of tangent :

(i) The equation of the tangent to the circle $x^2 + y^2 = a^2$ at its point (x_1, y_1) is, $x x_1 + y y_1 = a^2$.

(ii) The equation of the tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at its point (x_1, y_1) is : $xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$.

Note : In general the equation of tangent to any second degree curve at point (x_1, y_1) on it can be obtained by replacing x^2 by $x x_1$, y^2 by yy_1, x by $\frac{x + x_1}{2}$, y by $\frac{y + y_1}{2}$,

xy by $\frac{x_1y + xy_1}{2}$ and c remains as c.

Parametric form of tangent :

The equation of a tangent to circle $x^2 + y^2 = a^2$ at $(a\cos\alpha, a\sin\alpha)$ is $x\cos\alpha + y\sin\alpha = a$.

NOTE : The point of intersection of the tangents at the points $P(\alpha)$ & $Q(\beta)$ is $\left(\frac{a\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}, \frac{a\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}\right)$

Example # 10 Find the equation of the tangent to the circle $x^2 + y^2 - 2x - 2y - 11 = 0$ at (3, 4).

Solution. Equation of tangent is

$$3x + 4y - 2\left(\frac{x+3}{2}\right) - 2\left(\frac{y+4}{2}\right) - 11 = 0$$

or $2x + 3y - 18 = 0$

Hence, the required equation of the tangent is $2x + 3y - 18 = 0$

Example # 11 Find the equation of tangents to the circle $x^2 + y^2 - 4x + 2y = 0$ which are perpendicular to the line $x + 2y + 4 = 0$

Solution. Given circle is $x^2 + y^2 - 4x + 2y = 0$ ....(i)

and given line is $x + 2y + 4 = 0$ ....(ii)

Centre of circle (i) is (2,–1)) and its radius $\sqrt{5}$ is Equation of any line

$2x - y + k = 0$ perpendicular to the line (ii) ....(iii)

If line (iii) is tangent to circle (i) then

$$\frac{|4+1+k|}{\sqrt{5}} = \sqrt{5} \quad \text{or} \quad |k + 5| = 5 \quad \text{or} \quad k = 0, -10$$

Hence equation of required tangents are $2x - y = 0$ and $2x - y - 10 = 0$

Self practice problem :

(10) Find the equation of the tangents to the circle $x^2 + y^2 - 2x - 4y - 4 = 0$ which are

(i) parallel,

(ii) perpendicular to the line $3x - 4y - 1 = 0$

Answer.

(10) (i) $3x - 4y + 20 = 0$ and $3x - 4y - 10 = 0$ (ii) $4x + 3y + 5 = 0$ and $4x + 3y - 25 = 0$

Normal :

If a line is normal / orthogonal to a circle, then it must pass through the centre of the circle. Using this fact normal to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) is; $y - y_1 = \frac{y_1 + f}{x_1 + g}(x - x_1)$.

Example # 12 Two normals of a circle are $2x + 3y = 5$ and $3x - 4y + 1 = 0$. Find its equation having radius 2

Solution. Since point of intersection of normals is the centre of the circle

point of intersection of lines $2x + 3y = 5$ and $3x - 4y + 1 = 0$ is (1,1)

equation of circle having centre (1,1) and radius 2 is

$(x - 1)^2 + (y - 1)^2 = 4$

Self practice problem :

(11) Find the equation of the normal to the circle $x^2 + y^2 - 2x - 4y + 3 = 0$ at the point (2, 3).

Answer : (11) $x - y + 1 = 0$

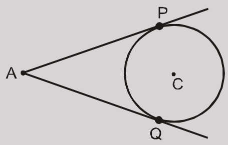
Pair of tangents from a point :

The equation of a pair of tangents drawn from the point A (x_1, y_1) to the circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ is : $SS_1 = T^2$.

Where $S \equiv x^2 + y^2 + 2gx + 2fy + c$; $S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

$T \equiv xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c$.



Example # 13 Find the equation of the pair of tangents drawn to the circle $x^2 + y^2 + 4x - 6y + 9 = 0$ from thepoint (2, 1)

Solution. Given circle is $S = x^2 + y^2 + 4x - 6y + 9 = 0$

Let $P \equiv (2, 1)$

For point P, $S_1 = 16$

Clearly P lies outside the circle

and $T \equiv 2x + y + 2(x + 2) - 3(y + 1) + 9 = 0$

i.e $T \equiv 2(2x - y + 5)$

Now equation of pair of tangents from P(2, 1) to circle (1) is $SS_1 = T^2$

or $16(x^2 + y^2 + 4x - 6y + 9) = 4(2x - y + 5)^2$ or $12y^2 - 16x - 56y + 16xy + 44 = 0$

or $3y^2 - 4x - 14y + 4xy + 11 = 0$

Self practice problems :

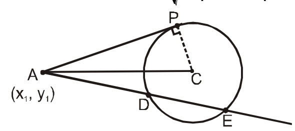
(12) Find the joint equation of the tangents through (7, 1) to the circle $x^2 + y^2 = 25$.

Answer : (12) $12x^2 - 12y^2 + 7xy - 175x - 25y + 625 = 0$

Length of a tangent and power of a point :

The length of a tangent from an external point (x_1, y_1) to the circle

$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ is given by $L = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2f_1y + c} = \sqrt{S_1}$.



AP = length of tangent
$AP^2 = AD \cdot AE$

Square of length of the tangent from the point A is also called the power of point w.r.t. a circle.
Power of a point w.r.t. a circle remains constant.
Power of a point P is positive, negative or zero according as the point 'A' is outside, inside or on the circle respectively.

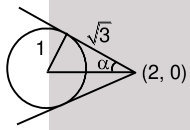
Example # 14 Find the angle between the tangents drawn from the point (2, 0) to the circle $x^2 + y^2 = 1$

Solution. Given circle is $x^2 + y^2 = 1$ ....(i)

Given point is (2, 0).

Now length of the tangent from (2, 0) to circle (i) = $\sqrt{2^2 + 0^2 - 1} = \sqrt{3}$

$\tan\alpha = \frac{1}{\sqrt{3}}$



$\alpha = \frac{\pi}{6}$

so angle between tangents = $2\alpha = \frac{\pi}{3}$

Self practice problems :

(13) The lenght of tangents from P(1, –1) & Q(3, 3) to a circle are $\sqrt{2}$ and $\sqrt{6}$ respectively. Then find the lenght of tangent from R (– 1, – 5) to the same circle.

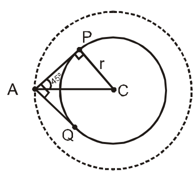
(14) Find the lenght of tangent drawn from any point on circle $x^2 + y^2 + 4x + 6y - 3 = 0$ to the circle $x^2 + y^2 + 4x + 6y + 4 = 0$.

Answer. (13) $\sqrt{38}$ (14) $\sqrt{7}$

Director circle :

The locus of the point of intersection of two perpendicular tangents is called the director circle of the given circle. The director circle of a circle is the concentric circle having radius equal to times the original circle.

Proof :



$AC = r \text{ cosec } 45^\circ = r\sqrt{2}$

Example # 15 Find the equation of director circle of the circle $x^2 + y^2 + 6x + 8y - 2 = 0$

Solution : Centre & radius of given circle are (–3, –4) & $\sqrt{27}$ respectively.

Centre and radius of the director circle will be (–3, –4) & $\sqrt{27}.\sqrt{2} = \sqrt{54}$ respectively.

$\therefore$ equation of director circle is $(x + 3)^2 + (y + 4)^2 = 54$

$\Rightarrow$ $x^2 + y^2 + 6x + 8y - 29 = 0$

Self practice problems :

(15) Find the angle between the tangents drawn from $(5, \sqrt{7})$ to the circle $x^2+y^2 = 16$

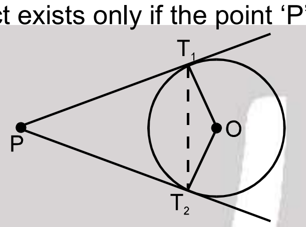
Answer (15) $\frac{\pi}{2}$

Chord of contact :

If two tangents PT_1 & PT_2 are drawn from the point $P(x_1, y_1)$ to the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, then the equation of the chord of contact T_1T_2 is:
$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$.

Note : Here R = radius; L = length of tangent.

(a) Chord of contact exists only if the point 'P' is not inside.



(b) Length of chord of contact $T_1T_2 = \frac{2LR}{\sqrt{R^2 + L^2}}$.

(c) Area of the triangle formed by the pair of the tangents & its chord of contact $= \frac{RL^3}{R^2 + L^2}$

(d) Tangent of the angle between the pair of tangents from $(x_1, y_1) = \left(\frac{2RL}{L^2 - R^2}\right)$

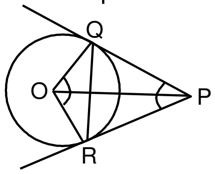
(e) Equation of the circle circumscribing the triangle PT_1T_2 is:
$(x - x_1)(x + g) + (y - y_1)(y + f) = 0$.

Example # 16 Find the equation of the chord of contact of the tangents drawn from (0, 1) to the circle $x^2 + y^2 - 2x + 4y = 0$

Solution. Given circle is $x^2 + y^2 - 2x + 4y = 0$ ....(i)
Let P = (0, 1)
For point P (0, 1), $T = x . 0 + y . 1 - (x + 0) + 2(y + 1)$
i.e. $T = x - 3y - 2$
Now equation of the chord of contact of point P(0, 1) w.r.t. circle (i) will be
$x - 3y - 2 = 0$

Example # 17 If the chord of contact of the tangents drawn from (α, β) to the circle $x^2 + y^2 = a^2$ subtends right angle at the centre then prove that $\alpha^2 + \beta^2 = 2a^2$.

Solution. $\angle QOR = \angle QPR = \frac{\pi}{2}$
so OQPR is a square



$OQ^2 = PQ^2$
$a^2 = \alpha^2 + \beta^2 - a^2$
$\alpha^2 + \beta^2 = 2a^2$

Self practice problems :

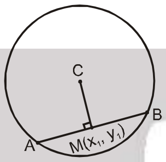
(16) Find the co-ordinates of the point of intersection of tangents at the points where the line $x - 2y + 1 = 0$ meets the circle $x^2 + y^2 = 25$

(17) If the chord of contact of the tangetns drawn from a point on circle $x^2 + y^2 = a^2$ to another circle $x^2 + y^2 = b^2$ touches the circle $x^2 + y^2 = c^2$ then prove that a,b,c are in G.P.

Answers : (16) (–25, 50) (17) $\frac{405\sqrt{3}}{52}$ sq. unit ; $4x + 6y - 25 = 0$

Equation of the chord with a given middle point:

The equation of the chord of the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ in terms of its mid point $M(x_1, y_1)$ is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ which is designated by $T = S_1$.



Notes : (i) The shortest chord of a circle passing through a point 'M' inside the circle is one chord whose middle point is M.

(ii) The chord passing through a point ' M ' inside the circle and which is at a maximum distance from the centre is a chord with middle point M.

Example # 18 Find the equation of the chord of the circle $x^2 + y^2 + 2x - 2y - 4 = 0$, whose middle point is (0, 0)

Solution. Equation of given circle is $S \equiv x^2 + y^2 + 2x - 2y - 4 = 0$

Let $L \equiv (0, 0)$

For point L(0, 0), $S_1 = -4$ and

$T \equiv x.0 + y(0) + (x + 0) - (y + 0) - 4$ i.e. $T \equiv x - y - 4$

Now equation of the chord of circle (i) whose middle point is L(0, 0) is

$T = S_1$ or $x - y = 0$

Second Method : Let C be the centre of the given circle, then $C \equiv (-1, 1)$. $L \equiv (0, 0)$ slope of CL = –1

$\therefore$ Equation of chord of circle whose middle point is L, is $y - 0 = 1(x - 0)$

($\because$ chord is perpendicular to CL) or $x - y = 0$

Self practice problems :

(18) Find the equation of that chord of the circle $x^2 + y^2 = 15$, which is bisected at (3, 2)

(19) A variable chord is drawn through the origin to the circle $x^2 + y^2 - 2ax = 0$. Find the locus of the centre of the circle drawn on this chord as diameter.

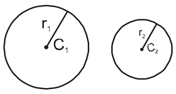
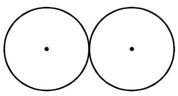
Answers : (18) $3x + 2y - 13 = 0$ (19) $x^2 + y^2 - ax = 0$

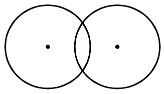
Equation of the chord joining two points of circle :

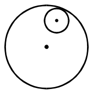
The equation of chord PQ to the circle $x^2 + y^2 = a^2$ joining two points $P(\alpha)$ and $Q(\beta)$ on it is given by the equation of a straight line joining two point α & β on the circle $x^2 + y^2 = a^2$ is

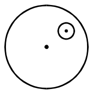
$$x \cos\frac{\alpha+\beta}{2} + y \sin\frac{\alpha+\beta}{2} = a \cos\frac{\alpha-\beta}{2}.$$

Common tangents to two circles:

	Case	Number of Tangents	Condition
(i)		4 common tangents (2 direct and 2 transverse)	$r_1 + r_2 < c_1 c_2$.
(ii)		3 common tangents.	$r_1 + r_2 = c_1 c_2$.

(iii)  2 common tangents. $|r_1 - r_2| < c_1 c_2 < r_1 + r_2$

(iv)  1 common tangent. $|r_1 - r_2| = c_1 c_2$.

(v)  No common tangent. $c_1 c_2 < |r_1 - r_2|$.

(Here C_1C_2 is distance between centres of two circles.)

Notes : (i) The direct common tangents meet at a point which divides the line joining centre of circles externally in the ratio of their radii.
Transverse common tangents meet at a point which divides the line joining centre of circles internally in the ratio of their radii.

(ii) Length of an external (or direct) common tangent & internal (or transverse) common tangent to the two circles are given by:
$L_{ext} = \sqrt{d^2 - (r_1 - r_2)^2}$ & $L_{int} = \sqrt{d^2 - (r_1 + r_2)^2}$, where d = distance between the centres of the two circles and r_1, r_2 are the radii of the two circles. Note that length of internal common tangent is always less than the length of the external or direct common tangent.

Example # 19 Examine if the two circles $x^2 + y^2 - 4x - 6y + 9 = 0$ and $x^2 + y^2 - 10x - 6y + 18 = 0$ intersect or not

Solution. Given circles are $x^2 + y^2 - 4x - 6y + 9 = 0$ ....(i)
and $x^2 + y^2 - 10x - 6y + 18 = 0$ ....(ii)
Let A and B be the centres and r_1 and r_2 the radii of circles (i) and (ii) respectively, then
$A \equiv (2, 3)$, $B \equiv (5, 3)$, $r_1 = 2$, $r_2 = 4$
Now AB = 3 and $r_1 + r_2 = 6$, $|r_1 - r_2| = 2$
Thus $|r_1 - r_2| < AB < r_1 + r_2$, hence the two circles intersect.

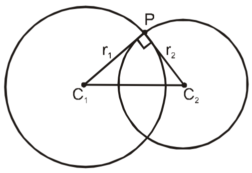
Self practice problems :

(20) Find the position of the circles $x^2 + y^2 - 10x + 4y - 20 = 0$ and $x^2 + y^2 + 14x - 6y + 22 = 0$ with respect to each other.

Answer : (20) touch externally

Orthogonality of two circles:

Two circles $S_1 = 0$ & $S_2 = 0$ are said to be orthogonal or said to intersect orthogonally if the tangents at their point of intersection include a right angle. The condition for two circles to be orthogonal is:
$2 g_1 g_2 + 2 f_1 f_2 = c_1 + c_2$.
Proof :
$(C_1C_2)^2 = (C_1P)^2 + (C_2P)^2$
$\Rightarrow \quad (g_1 - g_2)^2 + (f_1 - f_2)^2 = g_1^2 + f_1^2 - c_1 + g_2^2 + f_2^2 - c_2$
$\Rightarrow \quad 2g_1g_2 + 2f_1f_2 = c_1 + c_2$



Notes :
(a) The centre of a variable circle orthogonal to two fixed circles lies on the radical axis of two circles.
(b) If two circles are orthogonal, then the polar of a point 'P' on first circle w.r.t. the second circle passes through the point Q which is the other end of the diameter through P. Hence locus of a point which moves such that its polars w.r.t. the circles $S_1 = 0$, $S_2 = 0$ & $S_3 = 0$ are concurrent in a circle which is orthogonal to all the three circles.

(c) The centre of a circle which is orthogonal to three given circles is the radical centre provided the radical centre lies outside all the three circles.

Example # 20 If the circles $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $2x^2 + 2y^2 + 2g_2x + 2f_2y + c_2 = 0$ are orthogonal to each other then prove that $g_1g_2 + f_1f_2 = c_1 + \frac{c_2}{2}$

Solution. Given circles are $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ ....(i)

and $2x^2 + 2y^2 + 2g_2x + 2f_2y + c_2 = 0$

or $x^2 + y^2 + g_2x + f_2y + \frac{c_2}{2} = 0$ ....(ii)

Since circles (i) and (ii) cut orthogonally

$\therefore \quad 2g_1\left(\frac{g_2}{2}\right) + 2f_1\left(\frac{f_2}{2}\right) = c_1 + \frac{c_2}{2}$

$g_1g_2 + f_1f_2 = c_1 + \frac{c_2}{2}$

Self practice problems :

(21) For what value of λ the circles $x^2 + y^2 + 8x + 3y + 9 = 0$ and $x^2 + y^2 + 2x - y - \lambda = 0$ cut orthogonally.

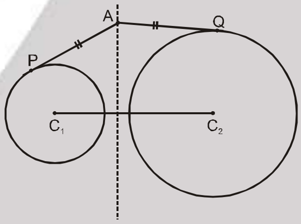
(22) Find the equation to the circle which passes through the origin and has its centre on the line $x - y = 0$ and cuts the circle $x^2 + y^2 - 4x - 6y + 10 = 0$ orthogonally.

Answer : (21) $\frac{5}{2}$ (22) $x^2 + y^2 - 2x - 2y = 0$

Radical axis and radical centre:

The radical axis of two circles is the locus of points whose powers w.r.t. the two circles are equal. The equation of radical axis of the two circles $S_1 = 0$ & $S_2 = 0$ is given by

$S_1 - S_2 = 0$ i.e. $2(g_1 - g_2)x + 2(f_1 - f_2)y + (c_1 - c_2) = 0$.



The common point of intersection of the radical axes of three circles taken two at a time is called the radical centre of three circles. Note that the length of tangents from radical centre to the three circles are equal.

Notes :

(a) If two circles intersect, then the radical axis is the common chord of the two circles.

(b) If two circles touch each other, then the radical axis is the common tangent of the two circlesat the common point of contact.

(c) Radical axis is always perpendicular to the line joining the centres of the two circles.

(d) Radical axis will pass through the mid point of the line joining the centres of the two circles only if the two circles have equal radii.

(e) Radical axis bisects a common tangent between the two circles.

(f) A system of circles, every two which have the same radical axis, is called a coaxial system.

(g) Pairs of circles which do not have radical axis are concentric.

Example # 21 Find the co-ordinates of the point from which the lengths of the tangents to the following three circles be equal.

$x^2 + y^2 = 1$

$x^2 + y^2 - 8x + 15 = 0$

$x^2 + y^2 + 10y + 24 = 0$

Solution : Here we have to find the radical centre of the three circles. First reduce them to standard form in which coefficients of x^2 and y^2 be each unity. Subtracting in pairs the three radical axes are

$x = 2$; $8x + 10y + 9 = 0$

$10y + 25 = 0$

solving any two, we get the point $\left(2, -\frac{5}{2}\right)$ which satisfies the third also. This point is called the radical centre and by definition the length of the tangents from it to the three circles are equal.

Self practice problem :

(23) Find the point from which the tangents to the three circles $x^2 + y^2 - 4x + 7 = 0$, $2x^2 + 2y^2 - 3x + 5y + 9 = 0$ and $x^2 + y^2 + y = 0$ are equal in length. Find also this length.

Answer : (23) (2, – 1) ; 2.

Family of Circles:

This article is aimed at obtaining the equation of a group of circles having a specific characteristic. For example, the equation $x^2 + y^2 + 4x + 2y + \lambda = 0$ where λ is arbitrary, represents a family of circles with fixed centre (–2, –1) but variable radius. We have the following results for some other families of circles.

(a) The equation of the family of circles passing through the points of intersection of two circles $S_1 = 0$ & $S_2 = 0$ is : $S_1 + K S_2 = 0$
($K \neq -1$, provided the co–efficient of x^2 & y^2 in S_1 & S_2 are same)

(b) The equation of the family of circles passing through the point of intersection of a circle S = 0 & a line L = 0 is given by S + KL = 0.

(c) The equation of a family of circles passing through two given points (x_1, y_1) & (x_2, y_2) can be written in the form:

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + K \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0, \text{ where K is a parameter.}$$

(d) The equation of a family of circles touching a fixed line $y - y_1 = m(x - x_1)$ at the fixed point (x_1, y_1) is $(x - x_1)^2 + (y - y_1)^2 + K(y - y_1 - m(x - x_1)) = 0$, where K is a parameter.

(e) Family of circles circumscribing a triangle whose sides are given by $L_1 = 0$, $L_2 = 0$ and $L_3 = 0$ is given by; $L_1L_2 + \lambda L_2L_3 + \mu L_3L_1 = 0$ provided co–efficient of xy = 0 and co–efficient of x^2 = co–efficient of y^2.

(f) Equation of circle circumscribing a quadrilateral whose side in order are represented by the lines $L_1 = 0$, $L_2 = 0$, $L_3 = 0$ & $L_4 = 0$ are $u L_1L_3 + \lambda L_2L_4 = 0$ where values of u & λ can be found out by using condition that co–efficient of x^2 = co–efficient of y^2 and co–efficient of xy = 0.

Example # 22 Find the equation of the circle passing through the point (1, 1) and points of intersection of the circles $x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$.

Solution. Any circle through the intersection of given circles is $S_1 + \lambda S_2 = 0$

or $x^2 + y^2 + 13x - 3y + \lambda(x^2 + y^2 + 2x - 7y/2 - 25/2) = 0$

This circle passes through (1, 1)

$1 + 1 + 13 - 3 + \lambda(1 + 1 + 2 - 7/2 - 25/2) = 0$

$\lambda = 1$

Putting the value of λ in (i) the required circle is $4x^2 + 4y^2 + 30x - 13y - 25 = 0$

Example # 23 Find the equations of smallest circle which passes through the points of intersection of the line x + y = 1 and the circle $x^2 + y^2 = 9$.

Solution. The required circle by $S + \lambda L = 0$ is

$x^2 + y^2 - 9 + \lambda(x + y - 1) = 0$ ....(i) centre (– g, – f) = $\left(-\frac{\lambda}{2}, -\frac{\lambda}{2}\right)$

centre lies on the line x + y = 1

$-\frac{\lambda}{2} - \frac{\lambda}{2} = 1$

$\lambda = -1$

Putting the value of λ in (i) the required circle is

$x^2 + y^2 - x - y - 8 = 0$

Example # 24 Find the equation of circle passing through the points A(1, 1) & B(0, 3) and whose radius is $\sqrt{\frac{5}{2}}$.

Solution. Equation of AB is 2x + y – 3 = 0

$\therefore$ equation of circle is

$(x - 1)(x) + (y - 1)(y - 3) + \lambda(2x + y - 3)$

$= 0$ or $x^2 + y^2 + (2\lambda - 1)x + (\lambda - 4)y + 3 - 3\lambda = 0$

$\sqrt{\left(\frac{2\lambda - 1}{2}\right)^2 + \left(\frac{\lambda - 4}{2}\right) + 3\lambda - 3} = \sqrt{\frac{5}{2}}$

$\lambda = 1$

$\therefore$ equation of circle is $x^2 + y^2 + x - 3y = 0$

Example # 25 A variable circle always touhces x + y = 2 at (1, 1), cuts the circle $x^2 + y^2 + 4x + 5y - 6 = 0$. Prove that all common chords pass through a fixed point. Also find the point.

Solution : Equation of circle is $(x - 1)^2 + (y - 1)^2 + \lambda(x + y - 2) = 0$

$x^2 + y^2 + x(\lambda - 2) + y(\lambda - 2) + 2 - 2\lambda = 0$

common chord of this circle with $x^2 + y^2 + 4x + 5y - 6 = 0$ is

$(\lambda - 6)x + (\lambda - 7)y + 8 - 2\lambda = 0$

$\lambda(x + y - 2) + (-6x - 7y + 8) = 0$

this chord passes through the point of intersection of the lines x + y – 2 = 0 and –6x – 7y + 8 = 0 which is (6, –4)

Example # 26 Find the equation of circle circumcscribing the triangle whose sides are 3x – y – 12 = 0, 5x – 3y – 28 = 0 & x + y – 4 = 0.

Solution : $L_1L_2 + \lambda L_2L_3 + \mu L_1L_3 = 0$

$(3x - y - 12)(5x - 3y - 28) + \lambda(5x - 3y - 28)(x + y - 4) + \mu(3x - y - 12)(x + y - 4) = 0$

coefficient of x^2 = coefficient of y^2

$\Rightarrow$ $5\lambda + 3\mu + 15 = 3 - 3\lambda - \mu$

$2\lambda + \mu + 3 = 0$ ....(ii)

coefficient of xy = 0

$\Rightarrow$ $\lambda + \mu - 7 = 0$ ....(iii)

Solving (ii) and (iii), we have

$\lambda = -10, \mu = 17$

Puting these values of λ & μ in equation (i), we get $2x^2 + 2y^2 - 9x + 11y + 4 = 0$

Self practice problems :

(24) Find the equation of the circle passing through the points of intersection of the circles $x^2 + y^2 - 6x + 2y + 4 = 0$ and $x^2 + y^2 + 2x - 4y - 6 = 0$ and with its centre on the line y = x.

(25) Find the equation of circle circumscribing the quadrilateral whose sides are x + y = 10, x – 7y + 50 = 0, 22x – 4y + 125 = 0 and 2x – 4y – 5

Answers : (24) $7x^2 + 7y^2 - 10x - 10y - 12 = 0$

(25) $x^2 + y^2 = \frac{125}{2}$

Exercise-1

✍ **Marked questions are recommended for Revision.**

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Equation of circle, parametric equation, position of a point

A-1. Find the equation of the circle that passes through the points (1, 0), (– 1, 0) and (0, 1).

A-2. ABCD is a square in first quadrant whose side is a, taking AB and AD as axes, prove that the equation to the circle circumscribing the square is $x^2 + y^2 = a(x + y)$.

A-3. Find the equation to the circle which passes through the origin and cuts off intercepts equal to 3 and 4 from the positive axes.

A-4. Find equation of circle which touches x & y axis & perpendicular distance of centre of circle from $3x + 4y + 11 = 0$ is 5. Given that circle lies in I[st] quadrant.

A-5.✍ Find the equation to the circle which touches the axis of x at a distance 3 from the origin and intercepts a distance 6 on the axis of y.

A-6. Find equation of circle whose cartesian equation are $x = -3 + 2 \sin \theta$, $y = 4 + 2 \cos \theta$

A-7. Find the values of p for which the power of a point (2, 5) is negative with respect to a circle $x^2 + y^2 - 8x - 12y + p = 0$ which neither touches the axes nor cuts them.

Section (B) : Line and circle, tangent, pair of tangent

B-1. If radii of the largest and smallest circle passing through the point (1, –1) and touching the circle $x^2 + y^2 + 2\sqrt{2}\, y - 2 = 0$ are r_1 and r_2 respectively, then find the sum of r_1 and r_2 .

B-2. Find the points of intersection of the line $x - y + 2 = 0$ and the circle $3x^2 + 3y^2 - 29x - 19y + 56 = 0$. Also determine the length of the chord intercepted.

B-3. Show that the line $7y - x = 5$ touches the circle $x^2 + y^2 - 5x + 5y = 0$ and find the equation of the other parallel tangent.

B-4. Find the equation of the tangents to the circle $x^2 + y^2 = 4$ which make an angle of 60º with the positive x-axis in anticlockwise direction.

B-5. Show that two tangents can be drawn from the point (9, 0) to the circle $x^2 + y^2 = 16$; also find the equation of the pair of tangents and the angle between them.

B-6. If the length of the tangent from (f, g) to the circle $x^2 + y^2 = 6$ be twice the length of the tangent from (f, g) to the circle $x^2 + y^2 + 3x + 3y = 0$, then will $f^2 + g^2 + 4f + 4g + 2 = 0$?

Section (C) : Normal, Director circle, chord of contact, chord with mid point

C-1. Find the equation of the normal to the circle $x^2 + y^2 = 5$ at the point (1, 2)

C-2. Find the equation of the normal to the circle $x^2 + y^2 = 2x$, which is parallel to the line $x + 2y = 3$.

C-3. Find the equation of director circle of the circle $(x + 4)^2 + y^2 = 8$

C-4.✍ Tangents are drawn from the point (h, k) to the circle $x^2 + y^2 = a^2$; prove that the area of the triangle formed by them and the straight line joining their points of contact is $\dfrac{a(h^2+k^2-a^2)^{3/2}}{h^2+k^2}$ c.

C-5. Find the equation of the chord of the circle $x^2 + y^2 + 6x + 8y + 9 = 0$ whose middle point is (– 2, – 3).

C-6.✍ Tangents are drawn to the circle $x^2 + y^2 = 12$ at the points where it is met by the circle $x^2 + y^2 - 5x + 3y - 2 = 0$; find the point of intersection of these tangents.

Section (D) : Position of two circle, Orthogonality, Radical axis and radical centre

D-1. Find the equations to the common tangents of the circles $x^2 + y^2 - 2x - 6y + 9 = 0$ and $x^2 + y^2 + 6x - 2y + 1 = 0$

D-2. Show that the circles $x^2 + y^2 - 2x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 4y - 6 = 0$ cut each other orthogonally.

D-3. Find the equation of the circle passing through the origin and cutting the circles $x^2 + y^2 - 4x + 6y + 10 = 0$ and $x^2 + y^2 + 12y + 6 = 0$ at right angles.

D-4. Given the three circles $x^2 + y^2 - 16x + 60 = 0$, $3x^2 + 3y^2 - 36x + 81 = 0$ and $x^2 + y^2 - 16x - 12y + 84 = 0$, find (1) the point from which the tangents to them are equal in length and (2) this length.

Section (E) : Family of circles , Locus, Miscellaneous

E-1. If $y = 2x$ is a chord of the circle $x^2 + y^2 - 10x = 0$, find the equation of a circle with this chord as diameter.

E-2. Find the equation of a circle which touches the line $2x - y = 4$ at the point $(1, -2)$ and
(i) Passes through (3, 4)
(ii) Radius = 5

E-3. Show that the equation $x^2 + y^2 - 2x - 2\lambda y - 8 = 0$ represents for different values of λ a system of circles passing through two fixed points A and B on the x-axis, and also find the equation of that circle of the system the tangent to which at A and B meet on the line $x + 2y + 5 = 0$.

E-4. Consider a family of circles passing through two fixed points A (3, 7) and B (6, 5). Show that the chords in which the circles $x^2 + y^2 - 4x - 3 = 0$ cuts the members of the family are concurrent at a point. Also find the co-ordinates of this point.

E-5. Find the equation of the circle circumscribing the triangle formed by the lines $x + y = 6$, $2x + y = 4$ and $x + 2y = 5$.

E-6. Prove that the circle $x^2 + y^2 + 2ax + c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$ touches each other if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Equation of circle, parametric equation, position of a point

A-1. The radius of the circle passing through the points (1, 2), (5, 2) & (5, – 2) is:

(A) $5\sqrt{2}$ (B) $2\sqrt{5}$ (C) $3\sqrt{2}$ (D) $2\sqrt{2}$

A-2. The centres of the circles $x^2 + y^2 - 6x - 8y - 7 = 0$ and $x^2 + y^2 - 4x - 10y - 3 = 0$ are the ends of the diameter of the circle
(A) $x^2 + y^2 - 5x - 9y + 26 = 0$ (B) $x^2 + y^2 + 5x - 9y + 14 = 0$
(C) $x^2 + y^2 + 5x - y - 14 = 0$ (D) $x^2 + y^2 + 5x + y + 14 = 0$

A-3. The circle described on the line joining the points (0, 1), (a, b) as diameter cuts the x–axis in points whose abscissa are roots of the equation:
(A) $x^2 + ax + b = 0$ (B) $x^2 - ax + b = 0$ (C) $x^2 + ax - b = 0$ (D) $x^2 - ax - b = 0$

A-4. The intercepts made by the circle $x^2 + y^2 - 5x - 13y - 14 = 0$ on the x-axis and y-axis are respectively
(A) 9, 13 (B) 5, 13 (C) 9, 15 (D) none

A-5. Equation of line passing through mid point of intercepts made by circle $x^2 + y^2 - 4x - 6y = 0$ on co-ordinate axes is
(A) $3x + 2y - 12 = 0$ (B) $3x + y - 6 = 0$ (C) $3x + 4y - 12 = 0$ (D) $3x + 2y - 6 = 0$

A-6. Two thin rods AB & CD of lengths 2a & 2b move along OX & OY respectively, when 'O' is the origin. The equation of the locus of the centre of the circle passing through the extremities of the two rods is:
(A) $x^2 + y^2 = a^2 + b^2$ (B) $x^2 - y^2 = a^2 - b^2$ (C) $x^2 + y^2 = a^2 - b^2$ (D) $x^2 - y^2 = a^2 + b^2$

A-7. Let A and B be two fixed points then the locus of a point C which moves so that (tan∠BAC) (tan ∠ABC)=1, $0 < \angle BAC < \frac{\pi}{2}$, $0 < \angle ABC < \frac{\pi}{2}$ is
(A) Circle (B) pair of straight line (C) A point (D) Straight line

A-8. **STATEMENT-1 :** The length of intercept made by the circle $x^2 + y^2 - 2x - 2y = 0$ on the x-axis is 2.
STATEMENT-2 : $x^2 + y^2 - \alpha x - \beta y = 0$ is a circle which passes through origin with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ and radius $\sqrt{\frac{\alpha^2+\beta^2}{2}}$
(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true

Section (B) : Line and circle, tangent, pair of tangent

B-1. Find the co-ordinates of a point p on line x + y = – 13, nearest to the circle $x^2 + y^2 + 4x + 6y - 5 = 0$
(A) (– 6, – 7) (B) (– 15, 2) (C) (– 5, – 6) (D) (– 7, – 6)

B-2. The number of tangents that can be drawn from the point (8, 6) to the circle $x^2 + y^2 - 100 = 0$ is
(A) 0 (B) 1 (C) 2 (D) none

B-3. Two lines through (2, 3) from which the circle $x^2 + y^2 = 25$ intercepts chords of length 8 units have equations
(A) 2x + 3y = 13, x + 5y = 17 (B) y = 3, 12x + 5y = 39
(C) x = 2, 9x – 11y = 51 (D) y = 0, 12x + 5y = 39

B-4. The line 3x + 5y + 9 = 0 w.r.t. the circle $x^2 + y^2 - 4x + 6y + 5 = 0$ is
(A) chord dividing circumference in 1 : 3 ratio (B) diameter
(C) tangent (D) outside line

B-5. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of the circle is
(A) 3 (B) 2 (C) 3/2 (D) 1

B-6. The tangent lines to the circle $x^2 + y^2 - 6x + 4y = 12$ which are parallel to the line 4x + 3y + 5 = 0 are given by:
(A) 4x + 3y – 7 = 0, 4x + 3y + 15 = 0 (B) 4x + 3y – 31 = 0, 4x + 3y + 19 = 0
(C) 4x + 3y – 17 = 0, 4x + 3y + 13 = 0 (D) 4x + 3y – 31 = 0, 4x + 3y – 19 = 0

B-7. The condition so that the line (x + g) cosθ + (y + f) sin θ = k is a tangent to $x^2 + y^2 + 2gx + 2fy + c = 0$ is
(A) $g^2 + f^2 = c + k^2$ (B) $g^2 + f^2 = c^2 + k$ (C) $g^2 + f^2 = c^2 + k^2$ (D) $g^2 + f^2 = c + k$

B-8. The tangent to the circle $x^2 + y^2 = 5$ at the point (1, –2) also touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$ at
(A) (–2, 1) (B) (–3, 0) (C) (–1, –1) (D) (3, –1)

B-9. The angle between the two tangents from the origin to the circle $(x - 7)^2 + (y + 1)^2 = 25$ equals
(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

B-10. A point A(2, 1) is outside the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ & AP, AQ are tangents to the circle. The equation of the circle circumscribing the triangle APQ is :
(A) (x + g) (x – 2) + (y + f) (y – 1) = 0 (B) (x + g) (x – 2) – (y + f) (y – 1) = 0
(C) (x – g) (x + 2) + (y – f) (y + 1) = 0 (D) (x – g) (x – 2) + (y – f) (y – 1) = 0

B-11. A line segment through a point P cuts a given circle in 2 points A & B, such that PA = 16 & PB = 9, find the length of tangent from points to the circle
(A) 7 (B) 25 (C) 12 (D) 8

B-12. The length of the tangent drawn from any point on the circle $x^2 + y^2 + 2gx + 2fy + p = 0$ to the circle $x^2 + y^2 + 2gx + 2fy + q = 0$ is:
(A) $\sqrt{q - p}$ (B) $\sqrt{p - q}$ (C) $\sqrt{q + p}$ (D) $\sqrt{2q + p}$

B-13. The equation of the diameter of the circle $(x - 2)^2 + (y + 1)^2 = 16$ which bisects the chord cut off by the circle on the line $x - 2y - 3 = 0$ is
(A) $x + 2y = 0$ (B) $2x + y - 3 = 0$ (C) $3x + 2y - 4 = 0$ (D) $3x - 2y - 4 = 0$

B-14. The locus of the point of intersection of the tangents to the circle $x^2 + y^2 = a^2$ at points whose parametric angles differ by $\frac{\pi}{3}$ is
(A) $x^2 + y^2 = \frac{4a^2}{3}$ (B) $x^2 + y^2 = \frac{2a^2}{3}$ (C) $x^2 + y^2 = \frac{a^2}{3}$ (D) $x^2 + y^2 = \frac{a^2}{9}$

Section (C) : Normal, Director circle, chord of contact, chord with mid point

C-1. The equation of normal to the circle $x^2 + y^2 - 4x + 4y - 17 = 0$ which passes through (1, 1) is
(A) $3x + y - 4 = 0$ (B) $x - y = 0$ (C) $x + y = 0$ (D) $3x - y - 4 = 0$

C-2. The normal at the point (3, 4) on a circle cuts the circle at the point (–1, –2). Then the equation of the circle is
(A) $x^2 + y^2 + 2x - 2y - 13 = 0$ (B) $x^2 + y^2 - 2x - 2y - 11 = 0$
(C) $x^2 + y^2 - 2x + 2y + 12 = 0$ (D) $x^2 + y^2 - 2x - 2y + 14 = 0$

C-3. The co-ordinates of the middle point of the chord cut off on $2x - 5y + 18 = 0$ by the circle $x^2 + y^2 - 6x + 2y - 54 = 0$ are
(A) (1, 4) (B) (2, 4) (C) (4, 1) (D) (1, 1)

C-4. The locus of the mid point of a chord of the circle $x^2 + y^2 = 4$ which subtends a right angle at the origin is:
(A) $x + y = 2$ (B) $x^2 + y^2 = 1$ (C) $x^2 + y^2 = 2$ (D) $x + y = 1$

C-5. The chords of contact of the pair of tangents drawn from each point on the line $2x + y = 4$ to the circle $x^2 + y^2 = 1$ pass through the point
(A) (1, 2) (B) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (C) (2, 4) (D) (4, 4)

C-6. The locus of the centers of the circles such that the point (2, 3) is the mid point of the chord $5x + 2y = 16$ is:
(A) $2x - 5y + 11 = 0$ (B) $2x + 5y - 11 = 0$ (C) $2x + 5y + 11 = 0$ (D) $2x - 5y - 11 = 0$

C-7. Find the locus of the mid point of the chord of a circle $x^2 + y^2 = 4$ such that the segment intercepted by the chord on the curve $x^2 - 2x - 2y = 0$ subtends a right angle at the origin.
(A) $x^2 + y^2 - 2x - 2y = 0$ (B) $x^2 + y^2 + 2x - 2y = 0$
(C) $x^2 + y^2 + 2x + 2y = 0$ (D) $x^2 + y^2 - 2x + 2y = 0$

Section (D) : Position of two circle, Orthogonality, Radical axis and radical centre

D-1. Number of common tangents of the circles $(x + 2)^2 + (y-2)^2 = 49$ and $(x - 2)^2 + (y + 1)^2 = 4$ is:
(A) 0 (B) 1 (C) 2 (D) 3

D-2. The equation of the common tangent to the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$ at their point of contact is
(A) $12x + 5y + 19 = 0$ (B) $5x + 12y + 19 = 0$ (C) $5x - 12y + 19 = 0$ (D) $12x - 5y + 19 = 0$

D-3. Equation of the circle cutting orthogonally the three circles $x^2 + y^2 - 2x + 3y - 7 = 0$, $x^2 + y^2 + 5x - 5y + 9 = 0$ and $x^2 + y^2 + 7x - 9y + 29 = 0$ is
(A) $x^2 + y^2 - 16x - 18y - 4 = 0$ (B) $x^2 + y^2 - 7x + 11y + 6 = 0$
(C) $x^2 + y^2 + 2x - 8y + 9 = 0$ (D) $x^2 + y^2 + 16x - 18y - 4 = 0$

D-4. If the length of a common internal tangent to two circles is 7, and that of a common external tangent is 11, then the product of the radii of the two circles is:
(A) 18 (B) 20 (C) 16 (D) 12

Section (E) : Family of circles , Locus, Miscellaneous

E-1. The locus of the centre of the circle which bisects the circumferences of the circles $x^2 + y^2 = 4$ & $x^2 + y^2 - 2x + 6y + 1 = 0$ is:
(A) a straight line (B) a circle (C) a parabola (D) pair of straight line

E-2. Equation of a circle drawn on the chord $x \cos \alpha + y \sin \alpha = p$ of the circle $x^2 + y^2 = a^2$ as its diameter, is
(A) $(x^2 + y^2 - a^2) - 2p\,(x\sin\alpha + y\cos\alpha - p) = 0$ (B) $(x^2 + y^2 - a^2) - 2p\,(x\cos\alpha + y\sin\alpha - p) = 0$
(C) $(x^2 + y^2 - a^2) + 2p\,(x\cos\alpha + y\sin\alpha - p) = 0$ (D) $(x^2 + y^2 - a^2) - p\,(x\cos\alpha + y\sin\alpha - p) = 0$

E-3. Find the equation of the circle which passes through the point (1, 1) & which touches the circle $x^2 + y^2 + 4x - 6y - 3 = 0$ at the point (2, 3) on it.
(A) $x^2 + y^2 + x - 6y + 3 = 0$ (B) $x^2 + y^2 + x - 6y - 3 = 0$
(C) $x^2 + y^2 + x + 6y + 3 = 0$ (D) $x^2 + y^2 + x - 3y + 3 = 0$

E-4. Find the equation of circle touching the line $2x + 3y + 1 = 0$ at (1, – 1) and cutting orthogonally the circle having line segment joining (0, 3) and (– 2, – 1) as diameter.
(A) $2x^2 + 2y^2 - 10x - 5y + 1 = 0$ (B) $2x^2 + 2y^2 - 10x + 5y + 1 = 0$
(C) $2x^2 + 2y^2 - 10x - 5y - 1 = 0$ (D) $2x^2 + 2y^2 + 10x - 5y + 1 = 0$

E-5. Equation of the circle which passes through the point (–1, 2) & touches the circle $x^2 + y^2 - 8x + 6y = 0$ at origin, is -
(A) $x^2 + y^2 - 2x - \frac{3}{2}y = 0$ (B) $x^2 + y^2 + x - 2y = 0$
(C) $x^2 + y^2 + 2x + \frac{3}{2}y = 0$ (D) $x^2 + y^2 + 2x - \frac{3}{2}y = 0$

E-6. Two circles are drawn through the point (a, 5a) and (4a, a) to touch the axis of 'y'. They intersect at an angle of θ then $\tan\theta$ is -
(A) $\frac{40}{9}$ (B) $\frac{9}{40}$ (C) $\frac{1}{9}$ (D) $\frac{1}{\sqrt{3}}$

PART - III : MATCH THE COLUMN

1.

Column – I		Column – II	
(A)	Number of values of a for which the common chord of the circles $x^2 + y^2 = 8$ and $(x - a)^2 + y^2 = 8$ subtends a right angle at the origin is	(p)	0
(B)	The number of circles touching all the three lines $3x + 7y = 2$, $21x + 49y = 5$ and $9x + 21y = 0$ are	(q)	2
(C)	The length of common chord of circles $x^2 + y^2 - x - 11y + 18 = 0$ and $x^2 + y^2 - 9x - 5y + 14 = 0$ is	(r)	5
(D)	Number of common tangents of the circles $x^2 + y^2 - 2x = 0$ and $x^2 + y^2 + 6x - 6y + 2 = 0$ is	(s)	3

2. **Column – I** | **Column – II**

	Column – I		Column – II
(A)	If director circle of two given circles C_1 and C_2 of equal radii touches each other, then ratio of length of internal common tangent of C_1 and C_2 to their radii equals to	(p)	13
(B)	Let two circles having radii r_1 and r_2 are orthogonal to each other. If length of their common chord is k times the square root of hormonic mean between squares of their radii, then k^4 equals to	(q)	7
(C)	The axes are translated so that the new equation of the circle $x^2 + y^2 - 5x + 2y - 5 = 0$ has no first degree terms and the new equation $x^2 + y^2 = \frac{\lambda^2}{4}$, then the value of λ is	(r)	4
(D)	The number of integral points which lie on or inside the circle $x^2 + y^2 = 4$ is	(s)	2

Exercise-2

PART - I : ONLY ONE OPTION CORRECT TYPE

1. If $\left(a, \frac{1}{a}\right), \left(b, \frac{1}{b}\right), \left(c, \frac{1}{c}\right)$ & $\left(d, \frac{1}{d}\right)$ are four distinct points on a circle of radius 4 units, then abcd is equal to:
(A) 4 (B) 16 (C) 1 (D) 2

2. From the point A (0, 3) on the circle $x^2 + 4x + (y - 3)^2 = 0$ a chord AB is drawn & extended to a point M such that AM = 2 AB. The equation of the locus of M is :
(A) $x^2 + 8x + y^2 = 0$ (B) $x^2 + 8x + (y - 3)^2 = 0$
(C) $(x - 3)^2 + 8x + y^2 = 0$ (D) $x^2 + 8x + 8y^2 = 0$

3. If tangent at (1, 2) to the circle c_1: $x^2 + y^2 = 5$ intersects the circle c_2: $x^2 + y^2 = 9$ at A & B and tangents at A & B to the second circle meet at point C, then the co–ordinates of C is
(A) (4, 5) (B) $\left(\frac{9}{15}, \frac{18}{5}\right)$ (C) (4, – 5) (D) $\left(\frac{9}{5}, \frac{18}{5}\right)$

4. A circle passes through point $\left(3, \sqrt{\frac{7}{2}}\right)$ touches the line pair $x^2 - y^2 - 2x + 1 = 0$. Centre of circle lies inside the circle $x^2 + y^2 - 8x + 10y + 15 = 0$. Co-ordinate of centre of circle is
(A) (4, 0) (B) (5, 0) (C) (6, 0) (D) (0, 4)

5. The length of the tangents from any point on the circle $15x^2 + 15y^2 - 48x + 64y = 0$ to the two circles $5x^2 + 5y^2 - 24x + 32y + 75 = 0$ and $5x^2 + 5y^2 - 48x + 64y + 300 = 0$ are in the ratio
(A) 1 : 2 (B) 2 : 3 (C) 3 : 4 (D) 2 : 1

6. The distance between the chords of contact of tangents to the circle; $x^2 + y^2 + 2gx + 2fy + c = 0$ from the origin & the point (g, f) is:
(A) $\sqrt{g^2 + f^2}$ (B) $\frac{\sqrt{g^2 + f^2 - c}}{2}$ (C) $\frac{g^2 + f^2 - c}{2\sqrt{g^2 + f^2}}$ (D) $\frac{\sqrt{g^2 + f^2 + c}}{2\sqrt{g^2 + f^2}}$

7. If from any point P on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, tangents are drawn to the circle $x^2 + y^2 + 2gx + 2fy + c\sin^2\alpha + (g^2 + f^2)\cos^2\alpha = 0$, then the angle between the tangents is:

(A) α (B) 2α (C) $\frac{\alpha}{2}$ (D) $\frac{\alpha}{3}$

8. The locus of the mid points of the chords of the circle $x^2 + y^2 + 4x - 6y - 12 = 0$ which subtend an angle of $\frac{\pi}{3}$ radians at its circumference is:

(A) $(x-2)^2 + (y+3)^2 = 6.25$ (B) $(x+2)^2 + (y-3)^2 = 6.25$
(C) $(x+2)^2 + (y-3)^2 = 18.75$ (D) $(x+2)^2 + (y+3)^2 = 18.75$

9. If the two circles, $x^2 + y^2 + 2g_1x + 2f_1y = 0$ & $x^2 + y^2 + 2g_2x + 2f_2y = 0$ touch each other then :

(A) $f_1 g_1 = f_2 g_2$ (B) $\frac{f_1}{g_1} = \frac{f_2}{g_2}$ (C) $f_1 f_2 = g_1 g_2$ (D) $f_1 + f_2 = g_1 + g_2$

10. A circle touches a straight line $\ell x + my + n = 0$ & cuts the circle $x^2 + y^2 = 9$ orthogonally. The locus of centres of such circles is:

(A) $(\ell x + my + n)^2 = (\ell^2 + m^2)(x^2 + y^2 - 9)$ (B) $(\ell x + my - n)^2 = (\ell^2 + m^2)(x^2 + y^2 - 9)$

(C) $(\ell x + my + n)^2 = (\ell^2 + m^2)(x^2 + y^2 + 9)$ (D) $(\ell x + my - n)^2 = (\ell^2 + m^2)(x^2 + y^2 - 9)$

11. The locus of the point at which two given unequal circles subtend equal angles is:
(A) a straight line (B) a circle (C) a parabola (D) an ellipse

12. A circle is given by $x^2 + (y-1)^2 = 1$. Another circle C touches it externally and also the x-axis, then the locus of its centre is
(A) $\{(x, y) : x^2 = 4y\} \cup \{(x, y) : y \le 0\}$ (B) $\{(x, y) : x^2 + (y-1)^2 = 4\} \cup \{(x, y) : y \le 0\}$
(C) $\{(x, y) : x^2 = y\} \cup \{(0, y) : y \le 0\}$ (D) $\{(x, y) : x^2 = 4y\} \cup \{(0, y) : y \le 0\}$

13. The locus of the centre of a circle touching the circle $x^2 + y^2 - 4y - 2x = 4$ internally and tangent on which from (1, 2) is making a 60° angle with each other.
(A) $(x-1)^2 + (y-2)^2 = 2$ (B) $(x-1)^2 + (y-2)^2 = 4$
(C) $(x+1)^2 + (y-2)^2 = 4$ (D) $(x+1)^2 + (y+2)^2 = 4$

14. **STATEMENT-1 :** If three circles which are such that their centres are non-collinear, then exactly one circle exists which cuts the three circles orthogonally.
STATEMENT-2 : Radical axis for two intersecting circles is the common chord.

(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true

15. The centre of family of circles cutting the family of circles $x^2 + y^2 + 4x\left(\lambda - \frac{3}{2}\right) + 3y\left(\lambda - \frac{4}{3}\right) - 6(\lambda + 2) = 0$ orthogonally, lies on

(A) $x - y - 1 = 0$ (B) $4x + 3y - 6 = 0$ (C) $4x + 3y + 7 = 0$ (D) $3x - 4y - 1 = 0$

16. The circle $x^2 + y^2 = 4$ cuts the circle $x^2 + y^2 + 2x + 3y - 5 = 0$ in A & B. Then the equation of the circle on AB as a diameter is:
(A) $13(x^2 + y^2) - 4x - 6y - 50 = 0$ (B) $9(x^2 + y^2) + 8x - 4y + 25 = 0$
(C) $x^2 + y^2 - 5x + 2y + 72 = 0$ (D) $13(x^2 + y^2) - 4x - 6y + 50 = 0$

PART-II: NUMERICAL VALUE QUESTIONS

INSTRUCTION :
- The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto two digit.
- If the numerical value has more than two decimal places **truncate/round-off** the value to **TWO** decimal placed.

1. Find maximum number of points having integer coordinates (both x, y integer) which can lie on a circle with centre at $\left(\sqrt{2}, \sqrt{3}\right)$ is (are)

2. If equation of smallest circle touching the circles $x^2 + y^2 - 2y - 3 = 0$ and $x^2 + y^2 - 8x - 18y + 93 = 0$ is $x^2 + y^2 - 4x - fy + c = 0$ then value of f + c is

3. A line meets the co–ordinate axes in A and B. A circle is circumscribed about the triangle OAB. If d_1 and d_2 are the distances of the tangent to the circle at the origin O from the points A and B respectively and diameter of the circle is $\lambda_1 d_1 + \lambda_2 d_2$, then find the value of $\lambda_1 + \lambda_2$.

4. A circle is inscribed (i.e. touches all four sides) into a rhombous ABCD with one angle 60º. The distance from the centre of the circle to the nearest vertex is equal to 1. If P is any point of the circle, then $|PA|^2 + |PB|^2 + |PC|^2 + |PD|^2$ is equal to :

5. Let x & y be the real numbers satisfying the equation $x^2 - 4x + y^2 + 3 = 0$. If the maximum and minimum values of $x^2 + y^2$ are M & m respectively, then find the numerical value of (M + m).

6. Find absolute value of 'c' for which the set, $\{(x, y) | x^2 + y^2 + 2x \le 1\} \cap \{(x, y) | 5x - 12y + c \ge 0\}$ contains only one point is common.

7. A rhombus is inscribed in the region common to the two circles $x^2 + y^2 - 4x - 12 = 0$ and $x^2 + y^2 + 4x - 12 = 0$ with two of its vertices on the line joining the centres of the circles then area of the rhombus is

8. If (α, β) is a point on the circle whose centre is on the x-axis and which touches the line x + y = 0 at (2, –2), then find the greatest value of 'α' is

9. Two circles whose radii are equal to 4 and 8 intersect at right angles, then length of their common chord is

10. A variable circle passes through the point A (a, b) & touches the x–axis and the locus of the other end of the diameter through A is $(x - a)^2 = \lambda by$, then find the value of λ

11. Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Suppose that the tangents at the points B (1, 7) & D (4, – 2) on the circle meet at the point C. Find the area of the quadrilateral ABCD.

12. If the complete set of values of a for which the point (2a, a + 1) is an interior point of the larger segment of the circle $x^2 + y^2 - 2x - 2y - 8 = 0$ made by the chord whose equation is 3x – 4y + 5 = 0 is (p,q) then value of p + q is

13. The circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q, then find the number of values of 'a' for which the line 5x + by – a = 0 passes through P and Q.

14. The circumference of the circle $x^2 + y^2 - 2x + 8y - q = 0$ is bisected by the circle $x^2 + y^2 + 4x + 12y + p = 0$, then find p + q

15. A circle touches the line y = x at a point P such that OP = $4\sqrt{2}$ where O is the origin. The circle contains the point (–10, 2) in its interior and the length of its chord on the line x + y = 0 is $6\sqrt{2}$. If the equation of the circle $x^2 + y^2 + 2gx + 2fy + 3c = 0$, then value of g + f + c is

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. The equation of circles passing through (3, –6) touching both the axes is
(A) $x^2 + y^2 - 6x + 6y + 9 = 0$ (B) $x^2 + y^2 + 6x - 6y + 9 = 0$
(C) $x^2 + y^2 + 30x - 30y + 225 = 0$ (D) $x^2 + y^2 - 30x + 30y + 225 = 0$

2. Equations of circles which pass through the points (1, –2) and (3, – 4) and touch the x-axis is
(A) $x^2 + y^2 + 6x + 2y + 9 = 0$ (B) $x^2 + y^2 + 10x + 20y + 25 = 0$
(C) $x^2 + y^2 - 6x + 4y + 9 = 0$ (D) $x^2 + y^2 + 10x + 20y - 25 = 0$

3. The centre of a circle passing through the points (0, 0), (1, 0) & touching the circle $x^2 + y^2 = 9$ is :
(A) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (B) $\left(\frac{1}{2}, \sqrt{2}\right)$ (C) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (D) $\left(\frac{1}{2}, -\sqrt{2}\right)$

4. The equation of the circle which touches both the axes and the line $\frac{x}{3} + \frac{y}{4} = 1$ and lies in the first quadrant is $(x - c)^2 + (y - c)^2 = c^2$ where c is
(A) 1 (B) 2 (C) 4 (D) 6

5. Find the equations of straight lines which pass through the intersection of the lines $x - 2y - 5 = 0$, $7x + y = 50$ & divide the circumference of the circle $x^2 + y^2 = 100$ into two arcs whose lengths are in the ratio 2 : 1.
(A) $3x - 4y - 25 = 0$ (B) $4x + 3y - 25 = 0$ (C) $4x - 3y - 25 = 0$ (D) $3x + 4y - 25 = 0$

6. Tangents are drawn to the circle $x^2 + y^2 = 50$ from a point 'P' lying on the x-axis. These tangents meet the y-axis at points 'P_1' and 'P_2'. Possible coordinates of 'P' so that area of triangle PP_1P_2 is minimum, is/are
(A) (10, 0) (B) $(10\sqrt{2}, 0)$ (C) (–10, 0) (D) $(-10\sqrt{2}, 0)$

7. If (a, 0) is a point on a diameter segment of the circle $x^2 + y^2 = 4$, then $x^2 - 4x - a^2 = 0$ has
(A) exactly one real root in (– 1, 0] (B) Exactly one real root in [2, 5]
(C) distinct roots greater than-1 (D) Distinct roots less than 5

8. The tangents drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ are perpendicular if
(A) $h = r$ (B) $h = -r$ (C) $r^2 + h^2 = 1$ (D) $r^2 = h^2$

9. The equation (s) of the tangent at the point (0, 0) to the circle where circle makes intercepts of length 2a and 2b units on the coordinate axes, is (are) -
(A) $ax + by = 0$ (B) $ax - by = 0$ (C) $x = y$ (D) $bx + ay = ab$

10. Consider two circles $C_1 : x^2 + y^2 - 1 = 0$ and $C_2 : x^2 + y^2 - 2 = 0$. Let A(1,0) be a fixed point on the circle C_1 and B be any variable point on the circle C_2. The line BA meets the curve C_2 again at C. Which of the following alternative(s) is/are correct ?
(A) $OA^2 + OB^2 + BC^2 \in [7, 11]$, where O is the origin.
(B) $OA^2 + OB^2 + BC^2 \in [4, 7]$, where O is the origin.
(C) Locus of midpoint of AB is a circle of radius $\frac{1}{\sqrt{2}}$.
(D) Locus of midpoint of AB is a circle of area $\frac{\pi}{2}$.

11. One of the diameter of the circle circumscribing the rectangle ABCD is $x - 3y + 1 = 0$.
If two verticles of rectangle are the points (– 2, 5) and (6, 5) respectively, then which of the following hold(s) good?
(A) Area of rectangle ABCD is 64 square units.
(B) Centre of circle is (2, 1)
(C) The other two vertices of the rectangle are (– 2, – 3) and (6, – 3)
(D) Equation of sides are x = – 2, y = – 3, x = 5 and y = 6.

12. Three concentric circles of which the biggest is $x^2 + y^2 = 1$, have their radii in A.P. If the line $y = x + 1$ cuts all the circles in real and distinct points. The permissible values of common difference of A.P. is/are
(A) 0.4 (B) 0.6 (C) 0.01 (D) 0.1

13. If $4\ell^2 - 5m^2 + 6\ell + 1 = 0$. Prove that $\ell x + my + 1 = 0$ touches a definite circle, then which of the following is/are true.
(A) Centre (0, 3) (B) centre (3, 0) (C) Radius $\sqrt{5}$ (D) Radius 5

14. If the circle $C_1: x^2 + y^2 = 16$ intersects another circle C_2 of radius 5 in such a manner that the common chord is of maximum length and has a slope equal to 3/4, then the co–ordinates of the centre of C_2 are:
(A) $\left(\frac{9}{5}, \frac{12}{5}\right)$ (B) $\left(\frac{9}{5}, \frac{-12}{5}\right)$ (C) $\left(\frac{-9}{5}, \frac{-12}{5}\right)$ (D) $\left(\frac{-9}{5}, \frac{+12}{5}\right)$

15. For the circles $x^2 + y^2 - 10x + 16y + 89 - r^2 = 0$ and $x^2 + y^2 + 6x - 14y + 42 = 0$ which of the following is/are true.
(A) Number of integral values of r are 14 for which circles are intersecting.
(B) Number of integral values of r are 9 for which circles are intersecting.
(C) For r equal to 13 number of common tangents are 3.
(D) For r equal to 21 number of common tangents are 2.

16. Which of the following statement(s) is/are correct with respect to the circles $S_1 \equiv x^2 + y^2 - 4 = 0$ and $S_2 \equiv x^2 + y^2 - 2x - 4y + 4 = 0$?
(A) S_1 and S_2 intersect at an angle of 90°.
(B) The point of intersection of the two circle are (2, 0) and $\left(\frac{6}{5}, \frac{8}{5}\right)$.
(C) Length of the common of chord of S_1 and S_2 is $\frac{4}{\sqrt{5}}$.
(D) The point (2, 3) lies outside the circles S_1 and S_2.

17. Two circles, each of radius 5 units, touch each other at (1, 2). If the equation of their common tangent is $4x + 3y = 10$. The equations of the circles are
(A) $x^2 + y^2 + 6x + 2y - 15 = 0$ (B) $x^2 + y^2 - 10x - 10y + 25 = 0$
(C) $x^2 + y^2 - 6x + 2y - 15 = 0$ (D) $x^2 + y^2 - 10x + 10y + 25 = 0$

18. $x^2 + y^2 = a^2$ and $(x - 2a)^2 + y^2 = a^2$ are two equal circles touching each other. Find the equation of circle (or circles) of the same radius touching both the circles.
(A) $x^2 + y^2 + 2ax + 2\sqrt{3}\,ay + 3a^2 = 0$ (B) $x^2 + y^2 - 2ax + 2\sqrt{3}\,ay + 3a^2 = 0$
(C) $x^2 + y^2 + 2ax - 2\sqrt{3}\,ay + 3a^2 = 0$ (D) $x^2 + y^2 - 2ax - 2\sqrt{3}\,ay + 3a^2 = 0$

19. The circle $x^2 + y^2 - 2x - 3ky - 2 = 0$ passes through two fixed points, (k is the parameter)
(A) $(1 + \sqrt{3}, 0)$ (B) $(-1 + \sqrt{3}, 0)$ (C) $(-\sqrt{3} - 1, 0)$ (D) $(1 - \sqrt{3}, 0)$

20. Curves $ax^2 + 2hxy + by^2 - 2gx - 2fy + c = 0$ and $a'x^2 - 2hxy + (a' + a - b)y^2 - 2g'x - 2f'y + c = 0$ intersect at four concyclic point A, B, C and D. If P is the point $\left(\frac{g'+g}{a'+a}, \frac{f'+f}{a'+a}\right)$, then which of the following is/are true
(A) P is also concyclic with points A, B, C, D (B) PA, PB, PC in G.P.
(C) $PA^2 + PB^2 + PC^2 = 3PD^2$ (D) PA, PB, PC in A.P.

PART - IV : COMPREHENSION

Comprehension # 1 (Q. No. 1 to 3)

Let S_1, S_2, S_3 be the circles $x^2 + y^2 + 3x + 2y + 1 = 0$, $x^2 + y^2 - x + 6y + 5 = 0$ and $x^2 + y^2 + 5x - 8y + 15 = 0$, then

1. Point from which length of tangents to these three circles is same is
(A) (1, 0) (B) (3, 2) (C) (10, 5) (D) (– 2, 1)

2. Equation of circle S_4 which cut orthogonally to all given circle is
(A) $x^2 + y^2 - 6x + 4y - 14 = 0$ (B) $x^2 + y^2 + 6x + 4y - 14 = 0$
(C) $x^2 + y^2 - 6x - 4y + 14 = 0$ (D) $x^2 + y^2 - 6x - 4y - 14 = 0$

3.✍ Radical centre of circles S_1, S_2, & S_4 is

(A) $\left(-\frac{3}{5}, -\frac{8}{5}\right)$ (B) (3, 2) (C) (1, 0) (D) $\left(-\frac{4}{5}, -\frac{3}{2}\right)$

Comprehension # 2 (Q. No. 4 to 6)✍

Two circles are $S_1 \equiv (x + 3)^2 + y^2 = 9$

$S_2 \equiv (x - 5)^2 + y^2 = 16$

with centres C_1 & C_2

4.✍ A direct common tangent is drawn from a point P (on x-axis) which touches S_1 & S_2 at Q & R, respectively. Find the ratio of area of ΔPQC_1 & ΔPRC_2.

(A) 3 : 4 (B) 9 : 16 (C) 16 : 9 (D) 4 : 3

5.✍ From point 'A' on S_2 which is nearest to C_1, a variable chord is drawn to S_1. The locus of mid point of the chord.

(A) circle (B) Diameter of s_1

(C) Arc of a circle (D) chord of s_1 but not diameter

6.✍ Locus obtained in question 5 cuts the circle S_1 at B & C, then line segment BC subtends an angle on the major arc of circle S_1 is

(A) $\cos^{-1}\frac{3}{4}$ (B) $\frac{\pi}{2} - \tan^{-1}\frac{4}{3}$ (C) $\frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{3}{4}$ (D) $\frac{\pi}{2}\cot^{-1}\left(\frac{4}{3}\right)$

Exercise-3

✍ Marked questions are recommended for Revision.

*** Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where k > 0, then the value of [k] is

[**Note** : [k] denotes the largest integer less than or equal to k] **[IIT-JEE - 2010, Paper-2, (3, 0), 79]**

2. The circle passing through the point (–1, 0) and touching the y-axis at (0, 2) also passes through the point **[IIT-JEE 2011, Paper-2, (3, –1), 80]**

(A) $\left(-\frac{3}{2}, 0\right)$ (B) $\left(-\frac{5}{2}, 2\right)$ (C) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (D) (–4, 0)

3. The straight line 2x – 3y = 1 divides the circular region $x^2 + y^2 \le 6$ into two parts.

If $S = \left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\}$, **[IIT-JEE 2011, Paper-2, (4, 0), 80]**

then the number of point(s) in S lying inside the smaller part is

4.✍ The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line 4x – 5y = 20 to the circle $x^2 + y^2 = 9$ is **[IIT-JEE 2012, Paper-1, (3, –1), 70]**

(A) $20(x^2 + y^2) - 36x + 45y = 0$ (B) $20(x^2 + y^2) + 36x - 45y = 0$

(C) $36(x^2 + y^2) - 20x + 45y = 0$ (D) $36(x^2 + y^2) + 20x - 45y = 0$

Paragraph for Question Nos. 5 to 6

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point $P(\sqrt{3}, 1)$. A straight line L, perpendicular to PT is a tangent to the circle $(x - 3)^2 + y^2 = 1$. **[IIT-JEE 2012, Paper-2, (3, –1), 66]**

5. A common tangent of the two circles is
(A) $x = 4$ (B) $y = 2$ (C) $x + \sqrt{3}y = 4$ (D) $x + 2\sqrt{3}y = 6$

6. A possible equation of L is
(A) $x - \sqrt{3}y = 1$ (B) $x + \sqrt{3}y = 1$ (C) $x - \sqrt{3}y = -1$ (D) $x + \sqrt{3}y = 5$

7*. Circle(s) touching x-axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ on y-axis is (are) **[JEE (Advanced) 2013, Paper-2, (3, –1)/60]**
(A) $x^2 + y^2 - 6x + 8y + 9 = 0$ (B) $x^2 + y^2 - 6x + 7y + 9 = 0$
(C) $x^2 + y^2 - 6x - 8y + 9 = 0$ (D) $x^2 + y^2 - 6x - 7y + 9 = 0$

8*. A circle S passes through the point (0, 1) and is orthogonal to the circles $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then **[JEE (Advanced) 2014, Paper-1, (3, 0)/60]**
(A) radius of S is 8 (B) radius of S is 7
(C) centre of S is (–7, 1) (D) centre of S is (–8, 1)

9*. The circle $C_1 : x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the y-axis, then **[JEE (Advanced) 2016, Paper-1, (4, –2)/62]**
(A) $Q_2Q_3 = 12$ (B) $R_2R_3 = 4\sqrt{6}$
(C) area of the triangle OR_2R_3 is $6\sqrt{2}$ (D) area of the triangle PQ_2Q_3 is $4\sqrt{2}$

10*. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point (1, 0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s) **[JEE (Advanced) 2016, Paper-1, (4, –2)/62]**
(A) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$ (B) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (C) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$ (D) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

11. For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points? **[JEE(Advanced) 2017, Paper-1,(3, 0)/61]**

PARAGRAPH "X" **[JEE(Advanced) 2018, Paper-1, (3, –1)/60]**
Let S be the circle in the xy-plane defined by the equation $x^2 + y^2 = 4$.
(***There are two questions based on PARAGRAPH "X", the question given below is one of them***)

12. Let E_1E_2 and F_1F_2 be the chords of S passing through the point $P_0(1, 1)$ and parallel to the x-axis and the y-axis, respectively. Let G_1G_2 be the chord of S passing through P_0 and having slope –1. Let the tangents to S at E_1 and E_2 meet at E_3, the tangents to S at F_1 and F_2 meet at F_3, and the tangents to S at G_1 and G_2 meet at G_3. Then, then, the points E_3, F_3, and G_3 lie on the curve
(A) $x + y = 4$ (B) $(x - 4)^2 + (y - 4)^2 = 16$ (C) $(x - 4)(y - 4) = 4$ (D) $xy = 4$

13. Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve

(A) $(x + y)^2 = 3xy$ (B) $x^{2/3} + y^{2/3} = 2^{4/3}$ (C) $x^2 + y^2 = 2xy$ (D) $x^2 + y^2 = x^2y^2$

14*. Let T be the line passing through the points P(–2, 7) and Q(2, –5). Let F_1 be the set of all pairs of circles (S_1, S_2) such that T is tangent to S_1 at P and tangent to S_2 at Q, and also such that S_1 and S_2 touch each other at a point, say, M. Let E_1 be the set representing the locus of M as the pair (S_1, S_2) varies in F_1. Let the set of all straight line segments joining a pair of distinct points of E_1 and passing through the point R(1, 1) be F_2. Let E_2 be the set of the mid-points of the line segments in the set F_2. Then, which of the following statement(s) is (are) TRUE **[JEE(Advanced) 2018, Paper-2, (4, –2)/60]**

(A) The point (–2, 7) lies in E_1 (B) The point $\left(\frac{4}{5}, \frac{7}{5}\right)$ does **NOT** lie in E_2

(C) The point $\left(\frac{1}{2}, 1\right)$ lies in E_2 (D) The point $\left(0, \frac{3}{2}\right)$ does **NOT** lie in E_1

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if **[AIEEE 2010, (4, –1), 144]**

(1) $-35 < m < 15$ (2) $15 < m < 65$ (3) $35 < m < 85$ (4) $-85 < m < -35$

2. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2 (c > 0)$ touch each other if : **[AIEEE-2011, I, (4, –1), 120]**

(1) $2|a| = c$ (2) $|a| = c$ (3) $a = 2c$ (4) $|a| = 2c$

3. The equation of the circle passing through the point (1, 0) and (0, 1) and having the smallest radius is - **[AIEEE-2011, II, (4, –1), 120]**

(1) $x^2 + y^2 - 2x - 2y + 1 = 0$ (2) $x^2 + y^2 - x - y = 0$

(3) $x^2 + y^2 + 2x + 2y - 7 = 0$ (4) $x^2 + y^2 + x + y - 2 = 0$

4. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is : **[AIEEE- 2012, (4, –1), 120]**

(1) $\frac{10}{3}$ (2) $\frac{3}{5}$ (3) $\frac{6}{5}$ (4) $\frac{5}{3}$

5. The circle passing through (1, –2) and touching the axis of x at (3, 0) also passes through the point **[AIEEE - 2013, (4, –1),120]**

(1) (–5, 2) (2) (2, – 5) (3) (5, – 2) (4) (–2, 5)

6. Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to : **[JEE(Main) 2014, (4, – 1), 120]**

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{\sqrt{3}}{\sqrt{2}}$ (4) $\frac{\sqrt{3}}{2}$

7. Locus of the image of the point (2, 3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \in R$, is a **[JEE(Main) 2015, (4, – 1), 120]**

(1) straight line parallel to x-axis (2) straight line parallel to y-axis

(3) circle of radius $\sqrt{2}$ (4) circle of radius $\sqrt{3}$

8. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is **[JEE(Main) 2015, (4, – 1), 120]**

(1) 1 (2) 2 (3) 3 (4) 4

9. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on : **[JEE(Main) 2016, (4, – 1), 120]**

(1) an ellipse which is not a circle (2) a hyperbola

(3) a parabola (4) a circle

10. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at (– 3, 2), then the radius of S is : **[JEE(Main) 2016, (4, – 1), 120]**

(1) $5\sqrt{3}$ (2) 5 (3) 10 (4) $5\sqrt{2}$

11. Let the orthocenter and centroid of a triangle be A (–3, 5) and B(3,3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segement AC as diameter , is :

[JEE(Main) 2018, (4, – 1), 120]

(1) $3\sqrt{\frac{5}{2}}$ (2) $\frac{3\sqrt{5}}{2}$ (3) $\sqrt{10}$ (4) $2\sqrt{10}$

12. Three circles of radii, a, b, c (a < b < c) touch each other externally, If they have x-axis as a common tangent, then : **[JEE(Main) 2019, Online (09-01-19),P-1 (4, – 1), 120]**

(1) a, b, c are in A.P. (2) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$ (3) $\sqrt{a}$, $\sqrt{b}$, $\sqrt{c}$ are in A.P. (4) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$

13. If a circle C passing through the point (4,0) touches the circle $x^2 + y^2 + 4x - 6y = 12$ externally at the point (1, –1), then the radius of C is: **[JEE(Main) 2019, Online (10-01-19),P-1 (4, – 1), 120]**

(1) $2\sqrt{5}$ (2) $\sqrt{57}$ (3) 4 (4) 5

14. If a variable line, $3x + 4y - \lambda = 0$ is such that the two circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 18x - 2y + 78 = 0$ are on its opposite sides, then the set of all values of λ is the interval:

[JEE(Main) 2019, Online (12-01-19),P-1 (4, – 1), 120]

(1) (2, 17) (2) [12, 21] (3) [13, 23] (4) (23,31)

15. If a circle of radius R passes through the origin O and intersects the coordinate axes at A and B, then the locus of the foot of perpendicular from O on AB is -

[JEE(Main) 2019, Online (12-01-19),P-2 (4, – 1), 120]

(1) $(x^2+y^2)(x + y) = R^2xy$ (2) $(x^2 + y^2)^3 = 4R^2x^2y^2$

(3) $(x^2 + y^2)^2 = 4Rx^2y^2$ (4) $(x^2 + y^2)^2 = 4R^2x^2y^2$

16. The sum of the squares of the lengths of the chords intercepted on the circle, $x^2 + y^2 = 16$, by the lines, $x + y = n$, $n \in N$, where N is the set of all natural numbers, is :

[JEE(Main) 2019, Online (08-04-19),P-1 (4, – 1), 120]

(1) 105 (2) 210 (3) 320 (4) 160

17. If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is : **[JEE(Main) 2019, Online (09-04-19),P-1 (4, – 1), 120]**

(1) $x^2 + y^2 - 4x^2y^2 = 0$ (2) $x^2 + y^2 - 16x^2y^2 = 0$ (3) $x^2 + y^2 - 2x^2y^2 = 0$ (4) $x^2 + y^2 - 2xy = 0$

18. The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y-axis and lie in the first quadrant, is : **[JEE(Main) 2019, Online (10-04-19),P-2 (4, – 1), 120]**

(1) $x = \sqrt{1+4y}, y \geq 0$ (2) $y = \sqrt{1+4x}, x \geq 0$ (3) $x = \sqrt{1+2y}, y \geq 0$ (4) $y = \sqrt{1+2x}, x \geq 0$

19. If a line, $y = mx + c$ is a tangent to the circle, $(x - 3)^2 + y^2 = 1$ and it is perpendicular to a line L_1, where L_1 is the tangent to the circle, $x^2 + y^2 = 1$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$; then :

[JEE(Main) 2020, Online (08-01-20),P-2 (4, –1), 120]

(1) $c^2 + 7c + 6 = 0$ (2) $c^2 + 6c + 7 = 0$ (3) $c^2 - 6c + 7 = 0$ (4) $c^2 - 7c + 6 = 0$

20. If the curves, $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0$, $(k > 0)$ touch each other at a point, then the largest value of k is ________ **[JEE(Main) 2020, Online (09-01-20),P-2 (4, 0), 120]**

Answers

EXERCISE - 1

PART - I

Section (A) :

A-1. $x^2 + y^2 = 1$
A-3. $x^2 + y^2 - 3x - 4y = 0$
A-4. $x^2 + y^2 - 4x - 4y + 4 = 0$
A-5. $x^2 + y^2 \pm 6\sqrt{2}y \pm 6x + 9 = 0$
A-6. $(x + 3)^2 + (y - 4)^2 = 4$
A-7. (36, 47)

Section (B) :

B-1. 2
B-2. (1, 3), (5, 7), $4\sqrt{2}$
B-3. $x - 7y - 45 = 0$
B-4. $\sqrt{3}x - y \pm 4 = 0$
B-5. $16x^2 - 65y^2 - 288x + 1296 = 0$, $\tan^{-1}\left(\frac{8\sqrt{65}}{49}\right)$
B-6. Yes

Section (C) :

C-1. $2x - y = 0$
C-2. $x + 2y - 1 = 0$
C-3. $(x + 4)^2 + y^2 = 16$
C-5. $x + y + 5 = 0$
C-6. $\left(6, -\frac{18}{5}\right)$

Section (D) :

D-1. $x = 0$, $3x + 4y = 10$, $y = 4$ and $3y = 4x$.

D-3. $2(x^2 + y^2) - 7x + 2y = 0$
D-4. $\left(\frac{33}{4}, 2\right)$; $\frac{1}{4}$

Section (E) :

E-1. $x^2 + y^2 - 2x - 4y = 0$.
E-2. (i) $(x - 1)^2 + (y + 2)^2 + 20\,(2x - y - 4) = 0$
(ii) $(x - 1)^2 + (y + 2)^2 \pm \sqrt{20}\,(2x - y - 4) = 0$
E-4. $\left(\frac{52}{3}, -\frac{23}{9}\right)$
E-5. $x^2 + y^2 - 17x - 19y + 50 = 0$

PART - II

Section (A) :

A-1. (D) **A-2.** (A) **A-3.** (B) **A-4.** (C) **A-5.** (D) **A-6.** (B) **A-7.** (A)
A-8. (C)

Section (B) :

B-1. (A) **B-2.** (B) **B-3.** (B) **B-4.** (B) **B-5.** (A) **B-6.** (B) **B-7.** (A)
B-8. (D) **B-9.** (C) **B-10.** (A) **B-11.** (C) **B-12.** (A) **B-13.** (B) **B-14.** (A)

Section (C) :

C-1. (A) **C-2.** (B) **C-3.** (A) **C-4.** (C) **C-5.** (B) **C-6.** (A) **C-7.** (A)

Section (D) :

D-1. (B) **D-2.** (B) **D-3.** (A) **D-4.** (A)

Section (E) :

E-1. (A) **E-2.** (B) **E-3.** (A) **E-4.** (A) **E-5.** (D) **E-6.** (A)

PART - III

1. (A) $\rightarrow$ (q), (B) $\rightarrow$ (p), (C) $\rightarrow$ (r), (D) $\rightarrow$ (s)
2. (A) $\rightarrow$ (s), (B) $\rightarrow$ (r), (C) $\rightarrow$ (q), (D) $\rightarrow$ (p)

EXERCISE - 2

PART - I

1.	(C)	**2.**	(B)	**3.**	(D)	**4.**	(A)	**5.**	(A)	**6.**	(C)	**7.**	(B)
8.	(B)	**9.**	(B)	**10.**	(A)	**11.**	(B)	**12.**	(D)	**13.**	(B)	**14.**	(D)
15.	(B)	**16.**	(A)										

PART - II

1.	01.00	**2.**	32.88 or 32.89	**3.**	02.00	**4.**	11.00	**5.**	10.00	**6.**	13.38
7.	13.85 or 13.86			**8.**	06.82 or 06.83			**9.**	07.15	**10.**	04.00
11.	75.00	**12.**	01.30	**13.**	00.00	**14.**	10.00	**15.**	18.66 or 18.67		

PART - III

1.	(AD)	**2.**	(BC)	**3.**	(BD)	**4.**	(AD)	**5.**	(CD)	**6.**	(AC)
7.	(ABCD)	**8.**	(ABD)	**9.**	(AB)	**10.**	(ACD)	**11.**	(ABC)	**12.**	(CD)
13.	(BC)	**14.**	(BD)	**15.**	(AC)	**16.**	(ACD)	**17.**	(AB)	**18.**	(BD)
19.	(AD)	**20.**	(BCD)								

PART - IV

1.	(B)	**2.**	(D)	**3.**	(A)	**4.**	(B)	**5.**	(C)	**6.**	(A)

EXERCISE - 3

PART - I

1.	3	**2.**	(D)	**3.**	2	**4.**	(A)	**5.**	(D)	**6.**	(A)	**7.**	(AC)
8.	(BC)	**9.**	(ABC)	**10.**	(A,C)	**11.**	(2)	**12.**	(A)	**13.**	(D)	**14.**	(BD)

PART - II

1.	(1)	**2.**	(2)	**3.**	(2)	**4.**	(1)	**5.**	(3)	**6.**	(2)	**7.**	(3)
8.	(3)	**9.**	(3)	**10.**	(1)	**11.**	(1)	**12.**	(2)	**13.**	(4)	**14.**	(2)
15.	(2)	**16.**	(2)	**17.**	(1)	**18.**	(4)	**19.**	(2)	**20.**	36		

High Level Problems (HLP)

Marked Questions may have for Revision Questions.

SUBJECTIVE QUESTIONS

1. Find the equation of the circle passing through the points A(4, 3), B(2, 5) and touching the axis of y. Also find the point P on the y–axis such that the angle APB has largest magnitude.

2. Let a circle be given by $2x(x - a) + y(2y - b) = 0$, $(a \neq 0, b \neq 0)$. Find the condition on a & b if two chords, each bisected by the x–axis, can be drawn to the circle from $\left(a\ ,\ \frac{b}{2}\right)$

3. A circle is described to pass through the origin and to touch the lines x = 1, x + y = 2. Prove that the radius of the circle is a root of the equation $\left(3 - 2\sqrt{2}\right) t^2 - 2\sqrt{2}\,t + 2 = 0$.

4. If (a, α) lies inside the circle $x^2 + y^2 = 9$: $x^2 - 4x - a^2 = 0$ has exactly one root in (– 1, 0), then find the area of the region in which (a, α) lies.

5. Let $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ be a given circle. Find the locus of the foot of the perpendicular drawn from the origin upon any chord of S which subtends right angle at the origin.

6. A ball moving around the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ in anti–clockwise direction leaves it tangentially at the point P(–2, –2). After getting reflected from a straight line it passes through the centre of the circle. Find the equation of this straight line if its perpendicular distance from P is $\frac{5}{2}$. You can assume that the angle of incidence is equal to the angle of reflection.

7. The lines 5x + 12 y – 10 = 0 and 5x – 12y – 40 = 0 touch a circle C_1 of diameter 6 unit. If the centre of C_1 lies in the first quadrant, find the equation of the circle C_2 which is concentric with C_1 and cuts of intercepts of length 8 on these lines.

8. The chord of contact of tangents drawn from a point on the circle $x^2 + y^2 = a^2$ to the circle $x^2 + y^2 = b^2$ touches the circle $x^2 + y^2 = c^2$. Show that a, b, c are in G.P.

9. Find the locus of the middle points of chords of a given circle $x^2 + y^2 = a^2$ which subtend a right angle at the fixed point (p, q).

10. Let $a\ell^2 - bm^2 + 2d\ell + 1 = 0$, where a, b, d are fixed real numbers such that $a + b = d^2$. If the line $\ell x + my + 1 = 0$ touches a fixed circle then find the equation of circle

11. The centre of the circle S = 0 lies on the line 2x – 2y + 9 = 0 and S = 0 cuts orthogonally the circle $x^2 + y^2 = 4$. Show that circle S = 0 passes through two fixed points and also find their co–ordinates.

12. Prove that the two circles which pass through the points (0, a), (0, – a) and touch the straight line y = m x + c will cut orthogonaly if $c^2 = a^2(2 + m^2)$.

13. Consider points A $(\sqrt{13}, 0)$ and B $(2\sqrt{13}, 0)$ lying on x-axis. These points are rotated in an-anticlockwise direction about the origin through an angle of $\tan^{-1}\left(\frac{2}{3}\right)$. Let the new position of A and B be A′ and B′ respectively. With A′ as centre and radius $\frac{2\sqrt{13}}{3}$ a circle C_1 is drawn and with B′ as a centre and radius $\frac{\sqrt{13}}{3}$ circle C_2 is drawn. Find radical axis of C_1 and C_2.

14. P(a, b) is a point in the first quadrant. If the two circles which pass through P and touch both the co–ordinate axes cut at right angles, then find condition in a and b.

15. Prove that the square of the tangent that can be drawn from any point on one circle to another circle is equal to twice the product of perpendicular distance of the point from the radical axis of two circles and distance between their centres.

16. Find the equation of the circle which cuts each of the circles, $x^2 + y^2 = 4$, $x^2 + y^2 - 6x - 8y + 10 = 0$ & $x^2 + y^2 + 2x - 4y - 2 = 0$ at the extremities of a diameter.

17. Show that if one of the circle $x^2 + y^2 + 2gx + c = 0$ and $x^2 + y^2 + 2g_1x + c = 0$ lies within the other, then gg_1 and c are both positive.

18. Let ABCD is a rectangle. Incircle of ΔABD touches BD at E. Incircle of ΔCBD toches BD at F.

If AB = 8 units, and BC = 6 units, then find length of EF.

19. Let circles S_1 and S_2 of radii r_1 and r_2 respectively ($r_1 > r_2$) touches each other externally. Circle S radii r touches S_1 and S_2 externally and also their direct common tangent. Prove that the triangle formed by joining centre of S_1, S_2 and S is obtuse angled triangle.

20. Circles are drawn passing through the origin O to intersect the coordinate axes at point P and Q such that m. OP + n. OQ is a constant. Show that the circles pass through a fixed point.

21. A triangle has two of its sides along the axes, its third side touches the circle $x^2 + y^2 - 2ax - 2ay + a^2 = 0$. Find the equation of the locus of the circumcentre of the triangle.

22. Let S_1 be a circle passing through A(0, 1), B(–2, 2) and S_2 is a circle of radius $\sqrt{10}$ units such that AB is common chord of S_1 and S_2. Find the equation of S_2.

23. The curves whose equations are
$S = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
$S' = a'x^2 + 2h'xy + b'y^2 + 2g'x + 2f'y + c' = 0$
intersect in four concyclic points then find relation in a, b, h, a′, b′, h′.

24. A circle of constant radius ‘r’ passes through origin O and cuts the axes of coordinates in points P and Q, then find the equation of the locus of the foot of perpendicular from O to PQ.

25. The ends A , B of a fixed straight line of length ‘a’ and ends A′ and B′ of another fixed straight line of length ‘b’ slide upon the axis of X & the axis of Y (one end on axis of X & the other on axis of Y). Find the locus of the centre of the circle passing through A, B, A′ and B′.

Answers

1. $x^2 + y^2 - 4x - 6y + 9 = 0$ OR $x^2 + y^2 - 20x - 22y + 121 = 0$, P(0, 3), $\theta = 45°$

2. $(a^2 > 2b^2)$

4. $4\left\{\sqrt{5} + \frac{9}{2}\cot^{-1}\left(\frac{2}{\sqrt{5}}\right)\right\}$

5. $x^2 + y^2 + gx + fy + \frac{c}{2} = 0$

6. $(4\sqrt{3} - 3)x - (4 + 3\sqrt{3})y - (39 - 2\sqrt{3}) = 0$

7. $x^2 + y^2 - 10x - 4y + 4 = 0$

9. $2x^2 + 2y^2 - 2px - 2qy + p^2 + q^2 - a^2 = 0$

10. $x^2 + y^2 - 2dx + d^2 - b = 0$

11. $(-4, 4)$; $\left(-\frac{1}{2}, \frac{1}{2}\right)$

13. $9x + 6y = 65$

14. $a^2 - 4ab + b^2 = 0$

16. $x^2 + y^2 - 4x - 6y - 4 = 0$

18. 2

21. $2(x + y) - a = \frac{2xy}{a}$

22. $x^2 + y^2 + 2x - 3y + 2 \pm \sqrt{7}(x + 2y - 2) = 0$

23. $\frac{a-b}{h} = \frac{a'-b'}{h'}$

24. $(x^2 + y^2)^2 (x^{-2} + y^{-2}) = 4r^2$

25. $(2ax - 2by)^2 + (2bx - 2ay)^2 = (a^2 - b^2)^2$

Exercise-1

✍ **Marked questions are recommended for Revision.**
✍ **चिन्हित प्रश्न दोहराने योग्य प्रश्न है।**

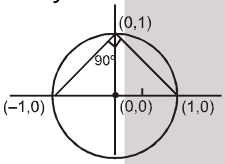
PART - I : SUBJECTIVE QUESTIONS

भाग - I : विषयात्मक प्रश्न (SUBJECTIVE QUESTIONS)

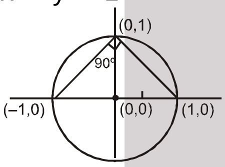
Section (A) : Equation of circle, parametric equation, position of a point
खण्ड (A) : वृत्त का समीकरण, प्राचलिक समीकरण, बिन्दु की स्थिति

A-1. Find the equation of the circle that passes through the points (1, 0), (– 1, 0) and (0, 1).
बिन्दुओं (1, 0), (– 1, 0) और (0, 1) से गुजरने वाले वृत्त का समीकरण ज्ञात कीजिए।
Ans. $x^2 + y^2 = 1$

Sol. Centre (0, 0), radius 1
$x^2 + y^2 = 1$

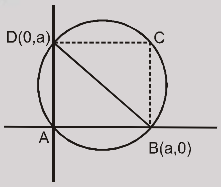


Hindi. केन्द्र (0, 0), त्रिज्या = 1
$x^2 + y^2 = 1$



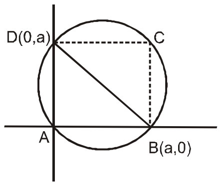
A-2. ABCD is a square in first quadrant whose side is a, taking AB and AD as axes, prove that the equation to the circle circumscribing the square is $x^2 + y^2 = a(x + y)$.
भुजा a वाला एक वर्ग ABCD प्रथम चतुर्थांश में है। भुजा AB और AD को अक्ष लेते हुए सिद्ध कीजिए कि वर्ग के परिगतवृत्त का समीकरण $x^2 + y^2 = a(x + y)$ है।

Sol. Since BD is diameter of circle



Hence $(x – a) (x – 0) + (y – 0) (y – a) = 0$
$\Rightarrow x^2 + y^2 = a (x + y)$

Hindi. चूँकि BD वृत्त का व्यास है।

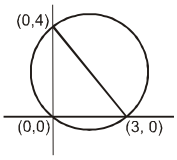


अतः $(x – a) (x – 0) + (y – 0) (y – a) = 0$
$\Rightarrow x^2 + y^2 = a (x + y)$

A-3. Find the equation to the circle which passes through the origin and cuts off intercepts equal to 3 and 4 from the positive axes.
उस वृत्त का समीकरण ज्ञात कीजिए जो मूलबिन्दु से गुजरता है तथा धनात्मक अक्षों पर क्रमशः 3 और 4 लम्बाई के अन्तःखण्ड काटता है।

Ans. $x^2 + y^2 - 3x - 4y = 0$

Sol.



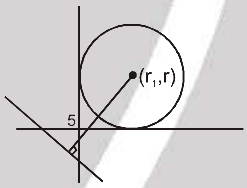
$(x - 3)(x - 0) + (y - 0)(y - 4) = 0$

A-4. Find equation of circle which touches x & y axis & perpendicular distance of centre of circle from $3x + 4y + 11 = 0$ is 5. Given that circle lies in I^{st} quadrant.
उस वृत्त का समीकरण ज्ञात कीजिए जो x व y अक्ष को स्पर्श करता है तथा रेखा $3x + 4y + 11 = 0$ से वृत्त के केन्द्र की लम्बवत् दूरी 5 है। दिया गया है कि वृत्त प्रथम चतुर्थांश में स्थित है।

Ans. $x^2 + y^2 - 4x - 4y + 4 = 0$

Sol. $\perp$ distance लम्बवत् दूरी $\left|\frac{3r + 4r + 11}{5}\right| = 5$

$7r + 11 = \pm 25$
$r = 2 , -36/7$
$\therefore$ circle is in I^{st} quadrant Hence $r = 2$
$\therefore$ वृत्त प्रथम चतुर्थांश में है अतः $r = 2$

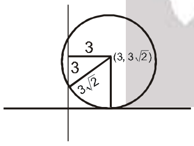


Equation समीकरण $(x - 2)^2 + (y - 2)^2 = 2^2 \Rightarrow x^2 + y^2 - 4x - 4y + 4 = 0$

A-5. Find the equation to the circle which touches the axis of x at a distance 3 from the origin and intercepts a distance 6 on the axis of y.
उस वृत्त का समीकरण ज्ञात कीजिए जो x-अक्ष को मूलबिन्दु से 3 इकाई दूरी पर स्पर्श करता है और y-अक्ष पर 6 इकाई का अन्तःखण्ड काटता हैं।

Ans. $x^2 + y^2 \pm 6\sqrt{2}y \pm 6x + 9 = 0$

Sol.



$$(x \pm 3)^2 + \left(y \pm 3\sqrt{2}\right)^2 = \left(3\sqrt{2}\right)^2$$

A-6. Find equation of circle whose cartesian equation are $x = -3 + 2 \sin\theta$, $y = 4 + 2\cos\theta$
उस वृत्त का समीकरण ज्ञात कीजिए जिसकी कार्तीय समीकरण $x = -3 + 2 \sin\theta$, $y = 4 + 2 \cos\theta$ है।

Ans. $(x + 3)^2 + (y - 4)^2 = 4$

Sol. $x = -3 + 2\sin\theta \Rightarrow x + 3 = 2 \sin\theta$
$y = 4 + 2\cos\theta \Rightarrow y - 4 = 2 \cos\theta$
Squarring and add $(x + 3)^2 + (y - 4)^2 = 4$

Hindi. $x = -3 + 2\sin\theta \Rightarrow x + 3 = 2 \sin\theta$
$y = 4 + 2\cos\theta \Rightarrow y - 4 = 2 \cos\theta$
वर्ग करके जोड़ने पर $(x + 3)^2 + (y - 4)^2 = 4$

A-7. Find the values of p for which the power of a point (2, 5) is negative with respect to a circle $x^2 + y^2 - 8x - 12y + p = 0$ which neither touches the axes nor cuts them.
वृत्त $x^2 + y^2 - 8x - 12y + p = 0$ जो न तो अक्षों को काटता हो न ही स्पर्श करता हो तथा बिन्दु (2, 5) के सापेक्ष बिन्दु–शक्ति ऋणात्मक होने के लिये p के मान ज्ञात कीजिए।
Ans. (36, 47)

Sol. $x^2 + y^2 - 8x - 12y + p = 0$
Power of (2, 5) is $S_1 = 4 + 25 - 16 - 60 + P = P - 47 < 0 \Rightarrow P < 47$
Circle neither touches nor cuts coordinate axes
$g^2 - c < 0 \Rightarrow 16 - p < 0 \Rightarrow p > 16$
$f^2 - c < 0 \Rightarrow 36 - p < 0 \Rightarrow p > 36$
taking intersection $P \in (36, 47)$

Sol. $x^2 + y^2 - 8x - 12y + p = 0$
(2, 5) का बिन्दु शक्ति है $S_1 = 4 + 25 - 16 - 60 + P = P - 47 < 0 \Rightarrow P < 47$
वृत्त न तो स्पर्श करता है न ही निर्देशांक अक्षों को स्पर्श करता है।
$g^2 - c < 0 \Rightarrow 16 - p < 0 \Rightarrow p > 16$
$f^2 - c < 0 \Rightarrow 36 - p < 0 \Rightarrow p > 36$
सर्वनिष्ठ लेने पर $P \in (36, 47)$

Section (B) : Line and circle, tangent, pair of tangent
खण्ड (B) : रेखा एवं वृत्त, स्पर्श रेखा, स्पर्श रेखा युग्म

B-1. If radii of the largest and smallest circle passing through the point (1, –1) and touching the circle $x^2 + y^2 + 2\sqrt{2}\,y - 2 = 0$ are r_1 and r_2 respectively, then find the sum of r_1 and r_2.
यदि बिन्दु (1, –1) से गुजरने वाले और वृत्त $x^2 + y^2 + 2\sqrt{2}\,y - 2 = 0$ को स्पर्श करने वाले सबसे बडे और सबसे छोटे वृत्त की त्रिज्याएं क्रमशः r_1 और r_2 है तब r_1 और r_2 के मध्य अन्तर ज्ञात कीजिए।

Ans. 2
Sol. $r_1 + r_2 = r = 2$

B-2. Find the points of intersection of the line $x - y + 2 = 0$ and the circle $3x^2 + 3y^2 - 29x - 19y + 56 = 0$. Also determine the length of the chord intercepted.
वृत्त $3x^2 + 3y^2 - 29x - 19y + 56 = 0$ तथा सरल रेखा $x - y + 2 = 0$ के प्रतिच्छेद बिन्दु ज्ञात कीजिए। प्रतिच्छेदन से प्राप्त जीवा की लम्बाई भी ज्ञात कीजिए।
Ans. (1, 3), (5, 7), $4\sqrt{2}$

Sol. On solving, points of intersection are (1,3) & (5, 7), length = $4\sqrt{2}$
Hindi. हल करने पर, प्रतिच्छेदी बिन्दु (1,3) व (5, 7) होंगे अतः जीवा की लम्बाई = $4\sqrt{2}$

B-3. Show that the line $7y - x = 5$ touches the circle $x^2 + y^2 - 5x + 5y = 0$ and find the equation of the other parallel tangent.
प्रदर्शित कीजिए कि सरल रेखा $7y - x = 5$, वृत्त $x^2 + y^2 - 5x + 5y = 0$ को स्पर्श करती है तथा इसके समान्तर अन्य स्पर्श रेखा की समीकरण ज्ञात कीजिए।
Ans. $x - 7y - 45 = 0$

Sol. Other tangent is $-x + 7y + \lambda = 0$ then $\left|\dfrac{-\frac{5}{2} - 7 \times \frac{5}{2} + \lambda}{\sqrt{50}}\right| = \dfrac{5}{\sqrt{2}} \Rightarrow \lambda = 45$ and -5

$\therefore$ other tangent is $x - 7y - 45 = 0$

Hindi अन्य स्पर्श रेखा है $-x + 7y + \lambda = 0$ तो $\left|\dfrac{-\frac{5}{2} - 7 \times \frac{5}{2} + \lambda}{\sqrt{50}}\right| = \dfrac{5}{\sqrt{2}} \Rightarrow \lambda = 45$ एवं -5

$\therefore$ अन्य स्पर्श रेखा है– $x - 7y - 45 = 0$

B-4. Find the equation of the tangents to the circle $x^2 + y^2 = 4$ which make an angle of 60º with the positive x-axis in anticlockwise direction .
वृत्त $x^2 + y^2 = 4$ की उस स्पर्श रेखा का समीकरण ज्ञात कीजिए जो कि वामावर्त दिशा में धनात्मक x अक्ष से 60º का कोण बनाती है।

Ans. $\sqrt{3}x - y \pm 4 = 0$

Sol. $m = \tan 60^\circ = \sqrt{3}x$; $y = mx\, a\sqrt{1+m^2}$ $y = x\sqrt{3} \pm 2\times 2$

B-5. Show that two tangents can be drawn from the point (9, 0) to the circle $x^2 + y^2 = 16$; also find the equation of the pair of tangents and the angle between them. **[16JM110493]**
प्रदर्शित कीजिए कि बिन्दु (9, 0) से वृत्त $x^2 + y^2 = 16$ पर दो स्पर्श रेखाएँ खींची जा सकती है। इस स्पर्श रेखा युग्म का समीकरण और उनके मध्य कोण भी ज्ञात कीजिए।

Ans. $16x^2 - 65y^2 - 288x + 1296 = 0$, $\tan^{-1}\left(\frac{8\sqrt{65}}{49}\right)$

Sol. $S_1 \equiv (9)^2 + (0)^2 - 16 = 65 > 0$
Since (9, 0) lies outside the circle. Hence two real tangents can be drawn.
Now $S \equiv x^2 + y^2 - 16$
$S_1 \equiv 9x - 16$. Hence pair of tangents $SS_1 = T^2$
$(x^2 + y^2 - 16)(65) = (9x - 16)^2$
$65x^2 + 65y^2 - 1040 = 81x^2 + 256 - 288x$
$16x^2 - 65y^2 - 288x + 1296 = 0$
Angle between these tangents = $\left|\frac{2\sqrt{h^2 - ab}}{(a+b)}\right| = \left|\frac{2\sqrt{0+16\times 65}}{16-65}\right| = \frac{8\sqrt{65}}{49}$

Hindi. $S_1 \equiv (9)^2 + (0)^2 - 16 = 65 > 0$
चूँकि (9, 0) वृत्त के बाहर स्थित है। अतः दो वास्तविक स्पर्श रेखाएँ खींची जा सकती है।
अब $S \equiv x^2 + y^2 - 16$
$S_1 \equiv 9x - 16$
अतः स्पर्शीयुग्म का समीकरण $SS_1 = T^2$
$(x^2 + y^2 - 16)(65) = (9x - 16)^2$
$65x^2 + 65y^2 - 1040 = 81x^2 + 256 - 288x$
$16x^2 - 65y^2 - 288x + 1296 = 0$
स्पर्श रेखाओं के मध्य कोण = $\left|\frac{2\sqrt{h^2 - ab}}{(a+b)}\right| = \left|\frac{2\sqrt{0+16\times 65}}{16-65}\right| = \frac{8\sqrt{65}}{49}$

B-6. If the length of the tangent from (f, g) to the circle $x^2 + y^2 = 6$ be twice the length of the tangent from (f, g) to the circle $x^2 + y^2 + 3x + 3y = 0$, then will $f^2 + g^2 + 4f + 4g + 2 = 0$?
यदि बिन्दु (f, g) से वृत्त $x^2 + y^2 = 6$ पर खींची गई स्पर्श रेखा की लम्बाई, बिन्दु (f, g) से वृत्त $x^2 + y^2 + 3x + 3y = 0$ पर खींची गई स्पर्श रेखा की लम्बाई से दुगनी हो, तो $f^2 + g^2 + 4f + 4g + 2 = 0$ सत्य है अथवा असत्य ?

Ans. Yes सत्य

Sol. given $\sqrt{f^2 + g^2 - 6} = 2\sqrt{f^2 + g^2 + 3g + 3f} \Rightarrow 3g^2 + 3f^2 + 12g + 12f + 6 = 0$
$\Rightarrow g^2 + f^2 + 4g + 4f + 2 = 0$

Hindi. दिया गया है $\sqrt{f^2 + g^2 - 6} = 2\sqrt{f^2 + g^2 + 3g + 3f} \Rightarrow 3g^2 + 3f^2 + 12g + 12f + 6 = 0$
$\Rightarrow$ $g^2 + f^2 + 4g + 4f + 2 = 0$

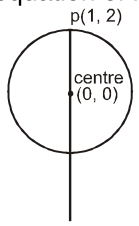
Section (C) : Normal, Director circle, chord of contact, chord with mid point
खण्ड (C) : अभिलम्ब, नियामक वृत्त, स्पर्श जीवा, मध्य बिन्दु वाली जीवा

C-1. Find the equation of the normal to the circle $x^2 + y^2 = 5$ at the point (1, 2)
बिन्दु (1,2) पर वृत्त $x^2 + y^2 = 5$ के अभिलम्ब का समीकरण ज्ञात कीजिए।

Ans. $2x - y = 0$

Sol. Normal passes through centre

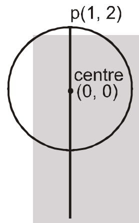
Hence equation of normal



$y = \frac{2}{1}x \Rightarrow y = 2x$

Hindi. अभिलम्ब वृत्त के केन्द्र से गुजरता है।

अतः अभिलम्ब का समीकरण



$y = \frac{2}{1}x \Rightarrow y = 2x$

C-2. Find the equation of the normal to the circle $x^2 + y^2 = 2x$, which is parallel to the line $x + 2y = 3$.

वृत्त $x^2 + y^2 = 2x$ के उस अभिलम्ब का समीकरण ज्ञात कीजिए जो सरल रेखा $x + 2y = 3$ के समान्तर हो।

Ans. $x + 2y - 1 = 0$

Sol. Equation of line parallel to $x + 2y - 3 = 0$ is $x + 2y + k = 0$

This is normal of $x^2 + y^2 - 2x = 0$. Hence centre of circle satisfies it $1 + 0 + k = 0 \Rightarrow k = -1$

$x + 2y - 1 = 0$

Hindi. $x + 2y - 3 = 0$ के समान्तर रेखा का समीकरण $x + 2y + k = 0$ है।

यह वृत्त $x^2 + y^2 - 2x = 0$, $1 + 0 + k = 0 \Rightarrow k = -1$ का अभिलम्ब है अतः वृत्त का केन्द्र इसे संतुष्ट करता है।

$x + 2y - 1 = 0$

C-3. Find the equation of director circle of the circle $(x + 4)^2 + y^2 = 8$

वृत्त $(x + 4)^2 + y^2 = 8$ के नियामक वृत्त का समीकरण ज्ञात कीजिए।

Ans. $(x + 4)^2 + y^2 = 16$

Sol. given circle $(x + 4)^2 + y^2 = 8$ centre of director circle $\equiv (-4, 0)$ radius of director circle $\equiv 4$

Hence equation of director circle $(x + 4)^2 + (y - 0)^2 = 4^2$

Hindi. दिया गया वृत्त $(x + 4)^2 + y^2 = 8$, नियामक वृत्त का केन्द्र $\equiv (-4, 0)$ तथा नियामक वृत्त की त्रिज्या $\equiv 4$

अतः नियामक वृत्त का समीकरण $(x + 4)^2 + (y - 0)^2 = 4^2$

C-4. Tangents are drawn from the point (h, k) to the circle $x^2 + y^2 = a^2$; prove that the area of the triangle formed by them and the straight line joining their points of contact is $\frac{a(h^2 + k^2 - a^2)^{3/2}}{h^2 + k^2}$ c.

बिन्दु (h,k) से वृत्त $x^2 + y^2 = a^2$ पर स्पर्श रेखाएँ खींची जाती है। सिद्ध कीजिए कि स्पर्श रेखाओं तथा स्पर्श बिन्दुओं को मिलाने वाली सरल रेखा से निर्मित त्रिभुज का क्षेत्रफल $\frac{a(h^2 + k^2 - a^2)^{3/2}}{h^2 + k^2}$ होता है।

Sol. Area of triangle formed by pair of tangents & chord of contact is = $\frac{RL^3}{R^2 + L^2}$

Here $R = a \Rightarrow L = \sqrt{h^2+k^2-a^2}$. Hence Area = $\frac{a\left(h^2+k^2-a^2\right)^{3/2}}{\left(h^2+k^2\right)}$

Hindi. स्पर्शी जीवा और स्पर्शी युग्म से बने त्रिभुज का क्षेत्रफल = $\frac{RL^3}{R^2+L^2}$

यहाँ $R = a \Rightarrow L = \sqrt{h^2+k^2-a^2}$. अतः क्षेत्रफल = $\frac{a\left(h^2+k^2-a^2\right)^{3/2}}{\left(h^2+k^2\right)}$

C-5. Find the equation of the chord of the circle $x^2 + y^2 + 6x + 8y + 9 = 0$ whose middle point is (– 2, – 3).
वृत्त $x^2 + y^2 + 6x + 8y + 9 = 0$ की उस जीवा का समीकरण ज्ञात कीजिए जिसका मध्य बिन्दु (– 2, – 3) है।

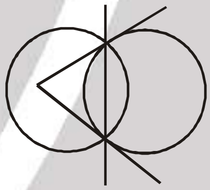
Ans. $x + y + 5 = 0$

Sol. $T = S_1 \Rightarrow -2x - 3y + 3(x-2) + 4(y-3) + 9 = 4 + 9 - 12 - 24 + 9 \Rightarrow x + y + 5 = 0$

C-6. Tangents are drawn to the circle $x^2 + y^2 = 12$ at the points where it is met by the circle $x^2 + y^2 - 5x + 3y - 2 = 0$; find the point of intersection of these tangents.
वृत्त $x^2 + y^2 = 12$ के उन बिन्दुओं पर स्पर्श रेखाएँ खींची जाती है जहाँ पर यह वृत्त $x^2 + y^2 - 5x + 3y - 2 = 0$ से मिलता है, तो इन स्पर्श रेखाओं का प्रतिच्छेद बिन्दु ज्ञात कीजिए।

Ans. $\left(6, -\frac{18}{5}\right)$

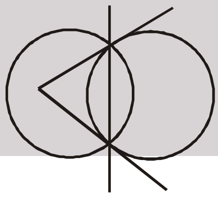
Sol. Equation of common chord is $S_1 - S_2 = 0 \Rightarrow 5x - 3y - 10 = 0$
This chord is also chord of contact.
Let point of intersection is p(h, k)



Then $hx + ky - 12 = 0$ compare both equations

$$\frac{h}{5} = \frac{k}{-3} = \frac{-12}{-10} \Rightarrow (h,k) \equiv \left(6, -\frac{18}{5}\right)$$

Hindi उभयनिष्ठ जीवा का समीकरण $S_1 - S_2 = 0 \Rightarrow 5x - 3y - 10 = 0$
यह जीवा स्पर्शी जीवा भी है। माना प्रतिच्छेद बिन्दु p(h, k) है।



तब $hx + ky - 12 = 0$
दोनों समीकरणों की तुलना करने पर $\Rightarrow$ (h,k)
दोनों समीकरणों की तुलना करने पर $\frac{h}{5} = \frac{k}{-3} = \frac{-12}{-10} \Rightarrow (h,k) \equiv \left(6, -\frac{18}{5}\right)$

Section (D) : Position of two circle, Orthogonality, Radical axis and radical centre

खण्ड (D) : दो वृत्तों की स्थिति, लम्बकोणीयता, मूलाक्ष एवं मूलाक्ष केन्द्र

D-1. Find the equations to the common tangents of the circles $x^2 + y^2 - 2x - 6y + 9 = 0$ and $x^2 + y^2 + 6x - 2y + 1 = 0$

वृत्तों $x^2 + y^2 - 2x - 6y + 9 = 0$ एवं $x^2 + y^2 + 6x - 2y + 1 = 0$ की उभयनिष्ठ स्पर्श रेखाओं के समीकरण ज्ञात कीजिए।

Ans. $x = 0, 3x + 4y = 10, y = 4$ and $3y = 4x$.

$x = 0, 3x + 4y = 10, y = 4$ एवं $3y = 4x$.

Sol. $S_1 : x^2 + y^2 - 2x - 6y + 9 = 0$ $\quad C_1(1, 3), r_1 = 1$

$S_2 : x^2 + y^2 + 6x - 2y + 1 = 0$ $\quad C_2(-3, 1), r_2 = 3$

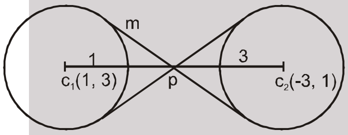
$C_1C_2 = \sqrt{16+4} = \sqrt{20} \Rightarrow n + r_2 = 4$

Hence $C_1C_2 > r_1 + r_2$ Both circles are non-intersecting.

Hence there are four common tangents.

Transverse common tangents :

coordinate of P $\left(\frac{3-3}{1+3}, \frac{1+9}{1+3}\right) \equiv \left(0, \frac{5}{2}\right)$



Let slope of these tangents is m

$y - \frac{5}{2} = m(x - 0) \Rightarrow mx - y + \frac{5}{2} = 0$

Now $\left|\frac{m - 3 + \frac{5}{2}}{\sqrt{1+m^2}}\right| = 1 \Rightarrow \left|m - \frac{1}{2}\right| = \sqrt{1+m^2}$

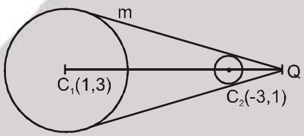
$\Rightarrow m^2 + \frac{1}{4} - m = 1 + m^2 \Rightarrow m = -\frac{3}{4}$, other tangents is vertical

Equation of tangents x = 0

$-\frac{3}{4}x - y + \frac{5}{2} = 0 \Rightarrow -3x - 4y + 10 = 0 \Rightarrow 3x + 3y = 10$

Direct common tangents

coordinate of Q $\left(\frac{-3-3}{1-3}, \frac{1-9}{1-3}\right) \equiv Q(3, 4)$



Hence equations $y - 4 = m(x - 3) \Rightarrow mx - y + (4 - 3m) = 0$

$\Rightarrow \left|\frac{m - 3 + 4 - 3m}{\sqrt{1+m^2}}\right| = 1 \Rightarrow |1 - 2m| = \sqrt{1+m^2} \Rightarrow 1 + 4m^2 - 4m = 1 + m^2 \Rightarrow 3m^2 - 4m = 0 \Rightarrow m = 0, \frac{4}{3}$

Hence equation $y - 4 = 0(x - 3) \Rightarrow y = 4 \Rightarrow y - 4 = \frac{4}{3}(x - 3) \Rightarrow 4x - 3y = 0$

Hindi. $S_1 : x^2 + y^2 - 2x - 6y + 9 = 0$ $\quad C_1(1, 3), r_1 = 1$

$S_2 : x^2 + y^2 + 6x - 2y + 1 = 0$ $\quad C_2(-3, 1), r_2 = 3$

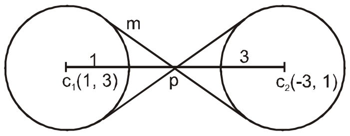
$C_1C_2 = \sqrt{16+4} = \sqrt{20} \Rightarrow n + r_2 = 4$

अतः $C_1C_2 > r_1 + r_2$ दोनों वृत्त प्रतिच्छेदी नहीं है।

अतः चार उभयनिष्ट स्पर्श रेखा है।

तिर्यक उभयनिष्ठ स्पर्श रेखाएँ **:**

P के निर्देशांक $\left(\frac{3-3}{1+3}, \frac{1+9}{1+3}\right) \equiv \left(0, \frac{5}{2}\right)$



माना स्पर्श रेखाओं की प्रवणता m है।

$y - \frac{5}{2} = m(x-0) \Rightarrow mx - y + \frac{5}{2} = 0$

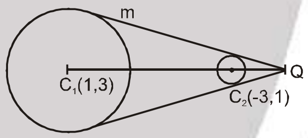
Now $\left|\frac{m - 3 + \frac{5}{2}}{\sqrt{1+m^2}}\right| = 1 \Rightarrow \left|m - \frac{1}{2}\right| = \sqrt{1+m^2}$

$\Rightarrow m^2 + \frac{1}{4} - m = 1 + m^2 \Rightarrow m = -\frac{3}{4}$, अतः स्पर्श रेखाएँ उर्ध्वाधर है। स्पर्श रेखाओं के समीकरण $x = 0$

$$-\frac{3}{4}x - y + \frac{5}{2} = 0 \Rightarrow -3x - 4y + 10 = 0 \Rightarrow 3x + 3y = 10$$

अनुक्रम उभयनिष्ठ स्पर्श रेखाएँ

Q के निर्देशांक $\left(\frac{-3-3}{1-3}, \frac{1-9}{1-3}\right) \equiv Q(3, 4)$



अतः समीकरणे $y - 4 = m(x-3) \Rightarrow mx - y + (4 - 3m) = 0$

$\Rightarrow \left|\frac{m - 3 + 4 - 3m}{\sqrt{1+m^2}}\right| = 1 \Rightarrow |1 - 2m| = \sqrt{1+m^2} \Rightarrow 1 + 4m^2 - 4m = 1 + m^2 \Rightarrow 3m^2 - 4m = 0 \Rightarrow m = 0, \frac{4}{3}$

अतः समीकरण $y - 4 = 0(x-3) \Rightarrow y = 4 \Rightarrow y - 4 = \frac{4}{3}(x-3) \Rightarrow 4x - 3y = 0$

D-2. Show that the circles $x^2 + y^2 - 2x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 4y - 6 = 0$ cut each other orthogonally.
सिद्ध कीजिए कि वृत्त $x^2 + y^2 - 2x - 6y - 12 = 0$ और $x^2 + y^2 + 6x + 4y - 6 = 0$ एक दूसरे को लम्बकोणीय प्रतिच्छेद करते हैं।

Sol. $C_1 \equiv (1, 3)\ C_2(-3, -2) \Rightarrow r_1 = \sqrt{22} \Rightarrow r_2 = \sqrt{19} \Rightarrow (C_1C_2)^2 = r_1^2 + r_2^2$

D-3. Find the equation of the circle passing through the origin and cutting the circles $x^2 + y^2 - 4x + 6y + 10 = 0$ and $x^2 + y^2 + 12y + 6 = 0$ at right angles.

उस वृत्त का समीकरण ज्ञात कीजिए जो मूल बिन्दु से गुजरता है तथा वृत्तों
$x^2 + y^2 - 4x + 6y + 10 = 0$ और $x^2 + y^2 + 12y + 6 = 0$ को समकोण पर काटता हैं।

Ans. $2(x^2 + y^2) - 7x + 2y = 0$

Sol. Equation of circle passing through origin is $x^2 + y^2 + 2gx + 2fy = 0$
This circle cuts the circle $x^2 + y^2 - 4x + 6y + 10 = 0$ orthogonally

$$2g(-2) + 2f(3) = 0 + 10 \Rightarrow -2g + 3f - 5 = 0 \quad \text{...(1)}$$

& $x^2 + y^2 + 12y + 6 = 0$ also

$2g(0) + 2f(6) = 6 + 0 \Rightarrow f = \frac{1}{2} \Rightarrow -2g + \frac{3}{2} - 5 = 0 \Rightarrow 2g = -\frac{7}{2} \Rightarrow g = -\frac{7}{4}$

Hence circle $x^2 + y^2 + 2\left(-\frac{7}{4}\right)x + 2\left(\frac{1}{2}\right)y = 0 \Rightarrow 2x^2 + 2y^2 - 7x + 2y = 0$

Hindi. मूल बिन्दु से गुजरने वाले वृत्त का समीकरण $x^2 + y^2 + 2gx + 2fy = 0$
यह वृत्त, वृत्त $x^2 + y^2 - 4x + 6y + 10 = 0$ को समकोणीय प्रतिच्छेद करता है।

$$2g(-2) + 2f(3) = 0 + 10 \Rightarrow -2g + 3f - 5 = 0 \quad ...(1)$$

तथा $x^2 + y^2 + 12y + 6 = 0$

$2g(0) + 2f(6) = 6 + 0 \Rightarrow f = \frac{1}{2} \Rightarrow -2g + \frac{3}{2} - 5 = 0 \Rightarrow 2g = -\frac{7}{2} \Rightarrow g = -\frac{7}{4}$

अतः वृत्त $x^2 + y^2 + 2\left(-\frac{7}{4}\right)x + 2\left(\frac{1}{2}\right)y = 0 \Rightarrow 2x^2 + 2y^2 - 7x + 2y = 0$

D-4. Given the three circles $x^2 + y^2 - 16x + 60 = 0$, $3x^2 + 3y^2 - 36x + 81 = 0$ and $x^2 + y^2 - 16x - 12y + 84 = 0$, find (1) the point from which the tangents to them are equal in length and (2) this length. **[16JM110497]**

तीन वृत्त $x^2 + y^2 - 16x + 60 = 0$, $3x^2 + 3y^2 - 36x + 81 = 0$ और $x^2 + y^2 - 16x - 12y + 84 = 0$ हो, तो ज्ञात कीजिए

(1) उस बिन्दु के निर्देशांक जिससे इन वृत्तों पर खींची गई स्पर्श रेखाओं की लम्बाइयाँ परस्पर बराबर हो।

(2) इन स्पर्श रेखाओं की लम्बाई।

Ans. $\left(\frac{33}{4}, 2\right)$; $\frac{1}{4}$

Sol. (1) $S_1 - S_2 = 0 \Rightarrow 4x - 33 = 0$

$S_3 - S_1 = 0 \Rightarrow -12y + 24 = 0 \Rightarrow \left(\frac{33}{4}, 2\right)$

(2) Length of tangent from $\left(\frac{33}{4}, 2\right)$ to any circle is $= \frac{1}{4}$.

Hindi. (1) $S_1 - S_2 = 0 \Rightarrow 4x - 33 = 0$

$S_3 - S_1 = 0 \Rightarrow -12y + 24 = 0 \Rightarrow \left(\frac{33}{4}, 2\right)$

(2) $\left(\frac{33}{4}, 2\right)$ से किसी भी वृत्त पर स्पर्श रेखा की लम्बाई $= \frac{1}{4}$.

Section (E) : Family of circles , Locus, Miscellaneous
खण्ड (E) : वृत्त निकाय, बिन्दुपथ, विविध

E-1. If $y = 2x$ is a chord of the circle $x^2 + y^2 - 10x = 0$, find the equation of a circle with this chord as diameter.

यदि $y = 2x$ वृत्त $x^2 + y^2 - 10x = 0$ की जीवा हो, तो उस वृत्त का समीकरण ज्ञात कीजिए जिसके लिए यह जीवा व्यास है।

Ans. $x^2 + y^2 - 2x - 4y = 0$.

Sol. $x^2 + y^2 - 10x + \lambda(2x - y) = 0 \quad ....(i)$

$x^2 + y^2 + 2x(\lambda - 5) - \lambda y = 0$

Centre $(-(\lambda - 5), \lambda/2)$

Using on $y = 2x \Rightarrow \frac{\lambda}{2} = -2(\lambda - 5) \Rightarrow \frac{5\lambda}{2} = 10$

Putting $\lambda = 4 \Rightarrow x^2 + y^2 - 2x - 4y = 0$

Hindi $x^2 + y^2 - 10x + \lambda(2x - y) = 0$ ....(i)

$x^2 + y^2 + 2x + (\lambda - 5) - \lambda y = 0$

केन्द्र $(-(\lambda - 5), \lambda/2) \Rightarrow y=2x$ का प्रयोग करने पर

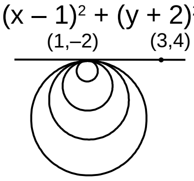
$\Rightarrow \frac{\lambda}{2} = -2(\lambda - 5) \Rightarrow \frac{5\lambda}{2} = 10$

$\lambda = 4$ रखने पर $\Rightarrow x^2 + y^2 - 2x - 4y = 0$

E-2. Find the equation of a circle which touches the line $2x - y = 4$ at the point $(1, -2)$ and
वृत्त का समीकरण ज्ञात कीजिए जो रेखा $2x - y = 4$ को बिन्दु $(1, -2)$ पर स्पर्श करती है।
(i) Passes through (3, 4) (i) (3, 4 से गुजरती है।
(ii) Radius = 5 (ii) त्रिज्या = 5

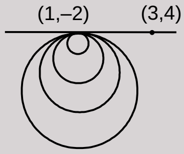
Ans. (i) $(x - 1)^2 + (y + 2)^2 + 20\,(2x - y - 4) = 0$
(ii) $(x - 1)^2 + (y + 2)^2 \pm \sqrt{20}\,(2x - y - 4) = 0$

Sol. (i) Equation of circle is $(x - 1)^2 + (y + 2)^2 + \lambda\,(2x - y - 4) = 0$



which passes through $(3,4) \Rightarrow \lambda = 20$
(ii) equation of circle is $(x - 1)^2 + (y + 2)^2 + \lambda\,(2x - y - 4) = 0$
whose radius is $5 \Rightarrow (\lambda - 1)^2 + \left(\frac{-\lambda + 4}{2}\right)^2 - (1 + 4 - 4\lambda) = 25 \Rightarrow \lambda = \pm\sqrt{20}$

Hindi (i) वृत्त का समीकरण $(x - 1)^2 + (y + 2)^2 + \lambda\,(2x - y - 4) = 0$ है –



जो (3,4) से गुजरती है $\Rightarrow \lambda = 20$
(ii) वृत्त का समीकरण $(x - 1)^2 + \sqrt{20}\,(y + 2)^2 + \lambda\,(2x - y - 4) = 0$ है
जिसकी त्रिज्या 5 है $\Rightarrow (\lambda - 1)^2 + \left(\frac{-\lambda + 4}{2}\right)^2 - (1 + 4 - 4\lambda) = 25 \Rightarrow \lambda = \pm\sqrt{20}$

E-3. Show that the equation $x^2 + y^2 - 2x - 2\lambda y - 8 = 0$ represents for different values of λ a system of circles passing through two fixed points A and B on the x-axis, and also find the equation of that circle of the system the tangent to which at A and B meet on the line $x + 2y + 5 = 0$.
दर्शाइये कि समीकरण $x^2 + y^2 - 2x - 2\lambda y - 8 = 0$ के लिए λ के भिन्न–भिन्न मानों के लिए वृत्तों का निकाय दो स्थिर बिन्दुओं A तथा B से गुजरता है जो x-अक्ष पर मिलता है। तथा वृत्त का समीकरण ज्ञात कीजिए जबकि स्पर्श रेखा निकाय, रेखा $x + 2y + 5 = 0$ को A तथा B पर मिलता है।

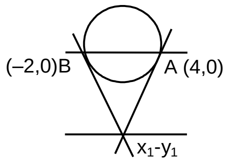
Sol. $(x^2 + y^2 - 2x - 8) - 2\lambda y = 0$
$S + \lambda L = 0$
solving (हल करने पर) $S = 0$ & $L = 0$
put $y = 0$ रखने पर) $\Rightarrow x^2 - 2x - 8 = 0$
$(x - 4)(x + 2) = 0$
$x = -2$ or 4
$A \equiv (4, 0)$
$B \equiv (-2, 0)$

Equation of AB (AB का समीकरण) :
C.O.C. $xx_1 + yy_1 - 1\,(x + x_1) - \lambda\,(y + y_1) - 8 = 0$
$\Rightarrow \quad x\,(x_1 - 1) + y\,(y - y_1) - (x_1 + \lambda y_1 + 8) = 0$ ...(1)
Also equation of AB is x-axis i.e
तथा AB का समीकरण x-अंक्ष है
$0.x + 1.y + 0.c = 0$ ...(2)
comparing (1) & (2) से

$$\frac{0}{x_1 - 1} = \frac{1}{y_1 - \lambda} = \frac{0}{x_1 + \lambda y_1 + 8}$$

$x_1 = 1$ & $\lambda y_1 + 9 = 0$

$$y_1 = \frac{-9}{\lambda}$$



Also इसलिए $x + 2y + 5 = 0 \Rightarrow 1 - \frac{18}{\lambda} + 5 = 0 \quad \Rightarrow \quad 6 = \frac{18}{\lambda} \Rightarrow \lambda = 3$

equation of circle is वृत्त का समीकरण $x^2 + y^2 - 2x - 6y - 8 = 0$

E-4. Consider a family of circles passing through two fixed points A (3, 7) and B (6, 5). Show that the chords in which the circles $x^2 + y^2 - 4x - 3 = 0$ cuts the members of the family are concurrent at a point. Also find the co-ordinates of this point.

माना कि वृत्तों का निकाय दो स्थिर बिन्दुओं A (3, 7) और B (6, 5) से गुजरता है। दर्शाइयें कि जीवाऐं जो वृत्त $x^2 + y^2 - 4x - 3 = 0$ को निकाय के सदस्य को एक बिन्दु पर संगामी होती है। तथा इस बिन्दु के निर्देशांक भी ज्ञात कीजिए।

Ans. $\left(\frac{52}{3}, -\frac{23}{9}\right)$

Sol. Family of circles passes through two fixed point is
दो स्थिर बिन्दुओं से गुजरने वाले वृत्तों का निकाय है –

$S + \lambda L = 0$ ...(1)

where जहाँ $S = (x - 3)(x - 6) + (y - 7)(y - 5) = 0$

$$L = \begin{vmatrix} x & y & 1 \\ 3 & 7 & 1 \\ 6 & 5 & 1 \end{vmatrix} = 0$$

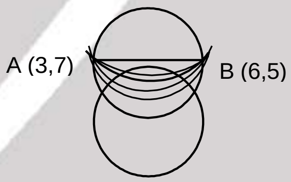
Equation of variable common chord is चर उभयनिष्ठ जीवा का समीकरण है –

$S + \lambda L - S' = 0$

$\Rightarrow \quad (S - S') + \lambda L = 0$

or या $L' + \lambda L = 0$ which represents family of lines concurrent at $L' = 0$ and $L = 0$

$L' + \lambda L = 0$ संगामी रेखा निकाय को व्यक्त रखता है $L' = 0$ तथा $L = 0$



$S = (x - 3)(x - 6) + (y - 7)(y - 5) = 0$

$x^2 + y^2 - ax - 12y + 53 = 0$

$x^2 + y^2 - 4x - 3 = 0$

$-5x - 12y + 56 = 0$

$4(2x + 3y - 27 = 0) \Rightarrow x = \frac{52}{3}, \; y = -\frac{23}{9}$

E-5. Find the equation of the circle circumscribing the triangle formed by the lines $x + y = 6$, $2x + y = 4$ and $x + 2y = 5$.

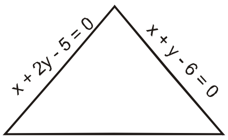
सरल रेखाओं $x + y = 6$, $2x + y = 4$ और $x + 2y = 5$ से निर्मित त्रिभुज के परिवृत्त का समीकरण ज्ञात कीजिए।

Ans. $x^2 + y^2 - 17x - 19y + 50 = 0$

Sol. Equation of circumcircle of this triangle

$L_1L_2 + \lambda L_2L_3 + \mu L_3L_1 = 0$

$(x + 2y - 5)(x + y - 6) + \lambda(x + y - 6)(2x + y - 4) + \mu(x + 2y - 5)(2x + y - 4) = 0$



coef. of xy = 0 $\Rightarrow$ 3 + 3λ + 5μ = 0 $\Rightarrow$ 3λ + 5μ + 3 = 0 ...(1)

coef. x^2 = coef. y^2 $\Rightarrow$ 1 + 2λ + 2μ = 2 + λ + 2μ

$\Rightarrow \lambda = 1 \qquad \mu = -\frac{6}{5}$

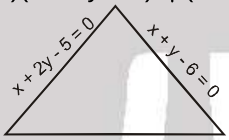
Hence $(x + 2y - 5)(x + y - 6) + (x + y - 6)(2x + y - 4) - \frac{6}{5}(x + 2y - 5)(2x + y - 4) = 0$

$\Rightarrow x^2 + y^2 - 17x - 19y + 50 = 0$

Hindi. इस त्रिभुज के परिगत वृत्त का समीकरण

$L_1L_2 + \lambda L_2L_3 + \mu L_3L_1 = 0$

$(x + 2y - 5)(x + y - 6) + \lambda(x + y - 6)(2x + y - 4) + \mu(x + 2y - 5)(2x + y - 4) = 0$



xy का गुणांक = 0 $\Rightarrow$ 3 + 3λ + 5μ = 0 $\Rightarrow$ 3λ + 5μ + 3 = 0 ...(1)

x^2 का गुणांक = y^2 का गुणांक $\Rightarrow$ 1 + 2λ + 2μ = 2 + λ + 2μ

$\Rightarrow \lambda = 1 \qquad \mu = -\frac{6}{5}$

अतः $(x + 2y - 5)(x + y - 6) + (x + y - 6)(2x + y - 4) - \frac{6}{5}(x + 2y - 5)(2x + y - 4) = 0$

$\Rightarrow x^2 + y^2 - 17x - 19y + 50 = 0$

E-6. Prove that the circle $x^2 + y^2 + 2ax + c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$ touches each other if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$

सिद्ध कीजिए कि वृत्त $x^2 + y^2 + 2ax + c^2 = 0$ और $x^2 + y^2 + 2by + c^2 = 0$ एक दूसरे को स्पर्श करते है

यदि $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$

Sol. Subtract to get common tangent and drop pependicular from centre on any one circle and equate it to its radius.

घटाने पर उभयनिष्ठ स्पर्श रेखा प्राप्त होती है। केन्द्र से डाला गया लम्ब त्रिज्या के बराबर होता है ।

Radical axes is ax – by = 0 which touches both the circle

मूलाक्ष ax – by = 0 जो दोनों वृत्तों को स्पर्श करती है

Now अब $x^2 + \left(\frac{ax}{b}\right)^2 + 2ax + c^2 = 0$

$\Rightarrow (b^2 + a^2)x^2 + 2ab^2 x + b^2c^2 = 0$

$\Rightarrow 4a^2b^2 - 4(b^2 + a^2) b^2c^2 = 0$

$\Rightarrow a^2b^2 = c^2 (b^2 + a^2)$

$\Rightarrow \frac{1}{c^2} = \frac{1}{a^2} + \frac{1}{b^2}$

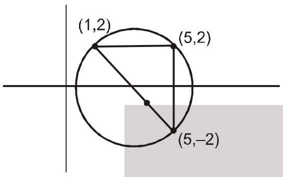
PART - II : ONLY ONE OPTION CORRECT TYPE

भाग - II : केवल एक सही विकल्प प्रकार (ONLY ONE OPTION CORRECT TYPE)

Section (A) : Equation of circle, parametric equation, position of a point
खण्ड (A) : वृत्त का समीकरण, प्राचलिक समीकरण, बिन्दु की स्थिति

A-1. The radius of the circle passing through the points (1, 2), (5, 2) & (5, – 2) is:
बिन्दुओं (1, 2), (5, 2) एवं (5, – 2) से गुजरने वाले वृत्त की त्रिज्या है –
(A) $5\sqrt{2}$ (B) $2\sqrt{5}$ (C) $3\sqrt{2}$ (D*) $2\sqrt{2}$

Sol.



diameter = $4\sqrt{2}$ व्यास = $4\sqrt{2}$ $\Rightarrow$ $r = 2\sqrt{2}$

A-2. The centres of the circles $x^2 + y^2 - 6x - 8y - 7 = 0$ and $x^2 + y^2 - 4x - 10y - 3 = 0$ are the ends of the diameter of the circle
वृत्तों $x^2 + y^2 - 6x - 8y - 7 = 0$ और $x^2 + y^2 - 4x - 10y - 3 = 0$ के केन्द्र निम्न में से किस वृत्त के व्यास के अन्तिम बिन्दु हैं।
(A*) $x^2 + y^2 - 5x - 9y + 26 = 0$ (B) $x^2 + y^2 + 5x - 9y + 14 = 0$
(C) $x^2 + y^2 + 5x - y - 14 = 0$ (D) $x^2 + y^2 + 5x + y + 14 = 0$

Sol. (3,4) & (2,5) are ends of diameter of circle
So, Equation $(x - 3)(x - 2) + (y - 4)(y - 5) = 0 \Rightarrow x^2 + y^2 - 5x - 9y + 26 = 0$

Hindi वृत्त के व्यास के अन्तिम सिरे (3,4) व (2,5) है।
अतः समीकरण $(x - 3)(x - 2) + (y - 4)(y - 5) = 0 \Rightarrow x^2 + y^2 - 5x - 9y + 26 = 0$

A-3. The circle described on the line joining the points (0, 1), (a, b) as diameter cuts the x–axis in points whose abscissa are roots of the equation:
बिन्दुओं (0, 1) और (a, b) को मिलाने वाली सरल रेखा को व्यास मानकर खींचा गया वृत्त, x–अक्ष को जिन बिन्दुओं पर काटता है, उनके भुज जिस समीकरण के मूल है, वह है –
(A) $x^2 + ax + b = 0$ (B*) $x^2 - ax + b = 0$ (C) $x^2 + ax - b = 0$ (D) $x^2 - ax - b = 0$

Sol. Equation of circle $(x - 0)(x - a) + (y - 1)(y - b) = 0$
it cuts x-axis put $y = 0$ $\Rightarrow$ $x^2 - ax + b = 0$
वृत्त का समीकरण $(x - 0)(x - a) + (y - 1)(y - b) = 0$
यह x-अक्ष को प्रतिच्छेद करता है तब $y = 0$ रखने पर $\Rightarrow$ $x^2 - ax + b = 0$

A-4. The intercepts made by the circle $x^2 + y^2 - 5x - 13y - 14 = 0$ on the x-axis and y-axis are respectively
(A) 9, 13 (B) 5, 13 (C*) 9, 15 (D)none
वृत्त $x^2 + y^2 - 5x - 13y - 14 = 0$ द्वारा x-अक्ष और y-अक्ष पर अन्तःखण्ड क्रमशः हैं –
(A) 9, 13 (B) 5, 13 (C) 9, 15 (D) इनमें से कोई नहीं

Sol. Length of intercept on x-axis = $2\sqrt{g^2 - c} = 2\sqrt{\frac{25}{4} + 14} = 2\sqrt{\frac{81}{4}} = 9$

on y-axis = $2\sqrt{f^2 - c} = 2\sqrt{\left(\frac{13}{2}\right)^2 + 14} = 2\sqrt{\frac{169 + 56}{4}} = 2\sqrt{\frac{225}{4}} = 15$

x-अक्ष पर अन्तखण्ड की लम्बाई = $2\sqrt{g^2 - c} = 2\sqrt{\frac{25}{4} + 14} = 2\sqrt{\frac{81}{4}} = 9$

y-अक्ष पर अन्तःखण्ड $= 2\sqrt{f^2 - c} = 2\sqrt{\left(\frac{13}{2}\right)^2 + 14} = 2\sqrt{\frac{169+56}{4}} = 2\sqrt{\frac{225}{4}} = 15$

A-5. Equation of line passing through mid point of intercepts made by circle $x^2 + y^2 - 4x - 6y = 0$ on co-ordinate axes is

वृत्त $x^2 + y^2 - 4x - 6y = 0$ द्वारा निर्देशी अक्षों पर काटे गये अन्तःखण्ड के मध्य बिन्दु से गुजरने वाली रेखा का समीकरण होगा–

(A) $3x + 2y - 12 = 0$ (B) $3x + y - 6 = 0$ (C) $3x + 4y - 12 = 0$ (D*) $3x + 2y - 6 = 0$

Sol. given circle $x^2 + y^2 - 4x - 6y = 0$ it cuts x-axis put $y = 0$, $x = 0, 4$
it cuts y-axis put $x = 0$, $y = 0, 6$. Hence mid points on x-axis $(2, 0)$ on y-axis $(0, 3)$

Equations of line $\frac{x}{2} + \frac{y}{3} = 1 \Rightarrow 3x + 2y - 6 = 0$

Hindi. दिया गया वृत्त $x^2 + y^2 - 4x - 6y = 0$
यह x-अक्ष को प्रतिच्छेद करता है $y = 0$, $x = 0, 4$ रखने पर
यह y-अक्ष को प्रतिच्छेद करता है $x = 0$, $y = 0, 6$. अतः x-अक्ष पर $(2, 0)$, y-अक्ष पर $(0, 3)$ मध्य बिन्दु है

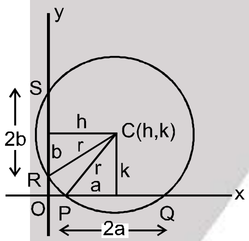
रेखा का समीकरण है $\frac{x}{2} + \frac{y}{3} = 1 \Rightarrow 3x + 2y - 6 = 0$

A-6. Two thin rods AB & CD of lengths 2a & 2b move along OX & OY respectively, when 'O' is the origin. The equation of the locus of the centre of the circle passing through the extremities of the two rods is:

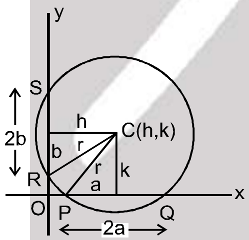
दो पतली छड़ें AB और CD जिनकी लम्बाइयाँ 2a और 2b है क्रमशः OX और OY अक्षों के अनुदिश गतिशील है, जबकि O मूलबिन्दु है। दोनों छड़ों के सिरों से गुजरने वाले वृत्त के केन्द्र का बिन्दुपथ होगा –

(A) $x^2 + y^2 = a^2 + b^2$ (B*) $x^2 - y^2 = a^2 - b^2$ (C) $x^2 + y^2 = a^2 - b^2$ (D) $x^2 - y^2 = a^2 + b^2$

Sol. $h^2 + b^2 = r^2 \Rightarrow k^2 + a^2 = r^2 \Rightarrow h^2 - k^2 = a^2 - b^2$



Hindi $h^2 + b^2 = r^2 \Rightarrow k^2 + a^2 = r^2 \Rightarrow h^2 - k^2 = a^2 - b^2$



$\therefore$ बिन्दुपथ है $x^2 - y^2 = a^2 - b^2$

A-7. Let A and B be two fixed points then the locus of a point C which moves so that $(\tan\angle BAC)(\tan\angle ABC)=1$,

$0 < \angle BAC < \frac{\pi}{2}$, $0 < \angle ABC < \frac{\pi}{2}$ is

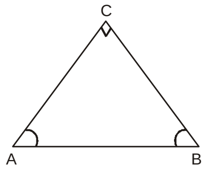
(A*) Circle (B) pair of straight line (C) A point (D) Straight line

माना A एवं B दो स्थिर बिन्दु है तब बिन्दु C का बिन्दुपथ जबकि बिन्दु C इस प्रकार है कि $(\tan\angle BAC)(\tan\angle ABC)=1$, 0 $0 < \angle BAC < \frac{\pi}{2}$, $0 < \angle ABC < \frac{\pi}{2}$ होगा–

(A*) वृत्त (B) सरल रेखा युग्म (C) एक बिन्दु (D) सरल रेखा

Sol. $\tan\alpha \tan\beta = 1 \Rightarrow \tan\alpha = \cot\beta \Rightarrow \tan\alpha = \tan\left(\frac{\pi}{2} - \beta\right)$

$\alpha + \beta = \frac{\pi}{2}$ $\Rightarrow$ $\angle ACB = \frac{\pi}{2}$ $\Rightarrow$ locus of C is a



circle as angle in a semicircle is $\frac{\pi}{2}$

अतः c का बिन्दुपथ एक वृत्त होगा जैसा कि अर्धवृत्त मे बना कोंण $\frac{\pi}{2}$ होता है।

A-8. **STATEMENT-1 :** The length of intercept made by the circle $x^2 + y^2 - 2x - 2y = 0$ on the x-axis is 2.

STATEMENT-2 : $x^2 + y^2 - \alpha x - \beta y = 0$ is a circle which passes through origin with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ and radius $\sqrt{\frac{\alpha^2+\beta^2}{2}}$

(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1

(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1

(C*) STATEMENT-1 is true, STATEMENT-2 is false

(D) STATEMENT-1 is false, STATEMENT-2 is true

कथन-1 : वृत्त $x^2 + y^2 - 2x - 2y = 0$ द्वारा x-अक्ष पर काटे गए अन्तःखण्ड की लम्बाई 2 है।

कथन-2 : $x^2 + y^2 - \alpha x - \beta y = 0$ मूलबिन्दु से गुजरने वाला एक वृत्त है जिसका केन्द्र $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ और त्रिज्या $\sqrt{\frac{\alpha^2+\beta^2}{2}}$ है।

(A) कथन–1 सत्य है, कथन–2 सत्य है ; कथन–2, कथन–1 का सही स्पष्टीकरण है।

(B) कथन–1 सत्य है, कथन–2 सत्य है ; कथन–2, कथन–1 का सही स्पष्टीकरण नहीं है।

(C) कथन–1 सत्य है, कथन–2 असत्य है।

(D) कथन–1 असत्य है, कथन–2 सत्य है।

Sol. Statement-1 is true and statement-2 is false as radius = $\frac{1}{2}\sqrt{\alpha^2+\beta^2}$

Hindi कथन-1 सत्य है और कथन-2 असत्य है क्योंकि त्रिज्या = $\frac{1}{2}\sqrt{\alpha^2+\beta^2}$

Section (B) : Line and circle, tangent, pair of tangent

खण्ड (B) : रेखा एवं वृत्त, स्पर्श रेखा, स्पर्श रेखा युग्म

B-1. Find the co-ordinates of a point p on line $x + y = -13$, nearest to the circle $x^2 + y^2 + 4x + 6y - 5 = 0$

रेखा $x + y = -13$ पर स्थित बिन्दु P के निर्देशांक जो कि वृत्त $x^2 + y^2 + 4x + 6y - 5 = 0$ के निकटतम हो, होगा–

(A*) (– 6, – 7) (B) (– 15, 2) (C) (– 5, – 6) (D) (– 7, – 6)

Sol. Point on the line $x + y + 13 = 0$ nearest to the circle $x^2 + y^2 + 4x + 6y - 5 = 0$ is foot of $\perp$ from centre

$$\frac{x+2}{1} = \frac{y+3}{1} = -\left(\frac{-2-3+13}{1^2+1^2}\right) = -4 \Rightarrow x = -6\ y = -7$$

Hindi. रेखा $x + y + 13 = 0$ पर स्थित बिन्दु के वृत्त $x^2 + y^2 + 4x + 6y - 5 = 0$ के नजदीक होने के लिए केन्द्र से लम्ब डालते है।

$$\frac{x+2}{1} = \frac{y+3}{1} = -\left(\frac{-2-3+13}{1^2+1^2}\right) = -4 \Rightarrow x = -6\ y = -7$$

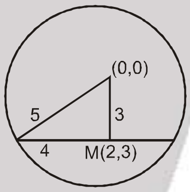
B-2. The number of tangents that can be drawn from the point (8, 6) to the circle $x^2 + y^2 - 100 = 0$ is
(A) 0 (B*) 1 (C) 2 (D) none
बिन्दु (8, 6) से वृत्त $x^2 + y^2 - 100 = 0$ पर खींची जा सकने वाली स्पर्श रेखाओं की संख्या है–
(A) 0 (B) 1 (C) 2 (D) इनमें से कोई नहीं

Sol. Point (8,6) lies on circle ; $S_1 = 0 \Rightarrow$ one tangent.

Hindi. बिन्दु (8,6) वृत्त पर स्थित है। $S_1 = 0 \Rightarrow$ अतः एक स्पर्श रेखा होगी।

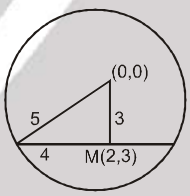
B-3. Two lines through (2, 3) from which the circle $x^2 + y^2 = 25$ intercepts chords of length 8 units have equations
(A) $2x + 3y = 13$, $x + 5y = 17$ (B*) $y = 3$, $12x + 5y = 39$
(C) $x = 2$, $9x - 11y = 51$ (D) $y = 0$, $12x + 5y = 39$
बिन्दु (2, 3) से गुजरने वाली उन दो सरल रेखाओं के समीकरण जो वृत्त $x^2 + y^2 = 25$ पर 8 इकाई लम्बाई के अन्तःखण्ड की जीवा हो, है–
(A) $2x + 3y = 13$, $x + 5y = 17$ (B) $y = 3$, $12x + 5y = 39$
(C) $x = 2$, $9x - 11y = 51$ (D) $y = 0$, $12x + 5y = 39$

Sol. Let slope of required line is m
$y - 3 = m(x - 2) \Rightarrow mx - y + (3 - 2m) = 0$
length of $\perp$ from origin = 3
$\Rightarrow 9 + 4m^2 - 12m = 9 + 9m^2 \Rightarrow 5m^2 + 12m = 0 \Rightarrow m = 0, -\frac{12}{5}$



Hence lines are $y - 3 = 0 \Rightarrow y = 3$
$y - 3 = -\frac{12}{5}(x - 2) \Rightarrow 5y - 15 = -12x + 24 \Rightarrow 12x + 5y = 39.$

Hindi. माना अभीष्ट रेखा की प्रवणता = m
$y - 3 = m(x - 2) \Rightarrow mx - y + (3 - 2m) = 0$
मूलबिन्दू से लम्ब की लम्बाई = 3
$\Rightarrow 9 + 4m^2 - 12m = 9 + 9m^2 \Rightarrow 5m^2 + 12m = 0 \Rightarrow m = 0, -\frac{12}{5}$



अतः रेखाएँ $y - 3 = 0 \Rightarrow y = 3$
$y - 3 = -\frac{12}{5}(x - 2) \Rightarrow 5y - 15 = -12x + 24 \Rightarrow 12x + 5y = 39.$

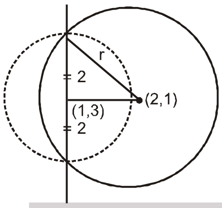
B-4. The line $3x + 5y + 9 = 0$ w.r.t. the circle $x^2 + y^2 - 4x + 6y + 5 = 0$ is
(A) chord dividing circumference in 1 : 3 ratio (B*) diameter
(C) tangent (D) outside line
रेखा $3x + 5y + 9 = 0$ वृत्त $x^2 + y^2 - 4x + 6y + 5 = 0$ के सापेक्ष हैं –
(A) जीवा जो परिधि को 1 : 3 में विभाजित करता है। (B) व्यास
(C) स्पर्श रेखा (D) वृत्त के बाहर से जाने वाली रेखा

Sol. From centre (2, –3), length of perpendicular on line $3x + 5y + 9 = 0$ is
$p = \frac{6 - 15 + 9}{\sqrt{25 + 9}} = 0$; line is diameter.

Hindi केन्द्र (2, –3) से रेखा $3x + 5y + 9 = 0$ पर डाले गये लम्ब की लम्बाई
$p = \frac{6 - 15 + 9}{\sqrt{25 + 9}} = 0 \Rightarrow$ रेखा व्यास है।

B-5. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of the circle is
यदि वृत्त $x^2 + y^2 - 2x - 6y + 6 = 0$ का एक व्यास उस वृत्त की जीवा है जिसका केन्द्र (2, 1) हो, तो वृत्त की त्रिज्या होगी–
(A*) 3 (B) 2 (C) 3/2 (D) 1

Sol. Clearly from the figure the radius of bigger circle
चित्र से स्पष्टतया बड़े वृत्त की त्रिज्या



$r^2 = 2^2 + \{(2-1)^2 + (1-3)^2\}$
$r^2 = 9$ or $r = 3$

B-6. The tangent lines to the circle $x^2 + y^2 - 6x + 4y = 12$ which are parallel to the line $4x + 3y + 5 = 0$ are given by:
(A) $4x + 3y - 7 = 0,\ 4x + 3y + 15 = 0$ (B*) $4x + 3y - 31 = 0,\ 4x + 3y + 19 = 0$
(C) $4x + 3y - 17 = 0,\ 4x + 3y + 13 = 0$ (D) $4x + 3y - 31 = 0,\ 4x + 3y - 19 = 0$
सरल रेखा $4x + 3y + 5 = 0$ के समान्तर वृत्त $x^2 + y^2 - 6x + 4y = 12$ की स्पर्श रेखाओं के समीकरण है–
(A) $4x + 3y - 7 = 0,\ 4x + 3y + 15 = 0$ (B) $4x + 3y - 31 = 0,\ 4x + 3y + 19 = 0$
(C) $4x + 3y - 17 = 0,\ 4x + 3y + 13 = 0$ (D) $4x + 3y - 31 = 0,\ 4x + 3y - 19 = 0$

Sol. Line parallel to given line $4x + 3y + 5 = 0$ is $4x + 3y + k = 0$
This is tangent to $x^2 + y^2 - 6x + 4y - 12 = 0$
$\left|\frac{12-6+k}{5}\right| = 5 \Rightarrow 6 + k = \pm 25 \Rightarrow k = 19, -31$
Hence required line $4x + 3y - 31 = 0,\ 4x + 3y + 19 = 0$

Hindi. दी गई रेखा $4x + 3y + 5 = 0$ के समान्तर रेखा $4x + 3y + k = 0$
यह वृत्त $x^2 + y^2 - 6x + 4y - 12 = 0$ की स्पर्श रेखा है।
$\left|\frac{12-6+k}{5}\right| = 5 \Rightarrow 6 + k = \pm 25 \Rightarrow k = 19, -31$
अतः अभीष्ट रेखा $4x + 3y - 31 = 0,\ 4x + 3y + 19 = 0$ है।

B-7. The condition so that the line $(x + g)\cos\theta + (y + f)\sin\theta = k$ is a tangent to $x^2 + y^2 + 2gx + 2fy + c = 0$ is

सरल रेखा $(x + g)\cos\theta + (y + f)\sin\theta = k$ के वृत्त $x^2 + y^2 + 2gx + 2fy + c = 0$ की स्पर्श रेखा होने के लिए प्रतिबन्ध है –
(A*) $g^2 + f^2 = c + k^2$ (B) $g^2 + f^2 = c^2 + k$ (C) $g^2 + f^2 = c^2 + k^2$ (D) $g^2 + f^2 = c + k$

Sol. $p = \left|\frac{(-g+g)\cos\theta + (-f+f)\sin\theta - k}{\sqrt{\cos^2\theta + \sin^2\theta}}\right| = \sqrt{g^2 + f^2 - c} \Rightarrow g^2 + f^2 = c + k^2$

B-8. The tangent to the circle $x^2 + y^2 = 5$ at the point (1, –2) also touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$ at
वृत्त $x^2 + y^2 = 5$ के बिन्दु (1,–2) पर स्पर्श रेखा, वृत्त $x^2 + y^2 - 8x + 6y + 20 = 0$ को भी स्पर्श करता हो, तो स्पर्श बिन्दु है–
(A) (–2, 1) (B) (–3, 0) (C) (–1, –1) (D*) (3, –1)

Sol. Equation of tangent $x - 2y = 5$
Let required point be (α, β)
$\alpha x + \beta y - 4(x + \alpha) + 3(y + \beta) + 20 = 0 \Rightarrow x(\alpha - 4) + y(\beta + 3) - 4\alpha + 3\beta + 20 = 0$
Comparing

$$\frac{\alpha-4}{1}=\frac{\beta+3}{-2}=\frac{4\alpha-3\beta-20}{5}$$

Similarly (α,β) $(3, -1)$

Hindi स्पर्श रेखा का समीकरण $x - 2y = 5$

माना कि अभीष्ट बिन्दु (α,β) है।

$\alpha x + \beta y - 4(x + \alpha) + 3(y + \beta) + 20 = 0 \Rightarrow x(\alpha - 4) + y(\beta + 3) - 4\alpha + 3\beta + 20 = 0$

तुलना करने पर

$$\frac{\alpha-4}{1}=\frac{\beta+3}{-2}=\frac{4\alpha-3\beta-20}{5}$$

इसी प्रकार (α,β) $(3, -1)$

B-9. The angle between the two tangents from the origin to the circle $(x-7)^2 + (y+1)^2 = 25$ equals

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C*) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

मूल बिन्दु से वृत्त $(x-7)^2 + (y+1)^2 = 25$ पर खींची गई स्पर्श रेखाओं के मध्य कोण है –

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C*) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

Sol. Let tangent be $y = mx \Rightarrow \left|\frac{7m+1}{\sqrt{1+m^2}}\right| = 5 \Rightarrow 49m^2 + 1 + 14m = 25(1+m^2)$

$24m^2 + 14m - 24 = 0 \Rightarrow m_1m_2 = -1 \Rightarrow$ angle $= 90^o$

Hindi मानाकि स्पर्श रेखा $y = mx$ है।

$\left|\frac{7m+1}{\sqrt{1+m^2}}\right| = 5 \Rightarrow \Rightarrow 49m^2 + 1 + 14m = 25(1+m^2)$

$24m^2 + 14m - 24 = 0 \Rightarrow m_1m_2 = -1 \Rightarrow$ कोण $= 90^o$

B-10. A point A(2, 1) is outside the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ & AP, AQ are tangents to the circle. The equation of the circle circumscribing the triangle APQ is :

(A*) $(x+g)(x-2) + (y+f)(y-1) = 0$ (B) $(x+g)(x-2) - (y+f)(y-1) = 0$

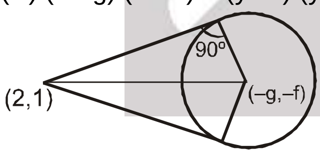
(C) $(x-g)(x+2) + (y-f)(y+1) = 0$ (D) $(x-g)(x-2) + (y-f)(y-1) = 0$

एक बिन्दु A(2, 1) वृत्त $x^2 + y^2 + 2gx + 2fy + c = 0$ के बाहर स्थित है तथा AP एवं AQ वृत्त की स्पर्श रेखाएँ हैं। त्रिभुज APQ के परिवृत्त का समीकरण हैं–

(A) $(x+g)(x-2) + (y+f)(y-1) = 0$ (B) $(x+g)(x-2) - (y+f)(y-1) = 0$

(C) $(x-g)(x+2) + (y-f)(y+1) = 0$ (D) $(x-g)(x-2) + (y-f)(y-1) = 0$

Sol.



$(x + g)(x - 2) + (y + f)(y - 1) = 0$

B-11. A line segment through a point P cuts a given circle in 2 points A & B, such that PA = 16 & PB = 9, find the length of tangent from points to the circle

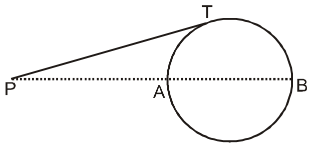
(A) 7 (B) 25 (C*) 12 (D) 8

बिन्दु P से गुजरने वाला रेखाखंड दिये गये वृत्त को दो बिन्दुओं A एवं B पर इस प्रकार काटता है कि PA = 16 व PB = 9 हो, तो बिन्दु से वृत्त पर खींची गयी स्पर्श रेखा की लम्बाई होगी–

(A) 7 (B) 25 (C*) 12 (D) 8

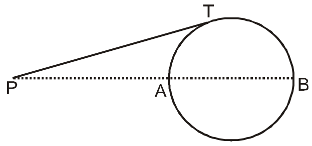
Sol. As we know

$PA.PB = PT^2 = (\text{Length of tangent})^2$



Length of tangent = $\sqrt{16 \times 9}$ = 12

Hindi. हम जानते है कि



PA.PB = PT^2 = (स्पर्श रेखा की लम्बाई)^2

स्पर्श रेखा की लम्बाई = $\sqrt{16 \times 9}$ = 12

B-12. The length of the tangent drawn from any point on the circle $x^2 + y^2 + 2gx + 2fy + p = 0$ to the circle $x^2 + y^2 + 2gx + 2fy + q = 0$ is:

(A*) $\sqrt{q - p}$ (B) $\sqrt{p - q}$ (C) $\sqrt{q + p}$ (D) $\sqrt{2q + p}$

वृत्त $x^2 + y^2 + 2gx + 2fy + p = 0$ के किसी बिन्दु से वृत्त $x^2 + y^2 + 2gx + 2fy + q = 0$ पर खींची गई स्पर्श रेखा की लम्बाई है –

(A) $\sqrt{q - p}$ (B) $\sqrt{p - q}$ (C) $\sqrt{q + p}$ (D) $\sqrt{2q + p}$

Sol. Let any point on the circle $x^2 + y^2 + 2gx + 2fy + p = 0$ (α, β)

This point satisfies $\alpha^2 + \beta^2 + 2g\alpha + 2f\beta + p = 0$

Length of tangent from this point to circle $x^2 + y^2 + 2gx + 2fy + q = 0$

length = $\sqrt{S_1}$ = $\sqrt{\alpha^2 + \beta^2 + 2g\alpha + 2f\beta + q}$ = $\sqrt{q - p}$

Hindi माना वृत्त $x^2 + y^2 + 2gx + 2fy + p = 0$ (α, β)

यह $\alpha^2 + \beta^2 + 2g\alpha + 2f\beta + p = 0$ संतुष्ट होता है।

इस बिन्दु से वृत्त $x^2 + y^2 + 2gx + 2fy + q = 0$ पर लम्ब की लम्बाई

लम्बाई = $\sqrt{S_1}$ = $\sqrt{\alpha^2 + \beta^2 + 2g\alpha + 2f\beta + q}$ = $\sqrt{q - p}$

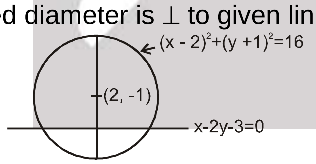
B-13. The equation of the diameter of the circle $(x - 2)^2 + (y + 1)^2 = 16$ which bisects the chord cut off by the circle on the line $x - 2y - 3 = 0$ is

(A) $x + 2y = 0$ (B*) $2x + y - 3 = 0$ (C) $3x + 2y - 4 = 0$ (D) $3x - 2y - 4 = 0$

वृत्त $(x - 2)^2 + (y + 1)^2 = 16$ के व्यास का वह समीकरण जो वृत्त द्वारा सरल रेखा $x - 2y - 3 = 0$ पर काटी गई जीवा को समद्विभाजित करता है –

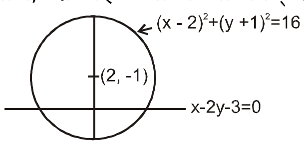
(A) $x + 2y = 0$ (B) $2x + y - 3 = 0$ (C) $3x + 2y - 4 = 0$ (D) $3x - 2y - 4 = 0$

Sol. Required diameter is $\perp$ to given line.



Hence $y + 1 = -2(x - 2) \Rightarrow 2x + y - 3 = 0$.

Hindi. अभीष्ट व्यास, दी गई रेखा के लम्बवत् है।



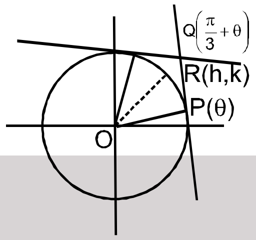
अतः $y + 1 = -2(x - 2)$ $\Rightarrow 2x + y - 3 = 0$

B-14. The locus of the point of intersection of the tangents to the circle $x^2 + y^2 = a^2$ at points whose parametric angles differ by $\frac{\pi}{3}$ is

वृत्त $x^2 + y^2 = a^2$ के उन बिन्दुओं जिनके प्राचलिक कोणों का अन्तर $\pi/3$ है, की स्पर्श रेखाओं के प्रतिच्छेद बिन्दु का बिन्दुपथ है।

(A*) $x^2 + y^2 = \frac{4a^2}{3}$ (B) $x^2 + y^2 = \frac{2a^2}{3}$ (C) $x^2 + y^2 = \frac{a^2}{3}$ (D) $x^2 + y^2 = \frac{a^2}{9}$

Sol. $\angle POQ = \frac{\pi}{3}$ and और $\angle POR = \frac{\pi}{6}$



OP = OR cos 30º

$a = \sqrt{h^2 + k^2}\ \frac{\sqrt{3}}{2} \Rightarrow x^2 + y^2 = \frac{4a^2}{3}$

Section (C) : Normal, Director circle, chord of contact, chord with mid point
खण्ड (C) : अभिलम्ब, नियामक वृत्त, स्पर्श जीवा, मध्य बिन्दु वाली जीवा

C-1. The equation of normal to the circle $x^2 + y^2 - 4x + 4y - 17 = 0$ which passes through (1, 1) is

(A*) $3x + y - 4 = 0$ (B) $x - y = 0$ (C) $x + y = 0$ (D) $3x - y - 4 = 0$

वृत्त $x^2 + y^2 - 4x + 4y - 17 = 0$ के अभिलम्ब का समीकरण जो बिन्दु (1, 1) से गुजरता है –

(A) $3x + y - 4 = 0$ (B) $x - y = 0$ (C) $x + y = 0$ (D) $3x - y - 4 = 0$

Sol. Normal to the circle $x^2 + y^2 - 4x + 4y - 17 = 0$ also passes through centre.
Hence its equation is line joining (2, –2) and (1, 1)

$(y - 1) = \frac{1+2}{1-2}(x - 1) \Rightarrow y - 1 = -3x + 3 \Rightarrow 3x + y - 4 = 0$

Hindi. वृत्त $x^2 + y^2 - 4x + 4y - 17 = 0$ का अभिलम्ब सदैव वृत्त के केन्द्र से गुजरता है।
अतः इसकी समीकरण बिन्दुओं (2, –2) और (1, 1) से मिलाने वाली रेखा है।

$(y - 1) = \frac{1+2}{1-2}(x - 1) \Rightarrow y - 1 = -3x + 3 \Rightarrow 3x + y - 4 = 0$

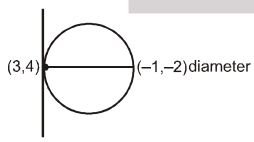
C-2. The normal at the point (3, 4) on a circle cuts the circle at the point (–1, –2). Then the equation of the circle is

वृत्त के बिन्दु (3, 4) पर अभिलम्ब वृत्त को बिन्दु (–1, –2) पर काटता हो, तो वृत्त का समीकरण है–

(A) $x^2 + y^2 + 2x - 2y - 13 = 0$ (B*) $x^2 + y^2 - 2x - 2y - 11 = 0$

(C) $x^2 + y^2 - 2x + 2y + 12 = 0$ (D) $x^2 + y^2 - 2x - 2y + 14 = 0$

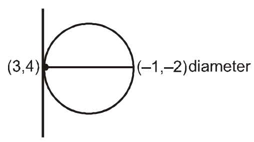
Sol.



$(x - 3)(x + 1) + (y - 4)(y + 2) = 0$

Equation $x^2 + y^2 - 2x - 2y - 11 = 0$

Hindi



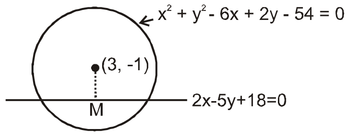
$(x - 3)(x + 1) + (y - 4)(y + 2) = 0$

अतः अभीष्ट समीकरण $x^2 + y^2 - 2x - 2y - 11 = 0$

C-3. The co-ordinates of the middle point of the chord cut off on $2x - 5y + 18 = 0$ by the circle $x^2 + y^2 - 6x + 2y - 54 = 0$ are
वृत्त $x^2 + y^2 - 6x + 2y - 54 = 0$ द्वारा सरल रेखा $2x - 5y + 18 = 0$ पर काटी गई जीवा के मध्य बिन्दु के निर्देशांक है–

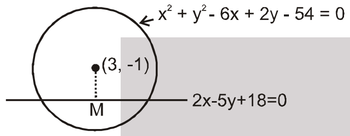
(A*) (1, 4) (B) (2, 4) (C) (4, 1) (D) (1, 1)

Sol. Required point is foot of $\perp$



$$\frac{x-3}{2} = \frac{y+1}{-5} = -\left(\frac{6+5+18}{4+25}\right) = -1 \Rightarrow x = 1, y = 4$$

Hindi. लम्बपाद के बिन्दु है

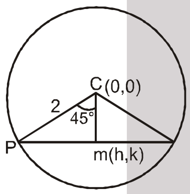


$$\frac{x-3}{2} = \frac{y+1}{-5} = -\left(\frac{6+5+18}{4+25}\right) = -1 \Rightarrow x = 1, y = 4$$

C-4. The locus of the mid point of a chord of the circle $x^2 + y^2 = 4$ which subtends a right angle at the origin is:
वृत्त $x^2 + y^2 = 4$ की उस जीवा के मध्य बिन्दु का बिन्दुपथ जो मूल बिन्दु पर समकोण बनातीहो,होगा–

(A) $x + y = 2$ (B) $x^2 + y^2 = 1$
(C*) $x^2 + y^2 = 2$ (D) $x + y = 1$

Sol. $\cos 45° = \dfrac{cm}{cp} = \dfrac{\sqrt{h^2 + k^2}}{2}$



Hence locus अतः बिन्दुपथ $x^2 + y^2 = 2$

C-5. The chords of contact of the pair of tangents drawn from each point on the line $2x + y = 4$ to the circle $x^2 + y^2 = 1$ pass through the point

(A) (1, 2) (B*) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (C) (2, 4) (D) (4, 4)

सरल रेखा $2x + y = 4$ के प्रत्येक बिन्दु से $x^2 + y^2 = 1$ पर खींचे गये स्पर्शी रेखा युग्म की स्पर्श जीवाएँ जिस बिन्दु से गुजरती हैं, वह है –

(A) (1, 2) (B) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (C) (2, 4) (D) (4, 4)

Sol. Let point on line be $(h, 4 - 2h)$ (chord of contact)

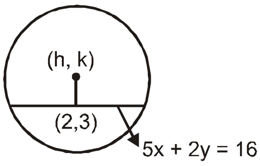
$hx + y(4 - 2h) = 1 \Rightarrow h(x - 2y) + 4y - 1 = 0 \Rightarrow$ Point $\left(\frac{1}{2}, \frac{1}{4}\right)$

Hindi. मानाकि रेखा पर बिन्दु $(h, 4 - 2h)$ (स्पर्शी जीवाएँ)

$hx + y(4 - 2h) = 1 \Rightarrow h(x - 2y) + 4y - 1 = 0 \Rightarrow$ अतः बिन्दु $\left(\frac{1}{2}, \frac{1}{4}\right)$

C-6. The locus of the centers of the circles such that the point (2, 3) is the mid point of the chord $5x + 2y = 16$ is:
(A*) $2x - 5y + 11 = 0$ (B) $2x + 5y - 11 = 0$ (C) $2x + 5y + 11 = 0$ (D) $2x - 5y - 11 = 0$
वृत्तों के केन्द्रों का बिन्दुपथ जबकि बिन्दु (2,3) जीवा $5x + 2y = 16$ का मध्य बिन्दु हैं–
(A) $2x - 5y + 11 = 0$ (B) $2x + 5y - 11 = 0$ (C) $2x + 5y + 11 = 0$ (D) $2x - 5y - 11 = 0$

Sol.



$\left(\frac{k-3}{h-2}\right)\left(-\frac{5}{2}\right) = -1 \Rightarrow 2x - 5y + 11 = 0$

C-7. Find the locus of the mid point of the chord of a circle $x^2 + y^2 = 4$ such that the segment intercepted by the chord on the curve $x^2 - 2x - 2y = 0$ subtends a right angle at the origin.
वृत्त $x^2 + y^2 = 4$ की जीवा के मध्य बिन्दु का बिन्दुपथ ज्ञात कीजिए जबकि जीवा द्वारा वक्र $x^2 - 2x - 2y = 0$ पर काटा गया अन्तःखण्ड मूलबिन्दु पर समकोण बनाता हैं।
(A*) $x^2 + y^2 - 2x - 2y = 0$ (B) $x^2 + y^2 + 2x - 2y = 0$
(C) $x^2 + y^2 + 2x + 2y = 0$ (D) $x^2 + y^2 - 2x + 2y = 0$

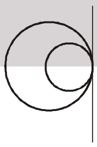
Sol. Let mid-point be (h, k) $\Rightarrow hx + ky = h^2 + k^2$
subtend right angle $\Rightarrow x^2 - 2(x + y)\left(\frac{hx+ky}{h^2+k^2}\right) = 0 \Rightarrow (h^2 + k^2)x^2 - 2(x + y)(hx + ky) = 0$
As angle 90º, Coefficient of x^2 + Coefficient of $y^2 = 0 \Rightarrow h^2 + k^2 - 2h - 2k = 0 \Rightarrow$
Locus $x^2 + y^2 - 2x - 2y = 0$

Hindi. मानाकि मध्य बिन्दु (h, k) है $\Rightarrow hx + ky = h^2 + k^2$
जो कि समकोण बनाता है $\Rightarrow x^2 - 2(x + y)\left(\frac{hx+ky}{h^2+k^2}\right) = 0 \Rightarrow (h^2 + k^2)x^2 - 2(x + y)(hx + ky) = 0$
कोण 90º होने के अनुसार x^2 का गुणांक + y^2 का गुणांक = 0
$h^2 + k^2 - 2h - 2k = 0 \Rightarrow$ बिन्दुपथ $x^2 + y^2 - 2x - 2y = 0$

Section (D) : Position of two circle, Orthogonality, Radical axis and radical centre
खण्ड (D) : दो वृत्तों की स्थिति, लम्बकोणीयता, मूलाक्ष एवं मूलाक्ष केन्द्र

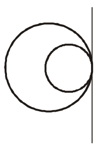
D-1. Number of common tangents of the circles $(x + 2)^2+(y-2)^2 = 49$ and $(x - 2)^2 + (y + 1)^2 = 4$ is:
वृत्त $(x + 2)^2+(y-2)^2 = 49$ और $(x - 2)^2 + (y + 1)^2 = 4$ की उभयनिष्ठ स्पर्श रेखाओं की संख्या हैं–
(A) 0 (B*) 1 (C) 2 (D) 3

Sol. $C_1C_2 = 5$, $r_1 = 7_1$ $r_2 = 2$



$C_1C_2 = |r_1 - r_2|$ one common tangent

Hindi $C_1C_2 = 5$, $r_1 = 7_1$ $r_2 = 2$



$C_1C_2 = |r_1 - r_2|$ एक उभयनिष्ठ स्पर्श रेखा

D-2. The equation of the common tangent to the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$ at their point of contact is

वृत्त $x^2 + y^2 - 4x - 6y - 12 = 0$ और $x^2 + y^2 + 6x + 18y + 26 = 0$ के स्पर्श बिन्दु पर उभयनिष्ठ स्पर्श रेखा का समीकरण है –

(A) $12x + 5y + 19 = 0$ (B*) $5x + 12y + 19 = 0$
(C) $5x - 12y + 19 = 0$ (D) $12x - 5y + 19 = 0$

Sol. Equation of common tangent at point of contact is $S_1 - S_2 = 0$

स्पर्श बिन्दु पर उभयनिष्ठ स्पर्श रेखा का समीकरण $S_1 - S_2 = 0$ है।

$\Rightarrow 10x + 24y + 38 = 0 \Rightarrow 5x + 12y + 19 = 0$

D-3. Equation of the circle cutting orthogonally the three circles $x^2 + y^2 - 2x + 3y - 7 = 0$, $x^2 + y^2 + 5x - 5y + 9 = 0$ and $x^2 + y^2 + 7x - 9y + 29 = 0$ is

(A*) $x^2 + y^2 - 16x - 18y - 4 = 0$ (B) $x^2 + y^2 - 7x + 11y + 6 = 0$
(C) $x^2 + y^2 + 2x - 8y + 9 = 0$ (D) $x^2 + y^2 + 16x - 18y - 4 = 0$

तीन वृत्तों $x^2 + y^2 - 2x + 3y - 7 = 0$, $x^2 + y^2 + 5x - 5y + 9 = 0$ एवं $x^2 + y^2 + 7x - 9y + 29 = 0$ को लम्बकोणीय काटने वाले वृत्त का समीकरण है –

(A) $x^2 + y^2 - 16x - 18y - 4 = 0$ (B) $x^2 + y^2 - 7x + 11y + 6 = 0$
(C) $x^2 + y^2 + 2x - 8y + 9 = 0$ (D) $x^2 + y^2 + 16x - 18y - 4 = 0$

Sol. $S_1 - S_2 = 0 \Rightarrow 7x - 8y + 16 = 0$

$S_2 - S_3 = 0 \Rightarrow 2x - 4y + 20 = 0$

$S_3 - S_1 = 0 \Rightarrow 9x - 12y + 36 = 0$

On solving centre (8, 9)

Length of tangent $= \sqrt{S_1} = \sqrt{64+81-16+27-7} = \sqrt{149} = (x - 8)^2 + (y - 9)^2 = 149$

$= x^2 + y^2 - 16x - 18y - 4 = 0$

Hindi $S_1 - S_2 = 0 \quad 7x - 3y + 16 = 0$

$S_2 - S_3 = 0 \quad 2x - 4y + 20 = 0$

$S_3 - S_1 = 0 \quad 9x - 12y + 36 = 0$

हल करने पर, केन्द्र (8,9) होगा

स्पर्श रेखा की लम्बाई $\sqrt{S_1} = \sqrt{64+81-16+27-7} = \sqrt{149} = (x - 8)^2 + (y - 9)^2 = 149$

$= x^2 + y^2 - 16x - 18y - 4 = 0$.

D-4. If the length of a common internal tangent to two circles is 7, and that of a common external tangent is 11, then the product of the radii of the two circles is:

यदि दो वृत्तों की उभयनिष्ठ आन्तरिक स्पर्श रेखाओं की लम्बाई 7 और उभयनिष्ठ बाह्य स्पर्श रेखा की लम्बाई 11 हो, तो दोनों वृत्तों की त्रिज्याओं का गुणनफल हैं :

(A*) 18 (B) 20 (C) 16 (D) 12

Sol. as we know $L_{int} = \sqrt{d^2 - (r_1 + r_2)^2} = 7 \Rightarrow L_{ext} = \sqrt{d^2 - (r_1 - r_2)^2} = 11$

squaring & subtact $r_1r_2 = 18$

Section (E) : Family of circles , Locus, Miscellaneous
खण्ड (E) : वृत्त निकाय, बिन्दुपथ, विविध

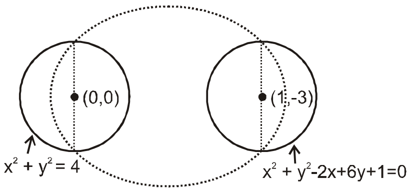
E-1. The locus of the centre of the circle which bisects the circumferences of the circles $x^2 + y^2 = 4$ & $x^2 + y^2 - 2x + 6y + 1 = 0$ is:

(A*) a straight line (B) a circle (C) a parabola (D) pair of straight line

वृत्तों $x^2 + y^2 = 4$ और $x^2 + y^2 - 2x + 6y + 1 = 0$ की परिधियों को समद्विभाजित करने वाले वृत्त के केन्द्र का बिन्दुपथ है –

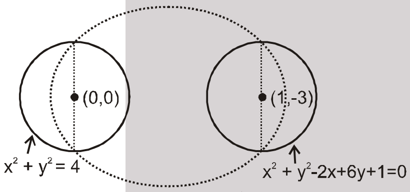
(A) एक सरल रेखा (B) वृत्त (C) परवलय (D) सरल रेखा युग्म

Sol. Let required circle is $x^2 + y^2 + 2gx + 2fy + c = 0$



Hence common chord with $x^2 + y^2 - 4 = 0$
is $2gx + 2fy + c + y = 0$
This is diameter of circle $x^2 + y^2 = 4$ hence $c = -4$.
Now again common chord with other circle
$2x(g + 1) + 2y(f - 3) + (c - 1) = 0$
This is diameter of $x^2 + y^2 - 2x + 6y + 1 = 0$
$2(g + 1) - 6(f - 3) + 5 = 0$
$2g - 6f + 15 = 0$
locus $2x - 3y - 15 = 0$ which is st. line.

Hindi. माना की वृत्त $x^2 + y^2 + 2gx + 2fy + c = 0$



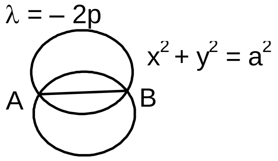
अतः $x^2 + y^2 - 4 = 0$ के साथ उभयनिष्ट जीवा $2gx + 2fy + c + y = 0$ है।
यह वृत्त $x^2 + y^2 = 4$ का व्यास है अतः $c = -4$
अब उभयनिष्ट जीवा दूसरे वृत्त के साथ
$2x(g + 1) + 2y(f - 3) + (c - 1) = 0$
यह वृत्त $x^2 + y^2 - 2x + 6y + 1 = 0$ का व्यास है
$2(g + 1) - 6(f - 3) + 5 = 0$
$2g - 6f + 15 = 0$
बिन्दुपथ $2x - 3y - 15 = 0$ जो कि सरल रेखा है।

E-2. Equation of a circle drawn on the chord $x \cos \alpha + y \sin \alpha = p$ of the circle $x^2 + y^2 = a^2$ as its diameter, is
वृत्त का समीकरण होगा जबकि वृत्त $x^2 + y^2 = a^2$ की जीवा $x \cos \alpha + y \sin \alpha = p$ इसका व्यास है।
(A) $(x^2 + y^2 - a^2) - 2p\, (x\sin\alpha + y\cos\alpha - p) = 0$
(B*) $(x^2 + y^2 - a^2) - 2p\, (x\cos\alpha + y\sin\alpha - p) = 0$
(C) $(x^2 + y^2 - a^2) + 2p\, (x\cos\alpha + y\sin\alpha - p) = 0$
(D) $(x^2 + y^2 - a^2) - p\, (x\cos\alpha + y\sin\alpha - p) = 0$

Sol. Equation of family of circles is $(x^2 + y^2 - a^2) + \lambda\, (x \cos \alpha + y \sin \alpha - p) = 0$
Now centre lies on the line $x \cos\alpha + y \sin\alpha = p$

वृत्तों के निकाय का समीकरण $(x^2 + y^2 - a^2) + \lambda\, (x \cos \alpha + y \sin \alpha - p) = 0$
अब केन्द्र रेखा पर स्थित है $x \cos\alpha + y \sin\alpha = p$
i.e. अर्थात् $-\dfrac{\lambda}{2} \cos^2\alpha - \dfrac{\lambda}{2} \sin^2 \alpha = p$

$\Rightarrow \quad \lambda = -2p$



$(x^2 + y^2 - a^2) - 2p\, (x\cos\alpha + y\sin\alpha - p) = 0$

E-3. Find the equation of the circle which passes through the point (1, 1) & which touches the circle $x^2 + y^2 + 4x - 6y - 3 = 0$ at the point (2, 3) on it.
उस वृत्त का समीकरण ज्ञात कीजिए जो बिन्दु (1,1) से गुजरता है तथा वृत्त $x^2 + y^2 + 4x - 6y - 3 = 0$ पर स्थित बिन्दु (2, 3) पर स्पर्श करता है।

(A*) $x^2 + y^2 + x - 6y + 3 = 0$ (B) $x^2 + y^2 + x - 6y - 3 = 0$
(C) $x^2 + y^2 + x + 6y + 3 = 0$ (D) $x^2 + y^2 + x - 3y + 3 = 0$

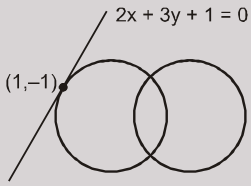
Sol. Equation of tangent to circle
$x^2 + y^2 + 4x - 6y - 3 = 0$ at (2,3) $\Rightarrow$ $2x + 3y + 2(x + 2) - 3(y + 3) - 3 = 0$
$4x - 8 = 0 \quad \Rightarrow \quad x - 2 = 0$
family of cirlce $\Rightarrow$ $S + \lambda L = 0$
$x^2 + y^2 + 4x - 6y - 3 + \lambda (x - 2) = 0$ ....(1)
Passes through (1, 1)
$-3 - \lambda = 0 \quad \Rightarrow \quad \lambda = -3$
Putting in (1)
$x^2 + y^2 + x - 6y + 3 = 0$

Hindi वृत्त की स्पर्श रेखा का समीकरण
$x^2 + y^2 + 4x - 6y - 3 = 0$ at (2,3) $\Rightarrow 2x + 3y + 2(x + 2) - 3(y + 3) - 3 = 0$
$4x - 8 = 0 \Rightarrow x - 2 = 0$
वृत्त निकाय $\Rightarrow$ $S + \lambda L = 0$
$x^2 + y^2 + 4x - 6y - 3 + \lambda (x - 2) = 0$ ....(1)
जो कि (1, 1) से गुजरता है
$-3 - \lambda = 0 \quad \Rightarrow \quad \lambda = -3$
(1) में रखने पर $x^2 + y^2 + x - 6y + 3 = 0$

E-4. Find the equation of circle touching the line $2x + 3y + 1 = 0$ at (1, – 1) and cutting orthogonally the circle having line segment joining (0, 3) and (– 2, – 1) as diameter.
उस वृत्त का समीकरण ज्ञात कीजिए जो कि रेखा $2x + 3y + 1 = 0$ को बिन्दु (1, – 1) पर स्पर्श करता हो तथा बिन्दुओं (0, 3) एवं (– 2, – 1) को मिलाने वाले रेखाखण्ड़ को व्यास मानकर बनाये गये वृत्त को लम्बकोणीय प्रतिच्छेद करता हो।

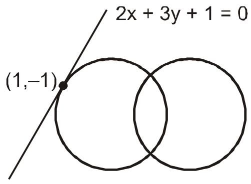
(A*) $2x^2 + 2y^2 - 10x - 5y + 1 = 0$ (B) $2x^2 + 2y^2 - 10x + 5y + 1 = 0$
(C) $2x^2 + 2y^2 - 10x - 5y - 1 = 0$ (D) $2x^2 + 2y^2 + 10x - 5y + 1 = 0$

Sol. The equation of circle having tangent $2x + 3y + 1 = 0$ at (1, – 1)
$\Rightarrow \quad (x - 1)^2 + (y + 1)^2 + \lambda(2x + 3y + 1) = 0$
$x^2 + y^2 + 2x(\lambda - 1) + y(3\lambda + 2) + (\lambda + 2) = 0$ ... (i)
equation of circle having end points of diameter (0, – 1) and (–2, 3) is
$x(x + 2) + (y + 1)(y - 3) = 0$ or $x^2 + y^2 + 2x - 2y - 3 = 0$ ... (ii)
since (i) & (ii) cut orthogonally



$\therefore \quad \dfrac{2(2\lambda - 2)}{2}.1 + \dfrac{2(3\lambda + 2)}{2}(-1) = \lambda + 2 - 3$
$\Rightarrow \quad 2\lambda - 2 - 3\lambda - 2 = \lambda - 1 \Rightarrow 2\lambda = -3 \Rightarrow \lambda = -3/2$
$\therefore$ from equation (i), equation of required circle is
$2x^2 + 2y^2 - 10x - 5y + 1 = 0$

Hindi वृत्त जिसके बिन्दु (1, –1) पर स्पर्श रेखा $2x + 3y + 1 = 0$ हो, का समीकरण
$(x - 1)^2 + (y + 1)^2 + \lambda(2x + 3y + 1) = 0 \Rightarrow x^2 + y^2 + 2x(\lambda - 1) + y(3\lambda + 2) + (\lambda + 2) = 0$ ... (i)
वृत्त का समीकरण जिसके व्यास के सिरे (0, – 1) तथा (–2, 3) हो
$x(x + 2) + (y + 1)(y - 3) = 0$ या $x^2 + y^2 + 2x - 2y - 3 = 0$ ... (ii)
चूँकि (i) तथा (ii) लम्बकोणीय काटते है।



$\therefore \quad \frac{2(2\lambda-2)}{2}.1+\frac{2(3\lambda+2)}{2}(-1) = \lambda + 2 - 3$

$\Rightarrow \quad 2\lambda - 2 - 3\lambda - 2 = \lambda - 1 \quad \Rightarrow \quad 2\lambda = -3 \quad \Rightarrow \quad \lambda = -3/2$

$\therefore$ समीकरण (i) से अभीष्ट वृत्त का समीकरण $2x^2 + 2y^2 - 10x - 5y + 1 = 0$.

E-5. Equation of the circle which passes through the point (–1, 2) & touches the circle $x^2 + y^2 - 8x + 6y = 0$ at origin, is -

वृत्त का समीकरण जो बिन्दु (–1, 2) से गुजरता है तथा वृत्त $x^2 + y^2 - 8x + 6y = 0$ को मूल बिन्दु पर स्पर्श करता है

(A) $x^2 + y^2 - 2x - \frac{3}{2}y = 0$ (B) $x^2 + y^2 + x - 2y = 0$

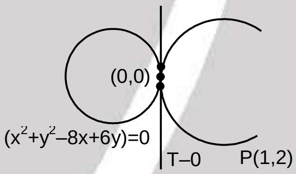
(C) $x^2 + y^2 + 2x + \frac{3}{2}y = 0$ (D*) $x^2 + y^2 + 2x - \frac{3}{2}y = 0$

Sol. Equations of tangent is : $4x - 3y = 0$

equation of the family of the cirlcle is

$(x - 0)^2 + (y - 0)^2 + \lambda (4x - 3y) = 0$

which passes through P (– 1,2) $\Rightarrow 1 + 4 \lambda (-4-6) = 0 \Rightarrow \lambda = \frac{1}{2}$



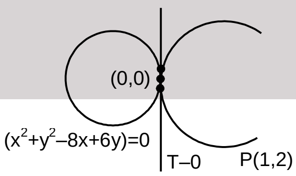
$\Rightarrow$ equation of circle is $x^2 + y^2 + 2x - \frac{3}{2}y = 0$

Hindi स्पर्श रेखा का समीकरण : $4x - 3y = 0$

वृत्तों के निकाय का समीकरण

$(x - 0)^2 + (y - 0)^2 + \lambda (4x - 3y) = 0$

वृत्त का समीकरण P (– 1,2) $\Rightarrow 1 + 4 \lambda (-4-6) = 0 \Rightarrow \lambda = \frac{1}{2}$ है।



$\Rightarrow$ वृत्त का समीकरण $x^2 + y^2 + 2x - \frac{3}{2}y = 0$ है।

E-6. Two circles are drawn through the point (a, 5a) and (4a, a) to touch the axis of 'y'. They intersect at an angle of θ then $\tan\theta$ is -

(a, 5a) और (4a, a) बिन्दुओं से गुजरने वाले दो वृत्त, 'y' अक्ष को स्पर्श करते है तथा वे θ कोण पर प्रतिच्छेद करते है तब $\tan\theta$ बराबर है –

(A*) $\frac{40}{9}$ (B) $\frac{9}{40}$ (C) $\frac{1}{9}$ (D) $\frac{1}{\sqrt{3}}$

Sol. Family of circle's passes through two fixed points is given by:

दो स्थिर बिन्दुओं से गुजरने वाले वृत्तों का निकाय है –

$(x - a)(x - 4a) + (y - 5a)(y - a) + \lambda L = 0$

L $4x + 3y = 19a$

$x^2 + y^2 - 5ax - 6ay + 4a^2 + 5a^2 + \lambda(4x + 13y - 19a) = 0$

touches y-axis $f^2 = 0$ & now proceed

y-अक्ष को स्पर्श करता है $f^2 = 0$ तथा अब

PART - III : MATCH THE COLUMN

भाग - III : कॉलम को सुमेलित कीजिए (MATCH THE COLUMN)

1. **Column – I** **Column – II**

(A) Number of values of a for which the common chord of the circles $x^2 + y^2 = 8$ and $(x - a)^2 + y^2 = 8$ subtends a right angle at the origin is — (p) 0

(B)✎ The number of circles touching all the three lines $3x + 7y = 2$, $21x + 49y = 5$ and $9x + 21y = 0$ are — (q) 2

(C) The length of common chord of circles $x^2 + y^2 - x - 11y + 18 = 0$ and $x^2 + y^2 - 9x - 5y + 14 = 0$ is — (r) 5

(D) Number of common tangents of the circles $x^2 + y^2 - 2x = 0$ and $x^2 + y^2 + 6x - 6y + 2 = 0$ is — (s) 3

Ans. (A) → (q), (B) → (p), (C) → (r), (D) → (s)

स्तम्भ – I **स्तम्भ – II**

(A) a के उन मानों की संख्या जिनके लिए वृत्तों $x^2 + y^2 = 8$ तथा $(x - a)^2 + y^2 = 8$ की उभयनिष्ठ जीवा मूल बिन्दु पर समकोण अन्तरित करती है – (p) 0

(B) सभी तीन रेखाओं $3x + 7y = 2$, $21x + 49y = 5$ तथा $9x + 21y = 0$ को स्पर्श करने वाले वृत्तों की संख्या है – (r) 2

(C) वृत्तों $x^2 + y^2 - x - 11y + 18 = 0$ और $x^2 + y^2 - 9x - 5y + 14 = 0$ की उभयनिष्ठ जीवा की लम्बाई — (r) 5

(D) वृत्तों $x^2 + y^2 - 2x = 0$ तथा $x^2 + y^2 + 6x - 6y + 2 = 0$ की उभयनिष्ठ स्पर्श रेखाओं की संख्या है — (s) 3

Ans. (A) → (q), (B) → (p), (C) → (r), (D) → (s)

Sol. (A) $S_1 - S_2 = 0$ is the required common chord i.e $2x = a$

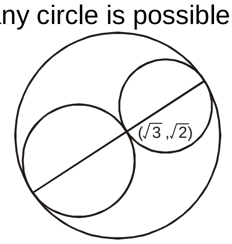
Make homogeneous, we get $x^2 + y^2 - 8.4\frac{x^2}{a^2} = 0$

As pair of lines substending angle of 90° at origin

$\therefore$ coefficient of x^2 + coefficient of $y^2 = 0$

$\therefore$ $a = \pm 4$

(B) Three lines are parallel so not any circle is possible



(C) Equation of common chord is $4x - 3y + 2 = 0$.

End points of common chord are (1,2) & (4,6)

Length of common chord is 5

(D) C_1 (1, 0), $r_1 = 1$ and C_2 (–3, 3), $r_2 = 4$
distance between centres C_1 and $C_2 = d = 5$
$d = r_1 + r_2 = 5 \quad \Rightarrow \quad$ 3 common tangents

Hindi (A) अभीष्ट उभयनिष्ठ जीवा $S_1 - S_2 = 0$ है अर्थात् $2x = a$

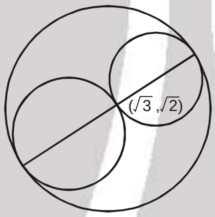
समघात बनाने पर $x^2 + y^2 - 8.4\ \frac{x^2}{a^2} = 0$

चूँकि रेखा युग्म मूल बिन्दु पर समकोण अन्तरित करता है।

$\therefore$ x^2 का गुणांक + y^2 का गुणांक = 0

$\therefore$ $a = \pm 4$

(B) तीनों रेखाएँ समान्तर है अतः कोई भी वृत्त विद्यमान नहीं होगा



(C) उभयनिष्ठ जीवा का समीकरण $4x - 3y + 2 = 0$.

उभयनिष्ठ जीवा के सिरे (1,2) & (4,6)

उभयनिष्ठ जीवा की लम्बाई Length of common chord is 5

(D) C_1 (1, 0), $r_1 = 1$ तथा C_2 (–3, 3), $r_2 = 4$
केन्द्रों C_1 तथा C_2 के मध्य दूरी $= d = 5$
$d = r_1 + r_2 = 5 \Rightarrow$ 3 उभयनिष्ठ स्पर्श रेखाएँ

2. **Column – I** **Column – II**

(A) If director circle of two given circles C_1 and C_2 of equal radii touches each other, then ratio of length of internal common tangent of C_1 and C_2 to their radii equals to (p) 13

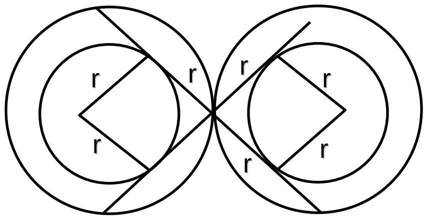
(B) Let two circles having radii r_1 and r_2 are orthogonal to each other. If length of their common chord is k times the square root of hormonic mean between squares of their radii, then k^4 equals to (q) 7

(C) The axes are translated so that the new equation of the circle $x^2 + y^2 - 5x + 2y - 5 = 0$ has no first degree terms and the new equation $x^2 + y^2 = \frac{\lambda^2}{4}$, then the value of λ is (r) 4

(D) The number of integral points which lie on or inside the circle $x^2 + y^2 = 4$ is (s) 2

Ans. (A) $\rightarrow$ (s), (B) $\rightarrow$ (r), (C) $\rightarrow$ (q), (D) $\rightarrow$ (p)

Sol. (A) Length of internal common tangent equals to 2r



(B) $\frac{1}{2}$ $r_1 r_2 = \frac{1}{2} \times \left(\frac{\ell}{2}\right) \times \sqrt{r_1^2 + r_2^2}$ {where ℓ is length of common chord)

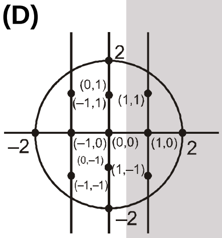
$\Rightarrow \ell = \sqrt{2}\ \sqrt{\frac{2r_1^2 r_2^2}{r_1^2 + r_2^2}} \Rightarrow k = \sqrt{2} \Rightarrow k^2 = 2$

(C) $x^2 + y^2 - 5x + 2y - 5 = 0 \Rightarrow \left(x - \frac{5}{2}\right)^2 + (y+1)^2 - 5 - \frac{25}{4} - 1 = 0$

$\Rightarrow \left(x - \frac{5}{2}\right)^2 + (y+1)^2 = \frac{49}{4}$ $\Rightarrow$ So the axes are shifted to $\left(\frac{5}{2}, -1\right)$

New equation of circle must be $x^2 + y^2 = \frac{49}{4}$

(D)



Hindi **स्तम्भ – I** **स्तम्भ – II**

(A) यदि दो दिए गए बराबर त्रिज्या के वृत्तों के नियामक वृत्त एक दुसरे को बाह्य स्पर्श करते है। तब C_1 और C_2 की आन्तरिक उभयनिष्ठ स्पर्श रेखा की लम्बाई का उनकी त्रिज्या से अनुपात बराबर है (p) 13

(B) माना दो वृत्त जिनकी त्रिज्याएं r_1 और r_2 एक दुसरे के लाम्बिक है। यदि उनकी उभयनिष्ठ जीवा की लम्बाई उनकी त्रिज्याओं के वर्गो के मध्य हरात्मक माध्य के वर्गमूल का k गुना है तब k^4 बराबर है– (q) 7

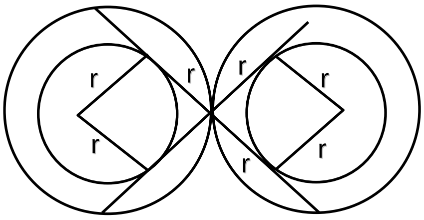
(C) अक्षों को इस प्रकार परिवर्तित किया जाता है कि वृत (r) 4

$x^2 + y^2 - 5x + 2y - 5 = 0$ की नई समीकरण में एक घात का कोई पद नहीं होता है तब नया समीकरण $x^2 + y^2 = \frac{\lambda}{4}$ से दी जाती है, तो λ का मान ज्ञात कीजिए।

(D) ऐसे पूर्णांक बिन्दुओं की संख्या ज्ञात कीजिए जो वृत्त $x^2 + y^2 = 4$ के ऊपर या अन्दर स्थित है। (s) 2

Ans. (A) $\rightarrow$ (s), (B) $\rightarrow$ (r), (C) $\rightarrow$ (q), (D) $\rightarrow$ (p)

Sol. (A) Length of internal common tangent equals to 2r



(B) $\frac{1}{2}$ $r_1 r_2 = \frac{1}{2} \times \left(\frac{\ell}{2}\right) \times \sqrt{r_1^2 + r_2^2}$ {where ℓ is length of common chord}

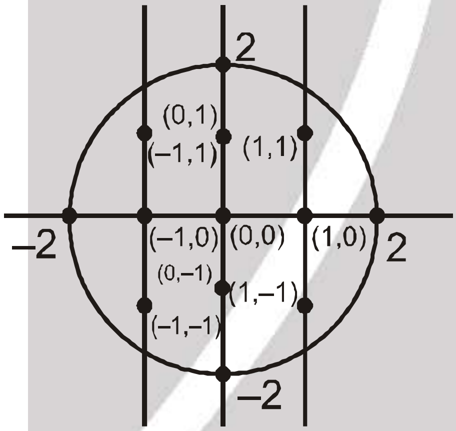
$\Rightarrow \ell = \sqrt{2}\sqrt{\frac{2r_1^2 r_2^2}{r_1^2 + r_2^2}} \Rightarrow k = \sqrt{2} \Rightarrow k^2 = 2$

(C) $x^2 + y^2 - 5x + 2y - 5 = 0 \Rightarrow \left(x - \frac{5}{2}\right)^2 + (y+1)^2 - 5 - \frac{25}{4} - 1 = 0$

$\Rightarrow \left(x - \frac{5}{2}\right)^2 + (y+1)^2 = \frac{49}{4}$ $\Rightarrow$ अक्षों को $\left(\frac{5}{2}, -1\right)$ पर प्रतिस्थापित करने पर

वृत्त की नयी समीकरण $x^2 + y^2 = \frac{49}{4}$ है।

(D)



Exercise-2

PART - I : ONLY ONE OPTION CORRECT TYPE

भाग-I : केवल एक सही विकल्प प्रकार (ONLY ONE OPTION CORRECT TYPE)

Single choice type
एकल विकल्पी प्रकार

1. If $\left(a, \frac{1}{a}\right), \left(b, \frac{1}{b}\right), \left(c, \frac{1}{c}\right)$ & $\left(d, \frac{1}{d}\right)$ are four distinct points on a circle of radius 4 units, then abcd is equal to:

(A) 4 (B) 16 (C*) 1 (D) 2

यदि 4 इकाई त्रिज्या के वृत्त पर 4 भिन्न बिन्दु $\left(a, \frac{1}{a}\right), \left(b, \frac{1}{b}\right), \left(c, \frac{1}{c}\right)$ और $\left(d, \frac{1}{d}\right)$ और हो, तो abcd बराबर है :

(A) 4 (B) 16 (C) 1 (D) 2

Sol. Point $\left(t, \frac{1}{t}\right)$ lies on $x^2 + y^2 = 16 \Rightarrow t^2 + \frac{1}{t^2} = 16 \Rightarrow t^4 - 16t^2 + 1 = 0$ ....(i)

If roots are t_1, t_2, t_3, t_4 then

$t_1t_2t_3t_4 = 1$ ....(ii)

Hindi. बिन्दु $\left(t, \frac{1}{t}\right)$ वृत्त $x^2 + y^2 = 16$ पर स्थित होगा $\Rightarrow t^2 + \frac{1}{t^2} = 16 \Rightarrow t^4 - 16t^2 + 1 = 0$ ....(i)

यदि t_1, t_2, t_3, t_4 मूल हो, तो

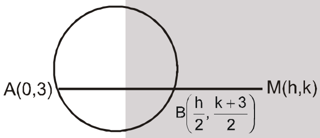
$t_1t_2t_3t_4 = 1$ ....(ii)

2. From the point A (0, 3) on the circle $x^2 + 4x + (y - 3)^2 = 0$ a chord AB is drawn & extended to a point M such that AM = 2 AB. The equation of the locus of M is :

वृत्त $x^2 + 4x + (y - 3)^2 = 0$ पर बिन्दु A (0,3) से एक जीवा AB खींची जाती है तथा इसे बिन्दु M तक इस प्रकार बढ़ाया जाता है कि AM = 2 AB हो, तो M के बिन्दुपथ का समीकरण है :

(A) $x^2 + 8x + y^2 = 0$
(B*) $x^2 + 8x + (y - 3)^2 = 0$
(C) $(x - 3)^2 + 8x + y^2 = 0$
(D) $x^2 + 8x + 8y^2 = 0$

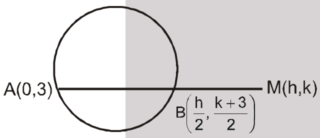
Sol.



B lies on circle $\left(\frac{h}{2}\right)^2 + 4\left(\frac{h}{2}\right) + \left(\frac{k+3}{2} - 3\right)^2 = 0 \Rightarrow \frac{h^2}{4} + 2h + \frac{(k-3)^2}{4} = 0$

Hence locus of (h, k) $\quad x^2 + 8x + (y - 3)^2 = 0$.

Sol.



B वृत्त पर स्थित है $\left(\frac{h}{2}\right)^2 + 4\left(\frac{h}{2}\right) + \left(\frac{k+3}{2} - 3\right)^2 = 0 \Rightarrow \frac{h^2}{4} + 2h + \frac{(k-3)^2}{4} = 0$

अतः (h, k) का बिन्दुपथ $x^2 + 8x + (y - 3)^2 = 0$

3. If tangent at (1, 2) to the circle c_1: $x^2 + y^2 = 5$ intersects the circle c_2: $x^2 + y^2 = 9$ at A & B and tangents at A & B to the second circle meet at point C, then the co–ordinates of C is

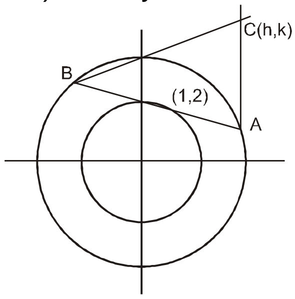
यदि बिन्दु (1,2) पर वृत्त c_1: $x^2 + y^2 = 5$ की स्पर्श रेखा, वृत्त c_2: $x^2 + y^2 = 9$ को बिन्दु A और B पर काटती है तथा द्वितीय वृत्त के बिन्दु A और B पर स्पर्श रेखाएँ बिन्दु C पर मिलती हो, तो C के निर्देशांक है –

(A) (4, 5)
(B) $\left(\frac{9}{15}, \frac{18}{5}\right)$
(C) (4, – 5)
(D*) $\left(\frac{9}{5}, \frac{18}{5}\right)$

Sol. Tangent at (1, 2) to the circle $x^2 + y^2 = 5$

$x + 2y - 5 = 0$

chord of contact from C(h, k) to $x^2 + y^2 = 9$

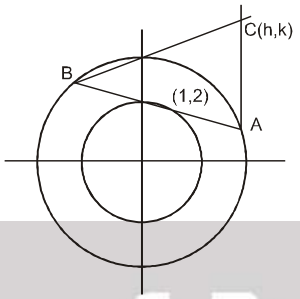


$hx + ky - 9 = 0$

compare both equations $\frac{h}{1} = \frac{k}{2} = \frac{9}{5}$

$$(h, k) \equiv \left(\frac{9}{5}, \frac{18}{5}\right)$$

Hindi. वृत्त $x^2 + y^2 = 5$ के बिन्दु (1, 2) पर स्पर्श रेखा का समीकरण
$x + 2y - 5 = 0$ है
वृत्त $x^2 + y^2 = 9$ के लिए C(h, k) से स्पर्श जीवा



$hx + ky - 9 = 0$

दोनो की तुलना करने पर $\frac{h}{1} = \frac{k}{2} = \frac{9}{5}$

$$(h, k) \equiv \left(\frac{9}{5}, \frac{18}{5}\right)$$

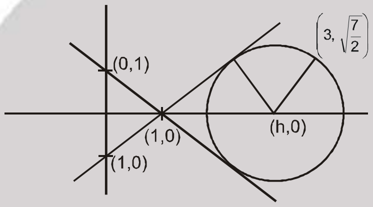
4. A circle passes through point $\left(3, \sqrt{\frac{7}{2}}\right)$ touches the line pair $x^2 - y^2 - 2x + 1 = 0$. Centre of circle lies inside the circle $x^2 + y^2 - 8x + 10y + 15 = 0$. Co-ordinate of centre of circle is

एक वृत्त बिन्दु $\left(3, \sqrt{\frac{7}{2}}\right)$ से गुजरता है तथा रेखा युग्म $x^2 - y^2 - 2x + 1 = 0$ को स्पर्श करता है। वृत्त का केन्द्र $x^2 + y^2 - 8x + 10y + 15 = 0$ के अन्दर स्थित हो, तो वृत्त के केन्द्र के निर्देशांक है –

(A*) (4, 0) (B) (5, 0) (C) (6, 0) (D) (0, 4)

Sol. $(x^2 - 2x + 1) - y^2 = 0 \Rightarrow (x + y - 1) = 0 \Rightarrow x - y - 1 = 0$

$$\left|\frac{h-0-1}{\sqrt{2}}\right| = \sqrt{(h-3)^2 + \frac{7}{2}}$$



$$h^2 + 1 - 2h = 2\left(h^2 + 9 - 6h + \frac{7}{2}\right) \Rightarrow h^2 - 10h + 24 = 0 \Rightarrow h = 6, 4$$

But centre lies inside the circle $x^2 + y^2 - 8x + 10y + 15 = 0$
Hence required point (4, 0).

5. The length of the tangents from any point on the circle $15x^2 + 15y^2 - 48x + 64y = 0$ to the two circles $5x^2 + 5y^2 - 24x + 32y + 75 = 0$ and $5x^2 + 5y^2 - 48x + 64y + 300 = 0$ are in the ratio

(A*) 1 : 2 (B) 2 : 3 (C) 3 : 4 (D) 2 : 1

वृत्त $15x^2 + 15y^2 - 48x + 64y = 0$ के किसी बिन्दु से दो वृत्त $5x^2 + 5y^2 - 24x + 32y + 75 = 0$ और $5x^2 + 5y^2 - 48x + 64y + 300 = 0$ पर खींची गई स्पर्श रेखाओं की लम्बाईयों का अनुपात हैं

(A) 1 : 2 (B) 2 : 3 (C) 3 : 4 (D) 2 : 1

Sol. Let any point $P(x_1, y_1)$ to the circle $x^2 + y^2 - \frac{16x}{5} + \frac{64y}{15} = 0 \Rightarrow x_1^2 + y_1^2 - \frac{16x}{5}x_1 + \frac{64y}{15}y_1 = 0$

Length of tangent from $P(x_1, y_1)$ to the circle are in ration

$$\frac{\sqrt{S_1}}{\sqrt{S_2}} = \frac{\sqrt{x_1^2 + y_1^2 - \frac{24}{5}x_1 + \frac{32}{5}y_1 + 15}}{\sqrt{x_1^2 + y_1^2 - \frac{48}{5}x_1 + \frac{64}{5}y_1 + 60}} = \sqrt{\frac{\frac{16}{5}x_1 - \frac{64}{15}y_1 - \frac{24}{5}x_1 + \frac{32}{5}y_1 + 15}{\frac{16}{5}x_1 - \frac{64}{15}y_1 - \frac{48}{5}x_1 + \frac{64}{5}y_1 + 60}}$$

$$= \sqrt{\frac{-24x_1 + 32y_1 + 225}{-96x_1 + 128y_1 + 900}} = \sqrt{\frac{-24x_1 + 32y_1 + 225}{4(-24x_1 + 32y_1 + 225)}} = \frac{1}{2}$$

Hindi. **Hindi.** अतः किसी बिन्दु $P(x_1, y_1)$ वृत्त $x^2 + y^2 - \frac{16x}{5} + \frac{64y}{15} = 0$ पर स्थित है

$P(x_1, y_1)$ से वृत्त पर स्पर्श रेखा की लम्बाईयों का अनुपात

$$\frac{\sqrt{S_1}}{\sqrt{S_2}} = \frac{\sqrt{x_1^2 + y_1^2 - \frac{24}{5}x_1 + \frac{32}{5}y_1 + 15}}{\sqrt{x_1^2 + y_1^2 - \frac{48}{5}x_1 + \frac{64}{5}y_1 + 60}} = \sqrt{\frac{\frac{16}{5}x_1 - \frac{64}{15}y_1 - \frac{24}{5}x_1 + \frac{32}{5}y_1 + 15}{\frac{16}{5}x_1 - \frac{64}{15}y_1 - \frac{48}{5}x_1 + \frac{64}{5}y_1 + 60}}$$

$$= \sqrt{\frac{-24x_1 + 32y_1 + 225}{-96x_1 + 128y_1 + 900}} = \sqrt{\frac{-24x_1 + 32y_1 + 225}{4(-24x_1 + 32y_1 + 225)}} = \frac{1}{2}$$

6. The distance between the chords of contact of tangents to the circle; $x^2 + y^2 + 2gx + 2fy + c = 0$ from the origin & the point (g, f) is:

मूलबिन्दु और बिन्दु (g,f) से वृत्त $x^2 + y^2 + 2gx + 2fy + c = 0$ पर खींचे गए स्पर्शी युग्म की स्पर्शी जीवाओं के बीच की दूरी है –

(A) $\sqrt{g^2 + f^2}$ (B) $\frac{\sqrt{g^2 + f^2 - c}}{2}$ (C*) $\frac{g^2 + f^2 - c}{2\sqrt{g^2 + f^2}}$ (D) $\frac{\sqrt{g^2 + f^2 + c}}{2\sqrt{g^2 + f^2}}$

Sol. Equation of chords of contact from (0, 0) & (g, f)

$gx + fy + c = 0 \Rightarrow gx + fy + g(x + g) + f(y + f) + c = 0 \Rightarrow gx + fy + \frac{(g^2 + f^2 + c)}{2} = 0$

Distance between these parallel lines = $\left|\frac{g^2 + f^2 - c}{2\sqrt{g^2 + f^2}}\right|$

Hindi. (0, 0) तथा (g, f) से स्पर्शी जीवा का समीकरण

$gx + fy + c = 0 \Rightarrow gx + fy + g(x + g) + f(y + f) + c = 0 \Rightarrow gx + fy + \frac{(g^2 + f^2 + c)}{2} = 0$

समान्तर रेखाओं के मध्य की दूरी = . $\left|\frac{g^2 + f^2 - c}{2\sqrt{g^2 + f^2}}\right|$

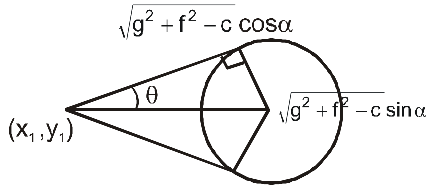
7. If from any point P on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, tangents are drawn to the circle $x^2 + y^2 + 2gx + 2fy + c\sin^2\alpha + (g^2 + f^2)\cos^2\alpha = 0$, then the angle between the tangents is:

(A) α (B*) 2α (C) $\frac{\alpha}{2}$ (D) $\frac{\alpha}{3}$

यदि वृत्त $x^2 + y^2 + 2gx + 2fy + c = 0$ पर स्थित किसी बिन्दु P से वृत्त $x^2 + y^2 + 2gx + 2fy + c\sin^2\alpha + (g^2 + f^2)\cos^2\alpha=0$ पर स्पर्श रेखाएँ खींची जाती हो, तो स्पर्श रेखाओं के मध्य कोण है–

(A) α (B) 2α (C) $\frac{\alpha}{2}$ (D) $\frac{\alpha}{3}$

Sol. $\tan\theta = \tan2\alpha \Rightarrow \theta = 2\alpha$



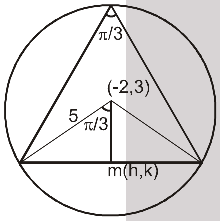
angle कोण $= 2\alpha$

8. The locus of the mid points of the chords of the circle $x^2 + y^2 + 4x - 6y - 12 = 0$ which subtend an angle of $\frac{\pi}{3}$ radians at its circumference is:

वृत्त $x^2 + y^2 + 4x - 6y - 12 = 0$ की जीवाओं जो इसकी परिधि पर $\frac{\pi}{3}$ रेडियन का कोण बनाती हो, के मध्य बिन्दुओं का बिन्दुपथ हैं–

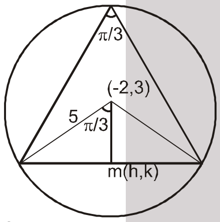
(A) $(x - 2)^2 + (y + 3)^2 = 6.25$
(B*) $(x + 2)^2 + (y - 3)^2 = 6.25$
(C) $(x + 2)^2 + (y - 3)^2 = 18.75$
(D) $(x + 2)^2 + (y + 3)^2 = 18.75$

Sol. $\cos \pi/3 = \frac{\sqrt{(h+2)^2+(k-3)^2}}{5}$



Locus $(x + 2)^2 + (y - 3)^2 = 6.25$

Hindi. $\cos \pi/3 = \frac{\sqrt{(h+2)^2+(k-3)^2}}{5}$



बिन्दुपथ $(x + 2)^2 + (y - 3)^2 = 6.25$

9. If the two circles, $x^2 + y^2 + 2g_1x + 2f_1y = 0$ & $x^2 + y^2 + 2g_2x + 2f_2y = 0$ touch each other then :

(A) $f_1 g_1 = f_2 g_2$
(B*) $\frac{f_1}{g_1} = \frac{f_2}{g_2}$
(C) $f_1 f_2 = g_1 g_2$
(D) $f_1 + f_2 = g_1 + g_2$

यदि दो वृत्त $x^2 + y^2 + 2g_1x + 2f_1y = 0$ एवं $x^2 + y^2 + 2g_2x + 2f_2y = 0$ एक दूसरे को स्पर्श करते हैं, तो :

(A) $f_1 g_1 = f_2 g_2$
(B) $\frac{f_1}{g_1} = \frac{f_2}{g_2}$
(C) $f_1 f_2 = g_1 g_2$
(D) $f_1 + f_2 = g_1 + g_2$

Sol. If two circles touch each other, then यदि दो वृत्त एक दूसरे को स्पर्श करते है तब

$C_1C_2 = r_1 + r_2$

$\sqrt{(-g_1+g_2)^2+(-f_1+f_2)^2} = \sqrt{g_1^2+f_1^2} + \sqrt{g_2^2+f_2^2}$ squaring both sides दोनों तरफ वर्ग करने पर

$-2g_1g_2 - 2f_1f_2 = 2\sqrt{\left(g_1^2+f_1^2\right)\left(g_2^2+f_2^2\right)} \Rightarrow (g_1 f_2)^2 + (g_2 f_1)^2 - 2g_1g_2f_1f_2 = 0 \Rightarrow \frac{g_1}{g_2} = \frac{f_1}{f_2}$

10. A circle touches a straight line $\ell x + my + n = 0$ & cuts the circle $x^2 + y^2 = 9$ orthogonally. The locus of centres of such circles is:

(A*) $(\ell x + my + n)^2 = (\ell^2 + m^2)(x^2 + y^2 - 9)$
(B) $(\ell x + my - n)^2 = (\ell^2 + m^2)(x^2 + y^2 - 9)$
(C) $(\ell x + my + n)^2 = (\ell^2 + m^2)(x^2 + y^2 + 9)$
(D) $(\ell x + my - n)^2 = (\ell^2 + m^2)(x^2 + y^2 - 9)$

एक वृत्त सरल रेखा $\ell x + my + n = 0$ को स्पर्श करता है तथा वृत्त $x^2 + y^2 = 9$ को लम्बकोणीय काटता हो, तो इस प्रकार के वृत्तों के केन्द्र का बिन्दुपथ हैं –

(A*) $(\ell x + my + n)^2 = (\ell^2 + m^2)(x^2 + y^2 - 9)$ (B) $(\ell x + my - n)^2 = (\ell^2 + m^2)(x^2 + y^2 - 9)$

(C) $(\ell x + my + n)^2 = (\ell^2 + m^2)(x^2 + y^2 + 9)$ (D) $(\ell x + my - n)^2 = (\ell^2 + m^2)(x^2 + y^2 - 9)$

Sol. Let required equation of circle is $x^2 + y^2 + 2gx + 2gx + 2fy + c = 0$
it cuts the circle $x^2 + y^2 - 9 = 0$ orthogonally
$\therefore\ 2g(0) + 2f(0) = c - 9 \Rightarrow c = 9$
It also touches straight line $\ell x + my + n = 0$

$$\therefore \left|\frac{\ell(-g) + m(-f) + n}{\sqrt{\ell^2 + m^2}}\right| = \sqrt{g^2 + f^2 - 9}$$

Locus of centre $(-g, -f)$ is $(\ell x + my + n)^2 = (x^2 + y^2 - 9)(\ell^2 + m^2)$

Hindi. माना वृत्त का अभीष्ट समीकरण $x^2 + y^2 + 2gx + 2gx + 2fy + c = 0$ है।
यह वृत्त $x^2 + y^2 - 9 = 0$ को लम्बकोणीय प्रतिच्छेद करता है।
$\therefore\ 2g(0) + 2f(0) = c - 9 \Rightarrow c = 9$
यह सरल रेखा $\ell x + my + n = 0$ को भी स्पर्श करता है

$$\therefore \left|\frac{\ell(-g) + m(-f) + n}{\sqrt{\ell^2 + m^2}}\right| = \sqrt{g^2 + f^2 - 9}$$

केन्द्र $(-g, -f)$ का बिन्दुपथ $(\ell x + my + n)^2 = (x^2 + y^2 - 9)(\ell^2 + m^2)$ है।

11. The locus of the point at which two given unequal circles subtend equal angles is:

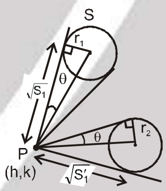
(A) a straight line (B*) a circle (C) a parabola (D) an ellipse

उस बिन्दु का बिन्दुपथ जिस पर दो दिये गए असमान वृत्त, सैमान कोण अन्तरित करते हैं, है–

(A) एक सरल रेखा (B*) एक वृत्त (C) एक परवलय (D) दीर्घवृत्त

Sol. Let two circles are $S = 0$ and $S' = 0$
having radius r_1 and r_2 respectively.

$$\Rightarrow \frac{r_1}{\sqrt{S_1}} = \frac{r_2}{\sqrt{S'_1}} \qquad \Rightarrow S'_1 r_1^2 = r_2^2 S_1$$

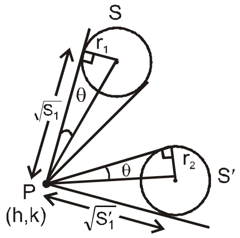


$$\Rightarrow S_1 - \left(\frac{r_1}{r_2}\right)^2 S'_1 = 0 \ \therefore \text{ Locus of } P(h,k)$$

$$S - \left(\frac{r_1}{r_2}\right)^2 S' = 0$$ which represents the equation of a circle.

Hindi. माना दो वृत्त $S = 0$ तथा $S' = 0$
जिनकी त्रिज्याऐं क्रमशः r_1 तथा r_2 है।

$$\Rightarrow \frac{r_1}{\sqrt{S_1}} = \frac{r_2}{\sqrt{S'_1}} \qquad \Rightarrow S'_1 r_1^2 = r_2^2 S_1$$



$\Rightarrow S_1 - \left(\frac{r_1}{r_2}\right)^2 \quad S'_1 = 0$ ∴ बिन्दु P(h,k) का बिन्दुपथ

$S - \left(\frac{r_1}{r_2}\right)^2 \quad S'_1 = 0$ ∴ बिन्दु P(h,k) का बिन्दुपथ

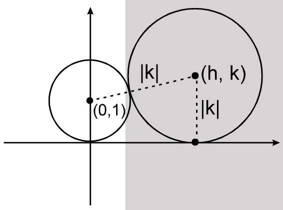
12. A circle is given by $x^2 + (y - 1)^2 = 1$. Another circle C touches it externally and also the x-axis, then the locus of its centre is

एक वृत्त का समीकरण $x^2 + (y - 1)^2 = 1$ है। एक अन्य वृत्त C इसको बाह्य स्पर्श करता हो तथा x-अक्ष को भी स्पर्श करता हो, तो इसके केन्द्र का बिन्दुपथ हैं –

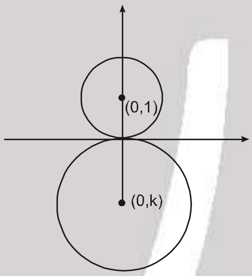
(A) $\{(x, y) : x^2 = 4y\} \cup \{(x, y) : y \le 0\}$ (B) $\{(x, y) : x^2 + (y - 1)^2 = 4\} \cup \{(x, y) : y \le 0\}$

(C) $\{(x, y) : x^2 = y\} \cup \{(0, y) : y \le 0\}$ (D*) $\{(x, y) : x^2 = 4y\} \cup \{(0, y) : y \le 0\}$

Sol. $\sqrt{(h-0)^2 + (k-1)^2} = 1 + |k|$



or



$\Rightarrow$ $h^2 + k^2 - 2k + 1 = 1 + 2|k| + k^2$ $\Rightarrow$ $h^2 = 2|k| + 2k$

$\Rightarrow$ $x^2 = 4y$ if $y \ge 0$ & $x = 0$ if $y \le 0$ $\Rightarrow$ $x^2 = 4y$ यदि $y \ge 0$ तथा $x = 0$ यदि $y \le 0$

13. The locus of the centre of a circle touching the circle $x^2 + y^2 - 4y - 2x = 4$ internally and tangent on which from (1, 2) is making a 60° angle with each other.

वृत्त $x^2 + y^2 - 4y - 2x = 4$ को आन्तरिक स्पर्श करने वाले वृत्त तथा बिन्दु (1, 2) से वृत्त पर खीचीं गई स्पर्श रेखाओं के मध्य कोण 60° है, तो वृत्त के केन्द्र का बिन्दुपथ है

(A) $(x - 1)^2 + (y - 2)^2 = 2$ (B*) $(x - 1)^2 + (y - 2)^2 = 4$

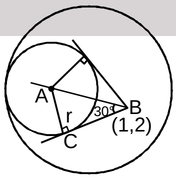
(C) $(x + 1)^2 + (y - 2)^2 = 4$ (D) $(x + 1)^2 + (y + 2)^2 = 4$

Sol. Let centre A(h, k) and r and R be radius of required and given circle
माना केन्द्र A(h, k) तथा अभीष्ट और दिए गए वृत्त की त्रिज्याएं r तथा R है।

Sol. Let centre A(h, k) and r and R be radius of required and given circle
माना केन्द्र A(h, k) तथा अभीष्ट और दिए गए वृत्त की त्रिज्याएं r तथा R है।

$\sqrt{(h-1)^2 + (k-2)^2} = R - r$ ....(i)

Now अब $\tan 30 = \frac{r}{AB}$



$r = (R - r)\frac{1}{2}$ ∴ $r = \frac{R}{3}$ ....(ii)

by (i) & (ii) से

$\sqrt{(h-1)^2 + (k-2)^2} = R - \frac{R}{3} = \frac{2R}{3}$ & $R = 3 \Rightarrow (x - 1)^2 + (y - 2)^2 = 4$

14. **STATEMENT-1 :** If three circles which are such that their centres are non-collinear, then exactly one circle exists which cuts the three circles orthogonally.

STATEMENT-2 : Radical axis for two intersecting circles is the common chord.

(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D*) STATEMENT-1 is false, STATEMENT-2 is true

कथन-1 : यदि तीन वृत्त, इस प्रकार है कि उनके केन्द्र असंरेखीय हो, तो तीनों वृत्तों को लम्बकोणीय प्रतिच्छेद करने वाला ठीक एक वृत्त सम्भव है।

कथन-2 : दो प्रतिच्छेदित वृत्तों की मूलाक्ष उनकी उभयनिष्ठ जीवा होती है।

(A) कथन–1 सत्य है, कथन–2 सत्य है ; कथन–2, कथन–1 का सही स्पष्टीकरण है।
(B) कथन–1 सत्य है, कथन–2 सत्य है ; कथन–2, कथन–1 का सही स्पष्टीकरण नहीं है।
(C) कथन–1 सत्य है, कथन–2 असत्य है।
(D) कथन–1 असत्य है, कथन–2 सत्य है।

Sol. Statement-1 : There is exactly one circle whose centre is the radical centre and the radius equal to the length of tangent drawn from the radical centre to any of the given circles.

Statement-2 is True But does not explain Statement-1.

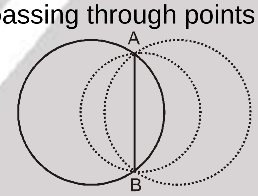
Hindi कथन -1 : ठीक एक वृत्त जिसका केन्द्र मूलाक्ष केन्द्र और त्रिज्या मूलाक्ष केन्द्र से किसी भी वृत्त पर खींची गयी स्पर्श रेखा की लम्बाई के बराबर होती है, ही होता है।
कथन–2 : सत्य है लेकिन कथन–2, कथन–1 का सही स्पष्टीकरण नहीं है।

15. The centre of family of circles cutting the family of circles $x^2 + y^2 + 4x\left(\lambda - \frac{3}{2}\right) + 3y\left(\lambda - \frac{4}{3}\right) - 6(\lambda + 2) = 0$ orthogonally, lies on

वृत्त निकाय का केन्द्र जो कि वृत्त निकाय $x^2 + y^2 + 4x\left(\lambda - \frac{3}{2}\right) + 3y\left(\lambda - \frac{4}{3}\right) - 6(\lambda + 2) = 0$ को लम्बकोणीय काटता है, पर स्थित होगा–

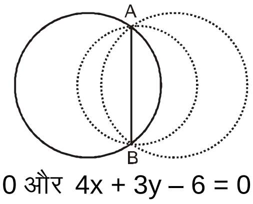
(A) $x - y - 1 = 0$ (B*) $4x + 3y - 6 = 0$ (C) $4x + 3y + 7 = 0$ (D) $3x - 4y - 1 = 0$

Sol. $(x^2 + y^2 - 6x - 4y - 12) + \lambda(4x + 3y - 6) = 0$
This is family of circle passing through points of intersection of circle



$x^2 + y^2 - 6x - 4y - 12 = 0$ and line $4x + 3y - 6 = 0$
other family will cut this family at A & B.
Hence locus of centre of circle of other family is this common chord $4x + 3y - 6 = 0$

Hindi. $(x^2 + y^2 - 6x - 4y - 12) + \lambda(4x + 3y - 6) = 0$
यह वृत्त निकाय वृत्त के दोनों प्रतिच्छेद बिन्दुओं से गुजरता है।



$x^2 + y^2 - 6x - 4y - 12 = 0$ और $4x + 3y - 6 = 0$
अन्य निकाय इसको A और B पर प्रतिच्छेद करते है।
अतः वृत्त के केन्द्र का बिन्दुपथ
यह उभयनिष्ठ जीवा $4x + 3y - 6 = 0$ है।

16. The circle $x^2 + y^2 = 4$ cuts the circle $x^2 + y^2 + 2x + 3y - 5 = 0$ in A & B. Then the equation of the circle on AB as a diameter is:

(A*) $13(x^2 + y^2) - 4x - 6y - 50 = 0$ (B) $9(x^2 + y^2) + 8x - 4y + 25 = 0$

(C) $x^2 + y^2 - 5x + 2y + 72 = 0$ (D) $13(x^2 + y^2) - 4x - 6y + 50 = 0$

यदि वृत्त $x^2 + y^2 = 4$ दूसरे वृत्त $x^2 + y^2 + 2x + 3y - 5 = 0$ को बिन्दु A एवं B पर काटता हो, तो AB व्यास वाले वृत्त का समीकरण हैं–

(A) $13(x^2 + y^2) - 4x - 6y - 50 = 0$ (B) $9(x^2 + y^2) + 8x - 4y + 25 = 0$

(C) $x^2 + y^2 - 5x + 2y + 72 = 0$ (D) $13(x^2 + y^2) - 4x - 6y + 50 = 0$

Sol. Common chord of given circle

$2x + 3y - 1 = 0$

family of circle passing through point of intersection of given circle

$(x^2 + y^2 + 2x + 3y - 5) + \lambda(x^2 + y^2 - 4) = 0 \Rightarrow (\lambda + 1)x^2 + (\lambda + 1)y^2 + 2x + 3y - (4\lambda + 5) = 0$

$x^2 + y^2 + \frac{2x}{\lambda+1} + \frac{3}{\lambda+1}y - \frac{(4\lambda+5)}{\lambda+1} = 0$

centre $\left(-\frac{1}{\lambda+1}, \frac{-3}{2(\lambda+1)}\right)$

This centre lies on AB

$2\left(-\frac{1}{\lambda+1}\right) + 3\left(\frac{-3}{2(\lambda+1)}\right) - 1 = 0 \Rightarrow -4 - 9 - 2\lambda - 2 = 0 \Rightarrow 2\lambda = -15 \Rightarrow \lambda = -15/2$

$\left(-\frac{15}{2}+1\right)x^2 + \left(-\frac{15}{2}+1\right)y^2 + 2x + 3y - \left(-4\times\frac{15}{2}+5\right) = 0 \Rightarrow -\frac{13x^2}{2} - \frac{13y^2}{2} + 2x + 3y + 25 = 0$

$\Rightarrow 13(x^2 + y^2) - 4x - 6y - 50 = 0.$

Hindi. दिए गए वृत्त की उभयनिष्ठ जीवा

$2x + 3y - 1 = 0$

दिए गए वृत्तों के प्रतिच्छेदन बिन्दुओं से गुजरने वाले वृत्तों का निकाय है।

$(x^2 + y^2 + 2x + 3y - 5) + \lambda(x^2 + y^2 - 4) = 0 \Rightarrow (\lambda + 1)x^2 + (\lambda + 1)y^2 + 2x + 3y - (4\lambda + 5) = 0$

$x^2 + y^2 + \frac{2x}{\lambda+1} + \frac{3}{\lambda+1}y - \frac{(4\lambda+5)}{\lambda+1} = 0$

केन्द्र $\left(-\frac{1}{\lambda+1}, \frac{-3}{2(\lambda+1)}\right)$

यह केन्द्र AB पर स्थित है।

$2\left(-\frac{1}{\lambda+1}\right) + 3\left(\frac{-3}{2(\lambda+1)}\right) - 1 = 0 \Rightarrow -4 - 9 - 2\lambda - 2 = 0 \Rightarrow 2\lambda = -15 \Rightarrow \lambda = -15/2$

$\left(-\frac{15}{2}+1\right)x^2 + \left(-\frac{15}{2}+1\right)y^2 + 2x + 3y - \left(-4\times\frac{15}{2}+5\right) = 0 \Rightarrow -\frac{13x^2}{2} - \frac{13y^2}{2} + 2x + 3y + 25 = 0$

$\Rightarrow 13(x^2 + y^2) - 4x - 6y - 50 = 0.$

PART-II: NUMERICAL VALUE QUESTIONS

भाग-II : संख्यात्मक प्रश्न (NUMERICAL VALUE QUESTIONS)

INSTRUCTION :

- The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto two digit.
- If the numerical value has more than two decimal places **truncate/round-off** the value to **TWO** decimal placed.

निर्देश :

- इस खण्ड में प्रत्येक प्रश्न का उत्तर **संख्यात्मक मान** के रूप में है जिसमें दो पूर्णांक अंक तथा दो अंक दशमलव के बाद में है।

❖ यदि संख्यात्मक मान में दो से अधिक दशमलव स्थान है, तो संख्यात्मक मान को दशमलव के **दो** स्थानों तक **ट्रंकेट/राउंड ऑफ (truncate/round-off)** करें।

1. Find maximum number of points having integer coordinates (both x, y integer) which can lie on a circle with centre at $\left(\sqrt{2}, \sqrt{3}\right)$ is (are)

पूर्णांक निर्देशांक वाले बिन्दुओं की अधिकतम संख्या होगी (दोनों x, y पूर्णांक है) जो $\left(\sqrt{2}, \sqrt{3}\right)$ केन्द्र के वृत्तों पर स्थित है:

Ans. **01.00**

Sol. Let equation of circle is $(x - \sqrt{2})^2 + (y - \sqrt{3}) = r^2$, (x_1, y_1) & (x_2, y_2) are integer points on circle

माना वृत्त का समीकरण $(x - \sqrt{2})^2 + (y - \sqrt{3}) = r^2$, (x_1, y_1) & (x_2, y_2) के पूर्णांक बिन्दु वृत्त पर स्थित है।

$(x_1 - \sqrt{2})^2 + (y_1 - \sqrt{3})^2 = (x_2 - \sqrt{2})^2 + (y_2 - \sqrt{3})^2 = r^2$

$(x_2 - x_1)(x_2 + x_1 - 2\sqrt{2}) + (y_2 - y_1)(y_2 + y_1 - 2\sqrt{3}) = 0$

$(x_2^2 - x_1^2) + (y_2^2 - y_1^2) = 2\sqrt{3}(y_2 - y_1) + 2\sqrt{2}(x_2 - x_1)$

$A = \sqrt{3}B + \sqrt{2}C$

इसलिए Therefore A = B = C = 0

$x_1 = x_2$ & $y_1 = y_2$

So, no distinct points are possible.

इसलिए कोई विभिन्न बिन्दु संभव नही है।

2. If equation of smallest circle touching the circles $x^2 + y^2 - 2y - 3 = 0$ and $x^2 + y^2 - 8x - 18y + 93 = 0$ is $x^2 + y^2 - 4x - fy + c = 0$ then value of f + c is

यदि वृत्तों $x^2 + y^2 - 2y - 3 = 0$ तथा $x^2 + y^2 - 8x - 18y + 93 = 0$ को स्पर्श करने वाले सबसे छोटे वृत्त का समीकरण $x^2 + y^2 - 4x - fy + c = 0$ हो तो f + c का मान होगा –

Ans. **32.88 or 32.89**

Sol. Ler r be the radius of new circle $C_1C_2 = 4\sqrt{5}$.

So $r = 2(\sqrt{5} - 1)$

Slope of line joining C_1 and C_2 i.e. $\tan\theta = 2$

$\therefore$ Equation of line joining C_1 and C_2 is

$$\frac{x - 0}{\cos\theta} = \frac{y - 1}{\sin\theta} = 2 + 2(\sqrt{5} - 1) = 2\sqrt{5}$$

x = 2 and y = 5 $\therefore$ Centre (2, 5)

hence equation of circle is $(x - 2)^2 + (y - 5)^2 = (2(\sqrt{5} - 1))^2$

$\Rightarrow x^2 + y^2 - 4x - 10y + (5 + 8\sqrt{5}) = 0$

Hindi. माना r नये वृत्त की त्रिज्या है $C_1C_2 = 4\sqrt{5}$.

अतः $r = 2(\sqrt{5} - 1)$

C_1 तथा C_2 को मिलाने वाली रेखा की प्रवणता अर्थात $\tan\theta = 2$

$\therefore$ C_1 तथा C_2 को मिलाने वाली रेखा का समीकरण

$$\frac{x - 0}{\cos\theta} = \frac{y - 1}{\sin\theta} = 2 + 2(\sqrt{5} - 1) = 2\sqrt{5}$$

x = 2 तथा y = 5 $\therefore$ केन्द्र (2, 5)

अतः वृत्त का समीकरण $(x - 2)^2 + (y - 5)^2 = (2(\sqrt{5} - 1))^2$ है

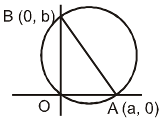
$\Rightarrow x^2 + y^2 - 4x - 10y + (5 + 8\sqrt{5}) = 0$

3. A line meets the co–ordinate axes in A and B. A circle is circumscribed about the triangle OAB. If d_1 and d_2 are the distances of the tangent to the circle at the origin O from the points A and B respectively and diameter of the circle is $\lambda_1 d_1 + \lambda_2 d_2$, then find the value of $\lambda_1 + \lambda_2$.

एक रेखा अक्षों को A तथा B पर मिलती है। त्रिभुज OAB पर परिवृत्त बनाया जाता है। यदि मूल बिन्दु O पर खींची गई स्पर्श रेखा की बिन्दुओं A और B से दूरी क्रमशः d_1 और d_2 है तथा वृत्त का व्यास $\lambda_1 d_1 + \lambda_2 d_2$, है तब $\lambda_1 + \lambda_2$ का मान है।

Ans. **02.00**

Sol.

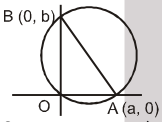


Equation of circum circle of triangle OAB $x^2 + y^2 - ax - by = 0$.

Equation of tangent at origin $ax + by = 0$.

$$d_1 = \frac{|a^2|}{\sqrt{a^2+b^2}} \text{ and } d_2 = \frac{|b^2|}{\sqrt{a^2+b^2}} \quad \Rightarrow \quad d_1 + d_2 = \sqrt{a^2+b^2} = \text{diameter}$$

Hindi.



त्रिभुज OAB के परिवृत्त का समीकरण $x^2 + y^2 - ax - by = 0$.

मूल बिन्दु पर स्पर्श रेखा का समीकरण $ax + by = 0$.

$$d_1 = \frac{|a^2|}{\sqrt{a^2+b^2}} \text{ तथा } d_2 = \frac{|b^2|}{\sqrt{a^2+b^2}} \quad \Rightarrow \quad d_1 + d_2 = \sqrt{a^2+b^2} = \text{व्यास}$$

4. A circle is inscribed (i.e. touches all four sides) into a rhombous ABCD with one angle 60º. The distance from the centre of the circle to the nearest vertex is equal to 1. If P is any point of the circle, then $|PA|^2 + |PB|^2 + |PC|^2 + |PD|^2$ is equal to :

एक वृत्त एक समचतुर्भुज ABCD जिसका एक कोण 60º है, में अर्न्तविष्ट है। वृत्त के केन्द्र की समीपवर्ती शीर्ष से दूरी 1 है। यदि वृत्त पर कोई बिन्दु P हो, तो $|PA|^2 + |PB|^2 + |PC|^2 + |PD|^2$ का मान होगा–

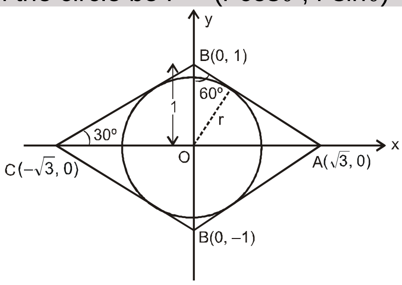
Ans. **11.00**

Sol. $\because$ $\tan 60^\circ = \frac{OA}{1} = \sqrt{3}$

$\therefore$ $A(\sqrt{3},0)$ and $C(-\sqrt{3},0)$

$\because$ $\sin 60^\circ = \frac{r}{1} = \frac{\sqrt{3}}{2}$

Let coordinates of any point P on the circle be $P \equiv (r\cos\theta, r\sin\theta)$



$\therefore$ $PA^2 = (\sqrt{3} - r\cos\theta)^2 + (r\sin\theta)^2$

$PB^2 = (r\cos\theta)^2 + (1 - r\sin\theta)^2$

$PC^2 = (r\cos\theta + \sqrt{3})^2 + (r\sin\theta)^2$

and $PD^2 = (r\cos\theta)^2 + (r\sin\theta + 1)^2$

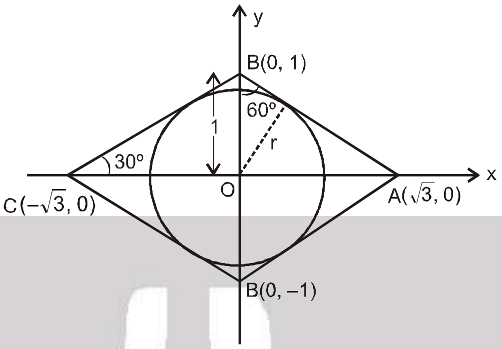
$\therefore \quad PA^2 + PB^2 + PC^2 + PD^2 = 4r^2 + 8 = 11 \qquad \because \quad r = \sqrt{3}/2$

Hindi. $\because \tan 60^o = \dfrac{OA}{1} = \sqrt{3}$

$\therefore A(\sqrt{3},0)$ and $C(-\sqrt{3},0)$

$\because \sin 60^o = \dfrac{r}{1} = \dfrac{\sqrt{3}}{2}$

मानाकि वृत्त पर स्थित किसी बिन्दु P के निर्देशांक $(r\cos\theta , r\sin\theta)$ है।



$\therefore PA^2 = (\sqrt{3} - r\cos\theta)^2 + (r\sin\theta)^2$

$PB^2 = (r\cos\theta)^2 + (1 - r\sin\theta)^2$

$PC^2 = (r\cos\theta + \sqrt{3})^2 + (r\sin\theta)^2$

तथा $PD^2 = (r\cos\theta)^2 + (r\sin\theta + 1)^2$

$\therefore \quad PA^2 + PB^2 + PC^2 + PD^2 = 4r^2 + 8 = 11 \qquad \because \quad r = \sqrt{3}/2$

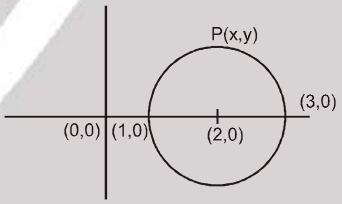
5. Let x & y be the real numbers satisfying the equation $x^2 - 4x + y^2 + 3 = 0$. If the maximum and minimum values of $x^2 + y^2$ are M & m respectively, then find the numerical value of $(M + m)$.

माना x और y वास्तविक संख्याएँ समीकरण $x^2 - 4x + y^2 + 3 = 0$ को सन्तुष्ट करती है। यदि $x^2 + y^2$ का अधिकतम और न्यूनतम मान क्रमशः M और m हो, तो $M + m$ का संख्यात्मक मान ज्ञात कीजिए।

Ans. **10.00**

Sol. $x^2 + y^2 - 4x + 3 = 0$

$\sqrt{x^2 + y^2}$ represents distance of p from origin



Hence $M = 3^2 + 0^2$

$M = 1^2 + 0^2$

$M - m = 10$.

6. Find absolute value of 'c' for which the set,

$\{(x, y) | x^2 + y^2 + 2x \le 1\} \cap \{(x, y) | 5x - 12y + c \ge 0\}$ contains only one point is common.

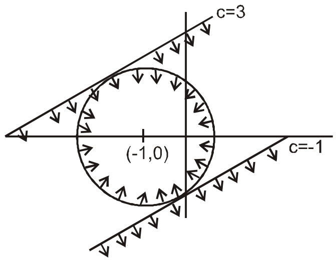
'c' का निरपेक्ष मान ज्ञात कीजिए जिसके लिए समुच्चय

$\{(x, y) | x^2 + y^2 + 2x \le 1\} \cap \{(x, y) | 5x - 12y + c \ge 0\}$ में केवल एक उभयनिष्ठ बिन्दु रखता है

Ans. **13.38**

Sol. $\left|\frac{-5-0+c}{13}\right| = \sqrt{2} \Rightarrow c - 5 = \pm 13\sqrt{2} \Rightarrow c = 5 \pm 13\sqrt{2}$

but $c \le 0$ hence $c = 5 - 13\sqrt{2}$



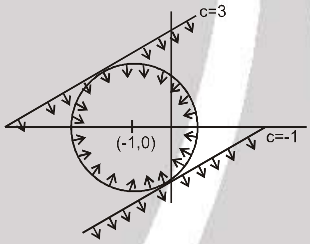
But $c = 5 - 13\sqrt{2}$ common point is one

$c = 5 + 13\sqrt{2}$ common point is infinite

Hence $c = 5 - 13\sqrt{2}$ is Answer.

Hindi. $\left|\frac{-5-0+c}{13}\right| = \sqrt{2} \Rightarrow c - 5 = \pm 13\sqrt{2} \Rightarrow c = 5 \pm 13\sqrt{2}$

लेकिन $c \le 0$ अतः $c = 5 - 13\sqrt{2}$



परन्तु $c = 5 - 13\sqrt{2}$ एक उभयनिष्ठ बिन्दु है।

$c = 5 + 13\sqrt{2}$ उभयनिष्ठ बिन्दु अनन्त है।

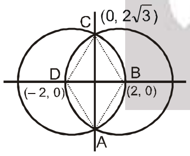
अतः $c = 5 - 13\sqrt{2}$ उत्तर

7. A rhombus is inscribed in the region common to the two circles $x^2 + y^2 - 4x - 12 = 0$ and $x^2 + y^2 + 4x - 12 = 0$ with two of its vertices on the line joining the centres of the circles then area of the rhombus is

दो वृत्तो $x^2 + y^2 - 4x - 12 = 0$ और $x^2 + y^2 + 4x - 12 = 0$ के उभयनिष्ठ क्षेत्र में एक समचतुर्भुज जिसके दो शीर्ष वृत्त के केन्द्रों को मिलाने वाली रेखा पर स्थित है, बनाया जाता है तब समचतुर्भुज का क्षेत्रफल होगा –

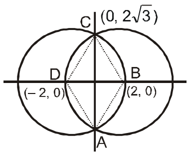
Ans. **13.85 or 13.86**

Sol.



Area of ABCD = $4\left(\frac{1}{2}.2.2\sqrt{3}\right)$.

Hindi.



ABCD का क्षेत्रफल $= 4 \times \left(\frac{1}{2}.2.2\sqrt{3}\right) = 8\sqrt{3}$

8. If (α, β) is a point on the circle whose centre is on the x-axis and which touches the line x + y = 0 at (2, –2), then find the greatest value of 'α' is

यदि वृत्त जिसका केन्द्र x-अक्ष पर स्थित हो, पर स्थित एक बिन्दु (α, β) है तथा जो रेखा x + y = 0 को (2, –2) बिन्दु पर स्पर्श करता हो, तो α का अधिकतम मान होगा–

Ans. **06.82 or 06.83**

Sol. $\because$ Equation of circle $(x-2)^2 + (y+2)^2 + \lambda(x+y) = 0$ ....(i)

$\because$ Centre lies on the x-axis

$\therefore$ $\lambda = -4$ put in (i)

$\therefore$ equation of circle is $x^2 + y^2 - 8x + 8 = 0$

(α, β) lies on it $\Rightarrow$ $\beta^2 = -\alpha^2 + 8\alpha - 8 \geq 0$

$\therefore$ greatest value of 'α' is $4 + 2\sqrt{2}$

Hindi. $\because$ वृत्त का समीकरण $(x-2)^2 + (y+2)^2 + \lambda(x+y) = 0$ ....(i)

$\because$ केन्द्र x अक्ष पर स्थित है।

$\therefore$ (i) में $\lambda = -4$ रखने पर

$\therefore$ वृत्त का समीकरण $x^2 + y^2 - 8x + 8 = 0$ है।

(α, β) इस पर स्थित है। $\Rightarrow \beta^2 = -\alpha^2 + 8\alpha - 8 \geq 0$

$\therefore$ α का अधिकतम मान $4 + 2\sqrt{2}$ है।

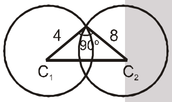
9. Two circles whose radii are equal to 4 and 8 intersect at right angles, then length of their common chord is

दो वृत्त जिनकी त्रिज्याएँ 4 और 8 हो, समकोण पर प्रतिच्छेद करते है, तब उभयनिष्ठ जीवा की लम्बाई होगी –

Ans. **07.15**

Sol. $C_1C_2 = \sqrt{80}$

Area क्षेत्रफल $= \frac{1}{2} \times 4 \times 8 = \frac{1}{2} \times \sqrt{80} \times \frac{\ell}{2}$



$\ell = \frac{64}{\sqrt{80}} = \frac{16}{\sqrt{5}}$

10. A variable circle passes through the point A (a, b) & touches the x–axis and the locus of the other end of the diameter through A is $(x-a)^2 = \lambda by$, then find the value of λ

एक चर वृत्त बिन्दु A (a, b) से गुजरता है तथा x–अक्ष को स्पर्श करता है। A से गुजरने वाले व्यास के दूसरे सिरे का बिन्दुपथ $(x-a)^2 = \lambda by$ हो, तो λ का मान ज्ञात कीजिये।

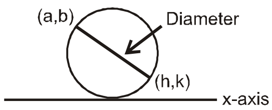
Ans. **04.00**

Sol. Equation of circle whose diameter's end points are (a, b) and (h, k)

$(x-a)(x-h) + (y-b)(y-k) = 0$

$x^2 + y^2 - x(a+h) - y(b+k) + ah + bk = 0$

it touches x-axis.



Hence $g^2 = c$ $\Rightarrow$ $\left(\frac{a+h}{2}\right)^2 = ah + bk \Rightarrow (h-a)^2 = 4bk$

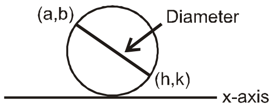
$\therefore$ Locus of (h, k) is $(x-a)^2 = 4by$.

Hindi उस वृत्त का समीकरण जिसके व्यास के अन्त सिरे (a, b) और (h, k) है।

$(x-a)(x-h) + (y-b)(y-k) = 0$

$x^2 + y^2 - x(a+h) - y(b+k) + ah + bk = 0$

यह x-अक्ष को स्पर्श करता है।



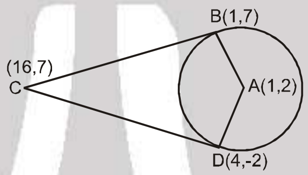
अतः $g^2 = c \quad \Rightarrow \left(\frac{a+h}{2}\right)^2 = ah + bk \Rightarrow (h-a)^2 = 4bk$

$\therefore$ (h, k) का बिन्दुपथ $(x-a)^2 = 4by$ है।

11. Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Suppose that the tangents at the points B (1, 7) & D (4, – 2) on the circle meet at the point C. Find the area of the quadrilateral ABCD.
माना वृत्त $x^2 + y^2 - 2x - 4y - 20 = 0$ का केन्द्र A है। माना वृत्त के बिन्दुओं B (1, 7) तथा D (4, – 2) पर खींची गई स्पर्श रेखाएँ बिन्दु C पर मिलती हो, तो चतुर्भुज ABCD का क्षेत्रफल ज्ञात कीजिए।

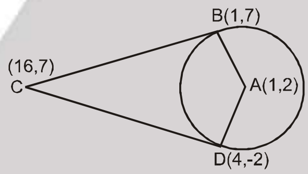
Ans. **75.00**

Sol. Given circle $x^2 + y^2 - 2x - 4y - 20 = 0$
Tangents at B(1, 7) is
$x + 7y - (x + 1) - 2(y + 7) - 20 = 0$
$5y - 35 = 0 \Rightarrow y = 7$



at D (4, –2)
$4x - 2y - (x + 4) - 2(y - 2) - 20 = 0$
$3x - 4y = 20$
Hence c(16, 7)
Area of quadrilateral ABCD = AB × BC = 5 × 15 = 75 square units.

Hindi दिया गया वृत्त $x^2 + y^2 - 2x - 4y - 20 = 0$ है।
बिन्दु B(1, 7) पर स्पर्श रेखा की समीकरण
$x + 7y - (x + 1) - 2(y + 7) - 20 = 0$
$5y - 35 = 0 \Rightarrow y = 7$

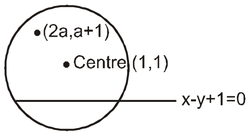


D (4, –2) पर
$4x - 2y - (x + 4) - 2(y - 2) - 20 = 0$
$3x - 4y = 20$
अतः c(16, 7)
ABCD चतुर्भुज का क्षेत्रफल = AB × BC = 5 × 15 = 75 वर्ग इकाई

12. If the complete set of values of a for which the point (2a, a + 1) is an interior point of the larger segment of the circle $x^2 + y^2 - 2x - 2y - 8 = 0$ made by the chord whose equation is $3x - 4y + 5 = 0$ is (p,q) then value of p + q is
यदि a के सभी मानों का समुच्चय जिसके लिए रेखा $3x - 4y + 5 = 0$ द्वारा वृत्त $x^2 + y^2 - 2x - 2y - 8 = 0$ को विभाजित करने पर बनने वाले बड़े वृत्त खण्ड (Larger segment) में बिन्दु (2a, a + 1) विद्यमान हो, (p,q) है तब p + q का मान होगा –

Ans. **01.30**

Sol. Point (2a, a + 1) lies inside circle $x^2 + y^2 - 2x - 2y - 8 = 0$
$4a^2 + (a + 1)^2 - 2(2a) - 2(a + 1) - 8 < 0 \Rightarrow 5a^2 - 4a - 9 < 0$
$5a^2 - 9a + 5a - 9 < 0 \Rightarrow a(5a - 9) + 1(5a - 9) < 0$



$(5a - 9)(a + 1) < 0 \Rightarrow a \in (-1, 9/5)$

centre & (2a, a + 1) lies on same side w.r.t. line $3x - 4y + 5 = 0$

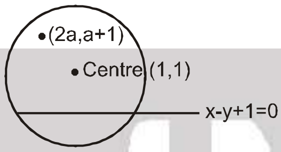
$6a - 4(a + 1) + 5 > 0 \Rightarrow a > -\frac{1}{2}$

Hence $a \in \left(-\frac{1}{2}, \frac{9}{5}\right)$

Hindi बिन्दु (2a, a + 1) वृत्त $x^2 + y^2 - 2x - 2y - 8 = 0$ के अन्दर है

$4a^2 + (a + 1)^2 - 2(2a) - 2(a + 1) - 8 < 0$

$5a^2 - 4a - 9 < 0 \Rightarrow 5a^2 - 9a + 5a - 9 < 0$



$a(5a - 9) + 1(5a - 9) < 0 \Rightarrow (5a - 9)(a + 1) < 0 \Rightarrow a \in (-1, 9/5)$

केन्द्र तथा (2a, a + 1) रेखा $3x - 4y + 5 = 0$ के एक ही और स्थित होगें

$6a - 4(a + 1) + 5 > 0 \Rightarrow a > -\frac{1}{2}$

अतः $a \in \left(-\frac{1}{2}, \frac{9}{5}\right)$

13. The circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q, then find the number of values of 'a' for which the line $5x + by - a = 0$ passes through P and Q.

यदि वृत्त $x^2 + y^2 + 2ax + cy + a = 0$ तथा $x^2 + y^2 - 3ax + dy - 1 = 0$ दो भिन्न–भिन्न बिन्दुओं P तथा Q पर काटते हो, तो a के मानों की संख्या ज्ञात कीजिए जिसके लिए रेखा $5x + by - a = 0$ बिन्दु P तथा Q से गुजरती है

Ans. **00.00**

Sol. Let $S_1 : x^2 + y^2 + 2ax + cy + a = 0$

$S_1 : x^2 + y^2 - 3ax + dy - 1 = 0$

common chord $S_1 - S_2 = 0 \Rightarrow 5ax + y(c - d) + (a + 1) = 0$

given line is $5x + by - a = 0$

compare both $\frac{5a}{5} = \frac{c-d}{b} = \frac{a+1}{-a} \quad \Rightarrow \quad a = \frac{c-d}{b} = -1 - \frac{1}{a}$

(i) (ii) (iii)

From (i) & (iii) $a^2 + a + 1 = 0 \Rightarrow a = \omega, \omega^2$ no real a.

Hindi माना $S_1 : x^2 + y^2 + 2ax + cy + a = 0 \quad \Rightarrow \quad S_1 : x^2 + y^2 - 3ax + dy - 1 = 0$

उभयनिष्ट जीवा $S_1 - S_2 = 0 \Rightarrow 5ax + y(c - d) + (a + 1) = 0$

दी गई रेखा $5x + by - a = 0$

तुलना करने पर $\frac{5a}{5} = \frac{c-d}{b} = \frac{a+1}{-a} \quad \Rightarrow \quad a = \frac{c-d}{b} = -1 - \frac{1}{a}$

(i) (ii) (iii)

(i) एवं (iii) से $a^2 + a + 1 = 0 \Rightarrow a = \omega, \omega^2$ किसी भी मान के लिए a वास्तविक नहीं है।

14. The circumference of the circle $x^2 + y^2 - 2x + 8y - q = 0$ is bisected by the circle $x^2 + y^2 + 4x + 12y + p = 0$, then find p + q

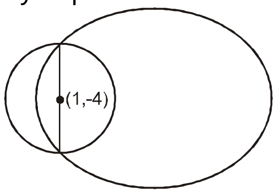
वृत्त $x^2 + y^2 - 2x + 8y - q = 0$ की परिधि को वृत्त $x^2 + y^2 + 4x + 12y + p = 0$ समद्विभाजित करता हो, तो p + q का मान ज्ञात कीजिए।

Ans. **10.00**

Sol. Common chord of given circle

$6x + 4y + (p + q) = 0$

This is diameter of $x^2 + y^2 - 2x + 8y - q = 0$



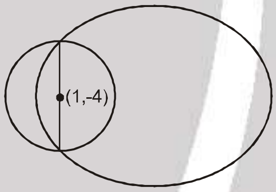
centre (1, –4)

$6 - 16 + (p + q) = 0 \Rightarrow p + q = 10$

Hindi दिए हुए वृत्त की उभयनिष्ट जीवा

$6x + 4y + (p + q) = 0$

यह वृत्त $x^2 + y^2 - 2x + 8y - q = 0$ का व्यास है



केन्द्र (1, –4)

$6 - 16 + (p + q) = 0 \Rightarrow p + q = 10$

15. A circle touches the line y = x at a point P such that OP = $4\sqrt{2}$ where O is the origin. The circle contains the point (–10, 2) in its interior and the length of its chord on the line x + y = 0 is $6\sqrt{2}$. If the equation of the circle $x^2 + y^2 + 2gx + 2fy + 3c = 0$, then value of g + f + c is

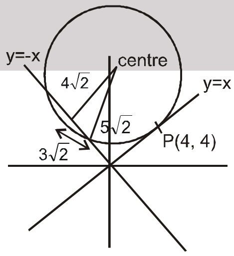
Ans. **18.66 or 18.67**

एक वृत्त सरल रेखा y = x को बिन्दु P पर इस प्रकार स्पर्श करता है कि OP = $4\sqrt{2}$ जहाँ O मूलबिन्दु है। (–10, 2) वृत्त के अन्दर कोई बिन्दु है तथा सरल रेखा x + y = 0 पर वृत्त की जीवा की लम्बाई $6\sqrt{2}$ है। यदि वृत्त का समीकरण $x^2 + y^2 + 2gx + 2fy + 3c = 0$ है, तब g + f + c का मान होगा –

Sol. By family of circle equation of circle touching y = x at p(4, 4)

$(x - 4)^2 + (y - 4)^2 + \lambda(x - y) = 0 \Rightarrow x^2 + y^2 + x(\lambda - 8) - y(\lambda + 8) + 32 = 0$

radius = $\sqrt{\left(\frac{\lambda-8}{2}\right)^2 + \left(\frac{\lambda+8}{2}\right)^2 - 32} = 5\sqrt{2}$



$2\lambda^2 + 128 - 128 = 200 \Rightarrow \lambda = \pm 10$

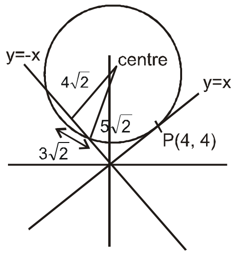
$\lambda = 10$ $x^2 + y^2 + 2x - 18y + 32 = 0$

$\lambda = -10$ $x^2 + y^2 - 18x + 2y + 32 = 0$

By family of circle equation of circle touching y = x at p(–4, –4)

$(x + 4)^2 + (y + 4)^2 + \lambda(x - y) = 0 \Rightarrow x^2 + y^2 + x(\lambda + 8) + y(8 - \lambda) + 32 = 0$

radius = $\sqrt{\left(\frac{\lambda+8}{2}\right)^2 + \left(\frac{\lambda-8}{2}\right)^2 - 32} = 5\sqrt{2} \Rightarrow \lambda^2 = 100 \Rightarrow \lambda = \pm 10$

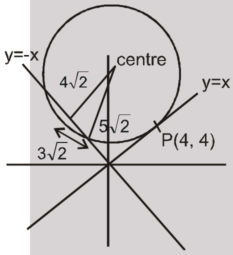


Hence $x^2 + y^2 + 18x - 2y + 32 = 0 \Rightarrow x^2 + y^2 - 2x - 18y + 32 = 0$
as (–10, 2) lies inside $x^2 + y^2 + 18x - 2y + 32 = 0$

$g = 9$, $f = -1$, $c = \dfrac{32}{3} \Rightarrow g + f + c = \dfrac{56}{3}$

Hindi वृत्तों के निकाय द्वारा y = x को p(4, 4) पर स्पर्श करने वाले वृत्त की समीकरण
$(x - 4)^2 + (y - 4)^2 + \lambda(x - y) = 0 \Rightarrow x^2 + y^2 + x(\lambda - 8) - y(\lambda + 8) + 32 = 0$

त्रिज्या $= \sqrt{\left(\dfrac{\lambda - 8}{2}\right)^2 + \left(\dfrac{\lambda + 8}{2}\right)^2 - 32} = 5\sqrt{2}$



$2\lambda^2 + 128 - 128 = 200 \Rightarrow \lambda = \pm 10$
$\lambda = 10$ $\quad x^2 + y^2 + 2x - 18y + 32 = 0$
$\lambda = -10$ $\quad x^2 + y^2 - 18x + 2y + 32 = 0$
वृत्तों के निकाय द्वारा y = x को p(–4, –4) पर स्पर्श करने वाले वृत्त की समीकरण
$(x + 4)^2 + (y + 4)^2 + \lambda(x - y) = 0 \Rightarrow x^2 + y^2 + x(\lambda + 8) + y(8 - \lambda) + 32 = 0$

त्रिज्या $= \sqrt{\left(\dfrac{\lambda + 8}{2}\right)^2 + \left(\dfrac{\lambda - 8}{2}\right)^2 - 32} = 5\sqrt{2}$

$\lambda^2 = 100 \Rightarrow \lambda = \pm 10$
अतः $x^2 + y^2 + 18x - 2y + 32 = 0 \Rightarrow x^2 + y^2 - 2x - 18y + 32 = 0$
जैसाकि (–10, 2) वृत्त $x^2 + y^2 + 18x - 2y + 32 = 0$ के अन्दर है।

$g = 9$, $f = -1$, $c = \dfrac{32}{3} \Rightarrow g + f + c = \dfrac{56}{3}$

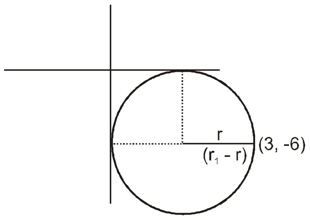
PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

भाग - III : एक या एक से अधिक सही विकल्प प्रकार

1. The equation of circles passing through (3, –6) touching both the axes is

बिन्दु (3, –6) से गुजरने वाले तथा दोनों अक्षों को स्पर्श करने वाले वृत्त का समीकरण हैं –
(A*) $x^2 + y^2 - 6x + 6y + 9 = 0$ (B) $x^2 + y^2 + 6x - 6y + 9 = 0$
(C) $x^2 + y^2 + 30x - 30y + 225 = 0$ (D*) $x^2 + y^2 - 30x + 30y + 225 = 0$

Sol. Now अब
$(r - 3)^2 + (-r + 6)^2 = r^2$
$r^2 - 18r + 45 = 0 \Rightarrow r = 3, 15$
Hence circle अतः वृत्त



$(x-3)^2 + (y+3)^2 = 3^2 \Rightarrow x^2 + y^2 - 6x + 6y + 9 = 0$

$(x-15)^2 + (y+15)^2 = (15)^2 \Rightarrow \Rightarrow x^2 + y^2 - 30x + 30y + 225 = 0$

2. Equations of circles which pass through the points (1, –2) and (3, – 4) and touch the x-axis is

(A) $x^2 + y^2 + 6x + 2y + 9 = 0$
(B*) $x^2 + y^2 + 10x + 20y + 25 = 0$
(C*) $x^2 + y^2 - 6x + 4y + 9 = 0$
(D) $x^2 + y^2 + 10x + 20y - 25 = 0$

बिन्दुओं (1, –2) एवं (3, – 4) से गुजरने वाले एवं x-अक्ष को स्पर्श करने वाले वृत्त का समीकरण है–

(A) $x^2 + y^2 + 6x + 2y + 9 = 0$
(B) $x^2 + y^2 + 10x + 20y + 25 = 0$
(C) $x^2 + y^2 - 6x + 4y + 9 = 0$
(D) $x^2 + y^2 + 10x + 20y - 25 = 0$

Sol. Let equation of required circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

it passes through (1, –2) & (3, –4)

$2g - 4f + c = -5$

$6g - 8f + c = -25$

$4g - 8f + 2c = -10$

$6g - 8f + c = -25$

$-2g + c = 15$

circle touches x-axis $g^2 = c \Rightarrow$

$g^2 - 2g - 15 = 0$

$g = 5, -3$

$g = 5, \quad c = 25, \quad f = 10 \quad \Rightarrow \quad x^2 + y^2 + 10x + 20y + 25 = 0$

$g = -3, \quad c = 9, \quad f = 2 \quad \Rightarrow \quad x^2 + y^2 - 6x + 4y + 9 = 0$

Hindi वृत्त का समीकरण

$x^2 + y^2 + 2gx + 2fy + c = 0$

यह (1, –2) तथा (3, –4) से गुजरता है।

$2g - 4f + c = -5$

$6g - 8f + c = -25$

$4g - 8f + 2c = -10$

$6g - 8f + c = -25$

$-2g + c = 15$

x-अक्ष को स्पर्श करता है $g^2 = c \Rightarrow g^2 - 2g - 15 = 0$

$g = 5, -3$

$g = 5, \quad c = 25, \quad f = 10 \quad \Rightarrow \quad x^2 + y^2 + 10x + 20y + 25 = 0$

$g = -3, \quad c = 9, \quad f = 2 \quad \Rightarrow \quad x^2 + y^2 - 6x + 4y + 9 = 0$

3. The centre of a circle passing through the points (0, 0), (1, 0) & touching the circle $x^2 + y^2 = 9$ is :

वृत्त जो कि बिन्दुओं (0, 0), (1, 0) से गुजरता हो तथा वृत्त $x^2 + y^2 = 9$ को स्पर्श करता हो, का केन्द्र है–

(A) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (B*) $\left(\frac{1}{2}, \sqrt{2}\right)$ (C) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (D*) $\left(\frac{1}{2}, -\sqrt{2}\right)$

Sol. Equation of circle passing through (0, 0) and (1, 0) is

$x^2 + y^2 - x + 2fy = 0$ ....(i)

$\because \quad x^2 + y^2 = 9$ ......(ii)

(i) & (ii) touch each other.

so equation of Radical axis is $x = 2fy + 9$ ......(iii)

line (iii) is also tangent to the circle (ii)

$\therefore$ on solving (ii) & (iii), we get

$(1 + 4f^2)y^2 + 36fy + 72 = 0$ ....(iv)

$\therefore \quad D = 0 \quad \Rightarrow \quad f = \pm \sqrt{2}$.

Hindi. (0, 0) तथा (1, 0) से गुजरने वाले वृत्त का समीकरण

$x^2 + y^2 - x + 2fy = 0$ ....(i)

$\because \quad x^2 + y^2 = 9 \qquad ......(ii)$

(i) तथा (ii) एक दूसरे को स्पर्श करते है।
अतः मूलाक्ष का समीकरण $x = 2fy + 9$ है। ......(iii)
(ii) तथा (iii) को हल करने पर
$(1 + 4f^2)y^2 + 36fy + 72 = 0$ ....(iv)
$\because$ (iii) वृत्त (ii) की स्पर्श रेखा है। अतः समीकरण (iv) का विवेचक = 0 अर्थात्
$D = 0 \Rightarrow f = \pm\sqrt{2}$.

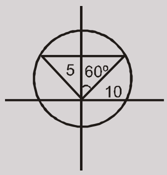
4. The equation of the circle which touches both the axes and the line $\frac{x}{3} + \frac{y}{4} = 1$ and lies in the first quadrant is $(x - c)^2 + (y - c)^2 = c^2$ where c is
प्रथम चतुर्थांश में दोनों अक्षों तथा सरल रेखा $\frac{x}{3} + \frac{y}{4} = 1$ को स्पर्श करने वाले वृत्त का समीकरण $(x - c)^2 + (y - c)^2 = c^2$ है, तो c का मान होगा–
(A*) 1 (B) 2 (C) 4 (D*) 6

Sol. $\left|\frac{4C + 3C - 12}{5}\right| = C \Rightarrow C = 1, 6.$

5. Find the equations of straight lines which pass through the intersection of the lines $x - 2y - 5 = 0$, $7x + y = 50$ & divide the circumference of the circle $x^2 + y^2 = 100$ into two arcs whose lengths are in the ratio 2 : 1.
सरल रेखाओं $x - 2y - 5 = 0$ और $7x + y = 50$ के प्रतिच्छेद बिन्दु से गुजरने वाली उन सरल रेखाओं का समीकरण ज्ञात कीजिए जो वृत्त $x^2 + y^2 = 100$ की परिधि को दो चापों जिनकी लम्बाइयों के अनुपात 2 : 1 हो, में विभाजित करता है।
(A) $3x - 4y - 25 = 0$ (B) $4x + 3y - 25 = 0$
(C*) $4x - 3y - 25 = 0$ (D*) $3x + 4y - 25 = 0$

Sol. Angle 360° is also divided into 2 : 1 = 240 : 120 in respect of arc
Point of intersection (7,1)
Here length of perpendicular from lines be 5
$y - 1 = m(x - 7)$ ....(1)

$\left|\frac{1 - 7m}{\sqrt{1 + m^2}}\right| = 5 \Rightarrow (1 - 7m)^2 = 25(1 + m^2)$



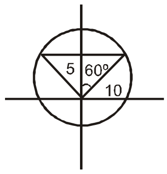
$24m^2 - 14m - 24 = 0 \Rightarrow (4m + 3)(6m - 8) = 0$

$m = -\frac{3}{4}, \; m = \frac{4}{3}$

Putting in (1) $\Rightarrow y - 1 = -\frac{3}{4}(x - 7), \quad y - 1 = \frac{4}{3}(x - 7)$

Sol. कोण 360°चाप के सापेक्ष 2 : 1 = 240 : 120 मे विभाजित होगा
प्रतिच्छेद बिन्दु (7,1) है।
यहाँ रेखा से डाले गये लम्ब की लम्बाई 5 है।
$y - 1 = m(x - 7)$ ....(1)

$\left|\frac{1 - 7m}{\sqrt{1 + m^2}}\right| = 5 \Rightarrow (1 - 7m)^2 = 25(1 + m^2)$



$24m^2 - 14m - 24 = 0 \quad \Rightarrow \quad (4m + 3)(6m - 8) = 0$

$m = -\frac{3}{4}, \; m = \frac{4}{3}$

(1) में रखने पर $\quad \Rightarrow \quad y - 1 = -\frac{3}{4}(x - 7), \quad y - 1 = \frac{4}{3}(x - 7)$

6. Tangents are drawn to the circle $x^2 + y^2 = 50$ from a point 'P' lying on the x-axis. These tangents meet the y-axis at points 'P_1' and 'P_2'. Possible coordinates of 'P' so that area of triangle PP_1P_2 is minimum, is/are

x- अक्ष पर स्थित बिन्दु 'P' से वृत्त $x^2 + y^2 = 50$ की स्पर्श रेखाऐं खीचीं जाती है। ये स्पर्श रेखाऐं y-अक्ष को 'P_1' तथा 'P_2' पर मिलती है। संभावित 'P' के निर्देशांक होगें जबकि त्रिभुज PP_1P_2 का क्षेत्रफल न्यूनतम है

(A*) (10, 0) (B) $(10\sqrt{2}, 0)$ (C*) (–10, 0) (D) $(-10\sqrt{2}, 0)$

Sol. $OP = 5\sqrt{2}\sec\theta, \; OP_1 = 5\sqrt{2}\,\text{cosec}\theta$

area (ΔPP_1P_2) का क्षेत्रफल $= \frac{100}{\sin 2\theta}$, area $(\Delta PP_1P_2)_{min}$ का क्षेत्रफल = 100

$\Rightarrow \theta = \pi/4 \Rightarrow OP = 10 \quad \Rightarrow \quad P = (10, 0), (-10, 0)$

Hence अतः (a), (c) are correct सही है।

7. If (a, 0) is a point on a diameter segment of the circle $x^2 + y^2 = 4$, then $x^2 - 4x - a^2 = 0$ has

(A*) exactly one real root in (– 1, 0] (B*) Exactly one real root in [2, 5]

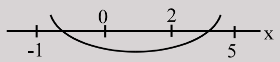
(C*) distinct roots greater than-1 (D*) Distinct roots less than 5

यदि (a, 0) वृत्त $x^2 + y^2 = 4$ के व्यास पर एक बिन्दु है तब $x^2 - 4x - a^2 = 0$ रखता है

(A*) (– 1, 0] मेंठीक एक वास्तविक मूल रखता है। (B*) [2, 5] में ठीक एक वास्तविक मूल रखता है।

(C*) असमान मूल –1 से बड़े है। (D*) भिन्न–भिन्न मूल 5 से कम है।

Sol.



Since (a, 0) is a point on the diameter of the circle $x^2 + y^2 = 4$,

So maximum value of a^2 is 4

Let $f(x) = x^2 - 4x - a^2$

clearly $f(-1) = 5 - a^2$ is 4

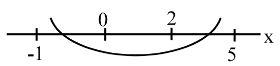
$f(2) = -(a^2 + 4) < 0$

$f(0) = -a^2 < 0$ and $f(5) = 5 - a^2 > 0$

so graph of f(x) will be as shown

Hence (a), (b), (c), (d) are the correct answer.

Hindi.



चूंकि (a, 0) वृत्त $x^2 + y^2 = 4$ के व्यास पर स्थित है।

इसलिए a^2 का अधिकतम मान 4 है।

माना $f(x) = x^2 - 4x - a^2$

स्पष्टतया $f(-1) = 5 - a^2$ is 4

$f(2) = -(a^2 + 4) < 0$

$f(0) = -a^2 < 0$ और $f(5) = 5 - a^2 > 0$

इसलिए f(x) का आरेख दर्शाया होगा

अतः (a), (b), (c), (d) सही उत्तर है।

8. The tangents drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ are perpendicular if

वृत्त $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ की मूल बिन्दु से खींची गई स्पर्श रेखाऐं लम्बवत् है यदि

(A*) $h = r$ (B*) $h = -r$ (C) $r^2 + h^2 = 1$ (D*) $r^2 = h^2$

Sol. Director circle is $(x - r)^2 + (y - R)^2 = \left(\sqrt{2}r\right)^2$. नियामक वृत्त $(x - r)^2 + (y - R)^2 = \left(\sqrt{2}r\right)^2$ है

satisfies by (0, 0) $\Rightarrow$ $r^2 + h^2 \;\; 2r^2 \Rightarrow r^2 = h^2$. (0, 0) सन्तुष्ट होता है $\Rightarrow$ $r^2 + h^2 \;\; 2r^2 \Rightarrow r^2 = h^2$.

9. The equation (s) of the tangent at the point (0, 0) to the circle where circle makes intercepts of length 2a and 2b units on the coordinate axes, is (are) -

वृत्त के (0, 0) पर स्पर्शरेखा का समीकरण होगा जहाँ वृत्त निर्देशांक अक्षों पर 2a तथा 2b लम्बाई का अन्तखण्ड काटता है–

(A*) $ax + by = 0$ (B*) $ax - by = 0$ (C) $x = y$ (D) $bx + ay = ab$

Sol. Equation of circle passing through origin and cutting off intercepts 2a and 2b units on the coordinate axes is $x^2 + y^2 \pm 2ax \pm 2by = 0$. Hence (a), (b) are correct answers.

मूल बिन्दु से गुजरने वाले और निर्देशांक अक्षों पर 2a तथा 2b लम्बाई के अन्तःखण्ड काटने वाले समीकरण $x^2 + y^2 \pm 2ax \pm 2by = 0$ है अतः (A), (B) सही है।

10. Consider two circles $C_1 : x^2 + y^2 - 1 = 0$ and $C_2 : x^2 + y^2 - 2 = 0$. Let A(1,0) be a fixed point on the circle C_1 and B be any variable point on the circle C_2. The line BA meets the curve C_2 again at C. Which of the following alternative(s) is/are correct ?

(A*) $OA^2 + OB^2 + BC^2 \in [7, 11]$, where O is the origin.

(B) $OA^2 + OB^2 + BC^2 \in [4, 7]$, where O is the origin.

(C*) Locus of midpoint of AB is a circle of radius $\frac{1}{\sqrt{2}}$.

(D*) Locus of midpoint of AB is a circle of area $\frac{\pi}{2}$.

माना कि वृत्त $C_1 : x^2 + y^2 - 1 = 0$ और $C_2 : x^2 + y^2 - 2 = 0$ है। माना A(1,0) वृत्त C_1 पर एक स्थिर बिन्दु है तथा वृत्त C_2 पर एक चर बिन्दु B है तथा रेखा BA, वक्र C_2 को पुनः C पर मिलता है। निम्न में से कौनसा विकल्प सही है ?

(A*) $OA^2 + OB^2 + BC^2 \in [7, 11]$, जहाँ O मूल बिन्दु है।

(B) $OA^2 + OB^2 + BC^2 \in [4, 7]$, जहाँ O मूल बिन्दु है।

(C*) AB के मध्य बिन्दु का बिन्दुपथ, त्रिज्या $\frac{1}{\sqrt{2}}$ का एक वृत्त है।

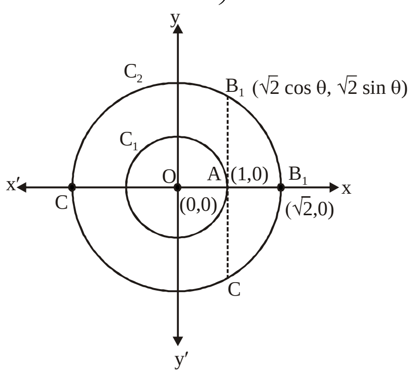
(D*) AB के मध्य बिन्दु का बिन्दुपथ, $\frac{\pi}{2}$ क्षेत्रफल का एक वृत्त है।

Sol. We have maximum BC = $2\sqrt{2}$ and minimum BC = 2

$\therefore OA^2 + OB^2 + BC^2 \in [7, 11]$

Let M be the midpoint of AB.

$\Rightarrow \quad M \equiv \left(\frac{1+\sqrt{2}\cos\theta}{2}, \frac{\sqrt{2}\sin\theta}{2}\right) = (h, k)$



$\therefore$ $\sin\theta = \frac{2k}{\sqrt{2}}$, $\cos\theta = \frac{2h-1}{\sqrt{2}}$

$\therefore$ Now on squaring & adding, we get

$\Rightarrow$ $4k^2 + (2h-1)^2 = 2$

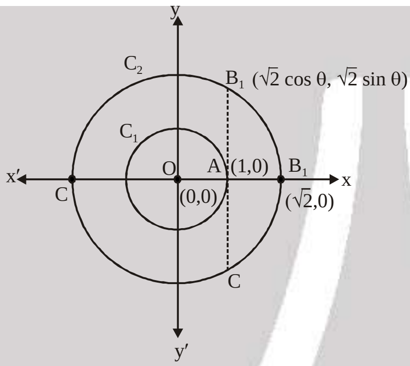
$\therefore$ Locus of M (h, k) is $\left(x - \frac{1}{2}\right)^2 + y^2 = \left(\frac{1}{\sqrt{2}}\right)^2$ **Ans.**

Hindi. अधिकतम BC = $2\sqrt{2}$ और न्यूनतम BC = 2

$\therefore$ $OA^2 + OB^2 + BC^2 \in [7, 11]$

माना M, AB का मध्य बिन्दु है।

$\Rightarrow$ $M \equiv \left(\frac{1+\sqrt{2}\cos\theta}{2}, \frac{\sqrt{2}\sin\theta}{2}\right) = (h, k)$



$\therefore$ $\sin\theta = \frac{2k}{\sqrt{2}}$, $\cos\theta = \frac{2h-1}{\sqrt{2}}$

$\therefore$ वर्ग करके, जोड़ने पर

$\Rightarrow$ $4k^2 + (2h-1)^2 = 2$

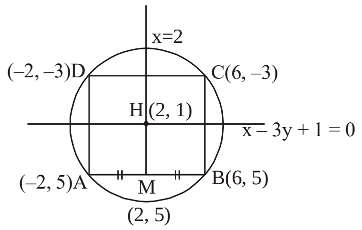
$\therefore$ M (h, k) का बिन्दुपथ $\left(x - \frac{1}{2}\right)^2 + y^2 = \left(\frac{1}{\sqrt{2}}\right)^2$ **Ans.**

11. One of the diameter of the circle circumscribing the rectangle ABCD is $x - 3y + 1 = 0$. If two verticles of rectangle are the points (– 2, 5) and (6, 5) respectively, then which of the following hold(s) good?

(A*) Area of rectangle ABCD is 64 square units.
(B*) Centre of circle is (2, 1)
(C*) The other two vertices of the rectangle are (– 2, – 3) and (6, – 3)
(D) Equation of sides are x = – 2, y = – 3, x = 5 and y = 6.

आयत ABCD के परिगत वृत्त का एक व्यास $x - 3y + 1 = 0$ है।
यदि आयत के दो शीर्ष (– 2, 5) और (6, 5) क्रमशः है तब निम्न में से कौनसा सही है।
(A*) आयत ABCD का क्षेत्रफल 64 वर्ग इकाई है।
(B*) वृत्त का केन्द्र (2, 1) है।
(C*) आयत के अन्य दो शीर्ष (– 2, – 3) और (6, – 3) है।
(D) भुजाओं के समीकरण x = – 2, y = – 3, x = 5 और y = 6. है।

Sol. Area of rectangle = (8)(8) = 64 sq. units.
आयत का क्षेत्रफल = (8)(8) = 64 वर्ग इकाई



Let माना D ((x, y)

$\therefore \quad \frac{x+6}{2} = 2$ and तथा $\frac{x+5}{2} = 1 \quad \therefore$ D (– 2, – 3) इसीप्रकार C(6, – 3).

12. Three concentric circles of which the biggest is $x^2 + y^2 = 1$, have their radii in A.P. If the line y = x + 1 cuts all the circles in real and distinct points. The permissible values of common difference of A.P. is/are

तीन संकेन्द्रीय वृत्त जिसमें सबसे बड़ा $x^2 + y^2 = 1$ है, इनकी त्रिज्यायें समान्तर श्रेढ़ी में है। यदि रेखा y = x + 1 वृत्तों को वास्तविक एवं भिन्न–भिन्न बिन्दुओं पर प्रतिच्छेद करती हो, तो समान्तर श्रेढ़ी के सार्वअन्तर के संभावित मान है–

(A) 0.4 (B) 0.6 (C*) 0.01 (D*) 0.1

Sol. Let 'd' be the common difference

$\therefore$ the radii of the three circles be 1 – 2d, 1 – d, 1

$\therefore$ equation of smallest circle is $x^2 + y^2 = (1 - 2d)^2$ ....(i)

$\therefore$ y = x + 1 intersect (i) at real and distinct points

$\therefore$ $x^2 + x + 2d - 2d^2 = 0$ ....(ii)

$\because$ D > 0 $\Rightarrow$ $8d^2 - 8d + 1 > 0 \Rightarrow$ $d > \frac{2+\sqrt{2}}{4}$ or $d < \frac{2-\sqrt{2}}{2}$

but d can not be greater than $\frac{2+\sqrt{2}}{2}$

$\therefore$ $d \in \left(0, \frac{2-\sqrt{2}}{4}\right)$

Hindi. मानाकि d सार्वअन्तर है।

$\therefore$ तीनों वृत्तों को त्रिज्यायें 1 – 2d, 1 – d, 1 होगी।

$\therefore$ सबसे छोटे वृत्त का समीकरण $x^2 + y^2 = (1 - 2d)^2$ ....(i)

$\therefore$ y = x + 1 रेखा (i) को वास्तविक एवं भिन्न बिन्दुओं पर प्रतिच्छेद करती है।

$\therefore$ $x^2 + x + 2d - 2d^2 = 0$ ....(ii)

$\because$ D > 0 $\Rightarrow$ $8d^2 - 8d + 1 > 0 \Rightarrow$ $d > \frac{2+\sqrt{2}}{4}$ or $d < \frac{2-\sqrt{2}}{2}$

परन्तु d का मान $\frac{2+\sqrt{2}}{2}$ परन्तु d का मान

$\therefore$ $d \in \left(0, \frac{2-\sqrt{2}}{4}\right)$

13. If $4\ell^2 - 5m^2 + 6\ell + 1 = 0$. Prove that $\ell x + my + 1 = 0$ touches a definite circle, then which of the following is/are true.

(A) Centre (0, 3) (B*) centre (3, 0) (C*) Radius $\sqrt{5}$ (D) Radius 5

यदि $4\ell^2 - 5m^2 + 6\ell + 1 = 0$ तो सिद्ध कीजिए कि $\ell x + my + 1 = 0$ एक निश्चित वृत्त को स्पर्श करती है। इस वृत्त का केन्द्र एवं त्रिज्या ज्ञात कीजिए।

(A) केन्द्र (0, 3) (B*) केन्द्र (3, 0) (C*) त्रिज्या $\sqrt{5}$ (D) त्रिज्या 5

Sol. $4\ell^2 - 5m^2 + 6\ell + 1 = 0 \Rightarrow (3\ell + 1)^2 = 5(\ell^2 + m^2) \Rightarrow \left|\frac{3\ell + 0.m + 1}{\sqrt{\ell^2 + m^2}}\right| = \sqrt{5}$

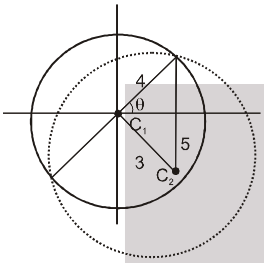
Hence centre (3, 0), radius = $\sqrt{5}$

Hindi $4\ell^2 - 5m^2 + 6\ell + 1 = 0 \Rightarrow (3\ell + 1)^2 = 5(\ell^2 + m^2) \Rightarrow \left|\frac{3\ell + 0.m + 1}{\sqrt{\ell^2 + m^2}}\right| = \sqrt{5}$

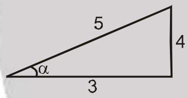
अतः केन्द्र (3, 0), त्रिज्या = $\sqrt{5}$.

14. If the circle C_1: $x^2 + y^2 = 16$ intersects another circle C_2 of radius 5 in such a manner that the common chord is of maximum length and has a slope equal to 3/4, then the co–ordinates of the centre of C_2 are:
यदि वृत्त C_1: $x^2 + y^2 = 16$, 5 इकाई त्रिज्या के दूसरे वृत्त C_2 को इस प्रकार प्रतिच्छेद करता है कि इनकी उभयनिष्ठ जीवा की लम्बाई अधिकतम तथा प्रवणता 3/4 हो, तो C_2 के केन्द्र के निर्देशांक हैं–

(A) $\left(\frac{9}{5}, \frac{12}{5}\right)$ (B*) $\left(\frac{9}{5}, \frac{-12}{5}\right)$ (C) $\left(\frac{-9}{5}, \frac{-12}{5}\right)$ (D*) $\left(\frac{-9}{5}, \frac{+12}{5}\right)$



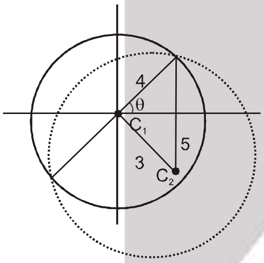
Sol.

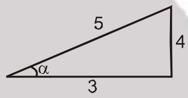


slope of C_1C_2 is $\tan\alpha = -\frac{4}{3}$

By using parametric coordinates C_2 (± 3 cos α , ± 3 sin α)
C_2 (± 3 (–3/5) , ± 3 (4/5)
C_2 (± 9/5 , 12/5)

Hinidi.





C_1C_2 की प्रवणता $\tan\alpha = -\frac{4}{3}$

प्राचलिक निर्देशांक की सहायता से C_2 (± 3 cos α , ± 3 sin α)
C_2 (± 3 (–3/5) , ± 3 (4/5)
C_2 (± 9/5 , 12/5)

15. For the circles $x^2 + y^2 - 10x + 16y + 89 - r^2 = 0$ and $x^2 + y^2 + 6x - 14y + 42 = 0$ which of the following is/are true.
(A*) Number of integral values of r are 14 for which circles are intersecting.
(B) Number of integral values of r are 9 for which circles are intersecting.
(C*) For r equal to 13 number of common tangents are 3.
(D) For r equal to 21 number of common tangents are 2.
वृत्तों $x^2 + y^2 - 10x + 16y + 89 - r^2 = 0$ और $x^2 + y^2 + 6x - 14y + 42 = 0$ के लिए, निम्न में से कौनसा/कौनसे सही है?
(A*) r के पूर्णांक मानों की संख्या 14 है जो वृत्तों को प्रतिच्छेद करती है।
(B) r के पूर्णांक मानों की संख्या 9 है जो वृत्तों को प्रतिच्छेद करती है।
(C*) r =13 के लिए उभयनिष्ठ स्पर्श रेखाओं की संख्या 3 है।
(D) r = 21 के लिए उभयनिष्ठ स्पर्श रेखाओं की संख्या 2 है।

[Sol. We have $(x-5)^2 + (y+8)^2 = 25 + 64 + r^2 - 89$

and $(x+3)^2 + (y-7)^2 = 49 + 9 - 42 = 16$

$\Rightarrow$ $(x-5)^2 + (y+8)^2 = r^2$

and $(x+3)^2 + (y-7)^2 = (4)^2$

$\underline{(5,-8) \qquad (-3,7)}$ $\Rightarrow$ $\sqrt{64+225} = \sqrt{289} = 17$ = distance between their centres

Now, $|r-4| < 17 < r+4$ $\Rightarrow$ $r + 4 > 17$ $\Rightarrow$ $r > 13$

and $-17 < r - 4 < 17$ $\Rightarrow$ $-13 < r < 21$

Hence $13 < r < 21$

$\therefore$ Possible values of 'r' can be 14, 15, 16, 17, 18, 19, 20

Hence required sum = 119

Hindi. यहाँ $(x-5)^2 + (y+8)^2 = 25 + 64 + r^2 - 89$

और $(x+3)^2 + (y-7)^2 = 49 + 9 - 42 = 16$

$\Rightarrow$ $(x-5)^2 + (y+8)^2 = r^2$

और $(x+3)^2 + (y-7)^2 = (4)^2$

$\underline{(5,-8) \qquad (-3,7)}$ $\Rightarrow$ $\sqrt{64+225} = \sqrt{289} = 17$ = केन्द्रों के मध्य दूरी

अब, $|r-4| < 17 < r+4$ $\Rightarrow$ $r + 4 > 17$ $\Rightarrow$ $r > 13$

तथा $-17 < r - 4 < 17$ $\Rightarrow$ $-13 < r < 21$ अतः $13 < r < 21$

$\therefore$ 'r' के संभावित मान हो सकते है 14, 15, 16, 17, 18, 19, 20

अतः अभीष्ट योगफल = 119

16. Which of the following statement(s) is/are correct with respect to the circles $S_1 \equiv x^2 + y^2 - 4 = 0$ and $S_2 \equiv x^2 + y^2 - 2x - 4y + 4 = 0$?

(A*) S_1 and S_2 intersect at an angle of 90°.

(B) The point of intersection of the two circle are (2, 0) and $\left(\frac{6}{5}, \frac{8}{5}\right)$.

(C*) Length of the common of chord of S_1 and S_2 is $\frac{4}{\sqrt{5}}$.

(D*) The point (2, 3) lies outside the circles S_1 and S_2.

वृत्त $S_1 \equiv x^2 + y^2 - 4 = 0$ और $S_2 \equiv x^2 + y^2 - 2x - 4y + 4 = 0$ के सापेक्ष निम्न में से कौनसे कथन सही है ?

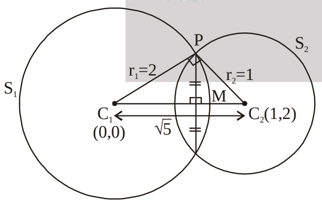
(A*) S_1 और S_2, 90° कोण पर प्रतिच्छेद करते है।

(B) दो वृत्त का प्रतिच्छेदन बिन्दु (2, 0) और $\left(\frac{6}{5}, \frac{8}{5}\right)$ है।

(C*) S_1 तथा S_2 की उभयनिष्ठ जीवा की लम्बाई $\frac{4}{\sqrt{5}}$ है।

(D*) बिन्दु (2, 3) की वृत्त S_1 तथा S_2 के बाहर स्थित है।

[Sol.



Clearly $PC_1^2 + PC_2^2 = (C_1C_2)^2$

$\Rightarrow$ Two circles intersect orthogonally.

Equation of common chord is $S_1 - S_2 = 0$

$\Rightarrow$ $-x + 2y + 4 = 0$

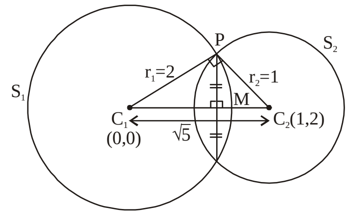
Now $C_1M = \frac{4}{\sqrt{5}}$

$\therefore$ Length of common chord = $2\sqrt{4 - \frac{16}{5}} = \frac{4}{\sqrt{5}}$

Clearly $S_1(2, 3) > 0$ and $S_2(2, 3) > 0$

So, the point (2, 3) lies outside the circles S_1 & S_2.]

Hindi.



स्पष्टतया $PC_1^2 + PC_2^2 = (C_1C_2)^2$

$\Rightarrow$ दोनों वृत्त समकोणीय प्रतिच्छेद करते है।

उभयनिष्ठ जीवा का समीकरण $S_1 - S_2 = 0$

$\Rightarrow \quad -x + 2y + 4 = 0$

अब $\quad C_1M = \dfrac{4}{\sqrt{5}}$

$\therefore$ उभयनिष्ठ जीवा की लम्बाई $2\sqrt{4-\dfrac{16}{5}} = \dfrac{4}{\sqrt{5}}$

स्पष्टतया S_1 (2, 3) > 0 तथा S_2 (2, 3) > 0

इसलिए बिन्दु (2, 3) वृत्त S_1 और S_2 के बाहर है।]

17. Two circles, each of radius 5 units, touch each other at (1, 2). If the equation of their common tangent is $4x + 3y = 10$. The equations of the circles are

5 इकाई त्रिज्या के दो वृत्त एक दूसरे को (1, 2) पर स्पर्श करते हैं। यदि उनकी उभयनिष्ठ स्पर्श रेखा $4x + 3y = 10$ है। वृत्तों के समीकरण है –

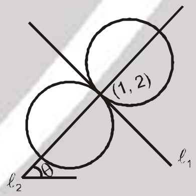
(A*) $x^2 + y^2 + 6x + 2y - 15 = 0$
(B*) $x^2 + y^2 - 10x - 10y + 25 = 0$
(C) $x^2 + y^2 - 6x + 2y - 15 = 0$
(D) $x^2 + y^2 - 10x + 10y + 25 = 0$

Sol. $\ell_1 \equiv 4x + 3y = 10 \quad ; \quad \ell_2 \equiv 3x - 4y = -5$

Let θ be the inclination of ℓ_2

$\therefore \tan\theta = \dfrac{3}{4}$

$\therefore$ equation of ℓ_2 in parametric form $\dfrac{x-1}{4/5} = \dfrac{y-2}{3/5} = \pm 5$



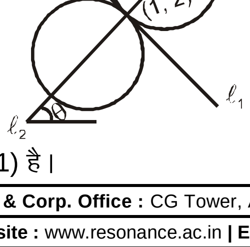
co-ordinates of centres are (5, 5), (–3, –1)

Hindi. $\ell_1 \equiv 4x + 3y = 10 \quad ; \quad \ell_2 \equiv 3x - 4y = -5$

माना सरल रेखा ℓ_2 का झुकाव θ है।

$\therefore \tan\theta = \dfrac{3}{4}$

$\therefore$ प्राचलिक रूप में सरल रेखा ℓ_2 का समीकरण $\dfrac{x-1}{4/5} = \dfrac{y-2}{3/5} = \pm 5$



$\therefore$ केन्द्रों के निर्देशांक (5, 5), (–3, –1) है।

18. $x^2 + y^2 = a^2$ and $(x - 2a)^2 + y^2 = a^2$ are two equal circles touching each other. Find the equation of circle (or circles) of the same radius touching both the circles.

$x^2 + y^2 = a^2$ और $(x - 2a)^2 + y^2 = a^2$ दो बराबर वृत्त एक दूसरे को स्पर्श करते है। समान त्रिज्या का वृत्त (या वृत्तों) के समीकरण ज्ञात कीजिए, जो दोनों वृत्त को स्पर्श करता है

(A) $x^2 + y^2 + 2ax + 2\sqrt{3}\,ay + 3a^2 = 0$ (B*) $x^2 + y^2 - 2ax + 2\sqrt{3}\,ay + 3a^2 = 0$

(C) $x^2 + y^2 + 2ax - 2\sqrt{3}\,ay + 3a^2 = 0$ (D*) $x^2 + y^2 - 2ax - 2\sqrt{3}\,ay + 3a^2 = 0$

Sol. Given circles are

$x^2 + y^2 = a^2$ ....(1)

and $(x - 2a)^2 + y^2 = a^2$ ....(2)

Let A and B be the centres and r_1 and r_2 the radii of the circles (1) and (2) respectively. Then

$A \equiv (0, 0)$, $B \equiv (2a, 0)$, $r_1 = a$, $r_2 = a$

Now $AB = \sqrt{(0-2a)^2 + 0^2} = 2a = r_1 + r_2$

Hence the two circles touch each other extenally.

Let the equation of the circle having same radius 'a' and touching the circles (1) and (2) be

$(x - \alpha)^2 + (y - \beta)^2 = a^2$ ....(3)

Its centre C is (α, β) and radius $r_3 = a$

Since circle (3) touches the circle (1),

$AC = r_1 + r_3 = 2a$. [Here $AC \neq |r_1 - r_3|$ as $r_1 - r_3 = a - a = 0$]

$\Rightarrow$ $AC^2 = 4a^2 \Rightarrow \alpha^2 + \beta^2 = 4a^2$ ....(4)

Again since circle (3) touches the circle (2)

$BC = r_2 + r_3 \Rightarrow BC^2 = (r_2)$

$\Rightarrow$ $(2a - \alpha)^2 + \beta^2 = (a + a)^2 \Rightarrow \alpha^2 + \beta^2 - 4a\,\alpha = 0$ $\Rightarrow$ $4a^2 - 4a\,\alpha = 0$ [from (4)]

$\Rightarrow$ $\alpha = a$ and from (4), we have $\beta = \pm\sqrt{3}\,a$.

Hence, the required circles are

$(x - a)^2 + (y \mp a\sqrt{3})^2 = a^2$

or $x^2 + y^2 - 2ax \mp 2\sqrt{3}\,ay + 3a^2 = 0$.

Hindi. दिये गए वृत्त

$x^2 + y^2 = a^2$ ....(1) और $(x - 2a)^2 + y^2 = a^2$ ....(2)

माना A तथा B वृत्त है तथा r_1 तथा r_2 वृत्तों (1) और (2) की क्रमशः त्रिज्याऐं है तब

$A \equiv (0, 0)$, $B \equiv (2a, 0)$, $r_1 = a$, $r_2 = a$ अब $AB = \sqrt{(0-2a)^2 + 0^2} = 2a = r_1 + r_2$

अतः दो वृत्त एक दूसरे बाह्य स्पर्श करते है।

माना समान त्रिज्या 'a' के वृत्त का समीकरण जो (1) और (2) को स्पर्श करता है।

$(x - \alpha)^2 + (y - \beta)^2 = a^2$ ....(3)

इसका केन्द्र C (α, β) है तथा त्रिज्या $r_3 = a$

चूंकि वृत्त (3) वृत्त की (1) को स्पर्श करता है।

$AC = r_1 + r_3 = 2a$. [यहाँ $AC \neq |r_1 - r_3|$ as $r_1 - r_3 = a - a = 0$]

$\Rightarrow$ $AC^2 = 4a^2 \Rightarrow \alpha^2 + \beta^2 = 4a^2$ ....(4)

पुनः वृत्त (3) वृत्त (2) को स्पर्श करता है।

$BC = r_2 + r_3 \Rightarrow BC^2 = (r_2)$

$\Rightarrow$ $(2a - \alpha)^2 + \beta^2 = (a + a)^2 \Rightarrow \alpha^2 + \beta^2 - 4a\,\alpha = 0$ $\Rightarrow$ $4a^2 - 4a\,\alpha = 0$ [(4) से]

$\Rightarrow$ $\alpha = a$ और (4) से यहाँ $\beta = \pm\sqrt{3}\,a$.

अतः अभीष्ट वृत्त

$(x - a)^2 + (y \pm a\sqrt{3})^2 = a^2$ या $x^2 + y^2 - 2ax \pm 2\sqrt{3}\,ay + 3a^2 = 0$.

19. The circle $x^2 + y^2 - 2x - 3ky - 2 = 0$ passes through two fixed points, (k is the parameter)

वृत्त $x^2 + y^2 - 2x - 3ky - 2 = 0$ जिन दो स्थिर बिन्दुओं से गुजरता है, वह हैं (k एक प्राचल है) –

(A*) $(1 + \sqrt{3}, 0)$ (B) $(-1 + \sqrt{3}, 0)$

(C) $(-\sqrt{3} - 1, 0)$ (D*) $(1 - \sqrt{3}, 0)$

Sol. Two fixed pts. are point of intersection of
$x^2 + y^2 - 2x - 2 = 0$ & $y = 0$
Point $x^2 - 2x - 2 = 0$
$(x-1)^2 - 3 = 0 \Rightarrow x - 1 = \sqrt{3}, \; x - 1 = -\sqrt{3} \Rightarrow (1+\sqrt{3}, 0)\ (1-\sqrt{3}, 0)$

Hindi. दो स्थिर बिन्दु $x^2 + y^2 - 2x - 2 = 0$ व $y = 0$ के प्रतिच्छेदी बिन्दु होगें
बिन्दु $x^2 - 2x - 2 = 0 \Rightarrow (x-1)^2 - 3 = 0$
$\Rightarrow \sqrt{3}\ x - 1 = , \quad x - 1 = -\sqrt{3} \Rightarrow (1+\sqrt{3}, 0)\ (1-\sqrt{3}, 0)$

20. Curves $ax^2 + 2hxy + by^2 - 2gx - 2fy + c = 0$ and $a'x^2 - 2hxy + (a' + a - b)y^2 - 2g'x - 2f'y + c = 0$ intersect at four concyclic point A, B, C and D. If P is the point $\left(\frac{g'+g}{a'+a}, \frac{f'+f}{a'+a}\right)$, then which of the following is/are true

(A) P is also concyclic with points A,B,C,D (B*) PA, PB, PC in G.P.

(C*) $PA^2 + PB^2 + PC^2 = 3PD^2$ (D*) PA, PB, PC in A.P.

वक्र $ax^2 + 2hxy + by^2 - 2gx - 2fy + c = 0$ और $a'x^2 - 2hxy + (a' + a - b)y^2 - 2g'x - 2f'y + c = 0$ चार समवृत्तीय बिन्दुओं A, B, C और D पर प्रतिच्छेद करते है। यदि P एक बिन्दु $\left(\frac{g'+g}{a'+a}, \frac{f'+f}{a'+a}\right)$ तब निम्न में से कौनसा सही है?

(A) P बिन्दु A,B,C,D के साथ समवृत्तीय है। (B*) PA, PB, PC गुणोत्तर श्रेणी में है।

(C*) $PA^2 + PB^2 + PC^2 = 3PD^2$ (D*) PA, PB, PC समान्तर श्रेणी में है।

Sol. Equation of a curve passing through the intersection points of the given curves
$ax^2 + 2hxy + by^2 - 2gx - 2fy + c = 0$ ....(1)
and $a'x^2 - 2hxy + (a' + a - b)y^2 - 2g'x - 2f'y + c = 0$ ....(2)
can be written as $\{a'x^2 - 2hxy + (a' + a - b)y^2 - 2g'x - 2f'y + c\}$
$+ \lambda\{ax^2 + 2hxy + by^2 - 2gx - 2fy + c\} = 0$
i.e. $(a' + \lambda a)x^2 + 2h(\lambda - 1)xy + (a' + a - b + \lambda b)y^2$
$- 2(g' + \lambda g)x - 2(f' + \lambda f)y + (1 + \lambda) c = 0$ ....(3)
According to the given condition equation (3) must represent a circle, therefore, we have
coeff. of x^2 = coeff. of y^2
i.e. $a' + \lambda a = a' + a - b + \lambda b$ i.e. $\lambda(a - b) = a - b$
gives $\lambda = 1$ and coeff. of xy = 0 i.e. $\lambda - 1 = 0$ gives $\lambda = 1$.
The identical values prove that the curve is a circle.
Putting the above value of λ in equation (3) gives the equation of the circle passing through the intersection points of the curves represented by equations (1) and (2) as $(a' + a)(x^2 + y^2) - 2(g' + g)x - 2(f' + f)y + 2c = 0$

which has its centre at the point $\left(\frac{g'+g}{a'+a}, \frac{f'+f}{a'+a}\right)$

We can see that the coordinates of the given point P is the same as the centre of the circle passing through the points A, B, C and D. Therefore, we have $PA^2 = PB^2 = PC^2 = PD^2$ = radius of the circle which gives the desired result $PA^2 + PB^2 + PC^2 = 3PD^2$.

Hindi. वक्र का समीकरण $ax^2 + 2hxy + by^2 - 2gx - 2fy + c = 0$ ....(1)
जो दिए गए वक्रों से गुजरता है।

तथा $a'x^2 - 2hxy + (a' + a - b)y^2 - 2g'x - 2f'y + c = 0$ ....(2)

को लिखा जा सकता है $\{a'x^2 - 2hxy + (a' + a - b)y^2 - 2g'x - 2f'y + c\}$

$+ \lambda\{ax^2 + 2hxy + by^2 - 2gx - 2fy + c\} = 0$

अर्थात् $(a' + \lambda a)x^2 + 2h(\lambda - 1)xy + (a' + a - b + \lambda b)y^2$

$- 2(g' + \lambda g)x - 2(f' + \lambda f)y + (1 + \lambda) c = 0$ ....(3)

दिए गए प्रतिबन्ध (3) के अनुसा एक वृत्त को अवश्य व्यक्त करेगा इसलिए

x^2 का गुणांक = y^2 का गुणांक

अर्थात् $a' + \lambda a = a' + a - b + \lambda b$ अर्थात् $\lambda(a - b) = a - b$

दिया है $\lambda = 1$ अर्थात् $xy = 0$ अर्थात् $\lambda - 1 = 0$ दिया है $\lambda = 1$.

सर्वसम मान से सिद्ध होता है कि वक्र एक वृत्त है।

λ के मान समीकरण (3) में रखने पर (1) व (2) समीकरणों से व्यक्त वक्रों के प्रतिच्छेद बिन्दुओं से गुजरने वाले वृत्त का समीकरण $(a' + a)(x^2 + y^2) - 2(g' + g)x - 2(f' + f)y + 2c = 0$ है।

जिस का केन्द्र $\left(\frac{g'+g}{a'+a}, \frac{f'+f}{a'+a}\right)$ पर है।

हम जानते है कि दिए गए बिन्दु के निर्देशांक, वृत्त जो A, B, C तथा D से गुजरता है, के समान है

इसलिए यहाँ $PA^2 = PB^2 = PC^2 = PD^2$ = वृत्त की त्रिज्या जो अभीष्ट परिणाम $PA^2 + PB^2 + PC^2 = 3PD^2$ देता है।

PART - IV : COMPREHENSION

भाग - IV : अनुच्छेद (COMPREHENSION)

Comprehension # 1 (Q. No. 1 to 3)

Let S_1, S_2, S_3 be the circles $x^2 + y^2 + 3x + 2y + 1 = 0$, $x^2 + y^2 - x + 6y + 5 = 0$ and $x^2 + y^2 + 5x - 8y + 15 = 0$, then

अनुच्छेद # 1 (Q. No. 1 to 3)

मानाकि S_1, S_2, S_3 तीन वृत्त क्रमशः $x^2 + y^2 + 3x + 2y + 1 = 0$, $x^2 + y^2 - x + 6y + 5 = 0$ तथा $x^2 + y^2 + 5x - 8y + 15 = 0$ हो, तो

1. Point from which length of tangents to these three circles is same is

वह बिन्दु जिससे तीनों वृत्तों पर स्पर्श रेखा की लम्बाई बराबर हो, होगा–

(A) (1, 0) (B*) (3, 2) (C) (10, 5) (D) (– 2, 1)

Sol. Point from which length of tangents to these circle is same is radical centre

इन वृत्तों पर बिन्दु से खीचीं गई स्पर्श रेखाओं की लम्बाई समान होती है तब वह बिन्दु मूलाक्ष बिन्दु होता है।

$S_1 - S_2 = 0 \Rightarrow 4x - 4y - 4 = 0 \Rightarrow x - y - 1 = 0$

$S_2 - S_3 = 0 \Rightarrow -6x + 14y - 10 = 0 \Rightarrow -3x + 7y - 5 = 0$

$3x - 3y - 3 = 0$

$4y - 8 = 0 \Rightarrow y = 2$ $x = 3$

2. Equation of circle S_4 which cut orthogonally to all given circle is

वृत्त S_4 जो कि दिये गये वृत्तों को लम्बकोणीय प्रतिच्छेद करता है, का समीकरण है–

(A) $x^2 + y^2 - 6x + 4y - 14 = 0$ (B) $x^2 + y^2 + 6x + 4y - 14 = 0$

(C) $x^2 + y^2 - 6x - 4y + 14 = 0$ (D*) $x^2 + y^2 - 6x - 4y - 14 = 0$

Sol. If circle be drawn taking radical centre as centre and length of tangents from radial centre to any circle as radius will cut all the three circles orthogonally

मूलाक्ष केन्द्र को वृत्त का केन्द्र मानकर और मूलाक्ष केन्द्र से किसी वृत्त पर स्पर्श रेखा की लम्बाई को त्रिज्या मानकर खीचा गया वृत्त इन वृत्तों को लम्बकोणीय प्रतिच्छेद करता है।

Length of tangent स्पर्श रेखा की लम्बाई = $\sqrt{9+4+9+4+1} = \sqrt{S_1} = \sqrt{27}$

Equation of circle वृत्त का समीकरण $(x - 3)^2 + (y - 2)^2 \Rightarrow S_4 : x^2 + y^2 - 6x - 4y - 14 = 0$

3. Radical centre of circles S_1, S_2, & S_4 is
S_1, S_2 एवं S_4 वृत्तों का मूलाक्ष है–

(A*) $\left(-\frac{3}{5},-\frac{8}{5}\right)$ (B) (3, 2) (C) (1, 0) (D) $\left(-\frac{4}{5},-\frac{3}{2}\right)$

Sol. $S_1 - S_2 = 0 \Rightarrow x - y - 1 = 0$
$S_1 - S_4 = 0 \Rightarrow 9x + 6y + 15 = 0 \Rightarrow 3x + 2y + 5 = 0 \Rightarrow 3x - 3y - 3 = 0$
$5y + 8 = 0 \Rightarrow y = -8/5 \quad x = -3/5$

Comprehension # 2 (Q. No. 4 to 6)
अनुच्छेद # 2

Two circles are $S_1 \equiv (x + 3)^2 + y^2 = 9$
$S_2 \equiv (x - 5)^2 + y^2 = 16$
with centres C_1 & C_2
दो वृत्त
$S_1 \equiv (x + 3)^2 + y^2 = 9$
$S_2 \equiv (x - 5)^2 + y^2 = 16$
जिनके केन्द्र C_1 व C_2 है।

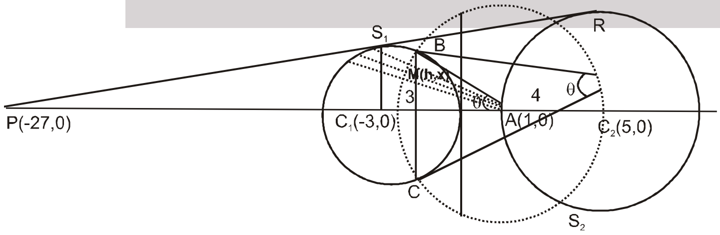
4. A direct common tangent is drawn from a point P (on x-axis) which touches S_1 & S_2 at Q & R, respectively. Find the ratio of area of ΔPQC_1 & ΔPRC_2. **[16JM110533]**
बिन्दु P जो x-अक्ष पर है, से अनुक्रम स्पर्श रेखा जो कि वृत्त S_1 व S_2 को क्रमशः Q व R पर स्पर्श करती है। ΔPQC_1 व ΔPRC_2 के क्षेत्रफलों का अनुपात होगा–
(A) 3 : 4 (B*) 9 : 16 (C) 16 : 9 (D) 4 : 3

5. From point 'A' on S_2 which is nearest to C_1, a variable chord is drawn to S_1. The locus of mid point of the chord.
(A) circle (B) Diameter of s_1 (C*) Arc of a circle (D) chord of s_1 but not diameter
वृत्त S_2 पर C_1 के निकटतम बिन्दु A से एक चर जीवा S_1 पर बनायी जाती है। जीवा के मध्य बिन्दु का बिन्दुपथ होगा–
(A) वृत्त (B) S_1 का व्यास (C*) वृत्त का चाप (D) S_1 की जीवा परन्तु व्यास नहीं

6. Locus obtained in question 5 cuts the circle S_1 at B & C, then line segment BC subtends an angle on the major arc of circle S_1 is
प्रश्न संख्या 7 का बिन्दुपथ, वृत्त S_1 को B एवं C पर काटता हो, तो BC रेखाखंड वृत्त S_1 के दीर्घ चाप पर कोण अंतरित करता है–

(A*) $\cos^{-1}\frac{3}{4}$ (B) $\frac{\pi}{2} - \tan^{-1}\frac{4}{3}$ (C) $\frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{3}{4}$ (D) $\frac{\pi}{2}\cot^{-1}\left(\frac{4}{3}\right)$

Sol. **4 to 6**
4. ΔPQC_1 and ΔPRC_2 are similar



$$\frac{\text{Area of } \Delta PQC_1}{\text{Area of } \Delta PRC_2} = \frac{r_1^2}{r_2^2} = \frac{9}{25}$$

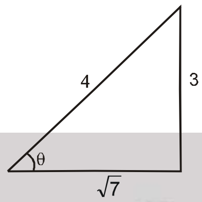
5. Let mid point m(h, k). Now equation of chord
$T = S_1$
$hx + ky + 3(x + h) = h^2 + k^2 + 6h$
it passes through (1, 0)

$h + 3(1 + h) = h^2 + k^2 + 6h$
locus $x^2 + y^2 + 2x - 3 = 0$
But clear from Geometry it will be arc of BC

6. Common chord of S_1 & answer of 5
$4x + 3 = 0 \Rightarrow x = -3/4$

at $x = -3/4$ $\left(-\frac{3}{4}+3\right)^2 + y^2 = 9 \Rightarrow y^2 = 9 - \frac{81}{16}$

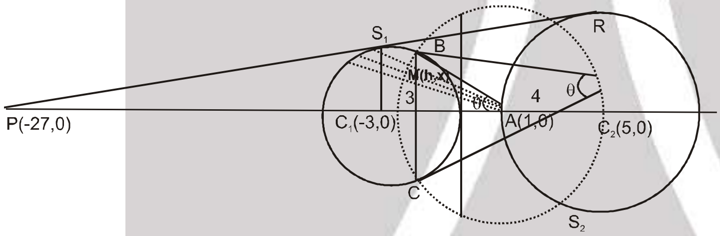
$y^2 = \frac{63}{16} \Rightarrow y = \pm \frac{3\sqrt{7}}{4}$



Hence $\tan\theta = \frac{\frac{3\sqrt{7}}{4}}{(1+3/4)} = \frac{3\sqrt{7}}{7} \Rightarrow \tan\theta = \frac{3}{\sqrt{7}}$

Sol. 4 to 6

4. ΔPQC_1 तथा ΔPRC_2 समरूप त्रिभुज है।



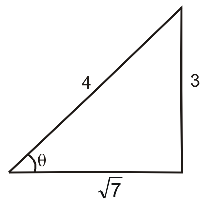
$$\frac{\text{Area of } \Delta PQC_1}{\text{Area of } \Delta PRC_2} = \frac{r_1^2}{r_2^2} = \frac{9}{25}$$

5. माना m(h, k) मध्य बिन्दु है। जीवा का समीकरण
$T = S_1$
$hx + ky + 3(x + h) = h^2 + k^2 + 6h$
यह (1, 0) से गुजरता है।
$h + 3(1 + h) = h^2 + k^2 + 6h$
बिन्दुपथ $x^2 + y^2 + 2x - 3 = 0$
परन्तु ज्यामिति से यह BC का चाप है

6. S_1 की उभयनिष्ठ जीवा औरा 7 का उत्तर
$4x + 3 = 0 \Rightarrow x = -3/4$

अतः $x = -3/4$ $\left(-\frac{3}{4}+3\right)^2 + y^2 = 9 \Rightarrow y^2 = 9 - \frac{81}{16}$

$y^2 = \frac{63}{16} \Rightarrow y = \pm \frac{3\sqrt{7}}{4}$



अतः $\tan\theta = \dfrac{\frac{3\sqrt{7}}{4}}{(1+3/4)} = \dfrac{3\sqrt{7}}{7} \Rightarrow \tan\theta = \dfrac{3}{\sqrt{7}}$

Exercise-3

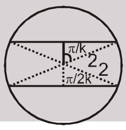
✍ Marked questions are recommended for Revision.
✍ चिन्हित प्रश्न दोहराने योग्य प्रश्न है।
*** Marked Questions may have more than one correct option.**
*** चिन्हित प्रश्न एक से अधिक सही विकल्प वाले प्रश्न है -**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

भाग - I : JEE (ADVANCED) / IIT-JEE (पिछले वर्षो) के प्रश्न

1. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the center, angles of $\dfrac{\pi}{k}$ and $\dfrac{2\pi}{k}$, where k > 0, then the value of [k] is
[**Note** : [k] denotes the largest integer less than or equal to k]
एक वृत्त जिसकी त्रिज्या 2 है, की दो समानान्तर जीवाओं के बीच की दूरी $\sqrt{3}+1$ है। यदि जीवाएँ केन्द्र पर $\dfrac{\pi}{k}$ तथा $\dfrac{2\pi}{k}$, k > 0 के कोण अन्तरित (subtend) करती हैं, तो [k] का मान है।
[IIT-JEE - 2010, Paper-2, (3, 0), 79]
[**नोट** : [k] अधिकतम पूर्णांक जो k से कम या समान है।]
Ans. 3
Sol. Since distance between parallel chords is greater than radius, therefore both chords lie on opposite side of centre.
चूंकि समान्तर जीवाओं के मध्य दूरी त्रिज्या से बडी है इसलिए दोनों जीवाएं केन्द्र के वितरीत और होगी।

$2\cos\dfrac{\pi}{2k} + 2\cos\dfrac{\pi}{k} = \sqrt{3}+1$



Let $\dfrac{\pi}{2k} = \theta$

$\therefore \quad 2\cos\theta + 2\cos 2\theta = \sqrt{3}+1 \quad \Rightarrow \quad 2\cos\theta + 2(2\cos^2\theta - 1) = \sqrt{3}+1$

$\Rightarrow \quad 4\cos^2\theta + 2\cos\theta - (3+\sqrt{3}) = 0$

$\therefore \quad \cos\theta = \dfrac{-2\pm\sqrt{4+16(3+\sqrt{3})}}{2(4)} = \dfrac{-2\pm 2\sqrt{1+12+4\sqrt{3}}}{2(4)} = \dfrac{-1\pm\sqrt{(\sqrt{12}+1)^2}}{4} = \dfrac{-1\pm(2\sqrt{3}+1)}{4}$

$\Rightarrow \quad \cos\dfrac{\pi}{2k} = \dfrac{\sqrt{3}}{2}, \dfrac{-(\sqrt{3}+1)}{2}$ Rejected अस्वीकार्य

$\Rightarrow \quad \dfrac{\pi}{2k} = \dfrac{\pi}{6} \quad \Rightarrow \quad k = 3 \quad \Rightarrow \quad [k] = 3$

2. The circle passing through the point (–1, 0) and touching the y-axis at (0, 2) also passes through the point

बिन्दु (–1, 0) से होकर जाने वाला और y-अक्ष को (0, 2) पर स्पर्श करने वाला वृत्त निम्न बिन्दु से भी होकर जाता है

(A) $\left(-\frac{3}{2}, 0\right)$ (B) $\left(-\frac{5}{2}, 2\right)$ (C) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (D) (–4, 0)

[IIT-JEE 2011, Paper-2, (3, –1), 80]

Ans. **(D)**

Sol. Let equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

as it passes through (-1,0) & (0,2)

$1 - 2g + c = 0$ and $4 + 4f + c = 0$

also $f^2 = c$ $\Rightarrow$ $f = -2,\ c = 4\ ;\ g = \frac{5}{2}$

equation of circle is $x^2 + y^2 + 5x - 4y + 4 = 0$

which passes through (–4, 0)

Hindi वृत्त का समीकरण

$x^2 + y^2 + 2gx + 2fy + c = 0$

(-1,0) & (0,2) से गुजरता है।

$1 - 2g + c = 0$ और $4 + 4f + c = 0$

तथा $f^2 = c$ $\Rightarrow$ $f = -2,\ c = 4\ ;\ g = \frac{5}{2}$

वृत्त का समीकरण $x^2 + y^2 + 5x - 4y + 4 = 0$

जो (–4, 0) से गुजरता है।

3. The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \le 6$ into two parts.

If S = $\left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\}$,

[IIT-JEE 2011, Paper-2, (4, 0), 80]

then the number of point(s) in S lying inside the smaller part is

रेखा $2x - 3y = 1$, वृत्तीय क्षेत्र $x^2 + y^2 \le 6$ को दो भागों में विभाजित करती है। यदि

S = $\left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\}$,

तो S में स्थित उन बिन्दुओं की संख्या जो लघुतर भाग में अन्दर है, निम्न है–

Ans. 2

Sol. $2x - 3y = 1$, $x^2 + y^2 \le 6$

S ≡ $\left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\}$

(I) (II) (III) (IV)

Plot the two curves

I, III, IV will lie inside the circle and point (I, III, IV) will lie on the P region

if (0, 0) and the given point will lie opposite to the line $2x - 3y - 1 = 0$

P(0, 0) = negative, $P\left(2, \frac{3}{4}\right)$ = positive, $P\left(\frac{1}{4}, -\frac{1}{4}\right)$ = positive $P\left(\frac{1}{8}, \frac{1}{4}\right)$ = negative

$P\left(\frac{5}{2}, \frac{3}{4}\right)$ = positive , but it will not lie in the given circle

so point $\left(2, \frac{3}{4}\right)$ and $\left(\frac{1}{4}, -\frac{1}{4}\right)$ will lie on the opp side of the line

so two point $\left(2, \frac{3}{4}\right)$ and $\left(\frac{1}{4}, -\frac{1}{4}\right)$

Further $\left(2, \frac{3}{4}\right)$ and satisfy $S_1\left(\frac{1}{4}, -\frac{1}{4}\right) < 0$

Hindi. $2x - 3y = 1,\ x^2 + y^2 \le 6$

$S \equiv \left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\}$

(I) (II) (III) (IV)

दो वक्रों के आरेख से

I, III, IV वृत्त के अन्दर स्थित है और (I, III, IV) क्षेत्र P पर स्थित है।

यदि (0, 0) और दिया गया बिन्दु रेखा $2x - 3y - 1 = 0$ के विपरीत ओर स्थित होगा।

$P(0, 0)$ = ऋणात्मक, $P\left(2, \frac{3}{4}\right)$ = धनात्मक, $P\left(\frac{1}{4}, -\frac{1}{4}\right)$ = धनात्मक $P\left(\frac{1}{8}, \frac{1}{4}\right)$ = ऋणात्मक

$P\left(\frac{5}{2}, \frac{3}{4}\right)$ = धनात्मक , परन्तु ये दिए गए वृत्त में स्थित नहीं है।

इसलिए बिन्दु $\left(2, \frac{3}{4}\right)$ और $\left(\frac{1}{4}, -\frac{1}{4}\right)$ रेखा के विपरीत ओर स्थित होंगे।

इसलिए दो बिन्दु $\left(2, \frac{3}{4}\right)$ और $\left(\frac{1}{4}, -\frac{1}{4}\right)$ है।

पुनः $\left(2, \frac{3}{4}\right)$ और $\left(\frac{1}{4}, -\frac{1}{4}\right)$ $S_1 < 0$ को सन्तुष्ट करते है

4. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is **[IIT-JEE 2012, Paper-1, (3, –1), 70]**

सरल रेखा $4x - 5y = 20$ के बिन्दुओं से वृत्त $x^2 + y^2 = 9$ पर डाली गयी स्पर्श रेखाओं की स्पर्श जीवा (chord of contact) के मध्य बिन्दु का बिन्दु पथ (locus) निम्न है

(A*) $20(x^2 + y^2) - 36x + 45y = 0$ (B) $20(x^2 + y^2) + 36x - 45y = 0$

(C) $36(x^2 + y^2) - 20x + 45y = 0$ (D) $36(x^2 + y^2) + 20x - 45y = 0$

Sol. **Ans (A)**

Circle $x^2 + y^2 = 9$; line $4x - 5y = 20$

$P\left(t, \frac{4t-20}{5}\right)$

equation of chord AB whose mid point is M (h, k)

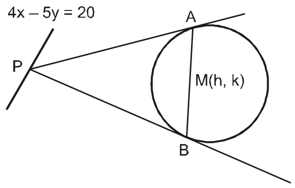
$T = S_1 \Rightarrow hx + ky = h^2 + k^2$ ....(1)

equation of chord of contact AB with respect to P.

$T = 0 \Rightarrow tx + \left(\frac{4t-20}{5}\right)y = 9$ ....(2)

comparing equation (1) and (2)

$$\frac{h}{t} = \frac{5k}{4t-20} = \frac{h^2+k^2}{9}$$



on solving

$45k = 36h - 20h^2 - 20k^2 \Rightarrow$ Locus is $20(x^2 + y^2) - 36x + 45y = 0$

Hindi. वृत्त है: $x^2 + y^2 = 9$; सरल रेखा है: $4x - 5y = 20$

$P\left(t, \frac{4t-20}{5}\right)$

जीवा AB जिसका मध्य बिन्दु M (h, k) है का समीकरण है :

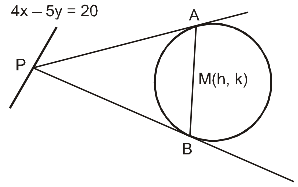
$T = S_1 \Rightarrow \quad hx + ky = h^2 + k^2$ ....(1)

बिन्दु P के सापेक्ष स्पर्श जीवा AB का समीकरण

$T = 0 \Rightarrow \quad tx + \left(\frac{4t-20}{5}\right)y = 9$ ....(2)

समीकरण (1) व (2) की तुलना करने पर

$$\frac{h}{t} = \frac{5k}{4t-20} = \frac{h^2+k^2}{9}$$



हल करने पर

$45k = 36h - 20h^2 - 20k^2 \quad \Rightarrow \quad$ बिन्दुपथ है : $20(x^2 + y^2) - 36x + 45y = 0$

Ans. (A)

Paragraph for Question Nos. 5 to 6

प्रश्न 5 से 6 के लिए अनुच्छेद

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point P($\sqrt{3}$, 1). A straight line L, perpendicular to PT is a tangent to the circle $(x - 3)^2 + y^2 = 1$. **[IIT-JEE 2012, Paper-2, (3, –1), 66]**

स्पर्श–रेखा PT वृत्त $x^2 + y^2 = 4$ को बिन्दु P($\sqrt{3}$, 1) पर स्पर्श करती है। सरल रेखा L, PT के लम्बवत् है और वृत्त $(x - 3)^2 + y^2 = 1$ की स्पर्श–रेखा है।

5. A common tangent of the two circles is

दोनों वृत्तो की एक उभयनिष्ठ स्पर्श–रेखा (common tangent) निम्न है

(A) $x = 4$ (B) $y = 2$ (C) $x + \sqrt{3}y = 4$ (D*) $x + 2\sqrt{3}y = 6$

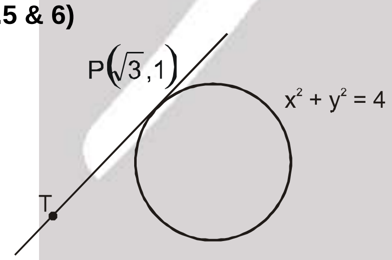
Ans. (D)

6. A possible equation of L is

L का एक सम्भावित समीकरण निम्न है –

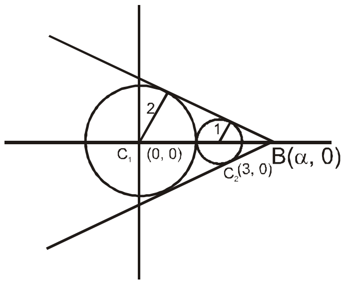
(A*) $x - \sqrt{3}y = 1$ (B) $x + \sqrt{3}y = 1$ (C) $x - \sqrt{3}y = -1$ (D) $x + \sqrt{3}y = 5$

Sol. (Q. No.5 & 6)



Equation of tangent at $(\sqrt{3}, 1) \Rightarrow \sqrt{3}x + y = 4$

5.



B divides C_1C_2 in 2 : 1 externally

$\therefore$ B(6, 0)

Hence let equation of common tangent is

$y - 0 = m(x - 6) \quad \Rightarrow \quad mx - y - 6m = 0$

length of $\perp^r$ dropped from center (0, 0) = radius

$$\left|\frac{6m}{\sqrt{1+m^2}}\right| = 2 \Rightarrow m = \pm\frac{1}{2\sqrt{2}}$$

$\therefore$ equation is $x + 2\sqrt{2}y = 6$ or $x - 2\sqrt{2}y = 6$

6. Equation of L is

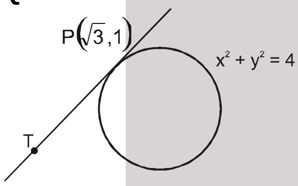
$x - y\sqrt{3} + c = 0$

length of perpendicular dropped from centre = radius of circle

$$\therefore \left|\frac{3+C}{2}\right| = 1 \quad \Rightarrow \quad C = -1, -5$$

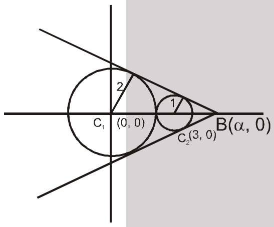
$\therefore x - \sqrt{3}y = 1 \qquad$ or $x - \sqrt{3}y = 5$

Hindi. Q.No. 5 to 6



स्पर्शरेखा का समीकरण $(\sqrt{3}, 1)$ पर $\Rightarrow \sqrt{3}x + y = 4$

5.



B,C_1 C_2 को 2 : 1 में बाह्य विभाजित करता है

$\therefore$ B(6, 0)

अतः उभयनिष्ट स्पर्शरेखा का समीकरण है

$y - 0 = m(x - 6) \quad \Rightarrow \quad mx - y - 6m = 0$

(0, 0) से स्पर्शरेखा पर लम्ब की लम्बाई = त्रिज्या

$$\left|\frac{6m}{\sqrt{1+m^2}}\right| = 2 \Rightarrow m = \pm\frac{1}{2\sqrt{2}}$$

इसलिए समीकरण है $x + 2\sqrt{2}y = 6$ या $x - 2\sqrt{2}y = 6$

6. L का समीकरण है $\qquad x - y\sqrt{3} + c = 0$

वृत्त के केन्द्र से डाले गए लम्ब की लम्बाई = वृत्त की त्रिज्या

$$\therefore \left|\frac{3+C}{2}\right| = 1 \quad \Rightarrow \quad C = -1, -5$$

$\therefore \quad x - \sqrt{3}y = 1$ या $\quad x - \sqrt{3}y = 5$

7.* Circle(s) touching x-axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ on y-axis is (are) **[JEE (Advanced) 2013, Paper-2, (3, –1)/60]**

x-अक्ष को मूलबिन्दु से 3 दूरी पर स्पर्श करने वाला (वाले) तथा y-अक्ष $2\sqrt{7}$ पर अन्तःखण्ड बनाने वाला (वाले) वृत्त है (हैं) **[JEE (Advanced) 2013, Paper-2, (3, –1)/60]**

(A*) $x^2 + y^2 - 6x + 8y + 9 = 0$ (B) $x^2 + y^2 - 6x + 7y + 9 = 0$

(C*) $x^2 + y^2 - 6x - 8y + 9 = 0$ (D) $x^2 + y^2 - 6x - 7y + 9 = 0$

Sol. **(AC)**

Let माना $x^2 + y^2 + 2gx + 2fy + c = 0$ $\Rightarrow$ $g^2 - c = 0$ $\Rightarrow$ $g^2 = c$ ...(i)

$2\sqrt{f^2 - c} = 2\sqrt{7}$ $\Rightarrow$ $f^2 - c = 7$ ...(ii)

$9 + 0 + 6g + 0 + c = 0$ $\Rightarrow$ $9 + 6g + g^2 = 0$ $\Rightarrow$ $(g + 3)^2 = 0$

$g = -3$ $\therefore$ $c = 9$

$f^2 = 16$ $\Rightarrow$ $f = \pm 4$

$\therefore$ $x^2 + y^2 - 6x \pm 8y + 9 = 0$

8.* A circle S passes through the point (0, 1) and is orthogonal to the circles $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then **[JEE (Advanced) 2014, Paper-1, (3, 0)/60]**

(A) radius of S is 8
(B*) radius of S is 7
(C*) centre of S is (–7, 1)
(D) centre of S is (–8, 1)

एक वृत्त S बिन्दु (0, 1) से गुजरता है तथा वृत्तों $(x - 1)^2 + y^2 = 16$ एवं $x^2 + y^2 = 1$ के लम्बकोणीय (orthogonal) है, तब

(A) S की त्रिज्या (radius) 8 है।
(B*) S की त्रिज्या 7 है।
(C*) S का केन्द्र (–7, 1) है।
(D) S का केन्द्र (–8, 1) है।

Ans. **(BC)**

Sol. Let the cirlce be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(1)

given circles

$x^2 + y^2 - 2x - 15 = 0$ ...(2)

$x^2 + y^2 - 1 = 0$ ....(3)

(1) & (2) are orthogonal

$\Rightarrow$ $-g + 0 = \frac{c - 15}{2}$

$0 + 0 = \frac{c - 1}{2}$

$\Rightarrow$ $c = 1$ & $g = 7$

so the cirle is

$x^2 + y^2 + 14x + 2fy + 1 = 0$ it passes thrgouh

(0, 1) $\Rightarrow$ $0 + 1 + 0 + 2f + 1 = 0$

$f = -1$

$\Rightarrow$ $x^2 + y^2 + 14x - 2y + 1 = 0$

Centre (–7, 1)

radius = 7

Hindi. माना कि वृत्त $x^2 + y^2 + 2gx + 2fy + c = 0$ ...(1)

दिये गये वृत्त $x^2 + y^2 - 2x - 15 = 0$ ...(2)

$x^2 + y^2 - 1 = 0$ ....(3)

(1) और (2) लाम्बिक हैं।

$\Rightarrow$ $-g + 0 = \frac{c - 15}{2}$

$0 + 0 = \frac{c - 1}{2}$

$\Rightarrow$ $c = 1$ & $g = 7$

इसलिए वृत्त है।

$x^2 + y^2 + 14x + 2fy + 1 = 0$ यह गुजरता है।

(0, 1)

$\Rightarrow$ $0 + 1 + 0 + 2f + 1 = 0$

$\Rightarrow$ $f = -1$ $\Rightarrow$ $x^2 + y^2 + 14x - 2y + 1 = 0$

केन्द्र (–7, 1) त्रिज्या = 7

9. The circle $C_1 : x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the y-axis, then **[JEE (Advanced) 2016, Paper-1, (4, –2)/62]**

(A) $Q_2Q_3 = 12$ (B) $R_2R_3 = 4\sqrt{6}$

(C) area of the triangle OR_2R_3 is $6\sqrt{2}$ (D) area of the triangle PQ_2Q_3 is $4\sqrt{2}$

वत्त $C_1 : x^2 + y^2 = 3$, जिसका केन्द्रबिन्दु O है, परवलय (parabola) $x^2 = 2y$ को प्रथम चतुर्थांा (first quadrant) में बिन्दु P पर प्रतिच्छेदित (intersect) करता है। माना कि वत्त C_1 के बिन्दु P पर खींची गई स्पर्शरेखा (tangent) अन्य दो वत्तों C_2 और C_3 को क्रमाः बिन्दुओं R_2 तथा R_3 पर स्पर्ा करती हैं। मान लीजिये कि C_2 तथा C_3 दोनों की त्रिज्याए 2के बराबर है और उनके केन्द्र बिन्दु क्रमाः Q_2 तथा Q_3 हैं। यदि Q_2 तथा Q_3 y-अक्ष पर स्थित है, तब

(A) $Q_2Q_3 = 12$ (B) $R_2R_3 = 4\sqrt{6}$

(C) त्रिभुज OR_2R_3 का क्षेत्रफल $6\sqrt{2}$ है (D) त्रिभुज PQ_2Q_3 का क्षेत्रफल $4\sqrt{2}$ है

Ans. **(A,B,C)**

Sol. $y^2 + 2y - 3 = 0$

$y = 1\ ,\ y = -3$

$p\left(\sqrt{2}, -1\right)$

tangent is $x\sqrt{2} + y = 3$

$C_2(0,\alpha)\ \perp$ distance = $2\sqrt{3}$

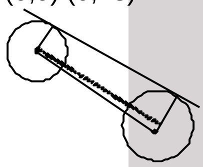
$\dfrac{|\alpha - 3|}{3} = 2\sqrt{3}$

$\alpha - 3 = \pm 6$

$\alpha = 3, \pm 6$

$\alpha = 9, -3$

(0,9) (0,–3)



$L_{DCT} = \sqrt{(C_2C_1)^2 - (R+r)^2} = \sqrt{144 - 16\times 3} = 4\sqrt{6}$

(C) A = $\frac{1}{2}\ R_3R_2 \times \perp$ from (0,0) = $2\sqrt{6} \times \frac{3}{\sqrt{3}} = 6\sqrt{2}$

(D) Area = $\begin{vmatrix} 0 & -3 & 1 \\ 0 & 9 & 1 \\ \sqrt{2} & 1 & 1 \end{vmatrix} = 6\sqrt{2}$

Area of ΔPQ_2Q_3 का क्षेत्रफल = $\frac{1}{2}\begin{vmatrix} \sqrt{2} & 1 & 1 \\ 0 & 9 & 1 \\ 0 & -3 & 1 \end{vmatrix} = \left|\frac{1}{2}\sqrt{2}(9+3)\right| = 6\sqrt{2}$

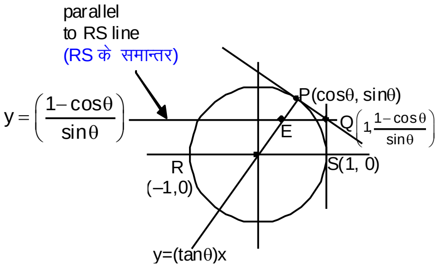
10*. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point (1, 0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s) **[JEE (Advanced) 2016, Paper-1, (4, –2)/62]**

माना कि RS वत्त $x^2 + y^2 = 1$ का व्यास (diameter) है, जहाँ कि S बिन्दु (1, 0) है। माना कि P (R और S से भिन्न) वत्त पर एक चर (variable) बिन्दु है और वत्त पर बिन्दुओं S और P पर खींची गई स्पर्शरेखाएँ (tangents) बिन्दु Q पर मिलती है। वत्त के बिन्दु P पर अभिलम्ब (normal) उस रेखा को, जो Q से गुजरती है तथा RS के समानान्तर (parallel) है, बिन्दु E पर प्रतिच्छेदित करता है। तब E का बिन्दुपथ (locus) निम्न बिन्दु(ओं) से गुजरता है–

(A) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$ (B) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (C) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$ (D) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

Ans. **(A,C)**

Sol.



$$E\left(\left(\frac{1-\cos\theta}{\sin\theta\tan\theta}\right), \left(\frac{1-\cos\theta}{\sin\theta}\right)\right) \Rightarrow E\left(\frac{\tan\frac{\theta}{2}}{\tan\theta}, \tan\frac{\theta}{2}\right)$$

Let माना $h = \frac{\tan\frac{\theta}{2}}{\tan\theta}$ and और $k = \tan\frac{\theta}{2}$ $\therefore h = \frac{k}{\tan\theta}$ $\therefore \tan\frac{\theta}{2} = \frac{k}{h}$

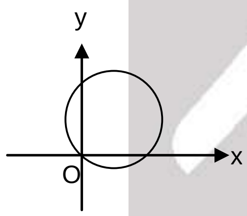
$$\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} = \frac{k}{h} \Rightarrow \left(\frac{2k}{1-k^2}\right) = \frac{k}{h} \quad \therefore \quad 2xy = y(1-y^2)$$

11. For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points? **[JEE(Advanced) 2017, Paper-1,(3, 0)/61]**

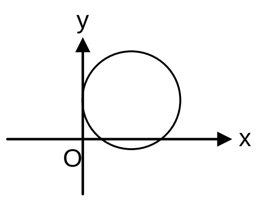
p के कितने मानों के लिये वृत्त (circle) $x^2 + y^2 + 2x + 4y - p = 0$ एवम् निर्देशांक अक्षों (coordinate axes) में केवल तीन बिन्दु उभयनिष्ठ (common) है?

Ans. **(2)**

Sol. **Case-I** Passing through origin $\Rightarrow p = 0$



Cass-II Touches y-axis and cuts x-axis



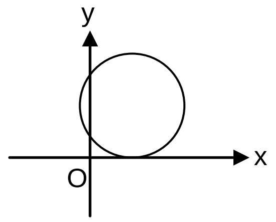
$f^2 - c = 0$ & $g^2 - c > 0$

$4 + p = 0$ $1 + p > 0$

$p = -4$

Not possible

Case-III Touches x-axis and cuts y-axis

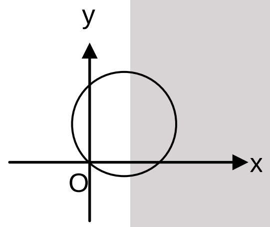


$f^2 - c > 0$ & $g^2 - c = 0$

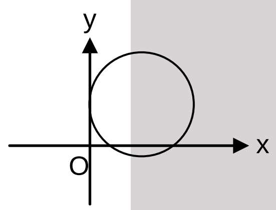
$4 + p > 0 \qquad 1 + p = 0$

So two value of p are possible

Hindi **Case-I** मूल बिन्दु से गुजरता है $\Rightarrow P = 0$



Cass-II y- अक्ष को स्पष्ट करता है, तथा x- अक्ष को काटता है।



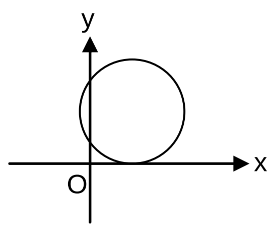
$f^2 - c = 0$ & $g^2 - c > 0$

$4 + p = 0 \qquad 1 + p > 0$

$p = -4$

संभव नही

Case-III x- अक्ष को स्पष्ट करता है, तथा y- अक्ष को काटता है।



$f^2 - c > 0$ & $g^2 - c = 0$

$4 + p > 0 \qquad 1 + p = 0$

अतः p के दो मान सम्भव है।

PARAGRAPH "X" अनुच्छेद "X" **[JEE(Advanced) 2018, Paper-1, (3, –1)/60]**

Let S be the circle in the xy-plane defined by the equation $x^2 + y^2 = 4$.

(***There are two questions based on PARAGRAPH "X", the question given below is one of them***)

मानकि S एक वृत्त (circle) है जो xy-समतल (plane) में समीकरण (equation) $x^2 + y^2 = 4$ के द्वारा परिभाषित है।

(अनुच्छेद "X" पर दो प्रशन आधारित है, नीचे दिया गया प्रश्न उनमें से एक है)

12. Let E_1E_2 and F_1F_2 be the chords of S passing through the point $P_0(1, 1)$ and parallel to the x-axis and the y-axis, respectively. Let G_1G_2 be the chord of S passing through P_0 and having slope –1. Let the tangents to S at E_1 and E_2 meet at E_3, the tangents to S at F_1 and F_2 meet at F_3, and the tangents to S at G_1 and G_2 meet at G_3. Then, then, the points E_3, F_3, and G_3 lie on the curve

माना कि E_1E_2 और F_1F_2 वृत्त S की ऐसी जीवाएं (chords) है जो बिन्दु $P_0(1, 1)$ से गुजरती है और क्रमशः x-अक्ष (axis) व y-अक्ष के समान्तर (parallel) है। मानाकि G_1G_2 , S की वह जीवा है जो P_0 से गुजरती है और जिसकी प्रवणता (slope) –1 है। मानाकि E_1 और E_2 पर S की स्पर्शियां (tangents) E_3 पर मिलती है, F_1 और F_2 पर S की स्पर्शियां F_3 पर मिलती है, तथा G_1 और G_2 पर S की स्पर्शियां G_3 पर मिलती है। तब वह वक्र (curve) जिस पर बिन्दु E_3, F_3 और G_3 स्थित है, हैं

(A) $x + y = 4$ (B) $(x – 4)^2 + (y – 4)^2 = 16$ (C) $(x – 4)(y – 4) = 4$ (D) $xy = 4$

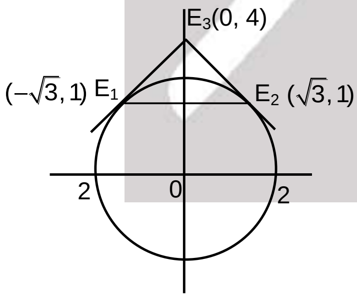
Ans. **(A)**

Sol. Tangent at E_1 and E_2 are $-\sqrt{3}x + y = 4$ and $\sqrt{3}x + y = 4$

E_1 और E_2 पर स्पर्श रेखाएं $-\sqrt{3}x + y = 4$ और $\sqrt{3}x + y = 4$

They intersect at E_3 (0, 4)

वे E_3 , (0, 4) पर मिलती है।



$F_1 (1, \sqrt{3})$, $F_2 (1, -\sqrt{3})$, $F_3 (4, 0)$

$G_1(0, 2)$, $G_2(2, 0)$, $G_3(2, 2)$

E_3, F_3, G_3 lie on line $x + y = 4$

E_3, F_3, G_3 रेखा $x + y = 4$ पर स्थित है।

13. Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve

मानाकि P वृत्त S पर स्थित एक ऐसा बिन्दु है जिसके दोनों निर्देशांक (coordinates) धनात्मक (positive) है। मानाकि वृत्त S के बिन्दु P पर स्पर्शी (tangent) निर्देशांक अक्षों (coordinate axes) को बिन्दुओं M और N पर प्रतिच्छेद (intersects) करती है। तब वह वक्र (curve) जिस पर रेखाखण्ड (line segment) MN का मध्य बिन्दु (mid-point) अनिवार्य रूप से स्थित है, हैं

(A) $(x + y)^2 = 3xy$ (B) $x^{2/3} + y^{2/3} = 2^{4/3}$
(C) $x^2 + y^2 = 2xy$ (D) $x^2 + y^2 = x^2y^2$

Ans. **(D)**

Sol. Let माना $P(2 \cos \theta, 2 \sin \theta)$

Tanget is $x \cos \theta + y \sin \theta = 2$

$M\left(\frac{2}{\cos\theta}, 0\right)$, $N\left(0, \frac{2}{\cos\theta}\right)$

$x = \frac{1}{\cos\theta}$ and और $y = \frac{1}{\sin\theta}$ $\Rightarrow$ $\frac{1}{x^2} + \frac{1}{y^2} = 1$ $\Rightarrow$ $x^2 + y^2 = x^2y^2$

14*. Let T be the line passing through the points P(–2, 7) and Q(2, –5). Let F_1 be the set of all pairs of circles (S_1, S_2) such that T is tangent to S_1 at P and tangent to S_2 at Q, and also such that S_1 and S_2 touch each other at a point, say, M. Let E_1 be the set representing the locus of M as the pair (S_1, S_2) varies in F_1. Let the set of all straight line segments joining a pair of distinct points of E_1 and passing through the point R(1, 1) be F_2. Let E_2 be the set of the mid-points of the line segments in the set F_2. Then, which of the following statement(s) is (are) TRUE **[JEE(Advanced) 2018, Paper-2, (4, –2)/60]**

(A) The point (–2, 7) lies in E_1

(B) The point $\left(\frac{4}{5}, \frac{7}{5}\right)$ does **NOT** lie in E_2

(C) The point $\left(\frac{1}{2}, 1\right)$ lies in E_2

(D) The point $\left(0, \frac{3}{2}\right)$ does **NOT** lie in E_1

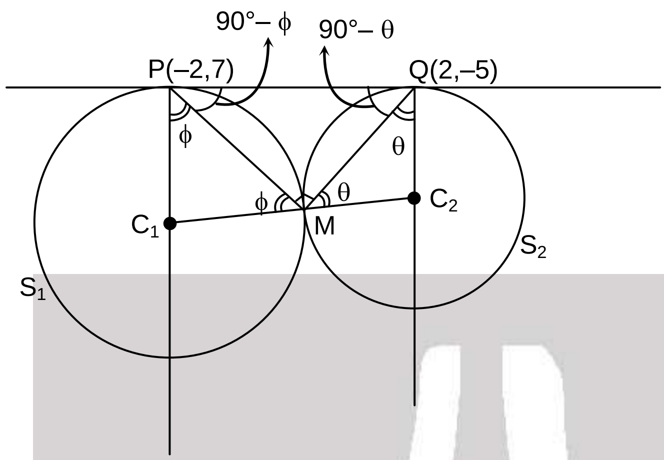
माना कि T, बिन्दुओं P(–2, 7) और Q(2, –5) से गुजरने वाली रेखा (line) है। माना कि F_1 उन सभी वृत्त युग्मो (pairs of circles) (S_1, S_2) का समुच्चय (set) है कि रेखा T, S_1 के बिन्दु P पर और S_2 के बिन्दु Q पर स्पर्शी (tangent) है तथा वृत्त S_1 व S_2 एक दूसरे को बिन्दु, माना कि M, पर स्पर्श करते हैं। जब युग्म (S_1, S_2), F_1 में विचरित (varies) करता है तो माना कि समुच्चय (set) E_1, बिन्दु M के बिन्दुपथ (locus) को दर्शाता है। माना कि F_2 उन सरल रेखा-खण्डो (straight line segments) का समुच्चय है, जो बिन्दु R(1, 1) से गुजरती है तथा E_1 के दो भिन्न बिन्दुओं के युग्म (pair of distinct points) को जोडती हैं माना कि E_2, समुच्चय F_2 के रेखाखण्डों के मध्य बिन्दुओं का समुच्चय है। तब निम्नलिखित में से कौन सा (से) कथन सत्य है (है) ?

(A) बिन्दु (–2, 7) समुच्चय E_1 में स्थित है। (B) बिन्दु $\left(\frac{4}{5}, \frac{7}{5}\right)$ समुच्चय E_2 में स्थित **नही** है।

(C) बिन्दु $\left(\frac{1}{2},1\right)$ समुच्चय E_2 में स्थित है। (D) बिन्दु $\left(0,\frac{3}{2}\right)$ समुच्चय E_1 में स्थित **नही** है।

Ans. (BD)

Sol.



Let C_1 and C_2 be the centre of circle S_1 and S_2 respectively

माना C_1 और C_2 क्रमशः वृत्त S_1 और S_2 के केन्द्र है।

Let माना $\angle C_2QM = \angle C_2MQ = \theta \Rightarrow \angle QC_2M = \pi - 2\theta$

Let माना $\angle C_1PM = \angle C_1MP = \phi \Rightarrow \angle PC_1M = \pi - 2\phi$

Now अब $\angle QC_2M + \angle PC_1M = \pi \Rightarrow \pi - 2\theta + \pi - 2\phi = \pi \Rightarrow \theta + \phi = \pi/2$

Now अब $\angle QMP = \pi - \angle QMC_2 - \angle PMC_1 = \pi - (\theta + \phi) = \pi - \pi/2 = \pi/2$

hence locus equation of variable point M is $(x + 2)(x - 2) + (y - 7)(y + 5) = 0$

अतः चर बिन्दु M का बिन्दुपथ समीकरण $(x + 2)(x - 2) + (y - 7)(y + 5) = 0$

but locus of M does not contains point P and Q because P is included when radius of S_1 is zero and circle S_2 becomes straight line which is impossible. Q is included when radius of S_2 is zero and circle S_1 becomes straight line which is also impossible.

परन्तु M का बिन्दुपथ P और Q को नहीं रखता है क्योंकि P शामिल होगा जब S_1 की त्रिज्या शून्य है और वृत्त S_2 एक सरल रेखा हो जाता है जो असंभव है। Q शामिल होगा जब S_2 की त्रिज्या शून्य है और वृत्त S_1 एक सरल रेखा हो जाता है जो असंभव है।

so set E_1 does not contain point P(–2, 7) and Q(2, – 5)

इसलिए समुच्चय E_1 बिन्दु P(–2, 7) और Q(2, – 5) को नहीं रखता है।

Locus of mid-points of chords passing through (1, 1) is $h + K - (1 + k) = h^2 + k^2 - 2K$

(1, 1) से गुजरने वाली जीवाओं के मध्य बिन्दु का बिन्दुपथ $h + K - (1 + k) = h^2 + k^2 - 2K$

$\Rightarrow h^2 + K^2 - 2K - h + 1 = 0 \Rightarrow x^2 + y^2 - x - 2y + 1 = 0$

Now equation of line passing through P(–2, 7) and R(1, 1) is $\frac{y-1}{x-1} = \frac{6}{-3} \Rightarrow y + 2x - 3 = 0$

अब रेखा का समीकरण जो बिन्दु P(–2, 7) और R(1, 1) से गुजरती है $\frac{y-1}{x-1} = \frac{6}{-3} \Rightarrow y + 2x - 3 = 0$

Let centre of $x^2 + y^2 - 2y - 39 = 0$ is C_3 (0, 1) $\Rightarrow$ centre of locus of M is C_3 (0, 1)

माना वृत्त $x^2 + y^2 - 2y - 39 = 0$ का केन्द्र C_3 (0, 1) है। $\Rightarrow$ बिन्दु M के बिन्दुपथ का केन्द्र C_3 (0, 1) है।

Now foot of $C_3(0, 1)$ on line $y + 2x - 3 = 0$ is $\left(\frac{4}{5}, \frac{7}{5}\right)$. which is mid-point of chord PR of circle $x^2 + y^2 - 2y - 39 = 0$

अब $C_3(0, 1)$ का लम्बपाद $\left(\frac{4}{5}, \frac{7}{5}\right)$ रेखा $y + 2x - 3 = 0$ पर स्थित हैं जो वृत्त $x^2 + y^2 - 2y - 39 = 0$ की जीवा PR का मध्य बिन्दु है।

But if P is not the part of locus of M then PQ is not the chord of locus of M.

परन्तु यदि P, M के बिन्दुपथ का भाग नहीं है, तब PQ, बिन्दु M के बिन्दुपथ की जीवा नहीं है।

So point $\left(\frac{4}{5}, \frac{7}{5}\right)$ does not lies in set E_2

इसलिए बिन्दु $\left(\frac{4}{5}, \frac{7}{5}\right)$ समुच्चय E_2 में स्थित नहीं है।

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

भाग - II : JEE (MAIN) / AIEEE (पिछले वर्षो) के प्रश्न

1. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if **[AIEEE 2010, (4, –1), 144]**

वृत्त $x^2 + y^2 = 4x + 8y + 5$ रेखा $3x - 4y = m$ को दो भिन्न बिन्दुओं पर प्रतिच्छेद करेगा यदि

(1*) $-35 < m < 15$ (2) $15 < m < 65$ (3) $35 < m < 85$ (4) $-85 < m < -35$

Ans. **(1)**

Sol. $r = \sqrt{4+16+5} = 5 \Rightarrow \left|\frac{6-16-m}{5}\right| < 5 \Rightarrow -25 < m + 10 < 25 \Rightarrow -35 < m < 15$

Hence correct option is (1) अतः सही विकल्प (1) है।

2. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2 (c > 0)$ touch each other if : **[AIEEE-2011, I, (4, –1), 120]**

दो वृत्त $x^2 + y^2 = ax$ तथा $x^2 + y^2 = c^2(c > 0)$ स्पर्श करते हैं यदि

(1) $2|a| = c$ (2*) $|a| = c$ (3) $a = 2c$ (4) $|a| = 2c$

Sol. **(2)**

$x^2 + y^2 = ax$ ....(1)

$\Rightarrow$ centre $c_1\left(-\frac{a}{2}, 0\right)$ and radius $r_1 = \left|\frac{a}{2}\right|$

$x^2 + y^2 = c^2$ ....(2)

$\Rightarrow$ centre c_2 (0, 0) and radius $r_2 = c$, both touch each other iff

$|c_1c_2| = r_1 \pm r_2 \quad \Rightarrow \frac{a^2}{4} = \left(\pm\frac{a}{2} \pm c\right)^2 \quad \Rightarrow \frac{a^2}{4} = \frac{a^2}{4} \pm |a|\,c + c^2 \quad \Rightarrow \quad |a| = c$

Hindi $x^2 + y^2 = ax$ ....(1)

$\Rightarrow$ केन्द्र $c_1\left(-\frac{a}{2}, 0\right)$ तथा त्रिज्या $r_1 = \left|\frac{a}{2}\right|$

$x^2 + y^2 = c^2$ ....(2)

$\Rightarrow$ केन्द्र c_2 (0, 0) तथा त्रिज्या $r_2 = c$

दोनों एक दूसरे को स्पर्श करेगें यदि और केवल यदि

$|c_1c_2| = r_1 \pm r_2 \quad \Rightarrow \frac{a^2}{4} = \left(\pm\frac{a}{2} \pm c\right)^2 \quad \Rightarrow \frac{a^2}{4} = \frac{a^2}{4} \pm |a|\,c + c^2 \quad \Rightarrow \quad |a| = c$

3. The equation of the circle passing through the point (1, 0) and (0, 1) and having the smallest radius is -

उस वृत्त का समीकरण जो बिन्दुओं (1, 0) तथा (0, 1) से होकर जाता है तथा जिसकी त्रिज्या न्यूनतम है, है :

(1) $x^2 + y^2 - 2x - 2y + 1 = 0$ (2*) $x^2 + y^2 - x - y = 0$ **[AIEEE-2011, II, (4, –1), 120]**

(3) $x^2 + y^2 + 2x + 2y - 7 = 0$ (4) $x^2 + y^2 + x + y - 2 = 0$

Sol. **(2)**

Circle whose diametric end points are (1, 0) and (0, 1) will be of smallest radius.

वह वृत्त जिसके व्यास के सिरे (1, 0) तथा (0, 1) है, न्यूनतम त्रिज्या वाला वृत्त होगा।

$(x - 1)(x - 0) + (y - 0)(y - 1) = 0 \quad \Rightarrow \quad x^2 + y^2 - x - y = 0$

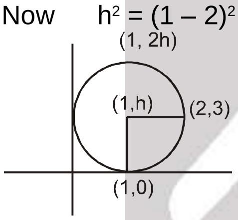
4. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is : **[AIEEE- 2012, (4, –1), 120]**

एक वृत्त जो x-अक्ष को बिंदु (1, 0) पर स्पर्श करता है तथा बिंदु (2, 3) से होकर जाता है, के व्यास की लंबाई है :

(1*) $\frac{10}{3}$ (2) $\frac{3}{5}$ (3) $\frac{6}{5}$ (4) $\frac{5}{3}$

Sol. **Ans. (1)**

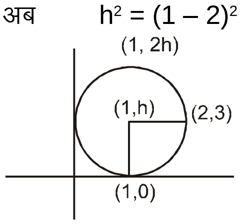
Now $h^2 = (1 - 2)^2 + (h - 3)^2 \quad \Rightarrow \quad 0 = 1 - 6h + 9$



$6h = 10 \quad \Rightarrow \quad h = \frac{5}{3}$

Now diameter is $2h = \frac{10}{3}$

Hindi. अब $h^2 = (1 - 2)^2 + (h - 3)^2 \quad \Rightarrow \quad 0 = 1 - 6h + 9$



$6h = 10 \quad \Rightarrow \quad h = \frac{5}{3}$

अब व्यास $2h = \frac{10}{3}$ है।

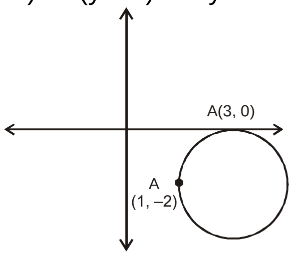
5. The circle passing through (1, –2) and touching the axis of x at (3, 0) also passes through the point **[AIEEE - 2013, (4, –1),120]**

एक वृत्त जो (1, –2) से होकर जाता है, तथा x-अक्ष को (3, 0) पर स्पर्श करता है, जिस अन्य बिन्दु से होकर जाता है, वह है– **[AIEEE - 2013, (4, –1),120]**

(1) (–5, 2) (2) (2, – 5) (3*) (5, – 2) (4) (–2, 5)

Sol. (3)

Let the equation of circle be $(x - 3)^2 + (y - 0)^2 + \lambda y = 0$



As it passes through (1, – 2)

$\therefore$ $(1 - 3)^2 + (-2)^2 + \lambda(-2) = 0 \Rightarrow \lambda = 4$

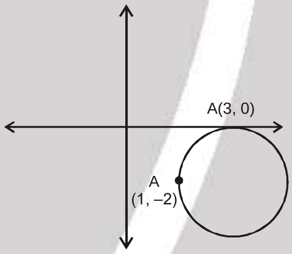
$\therefore$ equation of circle is

$(x - 3)^2 + y^2 - 8 = 0$

so (5, – 2) satisfies equation of circle

Hindi. (3)

माना वृत्त का समीकरण $(x - 3)^2 + (y - 0)^2 + \lambda y = 0$



दिया है कि यह (1, – 2) से गुजरता है

$\therefore$ $(1 - 3)^2 + (-2)^2 + \lambda(-2) = 0 \Rightarrow \lambda = 4$

$\therefore$ वृत्त का समीकरण है

$(x - 3)^2 + y^2 - 8 = 0$

(5, – 2) वृत्त के समीकरण को संतुष्ट करता है।

6. Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to :

माना C एक वृत्त है जिसका केंद्र (1, 1) पर है तथा त्रिज्या = 1 है। यदि T केंद्र (0, y) वाला वृत्त हैं जो मूल बिन्दु से हो कर जाता है तथा वृत्त C को बाह्य रूप से स्पर्श करता है तो T की त्रिज्या बराबर है : **[JEE(Main) 2014, (4, – 1), 120]**

(1) $\frac{1}{2}$ (2*) $\frac{1}{4}$ (3) $\frac{\sqrt{3}}{\sqrt{2}}$ (4) $\frac{\sqrt{3}}{2}$

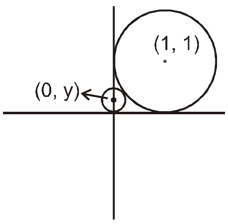
Sol. **Ans. (2)**

c_1 (1, 1) $r_1 = 1$

c_2 (0, y) $r_2 = |y|$

$c_1c_2 = r_1 + r_2$

$\sqrt{(1-0)^2 + (1-y)^2} = 1 + |y|$



$2 - 2y + y^2 = y^2 + 2|y| + 1$

$4|y| = 1$

$|y| = \frac{1}{4}$

$y = \frac{1}{4}$

7. Locus of the image of the point (2, 3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \in R$, is a

[JEE(Main) 2015, (4, – 1), 120]

(1) straight line parallel to x-axis (2) straight line parallel to y-axis

(3) circle of radius $\sqrt{2}$ (4) circle of radius $\sqrt{3}$

बिन्दु (2, 3) के रेखा $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \in R$ में प्रतिबिम्ब का बिन्दुपथ एक –

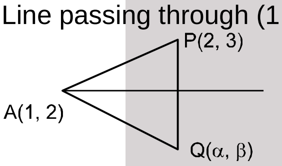
[JEE(Main) 2015, (4, – 1), 120]

(1) x-अक्ष के समान्तर रेखा है। (2) y-अक्ष के समान्तर रेखा है।

(3) $\sqrt{2}$ त्रिज्या का वृत्त है। (4) $\sqrt{3}$ त्रिज्या का वृत्त है।

Ans. **(3)**

Sol. Line passing through (1, 2)



AP = AQ

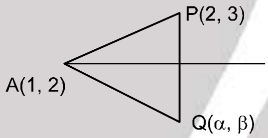
$(\alpha - 1)^2 + (\beta - 2)^2 = (2 - 1)^2 + (3 - 2)^2$

$\alpha^2 + \beta^2 - 2\alpha - 4\beta + 3 = 0$

$x^2 + y^2 - 2x - 4y + 3 = 0$

$r = \sqrt{1 + 4 - 3} = \sqrt{2}$

Hindi. (1, 2) से रेखा गुजरती है।



AP = AQ

$(\alpha - 1)^2 + (\beta - 2)^2 = (2 - 1)^2 + (3 - 2)^2$

$\alpha^2 + \beta^2 - 2\alpha - 4\beta + 3 = 0$

$x^2 + y^2 - 2x - 4y + 3 = 0$

$r = \sqrt{1 + 4 - 3} = \sqrt{2}$

8. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is

[JEE(Main) 2015, (4, – 1), 120]

(1) 1 (2) 2 (3*) 3 (4) 4

वृत्तों $x^2 + y^2 - 4x - 6y - 12 = 0$ तथा $x^2 + y^2 + 6x + 18y + 26 = 0$ की उभयनिष्ठ स्पर्श रेखाओं की संख्या है–

[JEE(Main) 2015, (4, – 1), 120]

(1) 1 (2) 2 (3*) 3 (4) 4

Ans. **(3)**

Sol. $C_1(2, 3)$ $r_1 = 5$

$C_2(-3, -9)$ $r_2 = 8$

$C_1C_2 = \sqrt{25 + 144} = 13$

$C_1C_2 = r_1 + r_2 \Rightarrow$ externally touch

$\Rightarrow$ 3 common tangents

Hindi. $C_1(2, 3)$ $r_1 = 5$

$C_2(-3, -9)$ $r_2 = 8$

$C_1C_2 = \sqrt{25 + 144} = 13$

$C_1C_2 = r_1 + r_2 \Rightarrow$ बाह्यय स्पर्श

$\Rightarrow$ 3 उभयनिष्ठ स्पर्श रेखाएं

9. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on : **[JEE(Main) 2016, (4, – 1), 120]**

(1) an ellipse which is not a circle (2) a hyperbola
(3) a parabola (4) a circle

उन वृत्तों के केन्द्र, जो वृत्त $x^2 + y^2 - 8x - 8y - 4 = 0$, को बाह्या रूप से स्पर्श करते हैं तथा x-अक्ष को भी स्पर्श करते है, स्थित हैं :

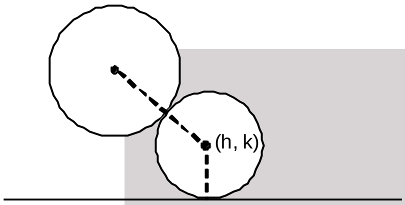
(1) एक दीर्घवत्त पर जो वृत्त नही है। (2) एक अतिपरवलय पर।
(3) एक परवलय पर। (4) एक वृत्त पर।

Ans. (3)

Sol. Parabola (परवलय)

Property : distance from a fixed point & fixed line is equal

गुणधर्म : स्थिर बिन्दु तथा स्थिर रेखा से दूरी समान है

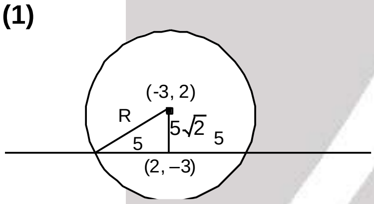


10. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at (– 3, 2), then the radius of S is : **[JEE(Main) 2016, (4, – 1), 120]**

यदि समीकरण, $x^2 + y^2 - 4x + 6y - 12 = 0$ द्वारा प्रदत्त एक वृत्त का एक व्यास एक अन्य वृत्त S, जिसका केन्द्र (– 3, 2) है, की जीवा है, तो वृत्त S की त्रिज्या है :

(1) $5\sqrt{3}$ (2) 5 (3) 10 (4) $5\sqrt{2}$

Ans. (1)



Sol.

$r_1 = \sqrt{4+9+12} = 5 \Rightarrow R = \sqrt{25+50} = 5\sqrt{3}$

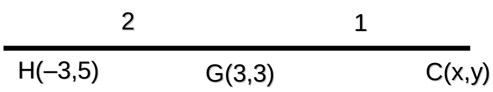
11. Let the orthocenter and centroid of a triangle be A (–3, 5) and B(3,3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segement AC as diameter , is :

[JEE(Main) 2018, (4, – 1), 120]

माना एक त्रिभुज का लम्ब केन्द्र तथा केन्द्रक क्रमशः A (–3, 5) तथा B(3,3) है। यदि इस त्रिभुज का परिकेन्द्र C है, तो रेखाखण्ड AC को व्यास मान कर बनाए जाने वाले वृत्त की त्रिज्या है :

(1*) $3\sqrt{\frac{5}{2}}$ (2) $\frac{3\sqrt{5}}{2}$ (3) $\sqrt{10}$ (4) $2\sqrt{10}$

Sol. (1)



$3 = \frac{2x-3}{3} \Rightarrow x = 6$

$3 = \frac{2y+5}{3} \Rightarrow y = 2$

$\frac{AC}{2} = \frac{1}{2}\sqrt{81+9} = \frac{1}{2}\sqrt{90} = \frac{3}{2}\sqrt{10}$

$r = 3\sqrt{\frac{5}{2}}$

12. Three circles of radii, a, b, c (a < b < c) touch each other externally, If they have x-axis as a common tangent, then : **[JEE(Main) 2019, Online (09-01-19),P-1 (4, – 1), 120]**

(1) a, b, c are in A.P.

(2) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$

(3) $\sqrt{a}$, $\sqrt{b}$, $\sqrt{c}$ are in A.P.

(4) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$

a, b, c (a < b < c) त्रिज्याओं वाले तीन वृत्त परस्पर बाह्म स्पर्श करते हैं। यदि x-अक्ष उनकी एक उभयनिष्ठ स्पर्श रेखा है, तो :

(1) a, b, c एक समांतर श्रेढ़ी में है।

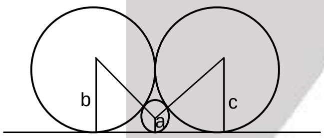
(2) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$

(3) $\sqrt{a}$, $\sqrt{b}$, $\sqrt{c}$ एक समांतर श्रेढ़ी में है।

(4) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$

Ans. **(2)**

Sol.



$\sqrt{(a+b)^2-(a-b)^2} + \sqrt{(a+c)^2-(a-c)^2} = \sqrt{(b+c)^2-(b-c)^2}$

$\sqrt{ab} + \sqrt{ac} = \sqrt{bc}$

$\frac{1}{\sqrt{c}} + \frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}}$

13. If a circle C passing through the point (4,0) touches the circle $x^2 + y^2 + 4x - 6y = 12$ externally at the point (1, –1), then the radius of C is: **[JEE(Main) 2019, Online (10-01-19),P-1 (4, – 1), 120]**

एक वृत्त C, बिंदु (4,0) से होकर जाता है तथा वृत्त $x^2 + y^2 + 4x - 6y = 12$ को बिंदु (1, –1) पर बाह्य स्पर्श करता है, तो C की त्रिज्या है:

(1) $2\sqrt{5}$ (2) $\sqrt{57}$ (3) 4 (4) 5

Ans. **(4)**

Sol. Tangent at (1,–1) is $x(1) + y(-1) + 2(x + 1) - 3(y - 1) - 12 = 0$

$\Rightarrow 3x - 4y = 7$

Required circle is

$(x - 1)^2 + (y + 1)^2 + \lambda(3x - 4y - 7) = 0$

It pass through (4,0)

$\Rightarrow 9 + 1 + \lambda(12 - 7) = 0 \Rightarrow \lambda = -2 \Rightarrow$ required circle is $x^2 + y^2 - 8x + 10y + 16 = 0$

$\Rightarrow$ Radius = $\sqrt{16+25-16} = 5$

Hindi. बिन्दु (1,–1) पर स्पर्श रेखा का समीकरण $x(1) + y(-1) + 2(x + 1) - 3(y - 1) - 12 = 0$

$\Rightarrow 3x - 4y = 7$

आवश्यक वृत्त, $(x - 1)^2 + (y + 1)^2 + \lambda(3x - 4y - 7) = 0$ है जो बिन्दु (4,0) से गुजरता है।

$\Rightarrow 9 + 1 + \lambda(12 - 7) = 0 \Rightarrow \lambda = -2 \Rightarrow$ आवश्यक वृत्त $x^2 + y^2 - 8x + 10y + 16 = 0$

$\Rightarrow$ त्रिज्या = $\sqrt{16+25-16} = 5$

14. If a variable line, $3x + 4y - \lambda = 0$ is such that the two circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 18x - 2y + 78 = 0$ are on its opposite sides, then the set of all values of λ is the interval:

यदि एक चर रेखा $3x + 4y - \lambda = 0$ इस प्रकार है कि दो वृत्त $x^2 + y^2 - 2x - 2y + 1 = 0$ तथा $x^2 + y^2 - 18x - 2y + 78 = 0$ इसके दोनों ओर (opposite sides) है, तो λ के सभी मानों का समुच्चय निम्न में से कौनसा अन्तराल है– **[JEE(Main) 2019, Online (12-01-19),P-1 (4, – 1), 120]**

(1) (2, 17) (2) [12, 21] (3) [13, 23] (4) (23,31)

Ans. **(2)**

Sol. $3x + 4y - \lambda = 0$

$(7 - \lambda)(31 - \lambda) < 0$ {since centres lie opposite side}

$(7 - \lambda)(31 - \lambda) < 0$ {चूंकि केन्द्र विपरीत भुजा पर स्थित है। }

$\lambda \in (7, 31)$ ……(1)

$\left|\frac{7-\lambda}{5}\right| \geq 1$ & $\left|\frac{31-\lambda}{5}\right| \geq 2$

$|7-\lambda| \geq 5$ &और $|31-\lambda| \geq 10$

$\lambda \leq 2$ or या $\lambda \geq 12$ ….(2) & और $\lambda \leq 21$ or या $\lambda \geq 41$ ….(3)

$(1) \cap (2) \cap (3)$

$\lambda \in [12, 21]$

15. If a circle of radius R passes through the origin O and intersects the coordinate axes at A and B, then the locus of the foot of perpendicular from O on AB is -

यदि R त्रिज्या का एक वृत्त मूल बिन्दु O से होकर जाता है तथा निर्देशांक अक्षों को A और B पर काटता है, तो O से रेखा AB पर डाले गये लम्ब के पाद का बिन्दुपथ है–**[JEE(Main) 2019, Online (12-01-19),P-2 (4, – 1), 120]**

(1) $(x^2 + y^2)(x + y) = R^2xy$

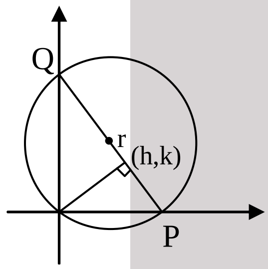
(2) $(x^2 + y^2)^3 = 4R^2x^2y^2$

(3) $(x^2 + y^2)^2 = 4Rx^2y^2$

(4) $(x^2 + y^2)^2 = 4R^2x^2y^2$

Ans. **(2)**

Sol.



Equation of line PQ

रेखा PQ का समीकरण

$$y - k = \frac{-h}{k}(x - h)$$

$$ky - k^2 = -hx + h^2$$

$$hx + ky = h^2 + k^2$$

$$A\left(\frac{h^2+k^2}{k}, 0\right), B\left(0, \frac{h^2+k^2}{k}\right)$$

O(0, 0)

AB = 2R

$$\frac{(h^2+k^2)^2}{k^2} + \frac{(h^2+k^2)^2}{h^2} = 4R^2 \Rightarrow (h^2+k^2)\left(\frac{h^2+k^2}{h^2k^2}\right) = 4R^2 ; (x^2+y^2)^3 = 4R^2x^2y^2$$

16. The sum of the squares of the lengths of the chords intercepted on the circle, $x^2 + y^2 = 16$, by the lines, $x + y = n$, $n \in N$, where N is the set of all natural numbers, is :

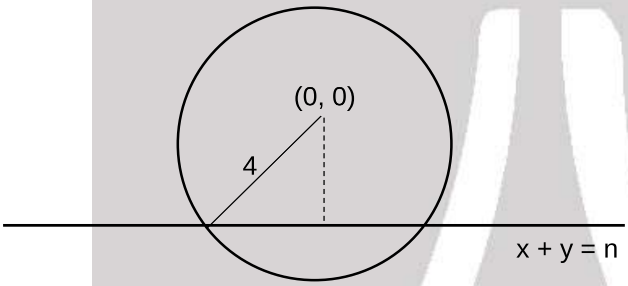
[JEE(Main) 2019, Online (08-04-19),P-1 (4, – 1), 120]

वत्त, $x^2 + y^2 = 16$, पर रेखाओं $x + y = n$, $n \in N$, जहां N सभी प्राकृत संख्याओं का समुच्चय है, द्वारा काटी गई जीवाओं की लम्बाईयों के वर्गो का योग है :

(1) 105 (2) 210 (3) 320 (4) 160

Ans. **(2)**

Sol. $p = \frac{n}{\sqrt{2}}$



to make the intercept (अन्तखण्ड बनाने के लिए) $\frac{n}{\sqrt{2}} < 4 \Rightarrow n < 4\sqrt{2}$

Length of intercepts अन्तखण्ड की लम्बाई = $2\sqrt{r^2 - p^2} = 2\sqrt{16 - n^2/2}$

Square of intercept अन्तखण्ड की लम्बाई का वर्ग = $4 \times (16 - \frac{n^2}{2})$, $n \in N$

Sum of squares of intercept अन्तखण्ड की लम्बाई के वर्गो का योग

$$= 4 \times \left(\left(16 - \frac{1}{2}\right) + \left(16 - \frac{4}{2}\right) + \left(16 - \frac{9}{2}\right) + \left(16 - \frac{16}{2}\right) + \left(16 - \frac{25}{2}\right)\right) = 2\left(80 - \frac{1}{2} \times 55\right) = 210$$

17. If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is : **[JEE(Main) 2019, Online (09-04-19),P-1 (4, – 1), 120]** **[Circle]**

यदि वृत्त $x^2 + y^2 = 1$ की एक स्पर्श रेखा निर्देशांक अक्षों को भिन्न बिन्दुओं P और Q पर प्रतिच्छेद करती है, तो PQ के मध्य बिन्दु का बिन्दुपथ (locus) है–

(1) $x^2 + y^2 - 4x^2y^2 = 0$ (2) $x^2 + y^2 - 16x^2y^2 = 0$ (3) $x^2 + y^2 - 2x^2y^2 = 0$ (4) $x^2 + y^2 - 2xy = 0$

Ans **(1)**

Sol. Let equation of tangent to the given circle be $x \cos\theta + y \sin\theta = 1$

The line meets x – axis at $(\sec\theta, 0)$ & y - axis at $(0, \text{cosec}\,\theta)$. If P (h, k) is the mid-point of this segment.

$\Rightarrow 2h = \sec\theta$ & $2k = \text{cosec}\,\theta$

$\Rightarrow \frac{1}{x^2} + \frac{1}{y^2} = 4 \Rightarrow x^2 + y^2 - 4x^2y^2 = 0$

18. The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y-axis and lie in the first quadrant, is :

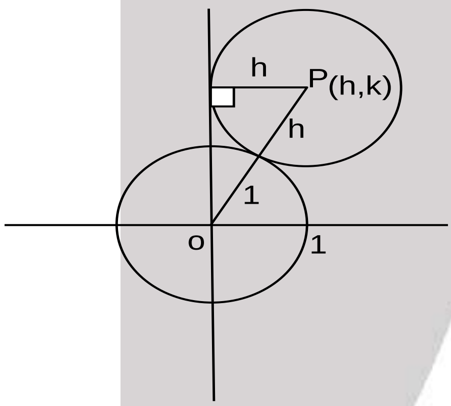
ऐसे वृत्तों, तो वृत्त $x^2 + y^2 = 1$ को बाह्य स्पर्श करते हैं, y-अक्ष को भी स्पर्श करते हैं तथा प्रथम चतुर्थांश में स्थित हैं, के केन्द्रों का बिदुपथ है–

(1) $x = \sqrt{1+4y}, y \geq 0$ (2) $y = \sqrt{1+4x}, x \geq 0$ (3) $x = \sqrt{1+2y}, y \geq 0$ (4) $y = \sqrt{1+2x}, x \geq 0$

Ans. **(4)** **[JEE(Main) 2019, Online (10-04-19),P-2 (4, – 1), 120]**

Sol. Let centre is (h, k) & radius is h (h, k > 0)

OP = h + 1



$\sqrt{h^2 + k^2} = h + 1 \Rightarrow h^2 + k^2 = h^2 + 2h + 1 \Rightarrow k^2 = 2h + 1$

Locus is $y^2 = 2x + 1$

19. If a line, $y = mx + c$ is a tangent to the circle, $(x - 3)^2 + y^2 = 1$ and it is perpendicular to a line L_1, where L_1 is the tangent to the circle, $x^2 + y^2 = 1$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$; then :

यदि एक रेखा $y = mx + c$, वृत्त $(x - 3)^2 + y^2 = 1$ की एक स्पर्श रेखा है तथा यह एक रेखा L_1 पर लम्ब हैं, जहाँ L_1 वृत्त $x^2 + y^2 = 1$ के बिन्दु $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ पर स्पर्श रेखा है, तो :

(1) $c^2 + 7c + 6 = 0$ (2) $c^2 + 6c + 7 = 0$ (3) $c^2 - 6c + 7 = 0$ (4) $c^2 - 7c + 6 = 0$

Ans. **(2)** **[JEE(Main) 2020, Online (08-01-20),P-2 (4, –1), 120]**

Sol. Slope of tangent to $x^2 + y^2 = 1$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$x^2 + y^2 = 1$ की $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ पर स्पर्श रेखा की प्रवणता

$x^2 + y^2 = 1$

$2x + 2yy' = 0$

$y' = -\frac{x}{y} = -1$

$y = mx + c$ is tangent of $x^2 + y^2 = 1$

$y = mx + c$ वृत्त $x^2 + y^2 = 1$ की स्पर्श रेखा है

so इसलिये $m = 1$

$y = x + c$

now distance of (3, 0) from $y = x + c$ is

अब (3, 0) की रेखा $y = x + c$ से दूरी है

$\left|\frac{c+3}{\sqrt{2}}\right| = 1$

$c^2 + 6c + 9 = 2$

$c^2 + 6c + 7 = 0$

20. If the curves, $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0$, $(k > 0)$ touch each other at a point, then the largest value of k is ________

एक वक्र $x^2 - 6x + y^2 + 8 = 0$ तथा $x^2 - 8y + y^2 + 16 - k = 0$, $(k > 0)$ एक दूसरे को एक बिन्दु पर स्पर्श करते हैं, तो k का अधिकतम मान हैं_________ । **[JEE(Main) 2020, Online (09-01-20),P-2 (4, 0), 120]**

Ans. **36**

Sol. Two circles touches each other if $C_1 C_2 = |r_1 \pm r_2|$

Distance between $C_2(3, 0)$ and $C_1(0, 4)$ is either $\sqrt{k}+1$ or $\left|\sqrt{k}-1\right|$ $(C_1 C_2 = 5)$

$\Rightarrow \sqrt{k}+1 = 5$ or $\left|\sqrt{k}-1\right| = 5$ $\Rightarrow k = 16$ or $k = 36 \Rightarrow$ maximum value of k is 36

Sol_H. दो वृत्त एक दूसरे को स्पर्श करते हैं यदि $C_1 C_2 = |r_1 \pm r_2|$

$C_2(3, 0)$ तथा $C_1(0, 4)$ के मध्य दूरी $\sqrt{k}+1$ या $\left|\sqrt{k}-1\right|$ है $(C_1 C_2 = 5)$

$\Rightarrow \sqrt{k}+1 = 5$ or $\left|\sqrt{k}-1\right| = 5$ $\Rightarrow k = 16$ or $k = 36$ $\Rightarrow$ k का अधिकतम मान 36 है

High Level Problems (HLP)

✍ **Marked Questions may have for Revision Questions.**
✍ **चिन्हित प्रश्न दोहराने योग्य प्रश्न है।**

SUBJECTIVE QUESTIONS

विषयात्मक प्रश्न (SUBJECTIVE QUESTIONS)

1. Find the equation of the circle passing through the points A(4, 3), B(2, 5) and touching the axis of y. Also find the point P on the y–axis such that the angle APB has largest magnitude.
बिन्दुओं A (4, 3) तथा B (2, 5) से गुजरने वाले तथा y-अक्ष को स्पर्श करने वाले वृत्त का समीकरण ज्ञात कीजिए। y–अक्ष पर बिन्दु P ज्ञात कीजिए ताकि कोण APB महत्तम परिमाण का हो।

Ans. $x^2 + y^2 - 4x - 6y + 9 = 0$ OR $x^2 + y^2 - 20x - 22y + 121 = 0$, P(0, 3), $\theta = 45°$

Sol. Equation of circle touching y - axis is $x^2 + y^2 + 2gx + 2fy + f^2 = 0$

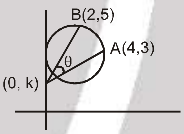
$\because$ it passes through (4, 3) & (2, 5)

so $25 + 8g + 6f + f^2 = 0 \Rightarrow 29 + 4g + 10f + f^2 = 0$

solving above two equations, we get $(g, f) \equiv (-2, -3)$ & $(-10, -11)$.

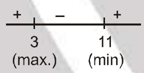
So equations of circles are $x^2 + y^2 - 4x - 6y + 9 = 0$ and $x^2 + y^2 - 20x - 22y + 121 = 0$

for circle $x^2 + y^2 - 4x - 6y + 9 = 0$.



$$\tan\theta = \frac{\frac{5-k}{2} - \frac{3-k}{4}}{1 + \frac{5-k}{2}\left(\frac{3-k}{4}\right)} = \frac{14-2k}{23+k^2-8k}$$

$$\frac{d(\tan\theta)}{dk} = \frac{2(k-11)(k-3)}{(k^2-8k+23)^2}$$



So $\tan\theta$ is max at k = 3 at k = 3, $\tan\theta = 1 \Rightarrow \theta = 45°$

Hindi. y - अक्ष को स्पर्श करने वाले वृत्त का समीकरण $x^2 + y^2 + 2gx + 2fy + f^2 = 0$ है।

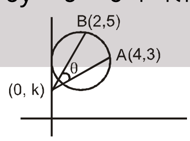
$\because$ यह (4, 3) & (2, 5) से गुजरता है

अतः $25 + 8g + 6f + f^2 = 0 \Rightarrow 2g + 4g + 10f + f^2 = 0$

दोनों समीकरणों को हल करने पर $(g, f) \equiv (-2, -3)$ & $(-10, -11)$.

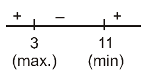
अतः वृत्त के समीकरण $x^2 + y^2 - 4x - 6y + 9 = 0$ तथा $x^2 + y^2 - 20x - 22y + 121 = 0$ है

वृत्त $x^2 + y^2 - 4x - 6y + 9 = 0$ के लिये



$$\tan\theta = \frac{\frac{5-k}{2} - \frac{3-k}{4}}{1 + \frac{5-k}{2}\left(\frac{3-k}{4}\right)} = \frac{14-2k}{23+k^2-8k}$$

$$\frac{d(\tan\theta)}{dk} = \frac{2(k-11)(k-3)}{(k^2-8k+23)^2}$$



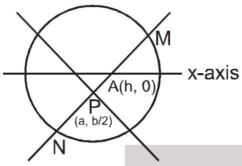
अतः k = 3 पर $\tan\theta$ महत्तम है। k = 3 पर $\tan\theta = 1 \Rightarrow \theta = 45°$

2. Let a circle be given by $2x(x - a) + y(2y - b) = 0$, $(a \neq 0, b \neq 0)$. Find the condition on a & b if two chords, each bisected by the x–axis, can be drawn to the circle from $\left(a\ ,\ \frac{b}{2}\right)$

मानाकि वृत्त का समीकरण $2x(x - a) + y(2y - b) = 0$, $(a \neq 0, b \neq 0)$ है यदि बिन्दु $\left(a\ ,\ \frac{b}{2}\right)$ से वृत्त पर दो जीवाएँ खींची जाए जो x–अक्ष द्वारा समद्विभाजित होती हो, तो a एवं b पर प्रतिबन्ध ज्ञात कीजिए।

Ans. $(a^2 > 2b^2)$

Sol.



Straight line MN bisected by x-aixs so A (h, 0) is midpoint of MN

So equation of chord of contact MN.

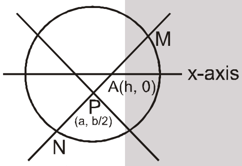
$T = S_1 \Rightarrow 2xh - a(x + h) - \frac{b(y+0)}{2} = 2h^2 - 2ah$

this line passes through (a, b/2)

$2ah - a(a + h) - \frac{b^2}{4} = 2h^2 - 2ah \Rightarrow 8h^2 - 12ah + 4a^2 + b^2 = 0$

$\because D > 0 \Rightarrow a^2 > 2b^2$

Sol.



सरल रेखा MN x-अक्ष द्वारा बिन्दु A (h, 0) पर समद्विभाजित होती है अतः बिन्दु A(h, 0), MN का मध्य बिन्दु है।

इस प्रकार स्पर्श जीवा MN का समीकरण

$T = S_1 \Rightarrow 2xh - a(x + h) - \frac{b(y+0)}{2} = 2h^2 - 2ah$

यह समीकरण बिन्दु (a, b/2) से गुजरता है तो

$2ah - a(a + h) - \frac{b^2}{4} = 2h^2 - 2ah \Rightarrow 8h^2 - 12ah + 4a^2 + b^2 = 0$

$\because \quad D > 0 \Rightarrow a^2 > 2b^2$

3. A circle is described to pass through the origin and to touch the lines x = 1, x + y = 2. Prove that the radius of the circle is a root of the equation $\left(3 - 2\sqrt{2}\right) t^2 - 2\sqrt{2}\, t + 2 = 0$.

एक वृत्त जो कि मूल बिन्दु से गुजरता है तथा रेखाओं x = 1 एवं x + y = 2 को स्पर्श करता है। सिद्ध कीजिए कि समीकरण $\left(3 - 2\sqrt{2}\right) t^2 - 2\sqrt{2}\, t + 2 = 0$ का एक मूल वृत्त की त्रिज्या है।

Sol. Let the centre of the circle be (h, k) and radius equal to 'r'

$\therefore \quad h^2 + k^2 = r^2$ ......(i)

and $\left|\frac{h+k-2}{\sqrt{2}}\right| = r \quad \Rightarrow \quad 2 - h - k = r\sqrt{2}$ .....(ii)

and h = 1 – r ....(iii)

put h = 1 – r in (ii), we get $k = r(1 - \sqrt{2}) + 1$

Now put the values of h and k in (i), we get

$(r(1 - \sqrt{2}) + 1)^2 + (1 - r)^2 = r^2 \quad \Rightarrow \quad r^2(3 - 2\sqrt{2}) - 2\sqrt{2}\, r + 2 = 0$

hence radius i.e. r is the root of the equation $(3 - 2\sqrt{2})\, t^2 - 2\sqrt{2}\, t + 2 = 0$

Hindi मानाकि वृत्त का केन्द्र (h, k) है तथा त्रिज्या r है।

$\therefore \quad h^2 + k^2 = r^2 \quad$ ......(i)

एवं $\left|\frac{h+k-2}{\sqrt{2}}\right| = r \quad \Rightarrow \quad 2 - h - k = r \quad$ .....(ii)

तथा $h = 1 - r \quad$ ....(iii)

समीकरण (ii) में $h = 1 - r$ रखने पर $k = r(1 - \sqrt{2}) + 1$

h एवं k का मान समीकरण (i) में रखने पर

$(r(1 - \sqrt{2}) + 1)^2 + (1 - r)^2 = r^2 \quad \Rightarrow \quad r^2(3 - 2\sqrt{2}) - 2\sqrt{2}\, r + 2 = 0$

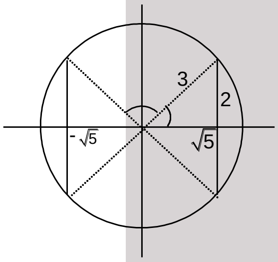
अतः त्रिज्या r समीकरण $(3 - 2\sqrt{2})\, t^2 - 2\sqrt{2}\, t + 2 = 0$ का मूल है।

4. If (a, α) lies inside the circle $x^2 + y^2 = 9 : x^2 - 4x - a^2 = 0$ has exactly one root in $(-1, 0)$, then find the area of the region in which (a, α) lies.

यदि (a, α) वृत्त के अन्दर स्थित है। $x^2 + y^2 = 9 : x^2 - 4x - a^2 = 0$ का ठीक एक मूल $(-1, 0)$ में स्थित है तब क्षेत्र का क्षेत्रफल ज्ञात कीजिए जिसमें (a, α) स्थित है

Ans. $4\left\{\sqrt{5} + \frac{9}{2}\cot^{-1}\left(\frac{2}{\sqrt{5}}\right)\right\}$

Sol.



$f(-1) \cdot f(0) < 0 \quad \Rightarrow \quad a^2 < 5 \quad \Rightarrow \quad -\sqrt{5} < a < \sqrt{5}$

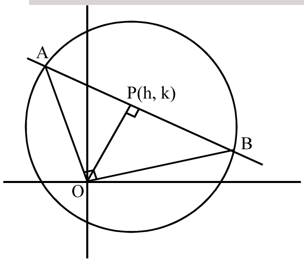
Required area $= 4.9\left(\frac{\pi}{2} - \tan^{-1}\frac{2}{\sqrt{5}}\right) + 4.\ \frac{1}{2}.\ \sqrt{5}\ .2 = 4\left\{\sqrt{5} + \frac{9}{2}\cot^{-1}\left(\frac{2}{\sqrt{5}}\right)\right\}$

5. Let $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ be a given circle. Find the locus of the foot of the perpendicular drawn from the origin upon any chord of S which subtends right angle at the origin.

माना $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ दिया गया वृत्त है। S की किसी जीवा जो मूल बिन्दू पर समकोण बनाती है, पर मूल बिन्दू से डाले गए लम्ब के लम्बपाद का बिन्दुपथ ज्ञात कीजिए।

Ans. $x^2 + y^2 + gx + fy + \frac{c}{2} = 0$

Sol.



AB is a variable chord such that $= \angle AOB = \frac{\pi}{2}$

Let P(h, k) be the foot of the perpendicular drawn from origin upon AB. Equation of the chord AB is

$$y - k = \frac{-h}{k}(x - h)$$

i.e. $hx + ky = h^2 + k^2$.......(1)

Equation of the pair of straight lines passingh through the origin and the intersection point of the given circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ ....(2)

and the variable chord AB is

$$x^2 + y^2 + 2(gx + fy)\left(\frac{hx+ky}{h^2+k^2}\right) + c\left(\frac{hx+ky}{h^2+k^2}\right)^2 = 0 \quad ....(3)$$

If equation (3) must represent a pair of perpendicular lines, then we have

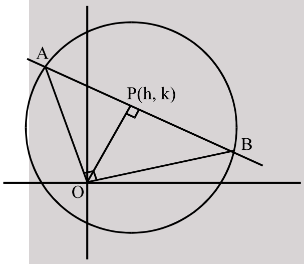
coeff. of x^2 + coeff. of y^2 = 0

i.e. $$\left(1+\frac{2gh}{h^2+k^2}+\frac{ch^2}{(h^2+k^2)^2}\right)+\left(1+\frac{2fk}{h^2+k^2}+\frac{ck^2}{(h^2+k^2)^2}\right) = 0.$$

Putting (x, y) in place of (h, k) gives the equation of the required locus as

$$x^2 + y^2 + gx + fy + \frac{c}{2} = 0.$$

Hindi.



AB एक चर जीवा इसप्रकार है कि = $\angle AOB = \frac{\pi}{2}$

माना P(h, k), AB पर मूल बिन्दू से खीचे गये लम्ब का लम्बपाद है। जीवा AB का समीकरण है। $y - k = \frac{-h}{k}(x - h)$

अर्थात् $hx + ky = h^2 + k^2$.......(1)

मूल बिन्दु से जाने वाली सरल रेखा युग्म का समीकरण ज्ञात कीजिए जो दिए गए वृत्त

$x^2 + y^2 + 2gx + 2fy + c = 0$ ....(2)

का प्रतिच्छेद बिन्दु है तथा चर जीवा AB है

$$x^2 + y^2 + 2(gx + fy)\left(\frac{hx+ky}{h^2+k^2}\right) + c\left(\frac{hx+ky}{h^2+k^2}\right)^2 = 0 \quad ....(3)$$

यदि समीकरण (3), लम्बवत् रेखाओं के युग्म को व्यक्त करता है तब x^2 + का गुणांक y^2 का गुणांक = 0

अर्थात् $$\left(1+\frac{2gh}{h^2+k^2}+\frac{ch^2}{(h^2+k^2)^2}\right)+\left(1+\frac{2fk}{h^2+k^2}+\frac{ck^2}{(h^2+k^2)^2}\right) = 0.$$

(x, y) को (h, k) से हटाने पर अभीष्ट बिन्दुपथ

$$x^2 + y^2 + gx + fy + \frac{c}{2} = 0.$$

6. A ball moving around the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ in anti–clockwise direction leaves it tangentially at the point P(–2, –2). After getting reflected from a straight line it passes through the centre of the circle. Find the equation of this straight line if its perpendicular distance from P is $\frac{5}{2}$. You can assume that the angle of incidence is equal to the angle of reflection.

वृत्त $x^2 + y^2 - 2x - 4y - 20 = 0$ के अनुदिश एक गेंद वामावर्त दिशा में गमन करती है तथा इसकी स्पर्श रेखा जो बिन्दु P(–2, –2) पर है, एक सरल रेखा पर परावर्तित होने के बाद वृत्त के केन्द्र से गुजरती है। सरल रेखा का समीकरण ज्ञात कीजिए यदि इसकी P से लम्बवत् दूरी $\frac{5}{2}$ हो। आप मान सकते है कि रेखा का झुकाव परावर्तन कोण के बराबर है।

Ans. $(4\sqrt{3} - 3)\,x - (4 + 3\sqrt{3})\,y - (39 - 2\sqrt{3}) = 0$

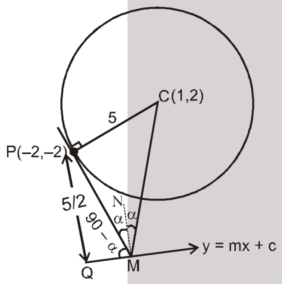
Sol. Let the equation of required straight line be y = mx + c.

$\Rightarrow \quad \frac{5}{2} = \frac{|-2+2m-c|}{\sqrt{1+m^2}}$ .....(i)

For ΔPCM $\quad \frac{PC}{PM} = \tan 2\alpha$.

$\Rightarrow \quad PM = 5\cot 2\alpha$ .....(ii)

For ΔPQM $\quad \frac{5}{2} = PM \sin(90 - \alpha) \quad \Rightarrow \quad \frac{5}{2} = \frac{5\cos 2\alpha}{\sin 2\alpha} \cos\alpha$.



on solving, we get $\alpha = 30°$. Equation of tangent at P(– 2, – 2) is $3x + 4y + 14 = 0$.

$\tan 60° = \left|\frac{m+3/4}{1-3m/4}\right| \quad \Rightarrow \quad \sqrt{3} = \frac{m+3/4}{1-3m/4} \quad \Rightarrow \quad m = \frac{4\sqrt{3}-3}{4+3\sqrt{3}}$

Now on substituting value of 'm' in equation (i), we get

$c = \frac{11+2\sqrt{3}}{4+3\sqrt{3}}$ or $\frac{-39+2\sqrt{3}}{4+3\sqrt{3}}$

but c should be (–ve)

So equation of line $\quad y = \frac{(4\sqrt{3}-3)}{4+3\sqrt{3}}\, x + \left(\frac{-39+2\sqrt{3}}{4+3\sqrt{3}}\right)$

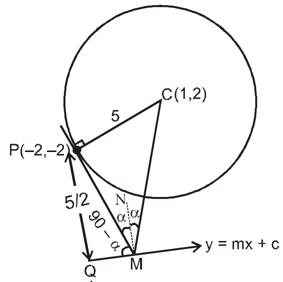
Hindi. माना अभीष्ट सरल रेखा का समीकरण y = mx + c.

$\frac{5}{2} = \frac{|-2+2m-c|}{\sqrt{1+m^2}}$ .....(i)

ΔPCM के लिए $\frac{PC}{PM} = \tan 2\alpha$.

$\Rightarrow \quad PM = 5\cot 2\alpha$ .....(ii)

ΔPQM के लिए $\frac{5}{2} = PM \sin(90 - \alpha) \quad \Rightarrow \quad \frac{5}{2} = \frac{5\cos 2\alpha}{\sin 2\alpha} \cos\alpha$.



हल करने पर $\alpha = 30°$ प्राप्त होता है। (– 2, – 2) पर स्पर्श रेखा का समीकरण

$3x + 4y + 14 = 0 \Rightarrow \quad \tan 60° = \left|\dfrac{m + 3/4}{1 - 3m/4}\right|$

$\tan 60° = \left|\dfrac{m + 3/4}{1 - 3m/4}\right| \quad \Rightarrow \quad \sqrt{3} = \dfrac{m + 3/4}{1 - 3m/4} \quad \Rightarrow \quad m = \dfrac{4\sqrt{3} - 3}{4 + 3\sqrt{3}}$

अब m का मान समीकरण (i) में रखने पर

$$c = \frac{11 + 2\sqrt{3}}{4 + 3\sqrt{3}} \text{ या } \frac{-39 + 2\sqrt{3}}{4 + 3\sqrt{3}}$$

परन्तु c ऋणात्मक होना चाहिए। रेखा का समीकरण $\quad y = \dfrac{(4\sqrt{3} - 3)}{4 + 3\sqrt{3}}\ x + \left(\dfrac{-39 + 2\sqrt{3}}{4 + 3\sqrt{3}}\right)$

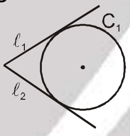
7. The lines 5x + 12 y – 10 = 0 and 5x – 12y – 40 = 0 touch a circle C_1 of diameter 6 unit. If the centre of C_1 lies in the first quadrant, find the equation of the circle C_2 which is concentric with C_1 and cuts of intercepts of length 8 on these lines.

रेखाऐं 5x + 12 y – 10 = 0 तथा 5x – 12y – 40 = 0 एक 6 इकाई व्यास वाले वृत्त C_1 को स्पर्श करती है। यदि C_1 का केन्द्र प्रथम चर्तुथांश में स्थित है तब वृत्त C_2 का समीकरण ज्ञात करो जो C_1 का संकेन्द्रीय है तथा जो इन रेखाओं पर 8 लम्बाई का अन्तःखण्ड काटता है।

Ans. $x^2 + y^2 - 10x - 4y + 4 = 0$

Sol. Centre of C_1 lies over angle bisector of ℓ_1 & ℓ_2

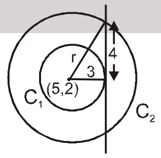
Equations of angle bisectors are



$\dfrac{5x + 12y - 10}{13} = \pm \dfrac{5x - 12y - 40}{13} \quad \Rightarrow x = 5 \text{ or } y = -\dfrac{5}{4}$

Since centre lies in first quadrant so it should be on x = 5.

So let centre be $(5, \alpha)$ $\quad \Rightarrow 3 = \dfrac{|25 + 12\alpha - 10|}{13} \quad \Rightarrow \quad \alpha = 2, -\dfrac{9}{2}$



From the figure $r = \sqrt{16 + 9} = 5$. But $\alpha \neq -\dfrac{9}{2}$ so $\alpha = 2$.

So equation of cirlce C_2 is

$(x - 5)^2 + (y - 2)^2 = 5^2 \quad \Rightarrow \quad x^2 + y^2 - 10x - 4y + 4 = 0.$

Hindi. वृत्त C_1 का केन्द्र ℓ_1 तथा ℓ_2 के कोण अर्धक पर स्थित है।

कोण अर्द्धकों का समीकरण $\dfrac{5x + 12y - 10}{13} = \pm \dfrac{5x - 12y - 40}{13} \quad \Rightarrow x = 5 \text{ or } y = -\dfrac{5}{4}$

चूँकि केन्द्र प्रथम चतुर्थांश में स्थित है।

अतः यह x = 5 पर होना चाहिए।

अतः माना केन्द्र (5, α) है। $\Rightarrow$ $3 = \frac{|25+12\alpha-10|}{13}$ $\Rightarrow$ $\alpha = 2, -\frac{9}{2}$

चित्रानुसार r = $\sqrt{16+9} = 5$. किन्तु $\alpha \neq -\frac{9}{2}$ अतः $\alpha = 2$.

$\therefore$ वृत्त C_2 का समीकरण

$(x-5)^2 + (y-2)^2 = 5^2$ $\Rightarrow$ $x^2 + y^2 - 10x - 4y + 4 = 0$.

8. The chord of contact of tangents drawn from a point on the circle $x^2 + y^2 = a^2$ to the circle $x^2 + y^2 = b^2$ touches the circle $x^2 + y^2 = c^2$. Show that a, b, c are in G.P.

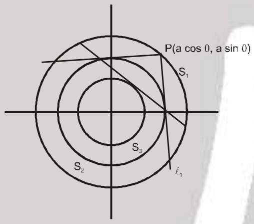
वृत $x^2 + y^2 = a^2$ के एक बिन्दु से वृत्त $x^2 + y^2 = b^2$ पर खींची गई स्पर्श रेखाओं की स्पर्शी जीवा, वृत्त $x^2 + y^2 = c^2$ को स्पर्श करती है। प्रदर्शित कीजिए कि a, b, c गुणोत्तर श्रेढ़ी में है।

Sol. $S_1 \equiv x^2 + y^2 = a^2$

$S_2 \equiv x^2 + y^2 = b^2$

$S_3 \equiv x^2 + y^2 = c^2$

equation of ℓ_1 is $ax\cos\theta + ay\sin\theta = b^2$



ℓ_1 is tangent to circle S_3

$c = \frac{|-b^2|}{\sqrt{a^2\cos^2\theta + a^2\sin^2\theta}}$ $\Rightarrow$ $ca = b^2$ Hence a,b,c are in G.P.

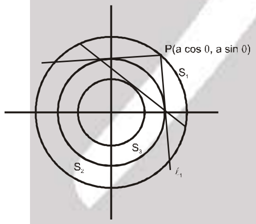
Hindi. $S_1 \equiv x^2 + y^2 = a^2$

$S_2 \equiv x^2 + y^2 = b^2$

$S_3 \equiv x^2 + y^2 = c^2$

ℓ_1 का समीकरण $ax\cos\theta + ay\sin\theta = b^2$

ℓ_1, वृत्त S_3 की स्पर्श रेखा है।



$\Rightarrow$ $c = \frac{|-b^2|}{\sqrt{a^2\cos^2\theta + a^2\sin^2\theta}}$ $\Rightarrow$ $ca = b^2$ अतः a,b,c गुणोत्तर श्रेढी में है।

9. Find the locus of the middle points of chords of a given circle $x^2 + y^2 = a^2$ which subtend a right angle at the fixed point (p, q).

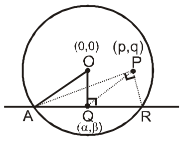
वृत्त $x^2 + y^2 = a^2$ की उन जीवाओं के मध्य बिन्दुओं का बिन्दुपथ ज्ञात कीजिए जो कि एक नियत बिन्दु (p, q) पर समकोण अन्तरित करता है।

Ans. $2x^2 + 2y^2 - 2px - 2qy + p^2 + q^2 - a^2 = 0$

Sol. OA = a and AQ = QP = QR

$\because$ OQ = $\sqrt{\alpha^2 + \beta^2}$

$\therefore$ AQ = $\sqrt{(p-\alpha)^2 + (q-\beta)^2}$ = PQ



$\because$ $(OA)^2 = (OQ)^2 + (AQ)^2$

$a^2 = \alpha^2 + \beta^2 + (p - \alpha)^2 + (q - \beta)^2$

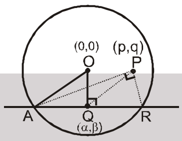
$2\alpha^2 + 2\beta^2 - 2p\alpha - 2q\beta + p^2 + q^2 - a^2 = 0.$

Locus of the middle point Q (α, β) is $2x^2 + 2y^2 - 2px - 2qy + p^2 + q^2 - a^2 = 0$

Hindi. OA = a तथा AQ = QP = QR

$\because$ $OQ = \sqrt{\alpha^2 + \beta^2}$

$\therefore$ $AQ = \sqrt{(p-\alpha)^2 + (q-\beta)^2} = PQ$



$\because$ $(OA)^2 = (OQ)^2 + (AQ)^2$

$a^2 = \alpha^2 + \beta^2 + (p - \alpha)^2 + (q - \beta)^2$

$2\alpha^2 + 2\beta^2 - 2p\alpha - 2q\beta + p^2 + q^2 - a^2 = 0.$

जीवाओं के मध्य बिन्दु Q (α, β) का बिन्दुपथ $2x^2 + 2y^2 - 2px - 2qy + p^2 + q^2 - a^2 = 0.$

10. Let $a\ell^2 - bm^2 + 2d\ell + 1 = 0$, where a, b, d are fixed real numbers such that $a + b = d^2$. If the line $\ell x + my + 1 = 0$ touches a fixed circle then find the equation of circle

माना $a\ell^2 - bm^2 + 2d\ell + 1 = 0$ जबकि a, b, d निश्चित वास्तविक संख्याएँ इस प्रकार है कि $a + b = d^2$, यदि रेखा $\ell x + my + 1 = 0$ एक निश्चित वृत्त को स्पर्श करती है तो वृत्त का समीकरण होगा –

Ans. $x^2 + y^2 - 2dx + d^2 - b = 0$

Sol. $\because$ $a\ell^2 - bm^2 + 2\ell d + 1 = 0$ ......(1)

and $a + b = d^2$ ....(2)

Put $a = d^2 - b$ in equation (1), we get $(\ell d + 1)^2 = b(\ell^2 + m^2)$

$\Rightarrow \frac{|\ell d + 1|}{\sqrt{\ell^2 + m^2}} = \sqrt{b}$ ......(3)

From (3) we can say that the line $\ell x + my + 1 = 0$ touches a fixed circle having centre at (d,0) and radius= $\sqrt{b}$

Hindi $\because$ $a\ell^2 - bm^2 + 2\ell d + 1 = 0$ ......(1)

तथा $a + b = d^2$ ....(2)

समीकरण (1) में $a = d^2 - b$ रखने पर $(\ell d + 1)^2 = b(\ell^2 + m^2)$

$\Rightarrow \frac{|\ell d + 1|}{\sqrt{\ell^2 + m^2}} = \sqrt{b}$ ......(3)

समीकरण (3) से हम कह सकते है कि रेखा $\ell x + my + 1 = 0$ एक निश्चित वृत्त को स्पर्श करती है जिसका केन्द्र (d,0) एवं त्रिज्या $\sqrt{b}$ है।

11. The centre of the circle S = 0 lies on the line $2x - 2y + 9 = 0$ and S = 0 cuts orthogonally the circle $x^2 + y^2 = 4$. Show that circle S = 0 passes through two fixed points and also find their co–ordinates.

वृत्त S = 0 का केन्द्र $2x - 2y + 9 = 0$ पर स्थित है तथा वृत्त S = 0 वृत्त $x^2 + y^2 = 4$ को लम्बकोणीय काटता है। दशाईये कि वृत्त S = 0 दो नियत बिन्दुओं से गुजरता है तथा उनके निर्देशांक भी ज्ञात कीजिए।

Ans. $(-4, 4)$; $\left(-\frac{1}{2}, \frac{1}{2}\right)$

Sol. $\because$ centre lies over the line $2x - 2y + 9 = 0$

So let coordinate of centre be $\left(h, \frac{2h+9}{2}\right)$

Let the radius of circle be 'r'. So equation of circle is $(x-h)^2 + \left(y - \frac{2h+9}{2}\right)^2 = r^2$

$x^2 + y^2 - 2hx - y(2h + 9) + 2h^2 + 9h - r^2 + \frac{81}{4} = 0$

$\because$ given circle cuts orthogonally to $x^2 + y^2 = 4$

so $2h^2 + 9h + \frac{65}{4} - r^2 = 0$ or $2h^2 + 9h - r^2 = -\frac{65}{4}$

so equation of required circle can be written as $x^2 + y^2 - 2hx - y(2h + 9) + 4 = 0$

$(x^2 + y^2 - 9y + 4) + h(-2y - 2x) = 0$

so this circle always passes through points of intersection of $x^2 + y^2 - 9y + 4 = 0$ and $x + y = 0$

so fixed points are $(-4, 4)$ and $\left(-\frac{1}{2}, \frac{1}{2}\right)$

Hindi. $\because$ केन्द्र रेखा $2x - 2y + 9 = 0$ पर स्थित है। अतः माना केन्द्र के निर्देशांक $\left(h, \frac{2h+9}{2}\right)$ है।

माना वृत्त की त्रिज्या r है। वृत्त का समीकरण $(x-h)^2 + \left(y - \frac{2h+9}{2}\right)^2 = r^2$

$x^2 + y^2 - 2hx - y(2h + 9) + 2h^2 + 9h - r^2 + \frac{81}{4} = 0$ ....(i)

$\because$ दिया गया वृत्त $x^2 + y^2 = 4$ को लम्बकोणीय काटता है।

अतः $2h^2 + 9h + \frac{65}{4} - r^2 = 0$ या $2h^2 + 9h - r^2 = -\frac{65}{4}$ ....(ii)

समीकरण (ii) का उपयोग (i) में करने पर $(x^2 + y^2 - 9y + 4) + h(-2y - 2x) = 0$

अतः यह वृत्त सदैव $x^2 + y^2 - 9y + 4 = 0$ तथा $x + y = 0$ के प्रतिच्छेद बिन्दुओं से गुजरता है।

अतः नियत बिन्दु $(-4, 4)$ तथा $\left(-\frac{1}{2}, \frac{1}{2}\right)$ है।

12. Prove that the two circles which pass through the points (0, a) , (0, – a) and touch the straight line y = m x + c will cut orthogonaly if $c^2 = a^2(2 + m^2)$.

सिद्ध कीजिए कि दो वृत्त जो बिन्दु (0, a) , (0, – a) से गुजरते है तथा सरल रेखा y = m x + c को स्पर्श करते है, परस्पर लम्बकोणीय प्रतिच्छेद करते है यदि $c^2 = a^2(2 + m^2)$ हो।

Sol. Let the equation of the circles be $x^2 + y^2 + 2gx + 2fy + d = 0$ ....(i)

$\because$ these circles pass through (0, a) and (0, –a)

$\therefore$ $a^2 + 2fa + d = 0$ ......(ii)

and $a^2 - 2fa + d = 0$ ......(iii)

solving (ii) and (iii), we get $f = 0$, $d = -a^2$

put these value of f and d in (i), we get

$x^2 + y^2 + 2gx - a^2 = 0$ ......(iv)

$\because$ y = mx + c touch these circles $\Rightarrow \left|\frac{-mg+c}{\sqrt{m^2+1}}\right| = \sqrt{g^2 + a^2}$

$\Rightarrow$ $g^2 + (2cm)g + a^2(1 + m^2) - c^2 = 0$ ......(v)

equation (v) is quadratic in 'g'

$\therefore$ Let g_1 and g_2 are its two roots

$\therefore$ $g_1g_2 = a^2(1 + m^2) - c^2$

$\because$ the two circles represented by (iv) are orthogonal

$\therefore$ $2g_1g_2 + 0 = -a^2 - a^2$ $\Rightarrow$ $g_1g_2 = -a^2$ $\Rightarrow$ $a^2(1 + m^2) - c^2 = -a^2$

$c^2 = a^2(2 + m^2)$ Hence proved

Hindi मानाकि वृत्त का समीकरण $x^2 + y^2 + 2gx + 2fy + d = 0$ है। ....(i)

$\because$ ये वृत्त (0, a) तथा (0, –a) से गुजरते है।

$\therefore$ $a^2 + 2fa + d = 0$ ......(ii)

तथा $a^2 - 2fa + d = 0$ ......(iii)

(ii) तथा (iii) को हल करने पर f = 0, d = – a^2 प्राप्त होता है।
f एवं d के मानों को (i) में रखने पर
$x^2 + y^2 + 2gx - a^2 = 0$ ......(iv)
$\because$ y = mx + c वृत्तों को स्पर्श करती है। $\Rightarrow \left|\frac{-mg+c}{\sqrt{m^2+1}}\right| = \sqrt{g^2+a^2}$

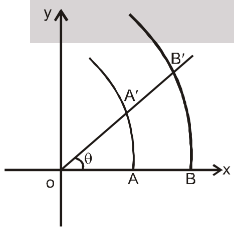
$\Rightarrow$ $g^2 + (2cm)\, g + a^2 (1 + m^2) - c^2 = 0$ ......(v)
समीकरण (v), g में द्विघात है।
$\therefore$ माना इसके दो मूल g_1 तथा g_2 है।
$\therefore$ $g_1g_2 = a^2 (1 + m^2) - c^2$
$\because$ समीकरण (iv) द्वारा प्रदर्शित दोनों वृत्त लम्बकोणीय है
$\therefore$ $2g_1g_2 + 0 = -a^2 - a^2$ $\Rightarrow$ $g_1g_2 = -a^2$ $\Rightarrow$ $a^2 (1 + m^2) - c^2 = -a^2$
$c^2 = a^2 (2 + m^2)$

13. Consider points A $(\sqrt{13}, 0)$ and B $(2\sqrt{13}, 0)$ lying on x-axis. These points are rotated in an-anticlockwise direction about the origin through an angle of $\tan^{-1}\left(\frac{2}{3}\right)$. Let the new position of A and B be A′ and B′ respectively. With A′ as centre and radius $\frac{2\sqrt{13}}{3}$ a circle C_1 is drawn and with B′ as a centre and radius $\frac{\sqrt{13}}{3}$ circle C_2 is drawn. Find radical axis of C_1 and C_2.

माना x-अक्ष पर स्थित बिन्दु A$(\sqrt{13}, 0)$ एवं बिन्दु B $(2\sqrt{13}, 0)$ है। इन बिन्दुओं को मूल बिन्दु के सापेक्ष दक्षिणावृत्त दिशा में $\tan^{-1}\left(\frac{2}{3}\right)$ कोण से घुमाया जाता है। माना कि A व B बिन्दुओं की नयी स्थिति A′ व B′ होगी। A′ को केन्द्र तथा $\frac{2\sqrt{13}}{3}$ त्रिज्या लेकर एक वृत्त C_1 बनाया जाता है तथा B′ को केन्द्र लेकर $\frac{\sqrt{13}}{3}$ एवं त्रिज्या लेकर वृत्त C_2 बनाया जाता है। C_1 व C_2 का मूलाक्ष होगा–

Ans. $9x + 6y = 65$

Sol. $\because$ $\theta = \tan^{-1}\left(\frac{2}{3}\right)$ $\Rightarrow$ $\tan\theta = \frac{2}{3}$ $\therefore$ $\sin\theta = \frac{2}{\sqrt{13}}$ and $\cos\theta = \frac{3}{\sqrt{13}}$
$\therefore$ $A' \equiv (OA\cos\theta, OA\sin\theta)$ $\Rightarrow$ $A' \equiv (3, 2)$
Similarly $B' \equiv (OB\cos\theta, OB\sin\theta) \equiv (6, 4)$
Now it can be checked that circles C_1 and C_2 touch each other.
Let the point of contact be C.
$\therefore$ $C \equiv \left(5, \frac{10}{3}\right)$



$\therefore$ required radical axis is a line perpendicular to A′B′ and passing through point C
$y - \frac{10}{3} = -\frac{3}{2}(x - 5)$

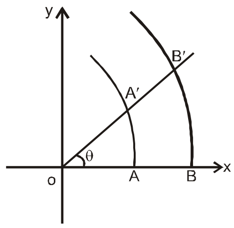
Hindi $\because$ $\theta = \tan^{-1}\left(\frac{2}{3}\right)$ $\Rightarrow$ $\tan\theta = \frac{2}{3}$ $\therefore$ $\sin\theta = \frac{2}{\sqrt{13}}$ and $\cos\theta = \frac{3}{\sqrt{13}}$
$\therefore$ $A' \equiv (OA\cos\theta, OA\sin\theta)$ $\Rightarrow$ $A' \equiv (3, 2)$

इसी प्रकार $B' \equiv (OB \cos\theta, OB \sin\theta) \equiv (6, 4)$

अब यह जाँच की जा सकती है कि वृत्त C_1 व C_2 परस्पर स्पर्श करते है।

मानाकि स्पर्श बिन्दु C है।

$\therefore \quad C \equiv \left(5, \frac{10}{3}\right)$



$\therefore$ अभीष्ट मूलाक्ष एक रेखा होगी जो कि A′B′ के लम्बवत् तथा C से गुजरती है।

$$y - \frac{10}{3} = -\frac{3}{2}(x - 5)$$

14. P(a, b) is a point in the first quadrant. If the two circles which pass through P and touch both the co–ordinate axes cut at right angles, then find condition in a and b.

प्रथम चतुर्थाश में एक बिन्दु P(a, b) है। यदि दो वृत्त जो P से गुजरते है तथा दोनों अक्षों को स्पर्श करते हो तथा एक दूसरे को समकोणीय काटते हो, तो–

Ans. $a^2 - 4ab + b^2 = 0$

Sol. Equations of two circles touching both the axes are

$x^2 + y^2 - 2c_1 x - 2c_1 y + c_1^2 = 0$ .....(i)

$x^2 + y^2 - 2c_2 x - 2c_2 y + c_2^2 = 0$ .....(ii)

$\because$ (i) & (ii) are orthogonal also

$\therefore$ $2c_1c_2 + 2c_1c_2 = c_1^2 + c_2^2$

or $6c_1 c_2 = (c_1 + c_2)^2$ ....(iii)

Now point P(a, b) lies over the circle

$x^2 + y^2 - 2cx - 2cy + c^2 = 0$.

so $c^2 - 2c(a + b) + a^2 + b^2 = 0$

$\because$ c_1 & c_2 are roots of this equation

so $c_1 + c_2 = 2(a + b)$ ....(iv)

and $c_1 c_2 = a^2 + b^2$ ....(v)

from (iii), (iv) & (v), we get

$6(a^2 + b^2) = 4(a + b)^2$.

Hindi. दोनों अक्षों को स्पर्श करने वाले दो वृत्तों के समीकरण

$x^2 + y^2 - 2c_1 x - 2c_1 y + c_1^2 = 0$ .....(i)

$x^2 + y^2 - 2c_2 x - 2c_2 y + c_2^2 = 0$ .....(ii)

$\because$ (i) एवं (ii) लम्बकोणीय भी है

अतः $2c_1c_2 + 2c_1c_2 = c_1^2 + c_2^2$

या $6c_1 c_2 = (c_1 + c_2)^2$ ....(iii)

अब बिन्दु P(a, b) वृत्त $x^2 + y^2 - 2cx - 2cy + c^2 = 0$ पर स्थित है।

अतः $c^2 - 2c(a + b) + a^2 + b^2 = 0$

c_1 एवं c_2 इस समीकरण के मूल हैं।

अतः $c_1 + c_2 = 2(a + b)$ ....(iv)

तथा $c_1 c_2 = a^2 + b^2$ ....(v)

(iii), (iv) तथा (v) से

$6(a^2 + b^2) = 4(a + b)^2$.

15. Prove that the square of the tangent that can be drawn from any point on one circle to another circle is equal to twice the product of perpendicular distance of the point from the radical axis of two circles and distance between their centres.

सिद्ध कीजिए कि एक वृत्त के किसी बिन्दु से दूसरे वृत्त पर खीचीं जाने वाली स्पर्श रेखा का वर्ग, दोनों वृत्तो की मूलाक्ष से बिन्दु की लम्बवत् दूरी और उनके केन्द्रों के मध्य दूरी के गुणनफल के दुगुने के बराबर है

Sol. Let us choose the circles, as $S_1 \equiv x^2 + y^2 - a^2 = 0$ ....(1)

and $S_2 \equiv (x - b)^2 + y^2 - c^2 = 0$ ....(2)

Let P(a cosθ, asinθ) be any point on circle S_1. The length of the tangent from P to circle S_2, is given by $PT^2 = S_2(a\cos\theta, a\sin\theta) = (a\cos\theta - b)^2 + (a\sin\theta)^2 - c^2 = a^2 + b^2 - c^2 - 2ab\cos\theta$

The distance between the centres of S_1 and S_2, is $C_1C_2 = b$

The radical axis of S_1 and S_2, is $2bx - a^2 - b^2 + c^2 = 0$ [equation (1) – equation (2)]

The perpendicular distance of P from the radical axis, is

$$PM = \frac{|2b(a\cos\theta) - a^2 - b^2 + c^2|}{2b}$$

Now, we have

$2.\ PM.\ C_1C_2 = 2b.\ \dfrac{|2ab\cos\theta - a^2 - b^2 + c^2|}{2b} = |a^2 + b^2 - c^2 - 2ab\cos\theta| = PT^2$ which proves the desired result.

Hindi. माना कि वृत्त $S_1 \equiv x^2 + y^2 - a^2 = 0$ ....(1)

तथा $S_2 \equiv (x - b)^2 + y^2 - c^2 = 0$.......(2)

माना P(a cosθ, asinθ) वृत्त S_1 पर कोई बिन्दु है। P से $S_2 = 0$ पर स्पर्श रेखा की लम्बाई $PT^2 = S_2(a\cos\theta, a\sin\theta)$ $= (a\cos\theta - b)^2 + (a\sin\theta)^2 - c^2 = a^2 + b^2 - c^2 - 2ab\cos\theta$

S_1 तथा S_2 के केन्द्रों के मध्य दूरी $C_1C_2 = b$ है।

S_1 तथा S_2 का मूलाक्ष $2bx - a^2 - b^2 + c^2 = 0$ है। [समीकरण (1) – समीकरण (2)]

मूलाक्ष की P से लम्बवत् दूरी है

$$PM = \frac{|2b(a\cos\theta) - a^2 - b^2 + c^2|}{2b}$$

यहँ

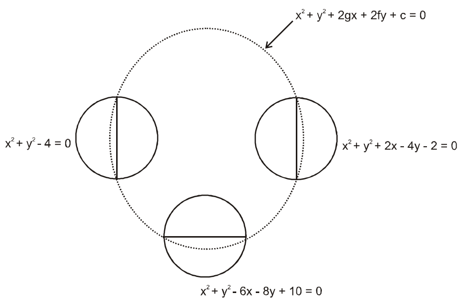
$2.\ PM.\ C_1C_2 = 2b.\ \dfrac{|2ab\cos\theta - a^2 - b^2 + c^2|}{2b} = |a^2 + b^2 - c^2 - 2ab\cos\theta| = PT^2$ जो कि अभीष्ट परिणाम है।

16. Find the equation of the circle which cuts each of the circles, $x^2 + y^2 = 4$, $x^2 + y^2 - 6x - 8y + 10 = 0$ & $x^2 + y^2 + 2x - 4y - 2 = 0$ at the extremities of a diameter. **(Revision Planner)**

उस वृत्त का समीकरण ज्ञात कीजिए जो वृत्तों $x^2 + y^2 = 4$, $x^2 + y^2 - 6x - 8y + 10 = 0$ और $x^2 + y^2 + 2x - 4y - 2 = 0$ को उनके व्यासों के अन्तिम सिरों पर काटता हो।

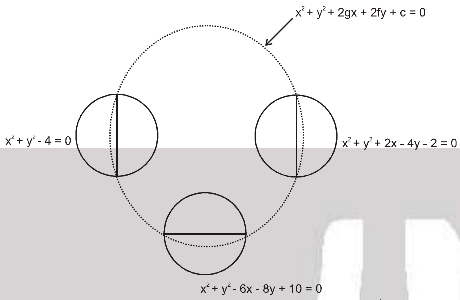
Ans. $x^2 + y^2 - 4x - 6y - 4 = 0$

Sol.



Let required equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$
Now common chord of given circle with required circle are
Common chord $2gx + 2fy + (c + 4) = 0$ it is also diameter of circle $x^2 + y^2 = 4$. Hence $c = -4$
similarly with $x^2 + y^2 - 6x - 8y + 10 = 0$ $\Rightarrow 2x(g + 3) + 2y(f + 4) - 14 = 0$
$\Rightarrow 6(g + 3) + 8(f + 4) - 14 = 0 \Rightarrow 6g + 8f + 36 = 0$
$\Rightarrow 3g + 4f + 18 = 0$
With circle $x^2 + y^2 + 2x - 4y - 2 = 0$ $\Rightarrow 2x(g - 1) + 2y(f + 2) - 2 = 0 \Rightarrow -2(g - 1) + 4(f + 2) - 2 = 0$
$\Rightarrow -2g + 4f + 8 = 0 \Rightarrow 2g - 4f - 8 = 0$
after simplification $g = -2$, $f = -3$, $c = -4$
Hence circle $x^2 + y^2 - 4x - 6y - 4 = 0$

Hindi.



माना वृत्त की समीकरण $x^2 + y^2 + 2gx + 2fy + c = 0$ है।
अब दिए गए वृत्त माने गए वृत्त उभयनिष्ट जीवा $2gx + 2fy + (c + 4) = 0$ है।
यह वृत्त $x^2 + y^2 = 4$ का व्यास भी है। अतः $c = -4$
इसी प्रकार $x^2 + y^2 - 6x - 8y + 10 = 0$ $\Rightarrow 2x(g + 3) + 2y(f + 4) - 14 = 0 \Rightarrow 6(g + 3) + 8(f + 4) - 14 = 0$
$\Rightarrow 6g + 8f + 36 = 0 \Rightarrow 3g + 4f + 18 = 0$
वृत्त $x^2 + y^2 + 2x - 4y - 2 = 0$ के साथ $\Rightarrow 2x(g - 1) + 2y(f + 2) - 2 = 0 \Rightarrow -2(g - 1) + 4(f + 2) - 2 = 0$
$\Rightarrow -2g + 4f + 8 = 0 \Rightarrow 2g - 4f - 8 = 0$
हल करने के बाद $g = -2$, $f = -3$, $c = -4$
अतः वृत्त $x^2 + y^2 - 4x - 6y - 4 = 0$.

17. Show that if one of the circle $x^2 + y^2 + 2gx + c = 0$ and $x^2 + y^2 + 2g_1x + c = 0$ lies within the other, then gg_1 and c are both positive.
प्रदर्शित कीजिए कि वृत्तों $x^2 + y^2 + 2gx + c = 0$ तथा $x^2 + y^2 + 2g_1x + c = 0$ में एक वृत्त दूसरे के अन्दर स्थित हो, तो gg_1 एवं c दोनों धनात्मक है।

Sol. $\because$ One circle lies within the other circle $\Rightarrow C_1C_2 < |r_1 - r_2| \Rightarrow$ $\sqrt{(g-g_1)^2} < \left|\sqrt{(g^2-c)} - \sqrt{g_1^2-c}\right|$
squaring both sides, we get
$-2gg_1 < -2\sqrt{g^2-c}\sqrt{g_1^2-c} - 2c$ $\Rightarrow$ $gg_1 > c + \sqrt{g^2-c}\sqrt{g_1^2-c}$.
$\Rightarrow$ $gg_1 - c > \sqrt{g^2-c}\sqrt{g_1^2-c}$ . ...(i)
$\Rightarrow$ $gg_1 - c > 0$ $\Rightarrow$ $gg_1 > c$
again squaring both sides of (i), we get
$-2cgg_1 > -c(g^2 + g_1^2)$ $\Rightarrow$ $c(g - g_1)^2 > 0$ $\Rightarrow$ $c > 0$ and from (i), we can say that
$\therefore$ gg_1 will also be > 0

Hindi $\because$ एक वृत्त दूसरे वृत्त में स्थित है।
$\Rightarrow C_1C_2 < |r_1 - r_2|$ $\sqrt{(g-g_1)^2} < \left|\sqrt{(g^2-c)} - \sqrt{g_1^2-c}\right|$
दोनों पक्षों का वर्ग करने पर
$-2gg_1 < -2\sqrt{g^2-c}\sqrt{g_1^2-c} - 2c$ $\Rightarrow$ $gg_1 > c + \sqrt{g^2-c}\sqrt{g_1^2-c}$.
$\Rightarrow$ $gg_1 - c > \sqrt{g^2-c}\sqrt{g_1^2-c}$ . ...(i)
$\Rightarrow$ $gg_1 - c > 0$ $\Rightarrow$ $gg_1 > c$

पुनः (i) के दोनों पक्षों का वर्ग करने पर

$-2cgg_1 > -c\,(g^2 + g_1^2)$ $\Rightarrow$ $c(g - g_1)^2 > 0$

$\Rightarrow$ $c > 0$ तथा (i) से हम कह सकते है कि

$\therefore$ gg_1 भी धनात्मक होगा।

18. Let ABCD is a rectangle. Incircle of ΔABD touches BD at E. Incircle of ΔCBD toches BD at F.

If AB = 8 units, and BC = 6 units, then find length of EF.

माना ABCD एक आयत है। ΔABD का अन्तवृत्तः BD को E पर स्पर्श करता है। ΔCBD का अन्तवृत्त BD को F पर स्पर्श करता है। यदि AB = 8 तथा BC = 6 तब EF की लम्बाई ज्ञात कीजिए।

Ans. 2

Sol. Let A is (0,0) , B is (8,0) , C is (8,6) and D is (0,6) Then incircle of ΔABD is (2,2) and similarly incircle of ΔCBD is (6,4)

Length of transverse common tangent is $\sqrt{(C_1C_2)^2 - (r_1 + r_2)^2} = \sqrt{(6-2)^2 + (4-2)^2 - (2+2)^2} = 2 = EF$.

Hindi माना A (0,0) , B (8,0) , C (8,6) और D (0,6) है तब ΔABD का अन्तवृत्त का केन्द्र (2,2) और ΔCBD का अन्तकेन्द्र (6,4) है।

तिर्यक उभयनिष्ठ स्पर्श रेखा की लम्बाई है $\sqrt{(C_1C_2)^2 - (r_1 + r_2)^2} = \sqrt{(6-2)^2 + (4-2)^2 - (2+2)^2} = 2 = EF$.

19. Let circles S_1 and S_2 of radii r_1 and r_2 respectively ($r_1 > r_2$) touches each other externally. Circle S radii r touches S_1 and S_2 externally and also their direct common tangent. Prove that the triangle formed by joining centre of S_1, S_2 and S is obtuse angled triangle.

माना S_1 और S_2 क्रमश r_1 और r_2 ($r_1 > r_2$) त्रिज्या के वृत्त है प्रत्येक एक दूसरे को बाह्य स्पर्श करते है S_1 तथा S_2 बाह्य तथा उनकी अनुस्पर्श उभयनिष्ठ स्पर्श रेखा को स्पर्श करने वाला r त्रिज्या का वृत्त S है। तब सिद्ध कीजिए की S_1, S_2 तथा S के केन्द्रों को मिलाने से बना त्रिभुज अधिककोण त्रिभुज है।

Sol. AP + PB = AB

$\Rightarrow \sqrt{(r_1+r)^2 - (r_1-r)^2} + \sqrt{(r_2+r)^2 - (r_2-r)^2} = \sqrt{(r_1+r_2)^2 - (r_1-r_2)^2}$

$\Rightarrow 2\sqrt{rr_1} + 2\sqrt{rr_2} = 2\sqrt{r_1r_2} \Rightarrow rr_1 + rr_2 + 2r\sqrt{r_1r_2} = r_1r_2$ ....(i)

$\Rightarrow \frac{1}{\sqrt{r_1}} + \frac{1}{\sqrt{r_2}} = \frac{1}{\sqrt{r}} \Rightarrow \sqrt{r_1}\,,\ 2\sqrt{r}\,,\ \sqrt{r_2}$ are in H.P.

$\Rightarrow \left(2\sqrt{r}\right)^2 < \sqrt{r_1}\ \sqrt{r_2} \Rightarrow 4r < \sqrt{r_1r_2} \Rightarrow 2\sqrt{r_1r_2} > 8r$

$\Rightarrow 2\sqrt{r_1r_2} - r > 7r > 0$ ....(2)

Now अब $(C_1C_2)^2 - (C_1C)^2 - (C_2C)^2$

$= r_1^2 + r_2^2 + 2r_1r_2 - r^2 - r_1^2 - 2rr_1 - r^2 - r_2^2 - 2rr_2$

$= -2rr_1 - 2rr_2 + 2r_1r_2 - 2r^2 = 4r\sqrt{r_1r_2} - 2r^2$

$= 2r\left(2\sqrt{r_1r_2} - r\right)$ = positive धनात्मक $\Rightarrow \Delta C_1CC_2$ isobtuse angle triangle अधिक कोण त्रिभुज है।

20. Circles are drawn passing through the origin O to intersect the coordinate axes at point P and Q such that m. OP + n . OQ is a constant. Show that the circles pass through a fixed point.

मूल बिन्दु से गुजरने वाले वृत्त खींचे जाते है जो निर्देशांक अक्ष को P और Q को इसप्रकार प्रतिच्छेदन करते है कि m, OP + n . OQ अचर है। दर्शाइये कि वृत्त एक स्थिर बिन्दु से गुजरते है।

Sol. Equation of a circle passing through the origin and having X and Y intercepts equal to a and b respectively is

$x^2 + y^2 - ax - by = 0$ ....(1)

According to the given condition, we have

ma + nb = k (constant)

i.e. $b = \frac{k - ma}{n}$ ......(2)

Putting the above value of b in equation (1) , we have, $x^2 + y^2 - ax - \left(\frac{k - ma}{n}\right) y = 0$

i.e. $\{n(x^2 + y^2) - ky\} - a(nx - my) = 0$

which represents the equation of a family of circles passing through the intersection points of the circle

$n(x^2 + y^2) - ky = 0$ ....(3)

and the line

$nx - my = 0$ ....(4)

Solving equation (3) and (4), gives the coordinates of the fixed point as $\left(\frac{mk}{m^2 + n^2}, \frac{nk}{m^2 + n^2}\right)$.

Sol. वृत्त का समीकरण जो मूल बिन्दु से गुजरता है तथा X और Y अन्तखण्ड क्रमशः a और b है

$x^2 + y^2 - ax - by = 0$ ....(1)

दिए गए प्रतिबन्ध के अनुसर यहाँ

ma + nb = k (अचर)

अर्थात् $b = \frac{k - ma}{n}$ ......(2)

समीकरण (1) में b का मान रखने पर $x^2 + y^2 - ax - \left(\frac{k - ma}{n}\right) y = 0$

अर्थात् $\{n(x^2 + y^2) - ky\} - a(nx - my) = 0$

वृत्तों के निकाय के समीकरण को व्यक्त करता है जो वृत्तों के प्रतिच्छेद बिन्दुओं से गुजरता है।

$n(x^2 + y^2) - ky = 0$ ....(3)

तथा रेखा

$nx - my = 0$ ....(4)

समीकरण (3) तथा (4) से स्थिर बिन्दुओं के निर्देशांक $\left(\frac{mk}{m^2 + n^2}, \frac{nk}{m^2 + n^2}\right)$ है।

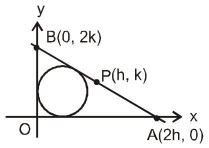
21. A triangle has two of its sides along the axes, its third side touches the circle $x^2 + y^2 - 2ax - 2ay + a^2 = 0$. Find the equation of the locus of the circumcentre of the triangle.

एक त्रिभुज की दो भुजाये अक्षों के अनुदिश है तथा तीसरी भुजा वृत्त $x^2 + y^2 - 2ax - 2ay + a^2 = 0$ को स्पर्श करती है। त्रिभुज के परिकेन्द्र का बिन्दुपथ ज्ञात कीजिए।

Ans. $2(x + y) - a = \frac{2xy}{a}$

Sol. Let the circumcentre be P(h, k) by using property circumcentre of rightangle triangle is lie on hypotenous

$\therefore$ Equation of AB is $\frac{x}{2h} + \frac{y}{2k} = 1 \Rightarrow a = \frac{\left|\frac{a}{2h} + \frac{a}{2k} - 1\right|}{\sqrt{\frac{1}{4h^2} + \frac{1}{4k^2}}}$

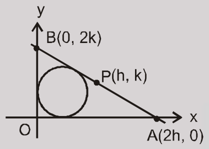


on solving $\frac{a(h+k)}{hk} = \frac{2hk+a^2}{2\,hk}$ $\Rightarrow$ $2(h + k) = \frac{2hk}{a} + a$

$\therefore$ locus of circumcentre P(h,k) is $2(x + y) - a = \frac{2xy}{a}$

Hindi. माना कि परिकेन्द्र P(h, k) है। हम जानते है कि समकोण त्रिभुज का परिकेन्द्र उसके कर्ण का मध्य बिन्दु होता है।

$\therefore$ AB का समीकरण है। $\frac{x}{2h} + \frac{y}{2k} = 1 \Rightarrow$ $a = \frac{\left|\frac{a}{2h} + \frac{a}{2k} - 1\right|}{\sqrt{\frac{1}{4h^2} + \frac{1}{4k^2}}}$



हल करने पर $\frac{a(h+k)}{hk} = \frac{2hk+a^2}{2\,hk}$ $\Rightarrow$ $2(h + k) = \frac{2hk}{a} + a$

$\therefore$ परिकेन्द्र P(h,k) का बिन्दुपथ $2(x + y) - a = \frac{2xy}{a}$.

22. Let S_1 be a circle passing through A(0, 1), B(–2, 2) and S_2 is a circle of radius $\sqrt{10}$ units such that AB is common chord of S_1 and S_2. Find the equation of S_2.

माना S_1 वृत्त है जो A(0, 1), B(–2, 2) बिन्दुओं से गुजरता है तथा S_2 , $\sqrt{10}$ त्रिज्या का एक वृत्त इस प्रकार है कि AB, S_1 और S_2 की उभयनिष्ठ जीवा है, तो S_2 का समीकरण ज्ञात कीजिए।

Ans. $x^2 + y^2 + 2x - 3y + 2 \pm \sqrt{7}\,(x + 2y - 2) = 0$

Sol. Equation of line AB is रेखा AB का समीकरण है

$y - 2 = \frac{2-1}{-2-0}(x + 2) = -\frac{1}{2}(x + 2)$ $\Rightarrow$ $x + 2y - 2 = 0$ ......(i)

Equation of circle whose diagonally opposite points are A and B:

$(x - 0)(x + 2) + (y - 1)(y - 2) = 0$

$\Rightarrow$ $x^2 + y^2 + 2x - 3y + 2 = 0$ .....(ii)

Family of circles passing through the points of intersection of Eqs. (i) and (ii)

$x^2 + y^2 + 2x - 3y + 2 + \lambda(x + 2y - 2) = 0$

$\Rightarrow$ $x^2 + y^2 + (2 + \lambda)x + (2\lambda - 3)y + 2 - 2\lambda = 0$ .....(iii)

Equation (iii), represents a circle of radius $\sqrt{10}$ units

$\Rightarrow$ $\sqrt{\left(-\frac{2+\lambda}{2}\right)^2 + \left(-\frac{2\lambda-3}{2}\right)^2 - 2 + 2\lambda} = \sqrt{10} \Rightarrow (4 + 4\lambda + \lambda^2) + (4\lambda^2 + 9 - 12\lambda) + 8\lambda - 8 = 40$

$\Rightarrow$ $\lambda = \pm\sqrt{7}$

Hence, required circles are

$x^2 + y^2 + 2x - 3y + 2 \pm \sqrt{7}\,(x + 2y - 2) = 0$

There are two such circles possible.

Hindi: रेखा AB का समीकरण है

$y - 2 = \frac{2-1}{-2-0}(x + 2) = -\frac{1}{2}(x + 2)$ $\Rightarrow$ $x + 2y - 2 = 0$ ......(i)

वृत्त का समीकरण जिसके व्यास के सिरे A तथा B है :

$(x - 0)(x + 2) + (y - 1)(y - 2) = 0$

$\Rightarrow$ $x^2 + y^2 + 2x - 3y + 2 = 0$ .....(ii)

समीकरण (i) तथा (ii) के प्रतिच्छेद बिन्दुओं से गुजरने वाले वृत्तों का निकाय है

$x^2 + y^2 + 2x - 3y + 2 + \lambda(x + 2y - 2) = 0$

$\Rightarrow \quad x^2 + y^2 + (2 + \lambda)x + (2\lambda - 3)y + 2 - 2\lambda = 0 \qquad .....(iii)$

समीरकण (iii), $\sqrt{10}$ इकाई त्रिज्या के वृत्त को व्यक्त करता है

$\Rightarrow \quad \sqrt{\left(-\frac{2+\lambda}{2}\right)^2 + \left(-\frac{2\lambda-3}{2}\right)^2 - 2 + 2\lambda} = \sqrt{10} \Rightarrow (4 + 4\lambda + \lambda^2) + (4\lambda^2 + 9 - 12\lambda) + 8\lambda - 8 = 40$

$\Rightarrow \quad \lambda = \pm\sqrt{7}$

अतः अभीष्ट वृत्त है

$x^2 + y^2 + 2x - 3y + 2 \pm \sqrt{7}\,(x + 2y - 2) = 0$

इस प्रकार के दो संभावित वृत्त है।

23. The curves whose equations are

$S = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$S' = a'x^2 + 2h'xy + b'y^2 + 2g'x + 2f'y + c' = 0$

intersect in four concyclic points then find relation in a, b, h, a′, b′, h′.

समीकरणों

$S = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$S' = a'x^2 + 2h'xy + b'y^2 + 2g'x + 2f'y + c' = 0$

से प्रदर्शित वक्र परस्पर चार समचक्रीय बिन्दुओं पर प्रतिच्छेद करते है, तो a, b, h, a′, b′, h′ में संबन्ध ज्ञात कीजिए।

Ans. $\frac{a-b}{h} = \frac{a'-b'}{h'}$

Sol. Equation of any curve passing through the four points of intersection of S = 0 and S′ = 0 is S + λS′ = 0. For this to be a circle, we must have coefficient of x^2 = coefficient of y^2 & coefficient of xy = 0.

S = 0 एवं S′ = 0 के चार प्रतिच्छेद बिन्दुओं से गुजरनें वाले किसी भी वक्र का समीकरण S + λS′ = 0 इस वक्र के वृत्त होनें के लिये x^2 का गुणांक = y^2 का गुणांक

तथा xy का गुणांक = 0

$\Rightarrow \quad a + \lambda a' = b + \lambda b'$

$a - b = -\lambda(a' - b') \qquad ....(1)$

and और $2h + \lambda 2h' = 0 \quad \Rightarrow \quad \lambda = -\frac{h}{h'} \qquad ....(2)$

$\Rightarrow$ from (1) and एवं (2) से $a - b = -\frac{h}{h'}(a' - b')$ or $\frac{a-b}{h} = \frac{a'-b'}{h'}$

24. A circle of constant radius 'r' passes through origin O and cuts the axes of coordinates in points P and Q, then find the equation of the locus of the foot of perpendicular from O to PQ.

एक निश्चित त्रिज्या r का एक वृत्त मूल बिन्दु O से गुजरता है तथा निर्देशी अक्षों को P व Q पर काटता हो, तो O से PQ पर डाले गये लम्ब पाद का बिन्दुपथ है –

Ans. $(x^2 + y^2)^2 (x^{-2} + y^{-2}) = 4r^2$

Sol. Let the coordinates of P and Q are (a, 0) and (0, b) respectively

$\therefore$ equation of PQ is $bx + ay - ab = 0$ ....(i)

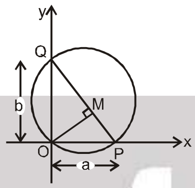
$\because$ $a^2 + b^2 = 4r^2$ ....(ii)

$\because$ $OM \perp PQ$

$\therefore$ equation of OM is $ax - by = 0$ ....(iii)

Let M(h, k)

$\therefore$ $bh + ak - ab = 0$ ....(iv) and $ah - bk = 0$ ....(v)



On solving equations (iv) and (v), we get

$a = \dfrac{h^2 + k^2}{h}$ and $b = \dfrac{h^2 + k^2}{k}$

put a and b in (ii), we get

$(h^2 + k^2)^2 (h^{-2} + k^{-2}) = 4r^2$

$\therefore$ locus of M(h, k) is $(x^2 + y^2)^2 (x^{-2} + y^{-2}) = 4r^2$

Hindi. मानाकि P व Q के निर्देशांक क्रमशः (a, 0) तथा (0, b) है।

$\therefore$ PQ का समीकरण $bx + ay - ab = 0$.......(i)

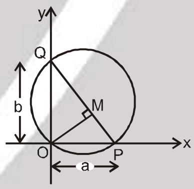
$\because$ $a^2 + b^2 = 4r^2$ .....(ii)

$\because$ $OM \perp PQ$

$\therefore$ OM का समीकरण $ax - by = 0$ ....(iii)

मानाकि M(h, k)

$\therefore$ $bh + ak - ab = 0$ ....(iv) तथा $ah - bk = 0$ ....(v)



(iv) व (v) को हल करने पर

$a = \dfrac{h^2 + k^2}{h}$ तथा $b = \dfrac{h^2 + k^2}{k}$

a व b को (ii) में रखने पर

$(h^2 + k^2)^2 (h^{-2} + k^{-2}) = 4r^2$

$\therefore$ M(h, k) का बिन्दुपथ $(x^2 + y^2)^2 (x^{-2} + y^{-2}) = 4r^2$ होगा।

25. The ends A , B of a fixed straight line of length 'a' and ends A′ and B′ of another fixed straight line of length 'b' slide upon the axis of X & the axis of Y (one end on axis of X & the other on axis of Y). Find the locus of the centre of the circle passing through A, B, A′ and B′.

a लम्बाई का नियत रेखाखण्ड के सिरे A एवं B तथा b लम्बाई का अन्य नियत रेखाखण्ड के सिरे A′ एवं B′ है, X-अक्ष व Y-अक्ष पर फिसलते है। (एक सिरा X-अक्ष पर व दूसरा सिरा Y-अक्ष पर). A, B, A′ एवं B′ से गुजरने वाले वृत्त के केन्द्र का बिन्दुपथ ज्ञात कीजिए।

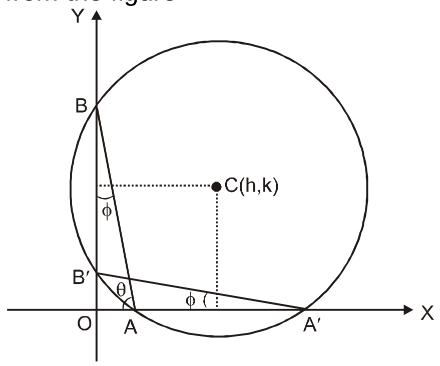
Ans. $(2ax - 2by)^2 + (2bx - 2ay)^2 = (a^2 - b^2)^2$

Sol. Let $\angle OA'B' = \phi$ and $\angle OAB = \theta$

$\Rightarrow$ $\theta + \phi = \dfrac{\pi}{2}$ and $\angle OBA = \phi$

$\because$ length of AB is 'a' and length of A′B′ is 'b'

∴ from the figure



A′ (b cos ϕ, 0) and A(a cos θ, 0)

similarly B(0, a sin θ) and B′ (0, b sin ϕ)

Let c(h, k) be the centre of circle ∴ $2h = a\cos\theta + b\cos\phi$

∵ $\phi = \frac{\pi}{2} - \theta$ ∴ $2h = a\cos\theta + b\sin\theta$ ....(i)

and $2k = a\sin\theta + b\sin\phi$ ∵ $\phi = \frac{\pi}{2} - \theta$

∴ $2k = a\sin\theta + b\cos\theta$ ....(ii)

on solving (i) and (ii), we get $\cos\theta = \frac{2ah-2bk}{a^2-b^2}$ and $\sin\theta = \frac{2ak-2bh}{a^2-b^2}$

∵ $\sin^2\theta + \cos^2\theta = 1$

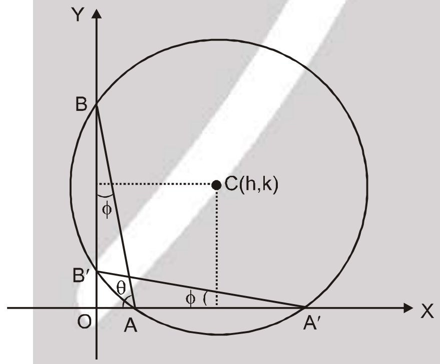
∴ locus of C(h, k) is $(2ax - 2by)^2 + (2bx - 2ay)^2 = (a^2 - b^2)^2$

Hindi माना कि ∠OA′B′ = ϕ तथा ∠OAB = θ

⇒ $\theta + \phi = \frac{\pi}{2}$ तथा ∠OBA = ϕ

∵ AB की लम्बाई a है तथा A′B′ की लम्बाई b है

∴ चित्र से



A′ (b cos ϕ, 0) तथा A(a cos θ, 0)

इसी प्रकार B(0, a sin θ) तथा B′ (0, b sin ϕ)

माना कि वृत्त का केन्द्र c(h, k) है।

∴ $2h = a\cos\theta + b\cos\phi$

∵ $\phi = \frac{\pi}{2} - \theta$

∴ $2h = a\cos\theta + b\sin\theta$ ....(i)

तथा $2k = a\sin\theta + b\sin\phi$ ∵ $\phi = \frac{\pi}{2} - \theta$

∴ $2k = a\sin\theta + b\cos\theta$ ....(ii)

(i) व (ii) को हल करने पर $\frac{2ah-2bk}{a^2-b^2}$ and $\sin\theta = \frac{2ak-2bh}{a^2-b^2}$

∵ $\sin^2\theta + \cos^2\theta = 1$

∴ C(h, k) का बिन्दुपथ $(2ax - 2by)^2 + (2bx - 2ay)^2 = (a^2 - b^2)^2$