competishun — Power of real gurus

# PHYSICS

TARGET : JEE- Advanced 2021

# CAPS-6

## CIRCULAR MOTION

## ANSWER KEY OF CAPS-6

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | (A) | **2.** | (D) | **3.** | (D) | **4.** | (D) | **5.** | (D) |
| **6.** | (A) | **7.** | (D) | **8.** | (A) | **9.** | (B) | **10.** | (B) |
| **11.** | (A) | **12.** | (D) | **13.** | (BCD) | **14.** | (BCD) | | |
| **15.** | (ACD) | **16.** | (D) | **17.** | (B) | **18.** | (C) | **19.** | 8 |

**20.** $u = \sqrt{\frac{48}{5}g}$ **21.** (A) q,s (B) p (C) p (D) q,r

**22.** (a) $K = \frac{mg}{R\ (3-2\sqrt{2})}$ (b) at intial instant $a_t = g$, $a_c = 0$, at bottommost position $a_t = 0$ $a_c = 0$

**23.** $F = -m\omega^2 r$, where r is the radius vector of the particle relative to the origin of coordinates;

$F = m\omega^2\sqrt{x^2+y^2}$ **24.** 2 rad/sec. **25.** (a) $\frac{\pi^2 R}{20\ T^2}(-3\hat{i} + 4\hat{j})$ m/s² (b) $\frac{\pi R}{\sqrt{2}\ T^2}$ m/s²

**SCQ (Single Correct Type) :**

**1.** A long horizontal rod has a bead, which can slide along its length and initially placed at a distance L from one end A of the rod. The rod is set into angular acceleration $\alpha$. If the coefficient of friction between the rod and the bead is $\mu$ and gravity is neglected, then time after which bead starts slipping is

(A) $\sqrt{\frac{\mu}{\alpha}}$ (B) $\frac{\mu}{\sqrt{\alpha}}$ (C) $\frac{1}{\sqrt{\mu\alpha}}$ (D) infinitesimal

**Ans.** (A)

**Sol.** Tangential acceleration $\alpha = L\alpha$

As there is no gravity, normal reaction is perpendicular to the tangential acceleration,

$N = M\alpha = ML\alpha$

Frictional force $F = \mu N = \mu ML\alpha$

For no sliding along the length

frictional force > centripetal force

$\mu ML\alpha \geq \cdot\ ML\omega^2$

$\omega = \omega_0 + \alpha t = \alpha t$ (as $\omega_0 = 0$)

For sliding $\mu ML\alpha \leq ML(\alpha t)^2$ For just sliding $t = \sqrt{\frac{\mu}{\alpha}}$

**2.** A particle is moving with constant angular acceleration ($\alpha$) in a circular path of radius $\sqrt{3}$ m. At t = 0, it was at rest and at t = 1 sec, the magnitude of its acceleration becomes $\sqrt{6}$ m/s$^2$, then $\alpha$ is :

(A) 2 rad/s$^2$ (B) $\sqrt{3}$ rad/s$^2$ (C) $\sqrt{2}$ rad/s$^2$ (D) 1 rad/s$^2$

**Ans.** **(D)**

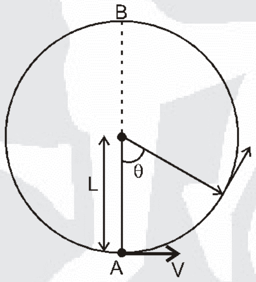
**Sol.** $\sqrt{6} = \sqrt{\left(\frac{v^2}{R}\right)^2 + (R\alpha)^2}$ (since $a_C = \frac{v^2}{R}$ ; $a_t = R\alpha$ )

$\sqrt{6} = \sqrt{\frac{((R\alpha)^2)^2}{R^2} + R^2\alpha^2}$ v = (R$\alpha$).1

$\therefore$ $\sqrt{6} = \sqrt{\frac{(\alpha R)^4}{R^2} + R^2\alpha^2}$ $\Rightarrow$ $3\alpha^4 + 3\alpha^2 = 6$ ; On solving $\alpha^2 = -2, 1$

So, the correct answer is 1 rad/s$^2$.

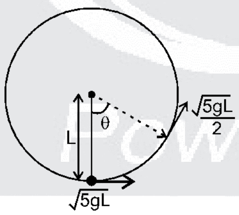
**3.** A bob of mass M is suspended by a massless string of length L. The horizontal velocity V at position A is just sufficient to make it reach the point B. The angle $\theta$ at which the speed of the bob is half of that at A, satisfies. Figure :




(A) $\theta = \frac{\pi}{4}$ (B) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$ (C) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$ (D) $\frac{3\pi}{4} < \theta < \pi$

**Ans.** **(D)**

**Sol.** By energy conservation,






$\frac{1}{2} mu^2 = \frac{1}{2} mv^2 + mg\ (1 - \cos\theta)$

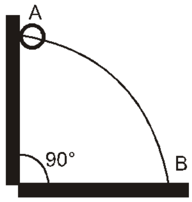
$V^2 = U^2 - 2g\ (L - L\cos\theta)$

$\frac{5gL}{4} = 5gL - 2gL\ (1 - \cos\theta)$

$5 = 20 - 8 + 8\cos\theta$

$\cos\theta = -\frac{7}{8}$ $\Rightarrow$ $\frac{3\pi}{4} < \theta < \pi$

**4.** A wire, which passes through the hole is a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is




(A) always radially outwards

(B) always radially inwards

(C) radially outwards initially and radially inwards later

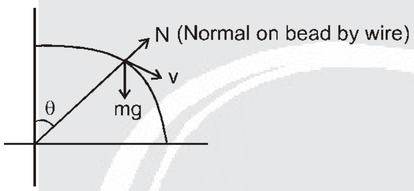
(D) radially inwards initially and radially outwards later.

**Ans.** **(D)**

**Sol.** Using conservation of energy : $mgR(1-\cos\theta) = \frac{1}{2}mv^2$

Radial force Equation : $mg\cos\theta - N = \frac{mv^2}{R}$

$\Rightarrow N = mg\cos\theta - \frac{mv^2}{R} = mg(3\cos\theta - 2)$

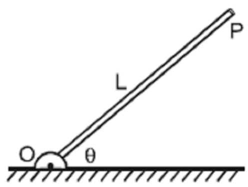



Normal act radially outward on bead if $\cos\theta > \frac{2}{3}$

Normal radially inward on bead if $\cos\theta < \frac{2}{3}$

$\therefore$ Normal on ring is opposite to reaction on bead.

**5.** A uniform pole of length L and mass M is pivoted on the ground with a frictionless hinge O. The pole is free to rotate without friction about an horizontal axis passing through O and normal to plane of the page.The pole makes an angle $\theta$ with the horizontal. The pole is released from rest in the position shown, then linear acceleration of the free end (P) of the pole just after its release would be :




(A) $\frac{2}{3}g\cos\theta$ (B) $\frac{2}{3}g$ (C) g (D) $\frac{3}{2}g\cos\theta$

**Ans.** **(D)**

**Sol.** About point O

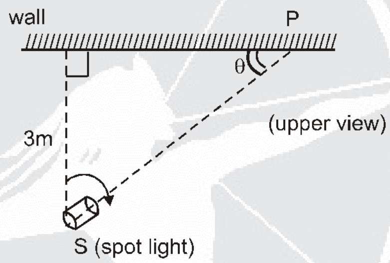
Torque $\tau = I\alpha$

$$Mg(\frac{L}{2}\cos\theta) = \frac{ML^2}{3}\alpha \Rightarrow \frac{3}{2}\frac{g}{L}\cos\theta = \alpha$$

Initially centripetal acceleration of point P is zero ( $a_c = \frac{v^2}{r} = \frac{0}{r} = 0$)

Acceleration of point P is $\sqrt{a_C^2 + a_t^2}$

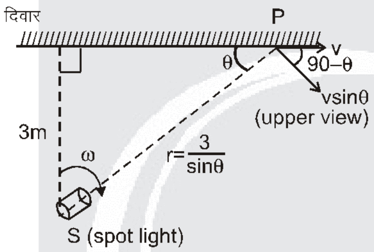
**6.** A spot light S rotates in a horizontal plane with a constant angular velocity of 0.1 rad/s. The spot of light P moves along the wall at a distance 3 m. What is the velocity of the spot P when $\theta = 45^0$?




(A) 0.6 m/s (B) 0.5 m/s (C) 0.4 m/s (D) 0.3 m/s

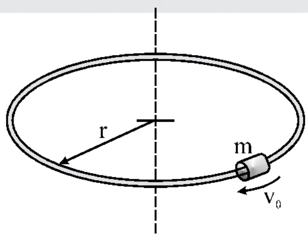
**Ans.** **(A)**

**Sol.**




$$\omega = \frac{v_\perp}{r} = \frac{v\sin\theta}{r} \Rightarrow v = \frac{\omega r}{\sin\theta} = \frac{3\omega}{\sin^2\theta} \Rightarrow v = \frac{0.1\times 3}{(1/\sqrt{2})^2} = 0.6 \text{ m/s}$$ **Ans.**

**7.** A small hoop of mass m is given an initial velocity of magnitude $v_0$ on the horizontal circular ring of radius 'r'. If the coefficient of kinetic friction is $\mu_k$ the tangential acceleration of the hoop immediately after its release is (assume the horizontal ring to be fixed and not in contact with any supporting surface)



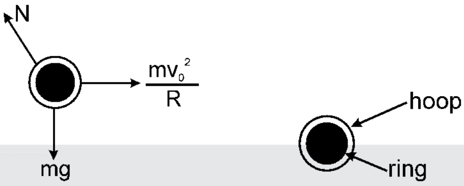

(A) $\mu_k g$ (B) $\mu_k \frac{v_0^2}{r}$ (C) $\mu_k\sqrt{g^2 + \frac{v_0^2}{r}}$ (D) $\mu_k\sqrt{g^2 + \frac{v_0^4}{r^2}}$

**Ans.** **(D)**

**Sol.** The free body daigram of hoop is

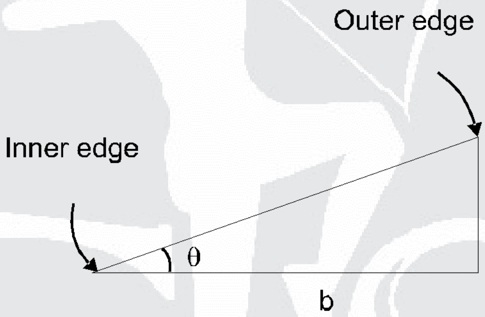
$\therefore$ The normal reaction $N = \sqrt{m^2g^2 + \frac{m^2{v_0}^4}{r^2}}$

$\therefore$ Frictional force = $\mu_k N = \mu_k \sqrt{m^2g^2 + \frac{m^2{v_0}^4}{r^2}}$




$\therefore$ tangential acceleration = $\frac{\mu_k N}{m} = \mu_k \sqrt{g^2 + \frac{{v_0}^4}{r^2}}$

**8.** A vehicle is moving with a speed v on a curved smooth road of width b and radius R. For counteracting the centrifugal force on the vehicle, the difference in elevation required in between the outer and inner edges of the road is :

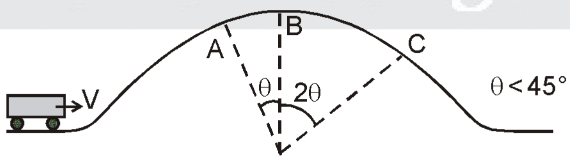



(A) $\frac{v^2b}{Rg}$ (B) $\frac{vb}{Rg}$ (C) $\frac{vb^2}{Rg}$ (D) $\frac{vb}{R^2g}$

**Ans.** **(A)**

**Sol.** $\tan\theta = \frac{v^2}{Rg} \Rightarrow \frac{h}{b} = \frac{v^2}{Rg} \Rightarrow h = \frac{v^2b}{Rg}$.

**9.** A self propelled vehicle (assume it as a point mass) runs on a track with constant speed V. It passes through three positions A, B and C on the circular part of the track.

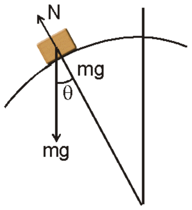



Suppose $N_A$, $N_B$ and $N_C$ are the normal forces exerted by the track on the vehicle when it is passing through points A, B and C respectively then

(A) $N_A = N_B = N_C$ (B) $N_B > N_A > N_C$ (C) $N_C > N_A > N_B$ (D) $N_B > N_C > N_A$

**Ans.** **(B)**

**Sol.**




$$mg\cos\theta - N = \frac{mv^2}{R}$$

$$N = mg\cos\theta - \frac{mv^2}{R}$$

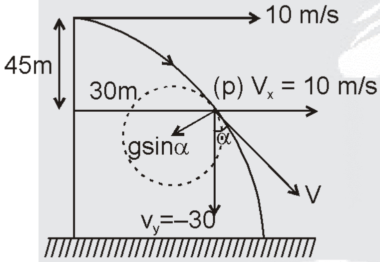
Hence N decrease as θ increases.

**10.** A stone is thrown horizontally under gravity with a speed of 10m/sec. Find the radius of curvature of it's trajectory at the end of 3 sec after motion began.

(A) $10\sqrt{10}$ m (B) $100\sqrt{10}$ m (C) $\sqrt{10}$ m (D) 100 m

**Ans.** **(B)**

**Sol.**




**Mathod (I)**

After 3 sec.

$V_y = u_y + gt = -30$ m/s

and $V_x = 10$ m/s $\quad \therefore V^2 = V_x^2 + V_y^2$

$\Rightarrow \quad V = 10\sqrt{10}$ m/s

Now, $\tan\alpha = \frac{V_x}{V_y} = \frac{1}{3} \quad \Rightarrow \sin\alpha = \frac{1}{\sqrt{10}}$

Radius of curvature $r = \frac{V_\perp^2}{g\sin\alpha}$

$r = 100\sqrt{10}$ m

**Mathod (II)**

Let horizontal and vertical position of point p be x & y respectively

$\therefore \quad x = Vt$ and $y = \frac{1}{2}gt^2$

$\therefore \quad$ equation of trajectory $y = \frac{gx^2}{2V^2}$

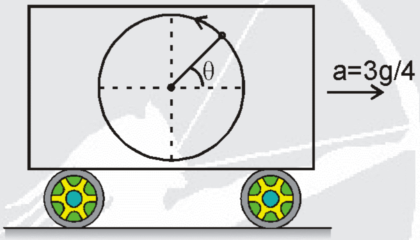
$\therefore \quad \frac{dy}{dx} = \frac{gx}{V^2}$ and $\frac{d^2y}{dx^2} = \frac{g}{V^2}$

Radius of curvature $r = \dfrac{\left\{1+\left(\dfrac{dy}{dx}\right)^2\right\}^{3/2}}{\dfrac{d^2y}{dx^2}} = \dfrac{\left(1+\dfrac{g^2x^2}{V^4}\right)^{3/2}}{g/v^2}$

Now after 3 s $x = Vt = 30$ m and $V = 10$ m/s

$\therefore$ $r = 100\sqrt{10}$ m.

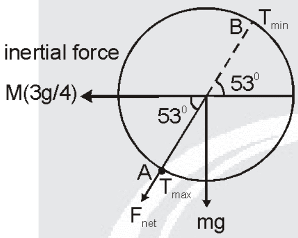
**11.** A bus is moving with a constant acceleration a = 3g/4 towards right. In the bus, a ball is tied with a rope and is rotated in vertical circle as shown. The tension in the rope will be minimum, when the rope makes an angle $\theta$ = ______ .




(A) 53° (B) 37° (C) 180 – 53° (D) 180 + 37°

**Ans.** **(A)**

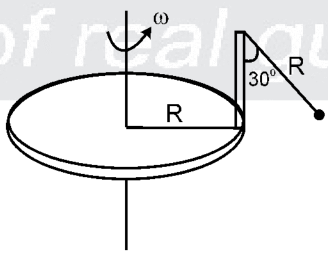
**Sol.**




$F_{net}$ is shown in the figure. So, tension will be max. at point A and will be min. at point B.

**12.** A disc of radius R has a light pole fixed perpendicular to the disc at the circumference which in turn has a pendulum of length R attached to its other end as shown in figure. The disc is rotated with a constant angular speed $\omega$. The string is making an angle $30^0$ with the rod. Then the angular speed $\omega$ of disc is:

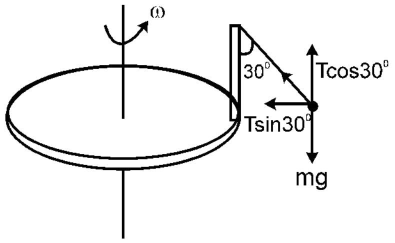





(A) $\left(\dfrac{\sqrt{3}}{R}\dfrac{g}{}\right)^{1/2}$ (B) $\left(\dfrac{\sqrt{3}}{2}\dfrac{g}{R}\right)^{1/2}$ (C) $\left(\dfrac{g}{\sqrt{3}\,R}\right)^{1/2}$ (D) $\left(\dfrac{2}{3\sqrt{3}}\dfrac{g}{R}\right)^{1/2}$

**Ans.** **(D)**

**Sol.** The bob of the pendulum moves in a circle of radius $(R + R\sin 30^0) = \frac{3R}{2}$



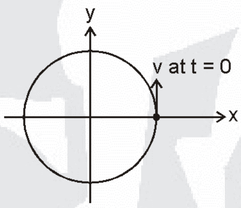

Force equations: $T\sin 30^0 = m\left(\frac{3R}{2}\right)\omega^2$

$T\cos 30^0 = mg$

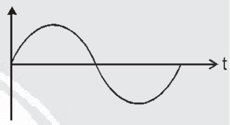
$\Rightarrow \quad \tan 30^0 = \frac{3}{2}\frac{\omega^2 R}{g} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \omega = \sqrt{\frac{2g}{3\sqrt{3}R}}$ **Ans.**

**MCQ (One or more than one correct) :**

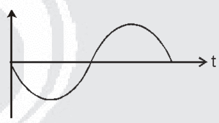
**13.** A particle is moving in a uniform circular motion on a horizontal surface. Particle position and velocity at time t = 0 are shown in the figure in the coordinate system. Which of the indicated variable on the vertical axis is/are correctly matched by the graph(s) shown alongside for particle's motion ?




(A) x component of velocity →

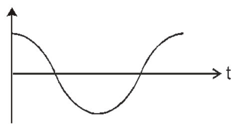



(B) y component of force keeping particle moving in a circle →



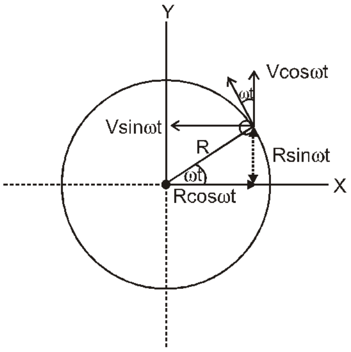

(C) Angular velocity of the particle →


(D) x coordinate of the particle →




**Ans.** **(BCD)**

**Sol.**



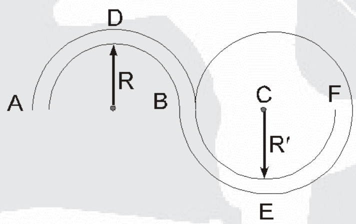

So X component of velocity $V_x = -V \sin\omega t$

y component of force $F_y = -mv^2/R \sin\omega t = -m\omega^2 R \sin\omega t$

Angular velocity of particle $\omega$ = constant.

X–coordinate of the particle $x = R\cos\omega t$. So B, C, D are correctly matched

**14.** In the figure shown ADB & BEF are two fixed circular paths. A block of mass m enters in the tube ADB through point A with minimum velocity to reach point B. From there it moves on another circular path of radius $R'$. There it is just able to complete the circle.




(A) velocity at A must be $\sqrt{4Rg}$

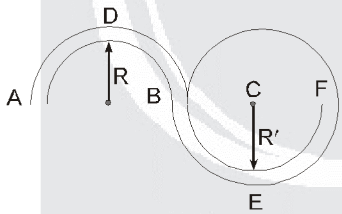
(B) velocity at A must be $\sqrt{2Rg}$

(C) $\frac{R'}{R} = \frac{2}{3}$

(D) the normal reaction at point E is 6 mg

**Ans.** **(BCD)**

**Sol.**




For minimum velocity. at A ;

$\frac{1}{2} mV_A^2 = mgR \Rightarrow V_A = \sqrt{2gR}$

Now; $\frac{1}{2} m V_B^2 + mgR' = \frac{1}{2} m\ V_E^2$

As, $V_B = \sqrt{2gR}$

For looping the loop ;

$V_E = \sqrt{5gR'}$

$\therefore \quad \frac{1}{2} m2gR + mgR' = \frac{1}{2} m\, 5gR'$

$\therefore \quad \frac{R'}{R} = \frac{2}{3}$

And also, $N - mg = \frac{mV_E^2}{R'}$

$N - mg = \frac{m \; 5gR'}{R'}$

N = 6mg

**15.** A particle is attached to an end of a rigid rod. The other end of the rod is hinged and the rod rotates always remaining horizontal. It's angular speed is increasing at constant rate. The mass of the particle is 'm'. The force exerted by the rod on the particle is $\vec{F}$, then :

(A) $F > mg$

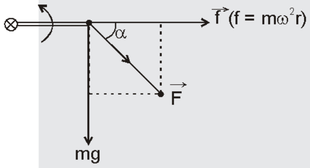
(B) F is constant

(C) The angle between $\vec{F}$ and horizontal plane decreases.

(D) The angle between $\vec{F}$ and the rod decreases.

**Ans.** **(ACD)**

**Sol.**



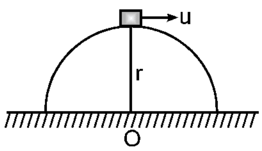

$F = \sqrt{f^2 + (mg)^2}$

Now when the angular speed of the rod is increasing at const. rate the resultant force will be more inclined towards $\vec{f}$.

Hence the angle between $\vec{F}$ and horizontal plane decreases so as with the rod.

**Comprehension Type Question:**

A small block of mass m is projected horizontally from the top of the smooth and fixed hemisphere of radius r with speed u as shown. For values of $u \geq u_0$, ($u_0 = \sqrt{gr}$) it does not slide on the hemisphere. [ i.e. leaves the surface at the top itself ]




**16.** For u = 2 $u_0$, it lands at point P on ground. Find OP.

(A) $\sqrt{2}$ r (B) 2 r (C) 4r (D) 2$\sqrt{2}$ r

**Ans.** **(D)**

**Sol.** $mg = \frac{mu_0^2}{r} \Rightarrow u_0 = \sqrt{gr}$

Now, along vertical ; $r = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2r}{g}}$

Along horizontal ; $OP = 2u_0t = 2\sqrt{2}\ r$

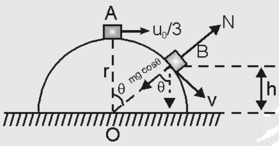
**17.** For $u = u_0/3$, find the height from the ground at which it leaves the hemisphere.

(A) $\frac{19\,r}{9}$ (B) $\frac{19\,r}{27}$ (C) $\frac{10\,r}{9}$ (D) $\frac{10\,r}{27}$

**Ans.** **(B)**

**Sol.** As at B it leaves the hemisphere,

$\therefore$ N = 0




$mg\cos\theta = \frac{mV^2}{r}$

$mg\frac{h}{r} = \frac{mV^2}{r}$

$mv^2 = mgh$ .............(1)

By energy conservation between A and B

$mgr + \frac{1}{2}m\left(\frac{u_0}{3}\right)^2 = mgh + \frac{1}{2}mv^2$

Put $u_0$ and $mv^2$ $\therefore$ $h = \frac{19r}{27}$

**18.** Find its net acceleration at the instant it leaves the hemisphere.

(A) –g (B) g/2 (C) g (D) g/3

**Ans.** **(C)**

**Sol.** As $a_c = \frac{v^2}{r} = g\cos\theta$

$\therefore$ $a_t = g\sin\theta$ $\therefore$ $a_{net} = g$

**Alternate Solution :**

when block leave only the force left is mg.

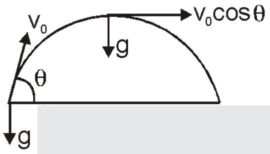
$\therefore$ $a_{net} = g.$ $\frac{mV^2}{r}$

**Numerical based Questions :**

**19.** A particle is projected from ground with an initial velocity 20 m/sec making an angle 60° with horizontal. If $R_1$ and $R_2$ are radius of curvatures of the particle at point of projection and highest point respectively, then find the value of $\frac{R_1}{R_2}$.

**Ans.** **8**

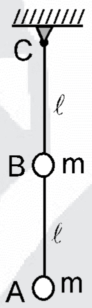
**Sol.**




$$R_1 = \frac{v_0^2}{g\cos\theta}$$

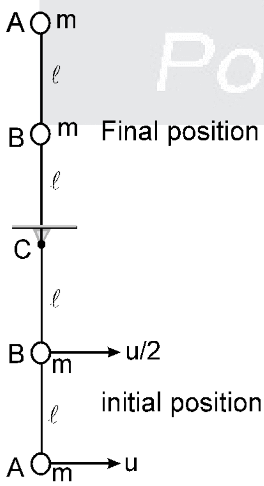
$$R_2 = \frac{(v_0\cos\theta)^2}{g} \qquad \therefore \qquad \frac{R_1}{R_2} = \frac{1}{(\cos\theta)^3} = 8$$

**20.** A weightless rod of length 2  carries two equal masses 'm', one secured at lower end A and the other at the middle of the rod at B. The rod can rotate in vertical plane about a fixed horizontal axis passing through C. What horizontal velocity must be imparted to the mass at A so that it just completes the vertical circle.




**Ans.** $u = \sqrt{\frac{48}{5}g}$

**Sol.** Let the initial velocity given to the mass at A be u.



Then the velocity of mass at B is u/2

As the system moves from initial the final position

Increase in potential energy is = 4 mg + 2mg

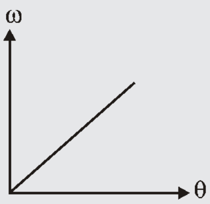
Decrease in kinetic energy = $\frac{1}{2}mu^2 + \frac{1}{2}m\left(\frac{u}{2}\right)^2$ = $mu^2$

From conservation of energy

$\frac{5}{8}$ $mu^2$ = 6 mgl or $u = \sqrt{\frac{48}{5}g}$

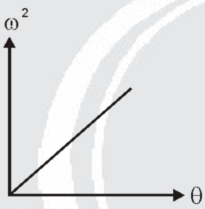
**Matrix Match Type :**

**21.** Each situation in column I gives graph of a particle moving in circular path. The variables $\omega, \theta$ and t represent angular speed (at any time t), angular displacement (in time t) and time respectively. Column II gives certain resulting interpretation. Match the graphs in column I with statements in column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the OMR.

(A) 

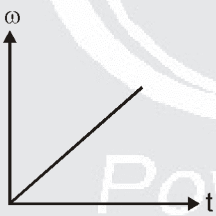
$\omega$ - $\theta$ graph

(p) Angular acceleration of particle is uniform

(B) 

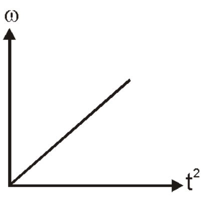
$\omega^2$ - $\theta$ graph

(q) Angular acceleration of particle is non-uniform

(C) 

$\omega$ - t graph

(r) Angular acceleration of particle is directly proportional to t.

(D) 

$\omega$ - $t^2$ graph

(s) Angular acceleration of particle is directly proportional to $\theta$.

**Ans.** (A) q,s (B) p (C) p (D) q,r

**Sol.** From graph (a) $\Rightarrow$ $\omega = k\theta$ where k is positive constant

angular acceleration = $\omega \dfrac{d\omega}{d\theta} = k\theta \times k = k^2\theta$

$\therefore$ angular acceleration is non uniform and directly proportional to $\theta$. $\therefore$ (A) q, s

From graph (b) $\Rightarrow$ $\omega^2 = k\theta$ . Differentiating both sides with respect to $\theta$.

$2\omega \dfrac{d\omega}{d\theta} = k$ or $\omega \dfrac{d\omega}{d\theta} = \dfrac{k}{2}$ Hence angular acceleration is uniform. $\therefore$ (B) p

From graph (c) $\Rightarrow$ $\omega = kt$

angular acceleration = $\dfrac{d\omega}{dt} = k$ Hence angular acceleration is uniform $\Rightarrow$ (C) p

From graph (d) $\Rightarrow$ $\omega = kt^2$

angular acceleration = $\dfrac{d\omega}{dt} = 2kt$ Hence angular acceleration is non uniform and directly proportional to t.

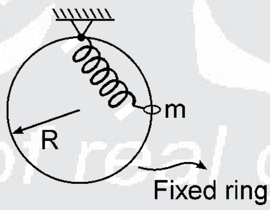
$\therefore$ (D) q,r

**Subjective Type Questions :**

**22.** A ring of radius R is placed such that it lies in a vertical plane. The ring is fixed. A bead of mass m is constrained to move along the ring without any friction. One end of the spring is connected with the mass m and other end is rigidly fixed with the topmost point of the ring. Initially the spring is in un-extended position and the bead is at a vertical distance R from the lowermost point of the ring. The bead is now released from rest.

(a) What should be the value of spring constant K such that the bead is just able to reach bottom of the ring.

(b)The tangential and centripetal accelerations of the bead at initial and bottommost position for the same value of spring constant K.




**Ans.** (a) $K = \dfrac{mg}{R\ (3-2\sqrt{2})}$ (b) at intial instant $a_t = g$, $a_c = 0$

at bottommost position $a_t = 0$ $a_c = 0$

**Sol.** (a) Applying conservation of energy between initial and final position is

Loss in gravitational P.E. of the bead of mass m = gain in spring P. E.

$\therefore$ $mg\,R = \dfrac{1}{2} K (2R - \sqrt{2}\,R)^2$ or $K = \dfrac{mg}{R\ (3-2\sqrt{2})}$

(b) At $t = 0$

$a_t = g$

$a_c = 0$

at lowest point

$a_t = 0$

$a_c = 0$

The centripetal acceleration of bead at the initial and final position is zero because its speed at both position is zero.

The tangential acceleration of the bead at initial position is g.

The tangential acceleration of the bead at lower most position is zero.

**23.** Find the magnitude and direction of the force acting on the particle of mass m during its motion in the plane xy according to the law $x = a \sin\omega t$, $y = b\cos\omega t$, where a, b and $\omega$ are constants.

**Ans.** $F = -m\omega^2 r$, where r is the radius vector of the particle relative to the origin of coordinates;

$F = m\omega^2\sqrt{x^2+y^2}$ ]

**Sol.** $F = ma$ or $F = m(a_x + a_y)$ ( $a_2 = 0$)

$$x = a \sin \omega t$$

$$v_x = \frac{dx}{dt} = aw \cos(\omega t)$$

$$a_x = \frac{d^2x}{dt^2} = -a\omega^2 \sin(\omega t)$$

$$v_y = \frac{dy}{dt} = -b\,\omega \sin(\omega t)$$

$$a_y = \frac{d^2y}{dt^2} = -b\,\omega^2 \cos(\omega t)$$

So $F = m(-a\omega^2 \sin\omega t\,\hat{i} - b\omega^2 \cos\omega t\,\hat{j})$

$$F = -m\omega^2(a\sin\omega t\,\hat{i} + b\cos\omega t\,\hat{j})$$

$$F = -m\omega^2(x\hat{i} + y\,\hat{j})$$

$$|F| = m\omega^2\sqrt{x^2+y^2}$$

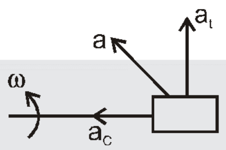
direction $\tan\alpha = \frac{y}{x} = \frac{b}{a}\cot(\omega t)$ (from x-axis)

or $[(x\hat{i} + y\hat{j})]$ is position vector of the particle in corrdinate system. Because of negative sign force is opposite to it and always acting towards the orzon.

**24.** A block of mass m is kept on a horizontal ruler. The friction coefficient between the ruler and the block is $\mu = 0.5$. The ruler is fixed at one end and the block is at a distance L =1 m from the fixed end. The ruler is rotated about the fixed end in the horizontal plane through the fixed end. If the angular speed of the ruler is uniformly increased from zero at a constant angular acceleration $\alpha$ = 3 rad/sec$^2$. Find the angular speed at which block will slip. (g = 10m/s$^2$)

**Ans.** 2 rad/sec.

**Sol.** Angular velocity increase with constant angular acceleration $\alpha$




$a_c = \omega^2 L$, $a_t = \alpha L$

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{(\omega^2 L)^2 + (\alpha L)^2} \quad ....(1)$$

for just slipping $\Rightarrow$ ma = $\mu$mg

$\Rightarrow$ a = $\mu$g .... (2)

form (1) & (2)

$$\mu g = \sqrt{(\omega^2 L)^2 + (\alpha L)^2}$$

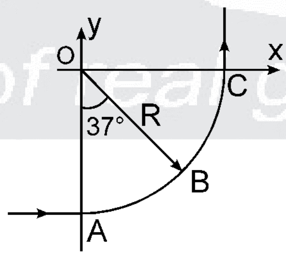
$$\omega = \left[\left(\frac{\mu g}{L}\right)^2 - \alpha^2\right]^{1/4} \quad \textbf{Ans.}$$

$\omega$ = 2 rad/sec.

**25.** A car initially traveling eastwards turns north by traveling in a quarter circular path of radius R metres at uniform speed as shown in figure. The car completes the turn in T second.

(a) What is the acceleration of the car when it is at B located at an angle of 37°. Express your answers in terms of unit vectors $\hat{i}$ and $\hat{j}$

(b) The magnitude of car's average acceleration during T second.




**Ans.** (a) $\frac{\pi^2}{20}\frac{R}{T^2}(-3\hat{i} + 4\hat{j})$ m/s$^2$

(b) $\frac{\pi}{\sqrt{2}}\frac{R}{T^2}$ m/s$^2$

**Sol.** Speed of car is $v = \dfrac{\pi R}{2T}$ m/s .....

(a) The acceleration of car is $\dfrac{v^2}{R} = \dfrac{\pi^2 R}{4\ T^2}$ at B and is directed from B to O.

$\therefore$ Acceleration vector of car at B is

$$a = \frac{v^2}{R}\ (-\sin 37°\ \hat{i} + \cos 37°\ \hat{j}) = \frac{\pi^2 R}{20\ T^2}\ (-3\hat{i} + 4\hat{j})\ \text{m/s}^2$$

(b) The magnitude of average acceleration of car is in time T is

$$\frac{\left|v_C - V_B\right|}{T} = \frac{\sqrt{2}\ v}{T} = \frac{\pi R}{\sqrt{2}\ T^2}\ \text{m/s}^2$$