competishun — Power of real gurus

PHYSICS

TARGET : JEE- Advanced 2021

CAPS-9

ROTATIONAL DYNAMICS-2

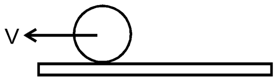
SCQ (Single Correct Type) :

1. A solid sphere of mass m and radius r is gently placed on a conveyer belt moving with constant velocity V. If the coefficient of friction between the belt and sphere is $\frac{2}{7}$, the distance travelled by the centre of the sphere before it starts pure rolling is

(A) $\frac{V^2}{7g}$ (B) $\frac{2V^2}{49g}$ (C) $\frac{2V^2}{5g}$ (D) $\frac{2V^2}{7g}$

Ans. (A)

Sol. Initial situation w.r.t. the conveyer belt :



For sphere : initial velocity(u) = V

$\mu \, mg \; R = \frac{2}{5} mR^2 \cdot \alpha \quad \Rightarrow \quad \alpha = = \frac{5}{7} \cdot \frac{g}{R}$

At pure rolling :

$v' = v - (\mu g)t$ and $\omega' = O + \left(\frac{5}{7} \cdot \frac{g}{R}\right) t$ (Since $v' = R\omega'$ at pure rolling)

$\Rightarrow \quad v - \mu g t = \frac{5}{7} \cdot g \cdot t \quad \Rightarrow \quad t = \frac{v}{g} \; (\because \mu = \frac{2}{7})$

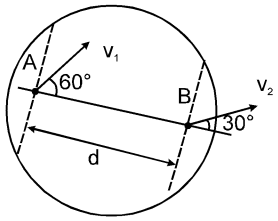
For centre of the sphere : $S = v\left(\frac{v}{g}\right) - \frac{1}{2} \cdot \left(\frac{2}{7} \cdot g\right)\left(\frac{v^2}{g^2}\right)$

$S = \frac{v^2}{g} - \frac{v^2}{7g}$ (w.r.t. belt)

In this time, the belt with speed 'v' will move a distance of $s_b = vt = v\left(\frac{v}{g}\right) = \frac{v^2}{g}$.

$\therefore$ Dist travelled w.r.t. ground will be : $S_g = |S - S_b| = \left|\left(\frac{v^2}{g} - \frac{v^2}{7g}\right) - \left(\frac{V^2}{g}\right)\right| = \frac{v^2}{7g}$

2. Two points A & B on a disc have velocities v_1 & v_2 at some moment. Their directions make angles 60° and 30° respectively with the line of separation as shown in figure. The angular velocity of disc is :



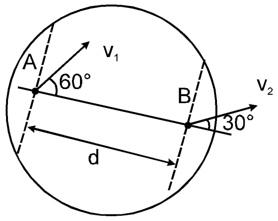
(A) $\frac{\sqrt{3}v_1}{d}$ (B) $\frac{v_2}{\sqrt{3}d}$ (C) $\frac{v_2 - v_1}{d}$ (D) $\frac{v_2}{d}$

Ans. **(D)**

Sol. For rigid body separation between two point remains same.

$v_1 \cos 60° = v_2 \cos 30°$

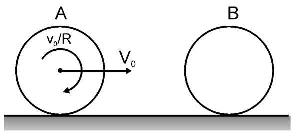
$\frac{v_1}{2} = \frac{\sqrt{3}\ v_2}{2} \Rightarrow \quad v_1 = \sqrt{3}\ v_2$



$$\omega_{disc} = \left|\frac{v_2 \sin 30° - v_1 \sin 60°}{d}\right| = \left|\frac{\frac{v_2}{2} - \frac{\sqrt{3}v_1}{2}}{d}\right| = \left|\frac{v_2 - \sqrt{3} \times \sqrt{3}v_2}{2d}\right| = \frac{2v_2}{2d} = \frac{v_2}{d}$$

$$\omega_{disc} = \frac{v_2}{d}$$

3. A hollow smooth uniform sphere A of mass 'm' rolls without sliding on a smooth horizontal surface. It collides elastically and head on with another stationary smooth solid sphere B of the same mass m and same radius. The ratio of kinetic energy of 'B' to that of 'A' just after the collision is :



(A) 5 : 2 (B) 1 : 1 (C) 2 : 3 (D) 3 : 2

Ans. **(D)**

Sol. Since the two bodies have same mass and collide head-on elastically, the linear momentum gets interchanged.

Hence just after the collision 'B' will move with velocity 'v_0' and 'A' becomes stationary but continues to rotate at the same initial angular velocity $\left(\frac{v_0}{R}\right)$.

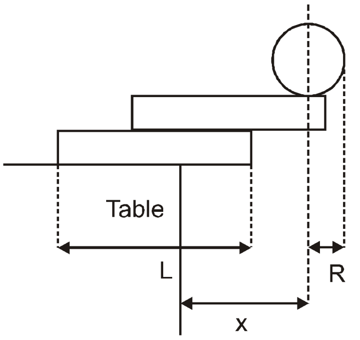
Hence, after collision.

$$(K.E.)_B = \frac{1}{2}mv_0^2 \qquad \text{and} \qquad (K.E.)_A = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{3}mR^2\right) . \left(\frac{v_0}{R}\right)^2$$

$\Rightarrow \quad \frac{(K.\ E.)_B}{(K.\ E.)_A} = \frac{3}{2}$ Hence (D).

Note : Sphere 'B' will not rotate, because there is no torque on 'B' during the collision as the collision is head-on

4. Two identical uniform rectangular blocks (with longest side L) and a solid sphere of radius R are to be balanced at the edge of a heavy table such that the centre of the sphere remains at the maximum possible horizontal distance from the vertical edge of the table without toppling as indicated in the figure.



If the mass of each block is M and of the sphere is M/2, then the maximum distance x that can be achieved is

(A) 8L/15 (B) 5L/6 (C) (3L/4 + R) (D) (7L/15 + R)

Ans. **(A)**

Sol. 2 + S system lie above dege of 1.

$$\frac{M}{2}y - M\left(\frac{L}{2} - y\right) = 0$$

$$\frac{y}{2} + y = \frac{L}{2}$$

$$y = \frac{L}{3}$$

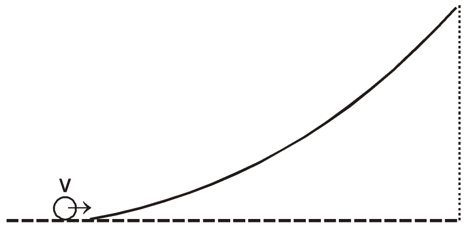
Now 1 + 2 + S centre of mass wili lie above the table

$$\frac{3M}{2}\left(x - \frac{L}{3}\right) + M\left(x - \frac{L}{3} - \frac{L}{2}\right) = 0$$

$$\frac{3x}{2} - \frac{L}{2} + x - \frac{L}{3} - \frac{L}{2} = 0$$

$$\frac{5x}{2} = \frac{4L}{3} \qquad x = \frac{8L}{15}$$

5. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is



(A) ring (B) solid sphere (C) hollow sphere (D) disc

Ans. **(D)**

Sol. From the conservation of energy

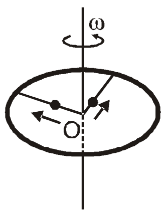
loss in KE of body = Gain in potential energy

$$\frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2 = mg\frac{3}{4}\frac{v^2}{g}$$

on solving

$$I = \frac{mr^2}{2} \qquad \therefore \text{ The body is a disc}$$

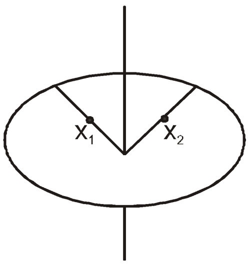
6. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is ata distance of $\frac{3}{5}R$ from O. At this instant the distance of the other mass from O is :



(A) $\frac{2}{3}R$ (B) $\frac{1}{3}R$ (C) $\frac{3}{5}R$ (D) $\frac{4}{5}R$

Ans. **(D)**

Ans. By conservation of angular momentum



$$MR^2\,\omega = \left(MR^2 + \frac{M}{8}\frac{9R^2}{25} + \frac{Md^2}{8}\right)\frac{8\omega}{9}$$

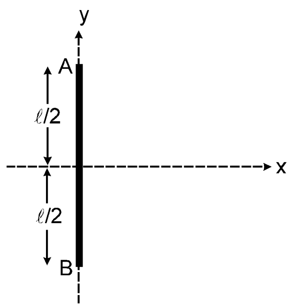
$$R^2 = \left(\frac{200R^2 + 9R^2 + 25d^2}{8 \times 25}\right)\frac{8}{9}$$

$$225\,R^2 - 209\,R^2 = 25\,d^2$$

$$d = \frac{16R^2}{25}$$

$$d = \frac{4R}{5}$$

7. A uniform rod of mass m, length ℓ is placed over a smooth horizontal surface along y-axis and is at rest as shown in figure. An impulsive force F is applied for a small time Δt along x-direction at point A after this rod moves freely. The x–coordinate of end A of the rod when the rod becomes parallel to x–axis for the first time is (initially the coordinate of centre of mass of the rod is (0, 0)) :



(A) $\frac{\pi\ell}{12}$ (B) $\frac{\ell}{2}\left(1+\frac{\pi}{12}\right)$ (C) $\frac{\ell}{2}\left(1-\frac{\pi}{6}\right)$ (D) $\frac{\ell}{2}\left(1+\frac{\pi}{6}\right)$

Ans. **(D)**

Sol. As torque = change in angular momentum

$\therefore$ $F\Delta t = mv$ (Linear) ..... (1)

and $\left(F \cdot \frac{\ell}{2}\right)\Delta t = \frac{m\ell^2}{12} \cdot \omega$ (Angular) ..... (2)

Dividing : (1) and (2)

$$2 = \frac{12v}{\omega\ell} \Rightarrow \omega = \frac{6v}{\ell}$$

Using S = ut :

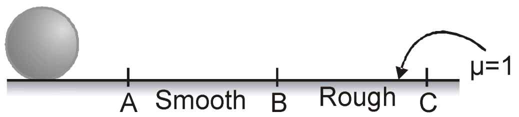
Displacement of COM is $\frac{\pi}{2} = \omega t = \left(\frac{6v}{\ell}\right)t$ and $x = vt$

Dividing $\frac{2x}{\pi} = \frac{\ell}{6}$

$\Rightarrow$ $x = \frac{\pi\ell}{12} \Rightarrow$ Coordinate of A will be $\left[\frac{\pi\ell}{12} + \frac{\ell}{2},\ 0\right]$

MCQ (One or more than one correct) :

8. A rigid body undergoing pure rolling encounters horizontal rigid tracks AB and BC as shown. AB is smooth surface and BC is rough surface with μ = 1. Which of the following statements is/are **correct** :



(A) Angular momentum of the rigid body is conserved only about a point on the horizontal surface.

(B) Angular momentum of the rigid body is conserved about every point in space.

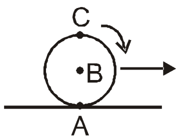
(C) In part BC, there will be no frictional force on the rigid body.

(D) In part BC, frictional force will act opposite to velocity of rigid body.

Ans. **(BC)**

Sol. Angular momentum will be conserved as external torque is zero.

9. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then,

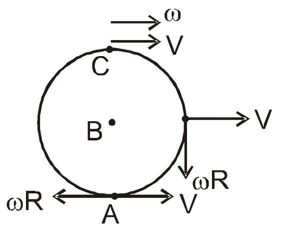


(A) $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B - \vec{V}_C)$ (B) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

(C) $|\vec{V}_C - \vec{V}_A| = 2\ |\vec{V}_B - \vec{V}_C|$ (D) $|\vec{V}_C - \vec{V}_A| = 4\ |\vec{V}_B|$

Ans. **(BC)**

Sol.



$\vec{V}_A = V(\hat{i}) + \omega R(-\hat{i})$; $\vec{V}_B = V\hat{i}$; $\vec{V}_C = V\hat{i} + \omega R\hat{i}$

$\vec{V}_C - \vec{V}_A = 2\omega R\hat{i}$

$2\left[\vec{V}_B - \vec{V}_C\right] = 2\ [\ V(\hat{i}) - V(\hat{i}) - \omega R(\hat{i})] = -2\omega R(\hat{i})$

Hence $\vec{V}_C - \vec{V}_A = -2(\vec{V}_B - \vec{V}_C)$

so $|\vec{V}_C - \vec{V}_A| = |2(\vec{V}_B - \vec{V}_C)|$

$\vec{V}_C - \vec{V}_B = \omega R(\hat{i})$

$\vec{V}_B - \vec{V}_A = \omega R(\hat{i})$

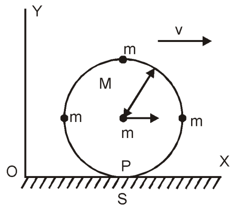
$\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

Hence $\vec{V}_C - \vec{V}_A = 2\omega R(\hat{i})$

$\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$; $4\vec{V}_B = 4V(\hat{i}) = 4\omega R\ (\hat{i})$

Hence $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B)$

10. A uniform circular disc of mass M. and radius R has four particles (each of mass m) rigidly attached to it. The disc rolls on the surface S without slipping. At the instant shown the centre of the disc has velocity v_0,



(A) The kinetic energy of the disc-particle assembly is $\left(\frac{9m}{2} + \frac{3M}{4}\right)v_0^2$

(B) The velocity of the centre of mass of the assembly is $\left(\frac{M+5m}{M+4m}\right)v_0\hat{i}$

(C) The angular momentum of the assembly about the point of contact P is $\frac{3}{2}$(M+ 6m) $(Rv_0(-\hat{k})$

(D) The maximum speed of a point on the disc with respect to S is $2v_0$

Ans. **(ABCD)**

Sol. The kinetic energy of a rolling body = $\frac{1}{2}I_P\omega_0^2$,

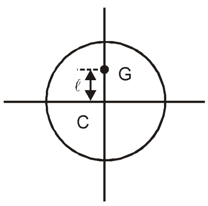
$I_P = \frac{3}{2}MR^2 + m(2R^2) \times 2 + mR^2 + m(2R)^2$

$= \frac{3}{2}MR^2 + 9mR^2$

$\therefore$ KE = $\left(\frac{9m}{2} + \frac{3}{4}M\right)v_0^2$ $(R\omega_0 = v_0)$

Location of the CM

$(M + 3m)\ell = m(R - \ell)$



$\ell = \frac{mR}{4m + M}$

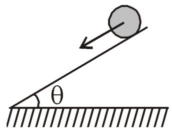
$\vec{v}_G = \ell\omega_0\hat{i} + v_0\hat{i} = \left(\frac{M+5m}{M+4m}\right)v_0\hat{i}$

Angular momentum about P

$\vec{L}_P = I_P\omega_0 = \left(\frac{3}{2}MR^2 + 9mR^2\right)\omega_0(-\hat{k}) = \frac{3}{2}\ (M + 6m)Rv_0(-\hat{k})$

The top most point on the disc has the maximum velocity = $2v_0\,\hat{i}$

11. A solid sphere is in pure rolling motion on an inclined surface having inclination θ.



(A) frictional force acting on sphere is f = μ mg cos θ.

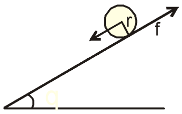
(B) f is dissipative force.

(C) friction will increase its angular velocity and decreases its linear velocity.

(D) If θ decreases, friction will decrease.

Ans. **(CD)**

Sol.



necessary torque for rolling $\tau = fr$, (frictional force provides this torque)

as $mg\sin\theta - f = ma$

but $a = r\alpha$ ⇒ $mg\sin\theta - f = mr\alpha$

as $\tau = fr = I\alpha$ ⇒ $\alpha = fr/I$

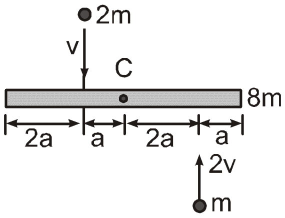
∴ $mg\sin\theta - f = mrfr/I = 5f/2$ $\left(I = \frac{2mr^2}{5}\right)$

∴ $mg\sin\theta = \frac{7f}{2}$

thus friction increases the torque in hence the angular velocity and decreases the linear velocity. If θ decreases friction will decrease.

Comprehension Type Question:

A uniform bar of length 6 a & mass 8 m lies on a smooth horizontal table. Two point masses m & 2 m moving in the same horizontal plane with speeds 2 v and v respectively strike the bar as shown & stick to the bar after collision.



12. Velocity of the centre of mass of the system is

(A) $\frac{v}{2}$ (B) v (C) $\frac{2v}{3}$ (D) Zero

Ans. **(D)**

13. Angular velocity of the rod about centre of mass of the system is

(A) $\frac{v}{5a}$ (B) $\frac{v}{15a}$ (C) $\frac{v}{3a}$ (D) $\frac{v}{10a}$

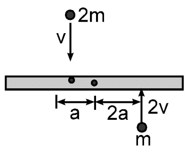
Ans. **(A)**

14. Total kinetic energy of the system, just after the collision is

(A) $\frac{3}{5}mv^2$ (B) $\frac{3}{25}mv^2$ (C) $\frac{3}{15}mv^2$ (D) $3\ mv^2$

Ans. **(A)**

Sol. (i) Cons. linear momentum

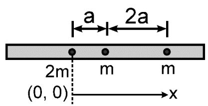


$-2m.v + 2v.m = 0 = MV_{cm}$

$V_{cm} = 0$

(ii) As ball sticks to Rod

Conserving angular momentum about C



$2v.m.\ 2a + 2mva = I\omega$

$$= \left(\frac{8m.\ \ 36a^2}{12} + 2m.\ a^2 + m.\ \ 4a^2\right)$$

$6mv.a = 30\ ma^2.\omega \ \Rightarrow\ \omega = \frac{v}{5a}$

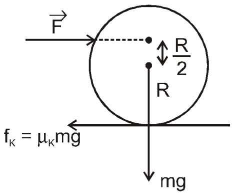
(iii) $KE = \frac{1}{2}\ I\omega^2 = \frac{1}{2}.\ 30\ ma^2 \times \frac{v^2}{25a^2} = \frac{3mv^2}{5}$.

Numerical based Questions :

15. A solid billiard ball of radius 'R' and mass 'm' initially at rest is given a sharp impulse by a cue, held horizontally at a distance $\frac{R}{2}$ above the centre. Just after the impulse, the velocity of centre of mass of the ball is v = 10 m/s. The coefficient of friction between ball and table is $\mu = \frac{1}{2}$. The ball starts rolling without slipping t seconds after impulse is given. Find value of $\frac{1}{t}$ (in sec^{-1})

Ans. **7**

Sol.



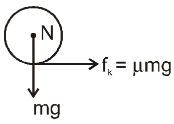
$\Delta\vec{p} = \vec{F}\Delta t$ = m (10 – 0) ....(i)

$\Delta\vec{L}\alpha$ centre $= \vec{F}\Delta t\frac{R}{2} = \frac{2}{5}mR^2$ (ω – 0) ....(ii)

Equation (ii) / equation (i)

$$\frac{\frac{2}{5}\omega R^2}{10} = \frac{R}{2} \qquad ....(iii)$$

V_{cm} = 10 & ωR = 12.5 kinetic friction acts



$a_{cm} = \frac{\mu mg}{m} = \mu g$ ....(iv)

$T_{\alpha c} = -\mu_k mgR = \frac{2}{5}mR^2\alpha$

Let after time t, $V'_{cm} = \omega' R$

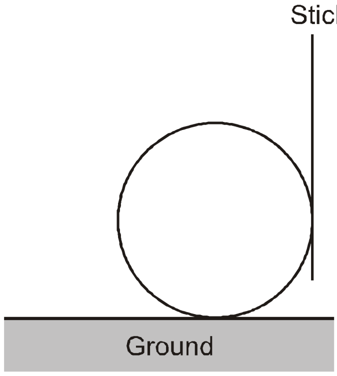
$$10 + \mu_k gt = \left[\frac{12.5}{R} - 2.5\frac{\mu_k g}{R}\right]R$$

$\Rightarrow 10 + \mu_k gt = 12.5 - 2.5\mu kg$

$\Rightarrow 3.5\ \mu kgt = 2.5$

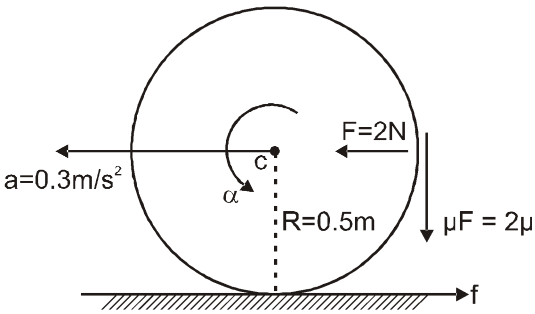
$\Rightarrow t = \frac{1}{7} \Rightarrow \frac{1}{t} = 7$

16. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a vertical stick as shown in the figure. The stick applies a normal force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P/10). The value of P is



Ans. 4

Sol.



II Law ⇒ $2 - f = 2\,[0.3]$

⇒ $f = 2 - 0.6$

$f - 1.4$ Nx ...(i)

$a = R\alpha$

⇒ $0.3 = \alpha\,[0.5]$

⇒ $\alpha = \frac{3}{5}$ rad/s ....(ii)

$\tau_c = I_c\,\alpha$

⇒ $fR - 2\mu R = mR^2\,\alpha$

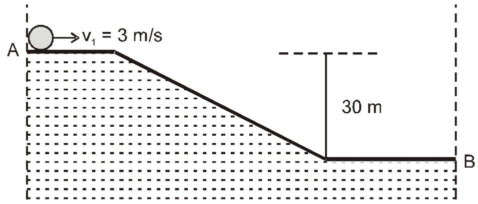
$f - 2\mu = mR\alpha$

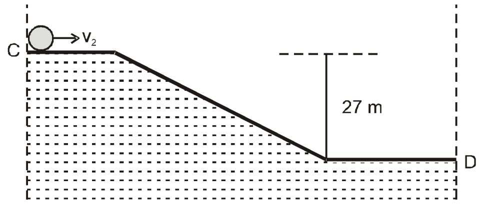
$1.4 - 2\mu = \frac{2}{2}\left(\frac{3}{5}\right)$

$1.4 - 0.6 = 2\mu$

$0.8 = 2\mu$ ⇒ $\mu = 0.4 = \frac{P}{10}$ ∴ P = 4 **Ans.**

17. Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with ths surfaces. If they reach B and D with the same linear speed and v_1 = 3 m/s, then v_2 in m/s is (g = 10 m/s²)





Ans. **7**

Sol. Final kinetic energy of both discs is same

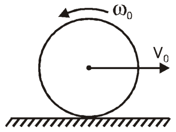
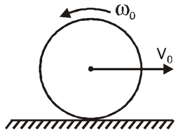
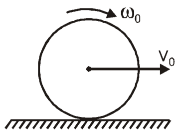
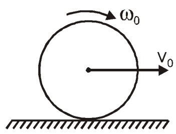
$$\left[\frac{3}{2}\right]\frac{1}{2}m(3)^2 + mg\,(30) = \frac{3}{2}\frac{1}{2}mv_2^2 + mg(27)$$

$$\frac{3}{4}.\ 9 + 300 = \frac{3}{4}v_2^2 + 270$$

$$\frac{27}{4} + 30 = \frac{3}{4}v_2^2 \Rightarrow \quad v_2^2 = 9 + 40 \Rightarrow v_2 = 7$$

Matrix Match Type :

18. A uniform disc of mass M and radius R lies on a fixed rough horizontal surface at time t = 0. Initial angular velocity ω_o of each disc (magnitude and sense of rotation) and horizontal velocity v_0 of centre of mass is shown for each situation of column-I. Match each situation in column-I with the results given in column-II.

Column-I	Column-II
(A)  It is given that $v_0 = 2R\omega_0$	(p) The angular speed keeps on decreasing till the disc stops slipping.
(B)  It is given that $2v_0 = R\omega_0$	(q) After the disc stops slipping, the angular velocity is nonzero and in clockwise sense
(C)  It is given that $v_0 = 2R\omega_0$	(r) After the disc stops slipping, the velocity of centre of disc is towards right
(D)  It is given that $2v_0 = R\omega_0$	(s) After the disc stops slipping, the kinetic energy of disc is less than its initial value.

Ans. (A) q,r,s (B) p,s (C) q,r,s (d) p,q,r,s

Sol. In all four situation of column-I, angular momentum of the disc about its point of contact on ground is conserved. Take angular momentum out of the paper as positive

(A) Initial angular momentum about its point of contact on ground $= \frac{1}{2}mR^2\omega_o - mR(2R\omega_o)$ = negative. Hence final state of the disc is as shown if figure B.

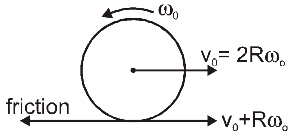


fig (a) initial state

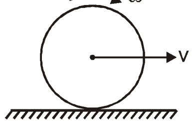


fig (b) final state

Hence angular velocity shall first decrease and then increase in opposite sense. The velocity of centre shall decrease till the disc starts rolling without slipping.

(B) The initial angular momentum about its point of contact on ground =0.

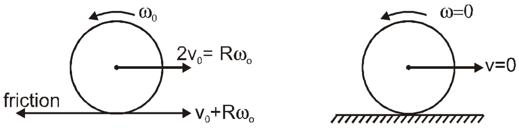


fig (a) initial state fig (b) final state

Hence angular speed and velocity of centre simultaneously reduce to zero without a change in direction.

(C) Because $v_0 > R\omega_0$, velocity of centre of mass will decrease and angular velocity will increase without a change in direction till disc starts rolling without slipping.

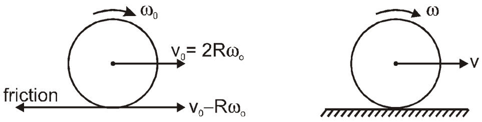


fig (a) initial state fig (b) final state

(D) Because $v_0 < R\omega_0$, velocity of centre of mass will increase and angular velocity will decrease without a change in direction till disc starts rolling without slipping.

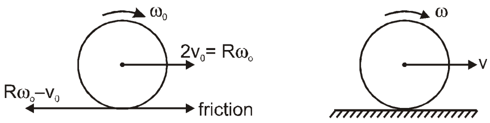
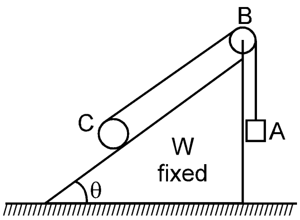


fig (a) initial state fig (b) final state

Subjective Type Questions :

19. In the figure shown there is a fixed wedge W of inclination 'θ'. A is a block, B is a disc and 'C' is a solid cylinder. A, B and C each has mass m. Assuming there there is no sliding anywhere and string to be of negligible mass find :

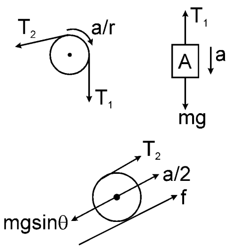


(a) the speed of A after descending distance 'x'.

(b) the friction force acting on the cylinder due to the wedge.

Ans.

Sol. Method - I



(force torque method)

F.B.D.

(a) $mg - T_1 = ma$ ...(1)

$\Rightarrow$ $T_2 + f - mg \sin\theta = m\ \frac{a}{2}$ ...(2)

$(T_2 - f)\ r = \frac{1}{2}\ mr^2 \times \frac{a}{2r}$ ...(3)

$\Rightarrow$ $(T_1 - T_2)\ r = \frac{1}{2}\ mr^2 \times \frac{a}{r}$ ...(4)

By (1), (2), (3) & (4)

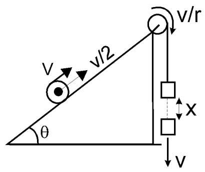
$$a = \frac{mg\left(1-\frac{\sin\theta}{2}\right)}{m\ \left(1+\frac{3}{98}+\frac{1}{2}\right)} = \frac{g(2-\sin\theta)\times 8}{2(8+3+4)} \quad \Rightarrow \quad a = \frac{4g(2-\sin\theta)}{15}$$

$\Rightarrow V_A^2 = u^2 + 2\ \frac{4g(2-\sin\theta)}{15}\ x.$ $\Rightarrow$ $\mathbf{V_A} = \sqrt{\frac{8gx}{15}(2-\sin\theta)}$ **.....Ans.**

(b) by (1), (2), (3), (4)

$f = \frac{mg}{15}\ (1+ 7\sin\theta)$

Method - II (Energy conservation)

(a) 

$$mgx = \frac{1}{2}mv^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}\left(\frac{mR^2}{2}\right)\left(\frac{v}{2R}\right)^2 + \frac{1}{2}\left(\frac{m\ \ r^2}{2}\right) + mg\ \frac{x}{2}\sin\theta$$

$Mgh\ \left(1-\frac{\sin\theta}{2}\right) = \frac{15}{16}mv^2$ .....(i)

$v = \sqrt{\frac{8g\ \ (2-\sin\theta)\ \ x}{15}}$ **....Ans.**

(b) Differentiating - (ii) w.r.t. time

$\frac{dV}{dt} = \frac{4g(2-\sin\theta)}{15} = a$

Now apply Newton's law on the cylinder

$T_2 + f = \frac{ma}{2}$ ...(1)

$T_2\ r - fr = \frac{mr^2}{2} \times \frac{a}{2r}$

$\Rightarrow$ $T_2 - f = \frac{ma}{4}$ ....(2)

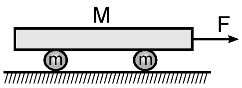
Now by Doing (1) & (2)

$2f - mg\sin\theta = \frac{ma}{4}$

$\Rightarrow \quad f = \frac{mg \sin\theta}{2} + \frac{ma}{8} = \frac{mg\sin\theta}{2} + \frac{mg\ (2-\sin\theta)}{30} = \frac{mg}{30}\ (15\sin\theta - 2 - \sin\theta)$

$\Rightarrow \quad$ **$f = \frac{mg}{15}\ (1 + 7\sin\theta)$ ....Ans.**

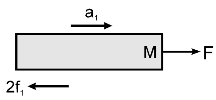
20. Two cylindrical rollers each of mass m are used to transport a log of mass M. If a horizontal force F acts on the log of wood find its acceleration. There is no slipping any where

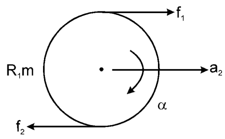


Ans. 4 F/(4M + 3m)

Sol. Each Rollers exert friction force f_1 on log f_2 is friction exerted by ground on rollers.

Equation of linear motions





$F - 2f_1 = Ma_1$ ....(1)

$f_1 - f_2 = ma_2$ ....(2)

Also, $a_1 = a_2 + R\alpha$ for no slipping & $a_2 = R\alpha$ at point of contact $a_1 = 2R\alpha$.

For Rotation of Rollear

$(f_1 + f_2)\ R = I\alpha.$

$\Rightarrow \quad f_1 + f_2 = \frac{I}{R}\alpha$ ....(3)

(2) + (3)

$2f_1 = ma_2 + \frac{I\alpha}{R}$

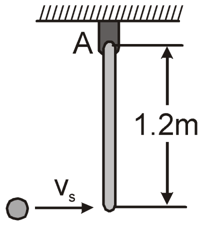
$F - ma_2 - \frac{I\alpha}{R} = Ma_1$

$F = 2MR\alpha + mR\alpha + \frac{I\alpha}{R}$

$\alpha = \frac{F}{2M\alpha + mR + \frac{I}{R}}$

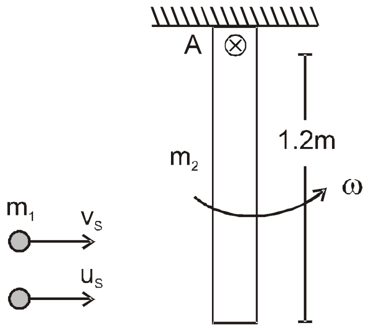
$a_1 = \frac{2RF}{2MR + mR + \frac{mR^2}{2R}} = \frac{4F}{4M + 3m}$

21. A 2 kg sphere moving horizontally to the right with an initial velocity of 5 m/s strikes the lower end of an 8 kg rigid rod AB. The rod is suspended from a hinge at A and is initially at rest. Knowing that the coefficient of restitution between the rod and sphere is 0.80, determine the angular velocity of the rod and the velocity of the sphere immediately after the impact.



Ans. $\omega = \frac{45}{14}$ = 3.21 rad/s (ccw), $v_s = \frac{1}{7}$ = 0.143 m/s ←

Sol.



Angular momentum about point A

$L_i = m_1 v_s \ell$ (u_s : Final velocity of ball after collision)

$L_f = \frac{m_2 \ell^2}{3}\omega + m_1 u_s \ell$

$L_i = L_f$

$(m_1 v_s \ell = \frac{m_2 \ell^2 . \omega}{3} + m_1 u_s \ell)$

$2 \times 5 = \frac{8 \times 1.2 \times \omega}{3} b + (2 \times u_s)$

$10 = \frac{32 \quad \omega}{10} + 2u_s$ .... (i)

Coefficient of restitution

$e = \frac{\omega \ell - u_s}{v_s}$

$0.8 = \frac{\omega \ell - u_s}{v_s}$

$\frac{4}{5} = \frac{\omega \quad (1.2) - u_s}{5}$

$4 = \frac{6\omega}{5} - u_s$

$$u_s = \left(\frac{6\omega - 20}{5}\right) \quad \text{............ (ii)}$$

Put equation (ii) in equation (i)

$$10 = \frac{32\omega}{10} + 2\left(\frac{6\omega - 20}{5}\right)$$

$$10 = \frac{32\omega}{10} + \frac{12\omega - 40}{5}$$

$$100 = 32\omega + 24\omega - 80$$

$$\omega = \frac{45}{14}$$

Put ω in equation (ii)

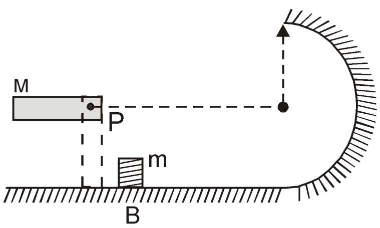
$$u_s = \left(\frac{6\omega - 20}{5}\right)$$

$$u_s = \frac{6\left(\frac{45}{14}\right) - 20}{5}$$

$$u_s = \frac{270 - 280}{14 \times 5} = -\frac{10}{14 \times 5} = \left(-\frac{1}{7}\right)$$

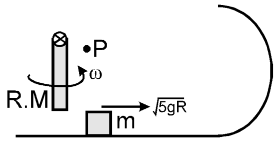
So direction is $(\leftarrow)$ $u_s \left(\frac{1}{7}\right)$

22. A rod of length R and mass M is free to rotate about a horizontal axis passing through hinge P as in figure. First it is taken aside such that it becomes horizontal and then released. At the lowest point the rod hits the small block B of mass m and stops. Find the ratio of masses such that the block B completes the circular track of radius R. Neglect any friction.



Ans. $\frac{M}{m} = \sqrt{15}$

Sol. Minimum velocity required by block 'm' to complete the motion in $\sqrt{5gR}$



conserving mech. energy

$$\frac{1}{2}I\omega^2 = Mg \cdot \frac{R}{2} \quad \Rightarrow \quad \omega = \sqrt{\frac{MgR}{I}}$$

Cons. angular momentum wrt P before & after collision.

$I\omega = m.R\sqrt{5gR}$

$I\sqrt{\frac{MgR}{I}} = mR\sqrt{5gR}$

$MgRI = m^2R^2\, 5gR$

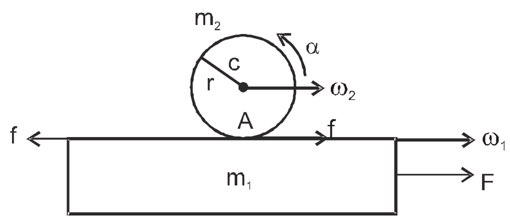
putting $I = \frac{ML^2}{3}$

$\frac{M}{m} = \sqrt{15}$

23. A plank of mass m_1 with a uniform sphere of mass m_2 placed on it rests on a smooth horizontal plane. A constant horizontal force F is applied to the plank. With what accelerations will the plank and the centre of the sphere move provided there is no sliding between the plank and the sphere ?

Ans. $w_1 = F/(m_1 + 2/7m_2)$; $w_2 = 2/7\, w_1$.

Sol.



α = angular acceleration

α = angular acceleration

For the plank

$F - f = m_1\omega_1$ .... (i)

For sphere torque about point C

$fr = I_c\alpha = \frac{2}{5}m_2r^2\alpha$ .... (ii)

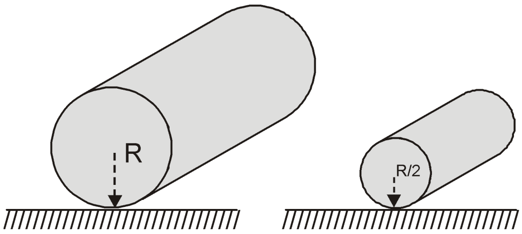
assuming ω_2 is the acceleration of COM of sphere at point A

$(\omega_1 = \omega_2 + \alpha r)$ .... (iii)

From equation (i), (ii) and (iii)

$\omega_1 = \frac{F}{\left(m_1 + \frac{2}{7}m_2\right)}$ and $\omega_2 = \left(\frac{2}{7}\omega_1\right)$

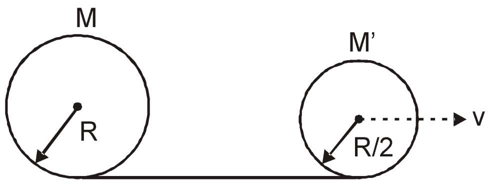
24. A carpet of mass 'M' made of inextensible material is rolled along its length in the form of a cylinder of radius 'R' and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. Calculate the horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to R/2.



Ans. $\upsilon = \sqrt{\dfrac{14gR}{3}}$

Sol. Let M be the mass of unwound carpet. Then ,

$$M' = \left(\frac{M}{\pi R^2}\right)\pi\left(\frac{R}{2}\right)^2 = \frac{M}{4}$$



From conservation of mechanical energy :

$$MgR - M'g\frac{R}{2} = \frac{1}{2}\left(\frac{M}{4}\right)v^2 + \frac{1}{2}I\omega^2$$

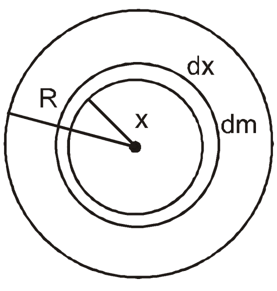
or $$MgR - \left(\frac{M}{4}\right)g\left(\frac{R}{2}\right) = \frac{Mv^2}{8} + \frac{1}{2}\left(\frac{1}{2}\times\frac{M}{4}\times\frac{R^2}{4}\right)\left(\frac{v}{R/2}\right)^2$$

or $$\frac{7}{8}MgR = \frac{3Mv^2}{16} \qquad \therefore \qquad v = \sqrt{\frac{14\,Rg}{3}}$$

25. The surface mass density (mass/area) of a circular disc of radius 'R' depends on the distance from the centre x given as, $\sigma(x) = \alpha + \beta x$. Where α and β are positive constant find its moment of inertia about the line perpendicular to the plane of the disc through its centre.

Ans. $2\pi\left(\dfrac{\alpha R^4}{4} + \dfrac{\beta R^5}{5}\right)$

Sol.



$$dm = (2\pi x dx)\sigma$$

$$I = \int dm \cdot x^2 = \int_o^R (2\pi x dx)\sigma . x^2$$

$$I = 2\pi \int_0^R x^3 \sigma dx$$

$$I = 2\pi \int_0^R x^3 \cdot (\alpha + \beta x) dx = 2\pi \left[\int_o^R \alpha \quad x^3 dx + \int_o^R \beta \quad x^4 dx \right.$$

$$I = 2\pi \left(\frac{\alpha R^4}{4} + \frac{\beta R^5}{5} \right)$$