SCQ (Single Correct Type) :

1. A cavity of radius R/2 is made inside a solid sphere of radius R. The centre of the cavity is located at a distance R/2 from the centre of the sphere. The gravitational force on a particle of mass 'm' at a distance R/2 from the centre of the sphere on the line joining both the centres of sphere and cavity is (opposite to the centre of cavity). [Here $g = GM/R^2$, where M is the mass of the solid sphere]

(A) $\frac{mg}{2}$ (B) $\frac{3\,mg}{8}$ (C) $\frac{mg}{16}$ (D) none of these

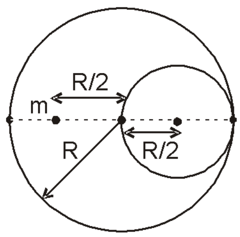
Ans. **(B)**

Sol. **(Tough)** Gravitation field at mass 'm' due to full solid sphere

$$\vec{E}_1 = \frac{\rho\vec{r}}{3\varepsilon_0} = \frac{\rho R}{6\varepsilon_0} .... \left[\varepsilon_0 = \frac{1}{4\pi G}\right]$$

Gravitational field at mass 'm' due to cavity ($-\rho$)

$$\vec{E}_2 = \frac{(-\rho)(R/2)^3}{3\varepsilon_0 R^2} \quad .... \quad \left[\frac{\rho a^3}{3\varepsilon_0 r^2}\right]$$



$$= -\frac{(-\rho)R^3}{24\varepsilon_0 R^2} = -\frac{-\rho R}{24\varepsilon_0}$$

Net gravitational field $\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho R}{6\varepsilon_0} - \frac{\rho R}{24\varepsilon_0} = \frac{\rho R}{8\varepsilon_0}$

Net force on 'm' $\rightarrow$ $F = m\vec{E} = \frac{m\rho R}{8\varepsilon_0}$

Here $\rho = \frac{M}{4/3\pi R^3}$ & $\varepsilon_0 = \frac{1}{4\pi G}$

then $F = \frac{3mg}{8}$

2. Maximum height reached by a rocket fired with a speed equal to 50% of the escape velocity from earths surface is given by (R is radius of earth) :

(A) $\frac{R}{3}$ (B) 3R (C) $\frac{2R}{3}$ (D) R

Ans. **(A)**

Sol. $v = \frac{50}{100}$ $\quad V_e = \frac{1}{2}\sqrt{\frac{2GM}{R}}$

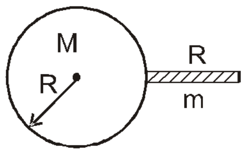
Applying energy conservation

$\Rightarrow -\frac{GMm}{R}+\frac{1}{2}mv^2 = -\frac{GMm}{(R+h)}$

$v^2 = \frac{2GM}{R}-\frac{2GM}{R+h} \quad \Rightarrow \quad \frac{1}{4}.\frac{2GM}{R} = 2GM\left(\frac{1}{R}-\frac{1}{R+h}\right) \quad \Rightarrow \quad \frac{1}{4R}=\frac{h}{R(R+h)}$

$\Rightarrow \quad R + h = 4h \quad \Rightarrow \quad h = R/3$

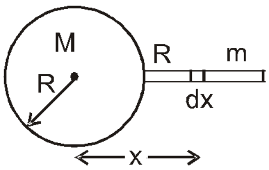
3. A uniform thin rod of mass m and length R is placed normally on surface of earth as shown. The mass of earth is M and its radius is R. Then the magnitude of gravitational force exerted by earth on the rod is



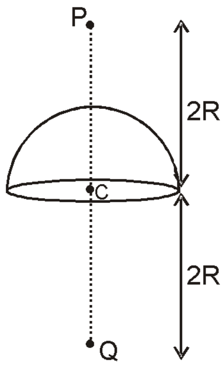
(A) $\frac{GMm}{2R^2}$ (B) $\frac{GMm}{4R^2}$ (C) $\frac{4GMm}{9R^2}$ (D) $\frac{GMm}{8R^2}$

Ans. **(A)**

Sol. $F = \int_R^{2R} \frac{GM\left(\frac{m}{R}\right)dx}{x^2} = \frac{GMm}{2R^2}$



4. If Gravitational field due to uniform thin hemispherical shell at point P is I, then the magnitude of gravitational field at Q is (Mass of hemisphere is M, radius R) -



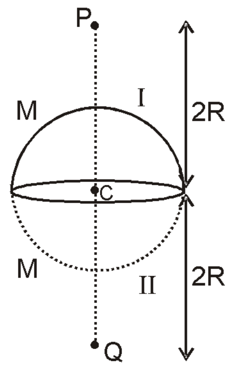
(A) $\frac{GM}{2R^2} - I$ (B) $\frac{GM}{2R^2} + I$ (C) $\frac{GM}{4R} - I$ (D) $2I - \frac{GM}{2R^2}$

Ans. **(A)**

Sol. |Gravitational field due to 2nd at P| = |Gravitation field due to 1st at Q|

at P

G.f. due to 1st + G.f due to 2nd = $\frac{G(2M)}{4R^2} = \frac{GM}{2R^2}$



G f due to 2nd at P = $\frac{GM}{2R^2} - \mathrm{I}$

5. Altitude at which acceleration due to gravity decreases by 0.1% approximately: (Radius of earth = 6400 km)

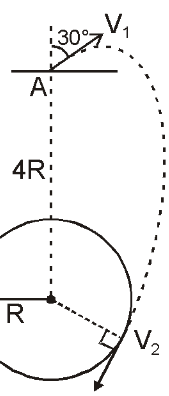
(A) 3.2 km (B) 6.4 km (C) 2.4 km (D) 1.6 km

Ans. **(A)**

Sol. $\frac{g\left(1-\frac{2h}{R_e}\right)-g}{g} = -0.1 \times \frac{1}{100}$

$\Rightarrow \quad -\frac{2h}{R_e} = -\frac{1}{1000} \Rightarrow \quad h = \frac{R_e}{2000} = \frac{6400}{2000}$ km = 3.2 km.

6. A particle is projected from point A, that is at a distance 4R from the centre of the earth, with speed V_1 in a direction making 30° with the line joining the centre of the earth and point A, as shown. Find the speed V_1 if particle passes grazing the surface of the earth. Consider gravitational interaction only between these two. (use $\frac{GM}{R} = 6.4 \times 10^7\ m^2/s^2$)



(A) $4\sqrt{2}$ km/s (B) $3\sqrt{2}$ km/s (C) $6\sqrt{2}$ km/s (D) $5\sqrt{2}$ km/s

Ans. **(A)**

Sol. Conserving angular momentum m.(V_1 cos60°). 4R = m.V_2.R ; $\frac{V_2}{V_1} = 2$

Conserving energy of the system

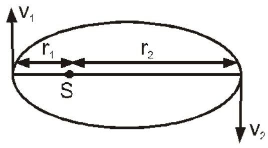
$$-\frac{G\ Mm}{4R} + \frac{1}{2}mV_1^2 = -\frac{GMm}{R} + \frac{1}{2}mV_2^2$$

$$\frac{1}{2}V_2{}^2 - \frac{1}{2}V_1{}^2 = \frac{3}{4}\frac{GM}{R} \qquad \text{or} \qquad V_1{}^2 = \frac{1}{2}\frac{GM}{R}$$

$$V_1 = \frac{1}{\sqrt{2}}\sqrt{64\times10^6} = \frac{8000}{\sqrt{2}}\ \text{m/s} = 4\sqrt{2}\ \text{km/s}$$

MCQ (One or more than one correct) :

7. A planet is revolving in an elliptical orbit around the sun as shown in figure. The areal velocity (area swapped by the radius vector with respect to sun in unit time) is :



(A) $r_1\ v_1$ (B) $r_2\ v_2$ (C) $\frac{1}{2}\frac{v_1\ r_1^2}{r_2}$

(D) dependent on the position of planet from sun

Ans. **(AB)**

Sol. As per Kepler's law, areal velocity remain constant

$$\frac{dA}{dt} = \frac{J}{2m} = \frac{mv_1r_1}{2m} = \frac{1}{2}r_1v_1 = \frac{1}{2}r_2\ v_2$$

8. Suppose the earth suddenly shrinks in size, still remaining spherical and mass unchanged (All gravitational forces pass through the centre of the earth).

(A) The days will become shorter.

(B) The kinetic energy of rotation about its own axis will increase

(C) The duration of the year will increase.

(D) The magnitude of angular momentum about its axis will increase.

Ans. **(AB)**

Sol. $I_1\omega_1 = I_1\omega_1$ (Angular momentum is conserved)

As I_2 decreases. ω_2 increases.

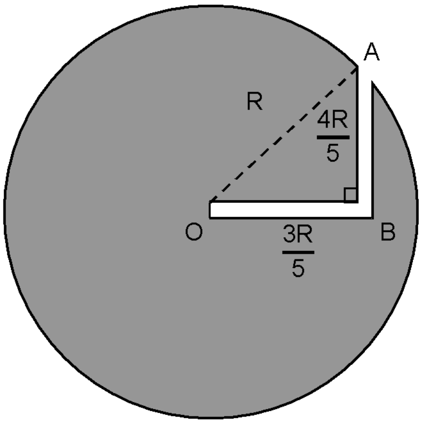
Thus T = $\frac{2\pi}{\omega}$ i.e. T decreases.

Therefore the earth is completing each circle around its own axis in lesser time.

K.E. = $\frac{1}{2}I\omega_2$ Therefore K.E. of rotation increases.

Duration of the year is dependent upon time taken to complete one revolution around the sun.

9. A narrow smooth tunnel of L–shape is made into earth up to center as shown in the figure. A small ball which just fits in tunnel is released from 'A'. The collision of ball is inelastic. (g is acceleration due to gravity on surface of planet)



(A) time to fall from A to B is $\frac{\pi}{2}\sqrt{\frac{R}{g}}$

(B) time to fall from B to O is $\frac{\pi}{4}\sqrt{\frac{R}{g}}$

(C) speed just before hitting B is $\frac{4R}{5}\sqrt{\frac{g}{R}}$

(D) speed just before hitting O is $\frac{R}{5}\sqrt{\frac{g}{R}}$

Ans. **(AC)**

Sol. Motion of the particle from A to B is SHM with $\omega = \sqrt{\frac{g}{R}}$ and amplitude = $\frac{4R}{5}$

$t_{AB} = \frac{\pi}{2\omega} = \frac{\pi}{2}\sqrt{\frac{R}{g}}$

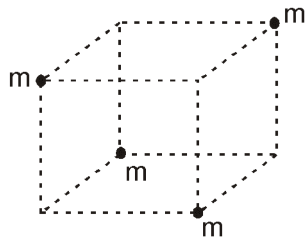
$V_1 = \frac{4R}{5}\sqrt{\frac{g}{R}}$

similarly motion of particle from OB is

also SHM with $\omega = \sqrt{\frac{g}{R}}$ and amplitude $\frac{3R}{5}$

$t_{BO} = \frac{\pi}{2}\sqrt{\frac{R}{g}}$, $V_2 = \frac{3R}{5}\sqrt{\frac{g}{R}}$

10. Four point masses each of mass m are kept at four corners of a cube of side length ‘a’. Then



(A) the gravitational potential energy of system is $= -\frac{3\sqrt{2}Gm^2}{a}$.

(B) the gravitational potential energy of system is $= -\frac{4\sqrt{2}Gm^2}{a}$.

(C) the magnitude of force on one of the particle due to the remaining masses is equal to $\frac{\sqrt{6}Gm^2}{a^2}$.

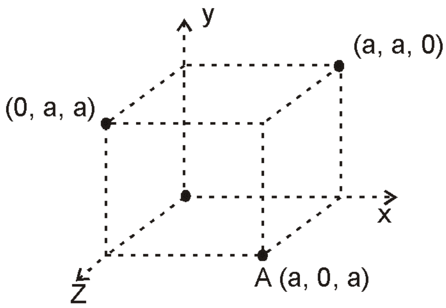
(D) the magnitude of force on one of the particle due to others is equal to $\sqrt{\frac{3}{2}}\frac{Gm^2}{a^2}$.

Ans. **(AD)**

Sol. $U = -6\,\frac{Gm^2}{a\sqrt{2}} = -\frac{3\sqrt{2}Gm^2}{a}$

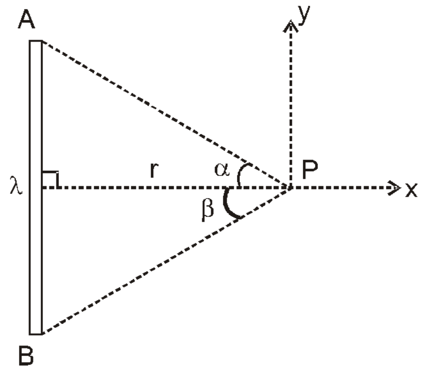
$\vec{F}$ on A $= \frac{-Gm^2}{(a\sqrt{2})^3}\left[a\hat{i}+a\hat{k}-a\hat{j}+a\hat{k}+a\hat{i}-a\hat{j}\right]$

$|F| = \frac{Gm^2}{a^3 2\sqrt{2}}\left[\sqrt{4a^2+4a^2+4a^2}\right] = \sqrt{\frac{3}{2}}\frac{Gm^2}{a^2}$



$\therefore$ **Ans.** (A) & (D)

11. A uniform rod AB lying in xy plane has linear mass density λ. A point P lies at perpendicular distance r from the rod and E_x and E_y are the components of the gravitational field at point P along x, y direction respectively then



(A) If $\alpha = \beta$ then $E_y = 0$

(B) If $\alpha = \beta = 45^o$, then $E_x = \frac{2G\lambda}{r}$

(C) Gravitational potential at point P is not defined if $\alpha = \beta = \frac{\pi}{2}$

(D) If $\alpha = \beta$ then $E_x = 0$ and $E_y = 0$

Ans. **(AC)**

Sol. $|E_x| = \frac{G\lambda}{r}(\sin\alpha + \sin\beta)$

$|E_y| = \frac{G\lambda}{r}(\cos\alpha - \cos\beta)$

If $\alpha = \beta$ then $E_y = 0$

If $\alpha = \beta = 45^o$ then $E_x = \frac{\sqrt{2}G\lambda}{r}$

Comprehension Type Question:

The minimum and maximum distances of a satellite from the center of earth are 2R and 4R respectively, here R is the radius of earth and M is the mass of the earth.

12. The minimum and maximum speeds are :

(A) $\sqrt{\frac{GM}{9R}}, \sqrt{\frac{2GM}{R}}$ (B) $\sqrt{\frac{GM}{5R}}, \sqrt{\frac{3GM}{2R}}$ (C) $\sqrt{\frac{GM}{6R}}, \sqrt{\frac{2GM}{3R}}$ (D) $\sqrt{\frac{GM}{3R}}, \sqrt{\frac{5GM}{2R}}$

Ans. **(C)**

13. The radius of curvature at the point of minimum distance is :

(A) $\frac{8R}{3}$ (B) $\frac{5R}{3}$ (C) $\frac{4R}{3}$ (D) $\frac{7R}{3}$

Ans. **(A)**

Sol. **(11 to 12)**

By principle of conservation of angular momentum :

$mv_1(2R) = mv_2(4R)$

$v_1 = 2v_2$ ....(i)

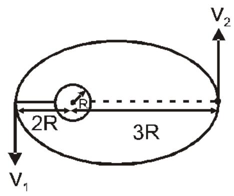
From conservation energy,

$$\frac{1}{2}mv_1^2 - \frac{GMm}{2R} = \frac{1}{2}mv_2^2 - \frac{GMm}{4R} \quad \text{............(ii)}$$

Solving (i) and (ii) we get

$$v_2 = \sqrt{\frac{GM}{6R}}, \quad v_1 = \sqrt{\frac{2GM}{3R}}$$

If r is the radius of curvature at point A



$$\frac{mv_1^2}{r} = \frac{GMm}{(2R)^2} \quad \Rightarrow \quad r = \frac{4R^2v_1^2}{GM} = \frac{8R}{3}$$

Numerical based Questions :

14. An earth satellite is revolving in a circular orbit of radius a with a velocity v_0. A gun in the satellite is directly aimed toward earth. A bullet is fired from the gun with muzzle velocity $\frac{v_0}{2}$. Find the ratio of distance of farthest and closest approach of bullet from centre of earth. (Assume that mass of the satellite is very-very large with respect to the mass of the bullet)

Ans. **3**

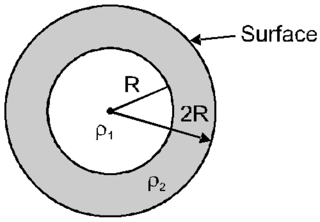
Sol. $v_0 = \sqrt{\frac{GM}{a}}$

$mv_0a = mvr$

$$\frac{1}{2}m\left(v_0^2 + \frac{v_0^2}{4}\right) - \frac{GMm}{a} = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

Solving, $\frac{r_{max}}{r_{min}} = \frac{2a}{2a/3} = 3$ **Ans.**

15. A planet is made of two materials of density ρ_1 and ρ_2 as shown in figure.



The acceleration due to gravity at surface of planet is same as a depth 'R'. The ratio of $\frac{\rho_1}{\rho_2}$ is $\frac{x}{3}$. Find the value of x.

Ans. 7

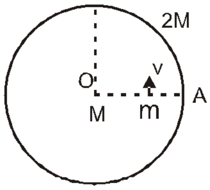
Sol. $\frac{GM}{(2R)^2} = \frac{GM'}{R^2} \Rightarrow \qquad \frac{M}{4} = M'$

$$\frac{4}{3}\pi R_3\ \rho_1 + \frac{4}{3}\pi(8R_3 - R_3)\rho_2 = 4\left(\frac{4}{3}.\pi R^3.\rho_1\right)$$

$$\rho_1 + 7\rho_2 = 4\rho_1$$

$$\frac{\rho_1}{\rho_2} = \frac{7}{3}$$

16. A particle of mass M is fixed at the centre (point O) of a uniform shell of mass 2M and radius R. Another particle of mass m (<<M) is projected from mid point of radius OA with velocity V perpendicular to the radius. The maximum value of v so that the particle does not strike the surface of the shell is $\sqrt{\frac{NGM}{3R}}$, calculate N.

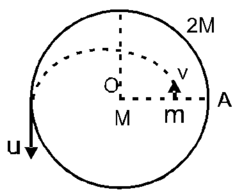


Ans. 8

Sol. $\frac{1}{2}mv^2 - \frac{GMm}{R/2} = \frac{1}{2}mu^2 - \frac{GMm}{R}$ [mechanical energy conservation]

$\frac{R}{2}mv = Rmv$ [Angular momentum conservation]

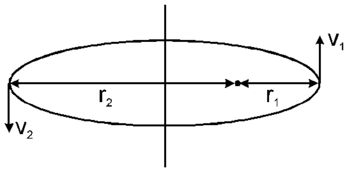
$\Rightarrow \qquad v = \sqrt{\frac{8GM}{3R}}$



Subjective Type Questions :

17. A planet of mass 'm' revolves around the sun (of mass M) in an elliptical orbit. The semi major axis of the orbit is 'a'. Find out the total mechanical energy of the planet.

Sol. Let v_1 and v_2 be the speeds of planet at nearest and farest positions from sun respectively.



$\therefore \quad r_1 = a\,(1 - e)$ and $r_2 = a\,(1 + e)$ ....(1)

Applying conservation of angular momentum

$$mv_1 r_1 = m\,v_2\,r_2$$

or $\quad v_1 r_1 = v_2\,r_2$ ....(2)

Applying conservation of energy, we get

$$\frac{1}{2}mv_1^2 - \frac{GMm}{r_1} = \frac{1}{2}mv_2^2 - \frac{GMm}{r_2} \quad ....(3)$$

Solving equation (1), (2) and (3)

we get

$$v_1 = \sqrt{\frac{GM}{a}\ \frac{(1-e)}{(1+e)}}$$

$\therefore \quad$ Total energy of planet

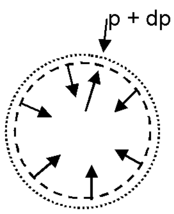
$$\frac{1}{2}m\,v_1^2 - \frac{GMm}{r_1} = \frac{1}{2}m\left[\frac{GM}{a}\frac{(1-e)}{(1+e)}\right] - \frac{GMm}{a\ (1-e)} = -\frac{GMm}{2a}$$

Ans. E = T + U = – GM$_s$/2a, where m$_s$ is the mass of the Sun.

18. A star in its prime age is said to be under equilibrium due to gravitational pull and outward radiation pressure (p). Consider the shell of thickness dr as in the figure of question (13). If the pressure on this shell id dp then the correct equation is (G is universal gravitational constant)

Ans. $\frac{dp}{dr} = -\frac{GM_r}{r^2}\rho_r$

Sol.



$$\left(\frac{GM_r}{r^2}\right) = \rho_r 4\pi r^2 dr$$

$$(P)(4\pi r^2) - (p + dp)(4\pi r^2) = \frac{GM_r}{r^2} = \Sigma_r 4\pi r^2 dr$$

$$-dp = \frac{GM_r\rho_r}{r^2}\,dr \quad \Rightarrow \quad \frac{dp}{dr} = -\frac{GM_r}{r^2}\rho_r$$