competishun Power of real gurus

PHYSICS

TARGET : JEE- Advanced 2021

CAPS-8

ROTATIONAL DYNAMICS-1

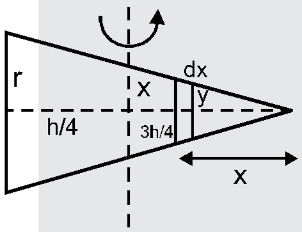
SCQ (Single Correct Type) :

1. Moment of inertia of a uniform solid cone about an axis passing through its centre of gravity and parallel to its base is (m is mass of cone, its height is h and its radius R = h)

(A) $\frac{10}{3}MR^2$ (B) $\frac{3}{5}MR^2$ (C) $\frac{3}{10}MR^2$ (D) $\frac{3}{16}MR^2$

Ans. **(D)**

Sol.



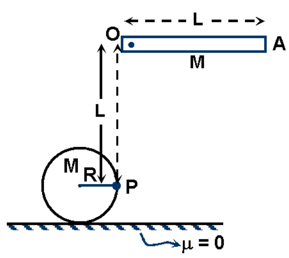
$$I_C = \int_0^R \frac{M}{\frac{1}{3}\pi R^2.R}.\pi x^2.dx\left[\frac{x^2}{3}+x^2\right]$$

$$= \frac{3M}{R^3}\int_0^R\left(\frac{x^4}{4}+x^4\right)dx \quad = \frac{3M}{R^3}\left(\frac{R^5}{20}+\frac{R^5}{5}\right) \quad = \frac{3}{4}MR^2$$

Applying parallel axis theorem,

$$\Rightarrow \quad I_C = I_{cm} + \left(\frac{3R}{4}\right)^2 = \frac{3}{4}MR^2 \qquad I_{cm} = \frac{3MR^2}{16}$$ **Ans.**

2. A slender rod of mass M and length L hinged at O is kept horizontal and then released. The other end of the rod strikes a solid sphere of mass M and radius R (at point P) kept on a smooth horizontal surface. The points O and P are on the same vertical line. After the collision, the rod comes to rest. The angular speed of the sphere after the collision is

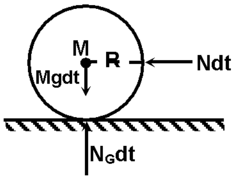


(A) $\sqrt{\frac{3g}{L}}$ (B) $\frac{R}{L}\sqrt{\frac{3g}{L}}$ (C) $\frac{L}{R}\sqrt{\frac{3g}{L}}$ (D) zero

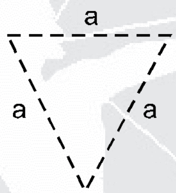
Ans. **(D)**

Sol. Draw the F.B.D of sphere.

Since all the impulses acting on the sphere are passing through the centre of the sphere, the angular velocity remains zero.



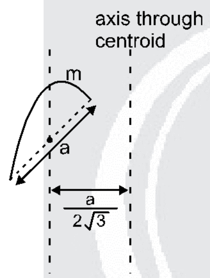
3. All sides of an equilateral triangle are diameter of three identical uniform semicircular rings each of mass m. Plane of each ring is perpendicular to the plane of paper. Then moment of inertia of this system of three semicircular rings about an axis through centroid of triangle and perpendicular to plane of paper is :

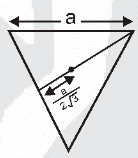


(A) $\frac{5ma^2}{24}$ (B) $\frac{5ma^2}{16}$ (C) $\frac{5ma^2}{8}$ (D) $\frac{5ma^2}{6}$

Ans. **(C)**

Sol.



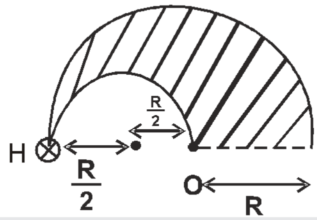


$$I' = I_{cm} + m\left(\frac{a}{2\sqrt{3}}\right)^2 = \frac{5ma^2}{24}$$

$$I_{cm} = \frac{m\left(\frac{a}{2}\right)^2}{2} = \frac{ma^2}{8}$$

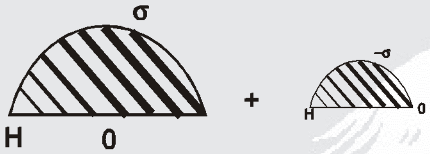
$$I = 3I' = \frac{5ma^2}{8}.$$

4. A uniform circular plate of radius R, from which a semicircular portion of radius $\frac{R}{2}$ is removed, shown in figure. The moment of inertia about an axis passing through its end 'H' and perpendicular to the plane of plate is [Assume mass of remaining portion of the plate is m]



(A) $\frac{13}{8}mR^2$ (B) $\frac{15}{8}mR^2$ (C) $\frac{3}{8}mR^2$ (D) $\frac{mR^2}{8}$

Ans. **(B)**



Sol. **(b)**

$$\sigma = \frac{8m}{3\pi R^2}$$

moment of inertia of larger plates ;

$$I_{H_1} = \frac{\sigma\left(\frac{\pi R^2}{2}\right).R^2}{2} - \sigma\left(\frac{\pi R^2}{2}\right)\left[\frac{4R}{3\pi}\right]^2 + \sigma\left(\frac{\pi R^2}{2}\right)\left[R^2 + \left(\frac{4R}{3\pi}\right)^2\right]$$

$$= \frac{3}{2}\sigma\left(\frac{\pi R^2}{2}\right).R^2$$

similarly ;

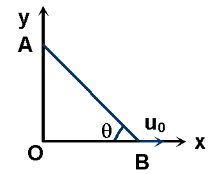
moment of inertia of smaller plates ;

$$I_{H_2} = = \frac{3}{2}\left\{-\sigma\left(\frac{\pi R^2}{8}\right)\right\}\frac{R^2}{4}$$

So

$$I_H = \frac{3}{2}\sigma\pi R^4.\left[\frac{1}{2} - \frac{1}{32}\right] = \frac{45}{64}\sigma\pi R^4 = \frac{15}{8}mR^2$$

5. The end B of the rod AB which makes angle θ with the floor is being pulled with a constant velocity u_0 as shown. The length of the rod is ℓ. Which of the following is/are correct?



(A) Velocity of end A is $\frac{7}{3}u_0$ downwards when $\theta = 37°$.

(B) Angular velocity of the rod is $\frac{5}{3}\frac{u_0}{\ell}$ when $\theta = 37°$.

(C) Angular velocity of the rod is constant.

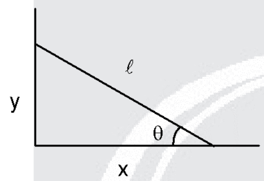
(D) Velocity of end A is constant.

Ans. **(B)**

Sol. $x^2 + y^2 = \ell^2$

$\Rightarrow\ 2x.\frac{dx}{dt} + 2y.\frac{dy}{dt} = 0$

$\therefore\ x.\frac{dx}{dt} + y.\frac{dy}{dt} = 0$

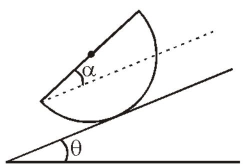


$\frac{dy}{dt} = -\frac{x}{y}.\frac{dx}{dt}$

Further $\ell \sin\theta = \ell\left(\frac{3}{5}\right)$

Applying $u_0 = \omega.\frac{3\ell}{5}$, $\omega = \frac{5}{3}\frac{u_0}{\ell}$

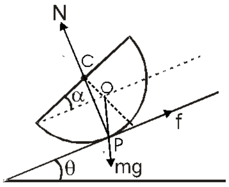
6. A uniform thin hemispherical shell is kept static on an inclined plane of angle θ = 30º as shown. If the surface of the inclined plane is sufficiently rough to prevent sliding then the angle α made by the plane of hemisphere with inclined plane is :

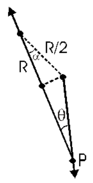


(A) value of μ is needed (B) 30º

(C) 45º (D) 60º

Ans. **(D)**

Sol. 



O is the centre of mass of the hollow hemisphere and is $\frac{R}{2}$ from C.

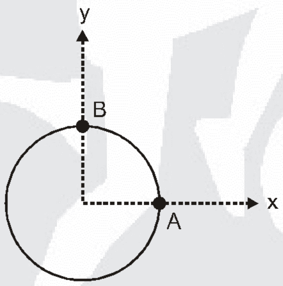
$f = mg \sin \theta$ .... (1)

$N = mg \cos \theta$ .... (2)

$N \times \frac{R}{2} \sin \alpha = \left[R - \frac{R}{2}\cos\alpha\right] f$ .....(3)

$\therefore \quad \tan \theta = \frac{\sin\alpha}{2-\cos\alpha} \Rightarrow \alpha = 60^o$

7. A non-uniform disc of mass m and radius R, hinged at some point and performing pure rotation with respect to hinge, in horizontal plane with an angular velocity ω. At certain instant center of the disc is at origin in the mentioned co-ordinate system and velocity of particle A is $\vec{V}_A = -\frac{\omega R}{4}\left(3\hat{i} - 4\hat{j}\right)$ m/s. Velocity of particle B at the given instant is –

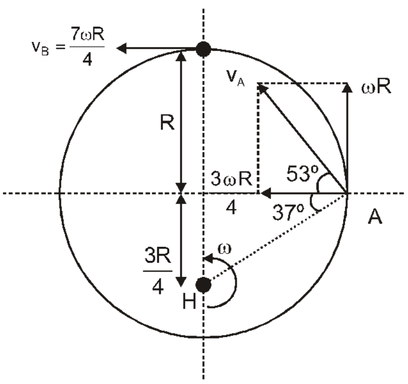


(A) $\frac{4}{5}\omega R\left(\hat{i} - \hat{j}\right)$ (B) $\frac{4}{5}\omega R\left(\hat{j} - \hat{i}\right)$ (C) $-\frac{7\omega R}{4}\hat{i}$ (D) $-\frac{4\omega R}{5}\hat{i}$

Ans. **(C)**

Sol. $v_A = \frac{\omega R}{4}\sqrt{9+16} = \frac{5\omega R}{4} = \omega r \Rightarrow r = \frac{5R}{4}$

$\vec{v}_B = -\frac{7\omega R}{4}\hat{i}$

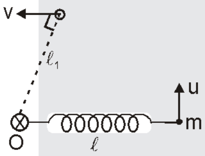


8. In the figure shown a particle of mass 'm' which is tied to an end of massless spring of natural length ℓ is given a velocity 'u' parpendicular to the spring. Initially the spring is undeformed. The maximum extension of the spring is 'ℓ'. The motion is on a horizontal smooth surface. The spring constant is

(A) $\frac{mu^2}{\ell^2}$ (B) $\frac{mu^2}{8\ell^2}$ (C) $\frac{mu^2}{4\ell^2}$ (D) $\frac{3mu^2}{4\ell^2}$

Ans. **(D)**

Sol.



At maximum extension, v is perpendicular to the spring.

Angular momentum about O will conserve.

$$mu.\ \ell = mv.\ \ell_1$$

$\Rightarrow$ $u\ell = v\ell_1$ ...(i)

Mechanical Energy will conserver

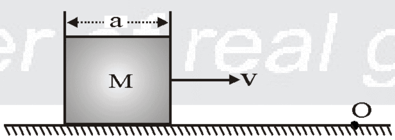
$$\frac{1}{2}mu^2\ \frac{1}{2}mv^2 + \frac{1}{2}k(\ell_1 - \ell)^2 \quad \text{...(ii)}$$

As $\ell_1 - \ell = \ell$ $\Rightarrow$ $\ell_1 = 2\ell$, we get

$$\frac{1}{2}mu^2 = \frac{1}{2}m\left(\frac{u}{2}\right)^2 + \frac{1}{2}k.\ \ell^2$$

$\Rightarrow$ $k = \frac{3}{4}\frac{mu^2}{\ell^2}$

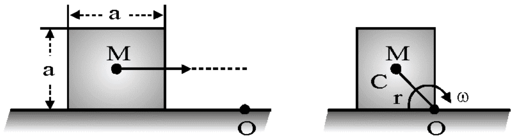
9. A cubical block of side 'a' moving with velocity v on a horizontal smooth plane as shown. It hits a ridge at point O. The angular speed of the block after it hits O is :-



(A) $\frac{3v}{4a}$ (B) $\frac{3v}{2a}$ (C) $\frac{\sqrt{3}}{\sqrt{2}a}$ (D) zero

Ans. **(A)**

Sol. $r = \sqrt{2}\dfrac{a}{2} \Rightarrow r^2 = \dfrac{a^2}{2}$



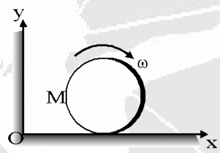
Net torque about O is zero. Therefore, angular momentum (L) about O will be conserved

$\Rightarrow L_i = L_f$

$Mv\left(\dfrac{a}{2}\right) = I_0\omega = (I_{CM} + Mr^2)\,\omega$

$= \left\{\left(\dfrac{Ma^2}{6}\right) + M\left(\dfrac{a^2}{2}\right)\right\}\omega = \dfrac{2}{3}Ma^2\omega \qquad \Rightarrow \omega = \dfrac{3v}{4a}$

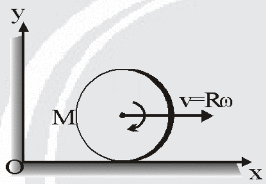
10. A disc of mass M and radius R is rolling (pure) with is angular speed ω on a horizontal plane as shown. The magnitude of angular momentum of the disc about the origin O is :-



(A) $\left(\dfrac{1}{2}\right)MR^2\omega$ (B) $MR^2\omega$ (C) $\left(\dfrac{3}{2}\right)MR^2\omega$ (D) $2MR^2\omega$

Ans. **(C)**

Sol. From the theorem



$\vec{L}_0 = \vec{L}_{CM} + M(\vec{r} \times \vec{v})$ ...(i)

We may write :

Angular momentum about O= Angular momentum about CM + Angular momentum of CM about origin.

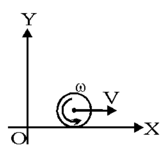
$\therefore L_0 = I\omega + MRv = \dfrac{1}{2}MR^2\omega + MR(R\omega) = \dfrac{3}{2}MR^2\omega$

Note that in this case both the terms in Equation (i) i.e. $\vec{L}_{CM}$ and have the same direction ⊗.

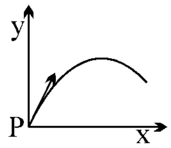
That is why we have used $M(\vec{r} \times \vec{v})$ $L_0 = I\omega + MRv$

We will use $L_0 = I\,\omega \sim MRv$

if they are in opposite direction as shown in figure.



11. At time t = 0, a ball of mass 'm' is thrown vertically from a point P (x = 0, y = 0) with initial velocity $(v_x\hat{i}+v_y\hat{j})$. The acceleration due to gravity is $\vec{g}=-g\hat{j}$. The angular momentum vector $\vec{L}(t)$ of the ball about the point P at time t is



(A) $-\dfrac{mv_xgt^2}{2}\hat{k}$ (B) $2mv_xv_yt\hat{k}$

(C) $(-2mv_xv_yt+\dfrac{v_xgt^2}{2})\hat{k}$ (D) $-2mv_xv_yt\hat{i}+\dfrac{mv_xgt^2}{2}\hat{j}$

Ans. **(A)**

Sol. $\Delta\vec{L}=\int T dt$

$\Delta\vec{L}=\int_0^t -mgv_x t dt\,\hat{k}$

$$\begin{cases} T=\vec{r}\times\vec{F} \\ \quad =(v_xt\hat{i})\times(-mg\hat{j})=-mgv_xt\hat{k} \end{cases}$$

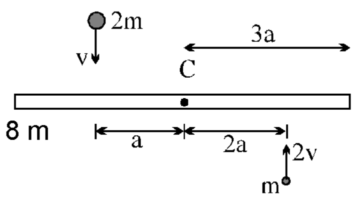
$\vec{L}_f-\vec{L}_i=-mgv_x\dfrac{t^2}{2}\hat{k}$

$\vec{L}_i=\vec{0}$

$\vec{L}_f=-mgv_x\dfrac{t^2}{2}\hat{k}$

MCQ (One or more than one correct) :

12. A uniform bar of length 6a and mass 8m lies on a smooth horizontal table. Two point masses m and 2m moving in the same horizontal plane with speeds 2v, and v, respectively, strike the bar (as shown in figure) and stick to the bar after collision. Denoting angular velocity, total energy and velocity of centre of mass by ω, E and V_c respectively, we have after collision



(A) $V_c=0$ (B) $\omega=\dfrac{3v}{5a}$ (C) $\omega=\dfrac{v}{5a}$ (D) $E=\dfrac{3mv^2}{5}$

Ans. **(ACD)**

Sol. Considering the two balls and bar as system

$P_b = 0$

$\therefore V_c = 0$

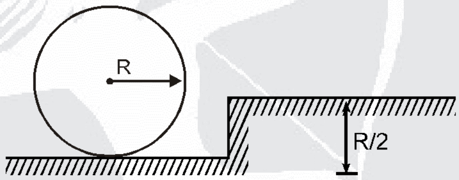
Taking C for conservation of angular momentum, we have

$$2mva + m \times 2v \times 2a = \left\{\frac{8m(6a)^2}{12} + 2ma^2 + m(2a)^2\right\}\omega$$

$$\omega = v/5a$$

$E = \frac{1}{2} \times 30ma^2 \times \omega^2 = \frac{3}{5}mv^2$

13. A wheel (to be considered as a ring) of mass m and radius R rolls without sliding on a horizontal surface with constant velocity v. It encounters a step of height R/2 at which it ascends without sliding.



(A) the angular velocity of the ring just after it comes in contact with the step is 3v/4R

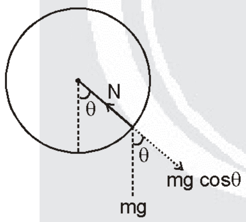
(B) the normal reaction due to the step on the wheel just after the impact is $\frac{mg}{2} + \frac{9\,mv^2}{16\,R}$

(C) the normal reaction due to the step on the wheel increases as the wheel ascends

(D) the friction will be absent during the ascent.

Ans. **(AC)**

Sol.



By angular momentum conservation ;

$L = I\omega \Rightarrow mv\frac{R}{2} + mvR = 2mR^2\omega$

$\frac{3}{2}mvR = 2mR^2\omega$

$\omega = \frac{3v}{4R}$

Also at the time of contact ;

$mg\cos\theta - N = \frac{mv^2}{R}$

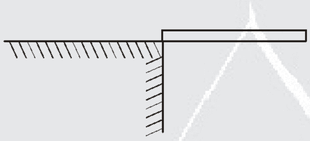
$\therefore$ N = mg cosθ – $\dfrac{mv^2}{R}$

when it ascends θ decreases so cosθ increases and v decreases.

$\therefore$ mgcosθ is increasing and $\dfrac{mv^2}{R}$ is decreasing

$\therefore$ we can say N increases as wheel ascends.

14. A uniform rod of length *l* is falling down with a velocity v_0 when one of it's end hits a fixed edge as shown.



(A) If the collision is elastic, the centre of mass may have upwards velocity just after the collision

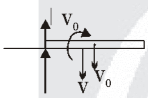
(B) If the collision is partially elastic, the centre of mass may come to rest just after the collision

(C) If the collision is perfectly inelastic, the centre of mass has a downward velocity just after the collision

(D) If the collision is elastic, the centre of mass has a downward velocity just after the collision

Ans. **(CD)**

Sol. $mv - mv_0 = \int N dt$



$$\frac{m\ell^2}{12}\omega = \frac{\ell}{2}\int N dt$$

$$v - (-ev_0) = \frac{\omega\ell}{2}$$

$$v + ev_0 = \frac{\omega\ell}{2}$$

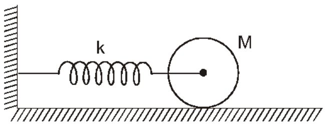
$$3 \times mv - mv_0 + \frac{m\omega\ell}{6} = 0$$

$$4v + (e - 3)\,v_0 = 0$$

$$v = \frac{(3-e)v_0}{4}$$

Comprehension Type Question:

A solid cylinder attached with a horizontal massless spring (see figure) can roll without slipping along a horizontal surface. If the system is released from rest at a position in which spring is stretched by 0.3 m.



15. Period of oscillation of c.o.m of cylinder is

(A) $2\pi\sqrt{\frac{M}{k}}$ (B) $2\pi\sqrt{\frac{3M}{2k}}$ (C) $2\pi\sqrt{\frac{2M}{3k}}$ (D) $2\pi\sqrt{\frac{M}{2k}}$

Ans. **(B)**

Sol. a = (kx – f)/m

f × R = mR^2 / 2 × a/R

16. If k = 20 N/m, Rotational K.E. of cylinder as it passes through mean position is

(A) 0.3 J (B) 0.45 J (C) 0.6 J (D) 0.9 J

Ans. **(A)**

Sol. by energy conservation

$$\frac{1}{2}\times 20\times 0.3\times 0.3 = \frac{1}{2}\times\frac{mR^2}{2}\times\frac{v^2}{R^2}+\frac{1}{2}mv^2$$

mv^2 = 1.2 J

Rotational K.E. = $\frac{mv^2}{4}$ = 0.3 J

17. Translation K.E. of cylinder as it passes through mean position is

(A) 0.3 J (B) 0.45 J (C) 0.6 J (D) 0.9 J

Ans. **(C)**

Sol. by energy conservation

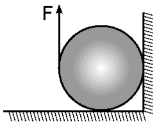
$$\frac{1}{2}\times 20\times 0.3\times 0.3 = \frac{1}{2}\times\frac{mR^2}{2}\times\frac{v^2}{R^2}+\frac{1}{2}mv^2$$

mv^2 = 1.2 J

Translation K.E. = $\frac{mv^2}{2}$ = 0.6 J

Numerical based Questions :

18. Figure shows a vertical force F that is applied tangentially to a uniform cylinder of weight W. The coefficient of static friction between the cylinder & all surfaces is 0.5. Find in terms of W. the maximum force that can be applied without causing the cylinder to rotate.



Ans. **3w/8**

Sol. When F is maximum equation. of rotational equilibrium.

$$F.R. = \mu (N_1 + N_2) R \qquad ....(1)$$

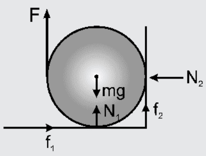
For equilibrium in horizontal direction

$$f_1 = N_2 = \mu N_1 \qquad ....(2)$$

In vertical direction

$$F + N_1 = mg$$

$$F = \mu [(mg - F) + \mu (mg - F)]$$

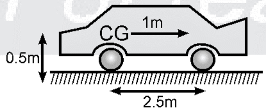


$$\frac{1}{2}\left[(mg-F)+\frac{1}{2}(mg-F)\right] \qquad \left[\text{putting } \mu=\frac{1}{2}\right]$$

$$F\left[1+\frac{1}{2}+\frac{1}{2}\right] = \frac{3}{4}mg$$

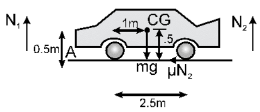
$$F = \frac{3}{8}mg = \frac{3}{8}w$$

19. A 2000 kg car is driven by an engine which provides torque to two rear driving wheels. The wheel base of the car is 2.5 m. The centre of the mass of the car is located 1 m from the front wheel at a height of 0.5 m from the ground. If μ = 0.7 is the friction with the road, calculate the minimum time taken to accelerate the car to 60 kmph. Neglect friction on front wheels.



Ans. **5.95 sec.**

Sol. For minimum time to accelerate, acceleration 'a' must be maximum. It's maximum when driving force friction on rear wheels is max (limiting static).



From FBD for vertical equilibrium

$N_1 + N_2 = Mg$

For rotational equation, taking torques about A

$Mg \times 1m = N_2 \times 2.5 \quad \Rightarrow \quad N_2 = \dfrac{Mg}{2.5}$

Now maximum acceleration = $\dfrac{\mu N_2}{M}$

$$= \frac{\mu Mg}{M \times 2.5} = \frac{\mu g}{2.5} = \frac{0.7g}{2.5}$$

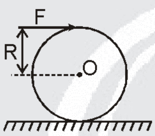
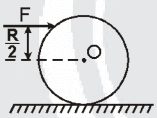
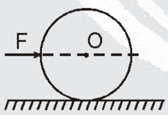
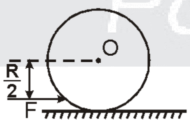
Now applying v = 4 + at

$$60 \times \frac{5}{18} = 0 + \frac{.7 \times gt}{2.5}$$

$$t = \frac{60 \times 5 \times 2.5}{18 \times .7 \times g} = 5.95 \text{ sec.}$$

Matrix Match Type :

20. A uniform solid cylinder of mass m and radius R is placed on a rough horizontal surface where friction is sufficient to provide pure rolling. A horizontal force of magnitude F is applied on cylinder at different positions with respect to its centre O in each of four situations of column-I, due to which magnitude of acceleration of centre of mass of cylinder is a. Match the appropriate results in column-II for conditions of column-I.

Column-I	**Column-II**
(A) 	(p) Friction force on cylinder will not be zero.
(B) 	(q) $a = \dfrac{F}{m}$
(C) 	(r) $a \neq \dfrac{F}{m}$
(D) 	(s) the direction of friction force acting on cylinder is towards left

Ans. (A) p,r (B) q (C) p,r,s (D) p,r,s

Sol. Assume friction to be absent and horizontal force F is applied at a distance x above centre.

$\therefore a = \dfrac{F}{m}$ .... (1)

and $Fx = \dfrac{mR^2}{2}\alpha$ or $R\alpha = \dfrac{2Fx}{mR}$ .... (2)

if $a = R\alpha$ or, from equation (1) and (2) $x = \frac{R}{2}$

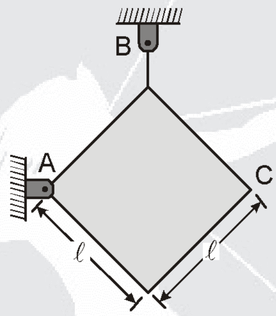
the friction force will be zero and $a = \frac{F}{m}$.

If $a > R\alpha$ or $x < \frac{R}{2}$ friction force is towards left and $a \neq \frac{F}{m}$

If $a < R\alpha$ or $x > \frac{R}{2}$ friction force is towards right and $a \neq \frac{F}{m}$.

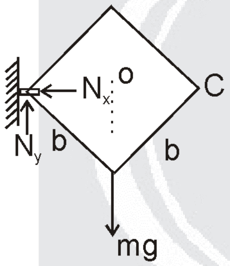
Subjective Type Questions :

21. A uniform square plate of mass m is supported as shown. If the cable suddenly breaks, determine just after that moment. The angular acceleration of the plate.



Ans. $\frac{3g}{2\sqrt{2}\,\ell}$ (cw)

Sol.



$$mg\left(b/\sqrt{2}\right) = I\alpha \text{ , } I = \frac{mb^2}{6} + m\left(\frac{b}{\sqrt{2}}\right)^2$$

$$\Rightarrow \quad I = \frac{mb^2}{6} + \frac{mb^2}{2} = \frac{mb^2}{2}\left(1+\frac{1}{3}\right)$$

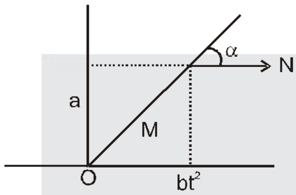
$$I = \frac{2mb^2}{3}$$

Hence $\frac{mgb}{\sqrt{2}} = \frac{2mb^2}{3}\alpha \Rightarrow \alpha = \frac{3g}{2\sqrt{2}b}$

22. The angular momentum of a particle relative to a certain' point O varies with time as $\vec{M} = \vec{a} + \vec{b}t^2$, where $\vec{a}$ and $\vec{b}$ are constant vectors, with $\vec{a} \perp \vec{b}$. Find the force moment N relative to the point O acting on the particle when the angle between the vectors N and M equals 45°.

Ans. $N = 2\vec{b}\sqrt{\frac{a}{b}}$

Sol.



Force moment relative to point O

$$\vec{N} = \frac{d\vec{M}}{dt} = 2\vec{b}t$$

Let the angle between $\vec{M}$ and $\vec{N}$

$$\alpha = 45^o \text{ at } t = t_0$$

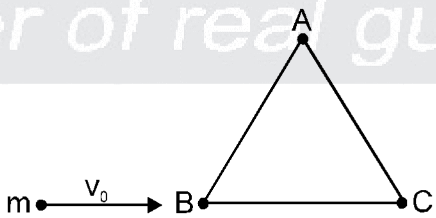
Then $\frac{1}{\sqrt{2}} = \frac{\vec{M} \;.\; \vec{N}}{|\,M\,|\;\;|\,N\,|} = \frac{(\vec{a} + \vec{b}t_0^2).2\vec{b}t_0}{\sqrt{a^2 + b^2t_0^4} \quad 2bt_0}$

$$= \frac{2b^2t_0^3}{\sqrt{a^2 + b^2t^4} \;.\; 2bt_0} = \frac{bt_0^2}{\sqrt{a^2 + b^2t_0^4}}$$

Solving, $t_0 = \sqrt{\frac{a}{b}}$ (as t_0 cannot be nagative)

Therefore $\vec{N} = 2\vec{b}t_0 = 2\vec{b}\sqrt{\frac{a}{b}}$

23. Three particles A, B, C of mass m each are joined to each other by massless rigid rods to form an equilateral triangle of side a. Another particle of mass m hits B with a velocity v_0 directed along BC as shown. The colliding particle stops immediately after impact.



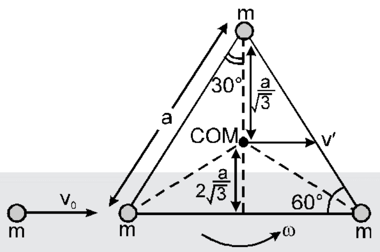
Calculate the time required by the triangle ABC to complete half revolution in its subsequent motion.

Ans. $t = \frac{6a\pi}{\sqrt{3}\,v_0}$

Sol. After collision, let COM move by velocity v' and system starts rotating by angular velocity ω about COM.

Using cons. of linear momentum

$$mv_0 = 3mv' \quad \Rightarrow \quad v' = \frac{v_0}{3}$$



conserving angular momentum about COM

$$mv_0 \cdot \frac{a}{2\sqrt{3}} = I\omega \qquad = \left(\frac{ma^2}{3} \times 3\right) . \omega$$

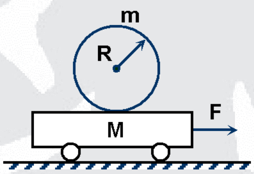
$$= ma^2\omega$$

$$\omega = \frac{v_0}{2\sqrt{3}\ a}$$

Time to complete half revolution.

$$t = \frac{\pi}{\omega} = \frac{2\sqrt{3}\ a\pi}{v_0}$$

24. Determine the maximum horizontal force F that can be applied to the plank of mass M for which the solid sphere does not slip as it begins to roll on the plank. The sphere has mass m and radius R. The coefficient of static and kinetic friction between the sphere and the plank are μ_s and μ_k respectively.



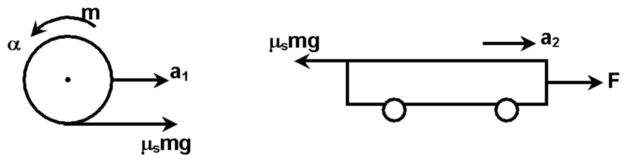
Ans. $F = \mu_s g\left(m + \frac{7}{2}M\right)$

Sol. For the sphere

$$a_1 = \frac{\mu_s mg}{m} = \mu_s g$$

Angular acceleration,

$$\alpha = \mu_s mgR \Big/ \frac{2}{5}mR^2 = \frac{5}{2}\frac{\mu_s g}{R}$$



For the plank,

$$a_2 = \frac{F - \mu_s mg}{M}$$

For no slipping $a_2 = a_1 + R\alpha$

Solving the above equation

$$F = \mu_s g\left(m + \frac{7}{2}M\right)$$

25. A right circular uniform solid cone of mass m and semi vertical angle α is placed over a rough horizontal surface. Assuming friction to be sufficient to prevent slipping, find the minimum force that can be applied at the tip of the cone so that it just topples.

Ans. $\therefore\ F_{min} = \dfrac{mg\tan\alpha}{\cos\alpha + \dfrac{\sin^2\alpha}{\cos\alpha}}$ = mg sin α

Sol. Assume the force applied at an angle θ from the horizontal as shown

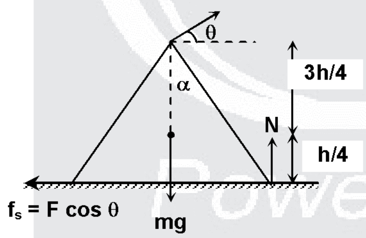
$\therefore$ N = mg − F sin θ

For rotational equilibrium, $\tau_{Cm} = 0$

$$F\cos\theta.\frac{H}{4} + F\cos\theta.\frac{3H}{4} - N.H\tan\alpha = 0$$

By putting the value of N

$$F = \frac{mg\tan\alpha}{\cos\theta + \tan\alpha.\sin\theta}$$



For maximum or minimum, F, $\dfrac{dF}{d\theta} = 0 \Rightarrow \theta = \alpha$

$$\therefore\ F_{min} = \frac{mg\tan\alpha}{\cos\alpha + \dfrac{\sin^2\alpha}{\cos\alpha}} = mg\sin\alpha$$