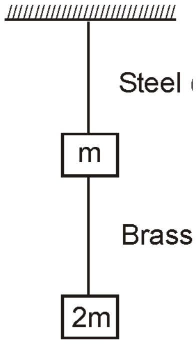
competishun — Power of real gurus **PHYSICS** TARGET : JEE- Advanced 2021	**CAPS-24** **ELASTICITY & VISCOSITY**

SCQ (Single Correct Type) :

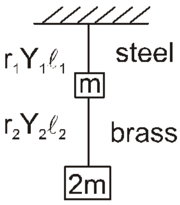
1. If the ratio of lengths, radii and Young's modulii of steel and brass wires in the figure are a, b and c respectively. Then the corresponding ratio of increase in their lengths would be :



(A) $\frac{2ac}{b^2}$ (B) $\frac{3a}{2b^2c}$ (C) $\frac{3c}{2ab^2}$ (D) $\frac{2a^2c}{b}$

Ans. **(B)**

Sol.



$\frac{r_1}{r_2} = b$

$\frac{\ell_1}{\ell_2} = a$

$\frac{Y_1}{Y_2} = c$

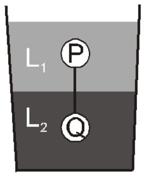
$$\Delta\ell_1 = \frac{(3\ \ mg)\ \ \ell_1}{A_1Y_1}$$

$$\Delta\ell_2 = \frac{(2mg)\ \ \ell_2}{A_2Y_2}$$

$$\frac{\Delta\ell_1}{\Delta\ell_2} = \frac{3\ell_1}{2\ell_2\ \ A_1Y_1} \times\ A_2Y_2 = \frac{3}{2}\ \frac{a}{b^2c}\ \ = \frac{3a}{2b^2c}$$

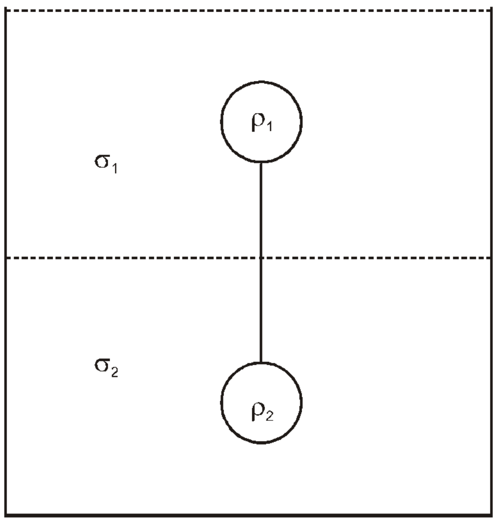
MCQ (One or more than one correct) :

2. Two spheres P and Q of equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities $\sigma_1$and andσ_2 viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string being taut (see figure). If sphere P alone in L_2 has terminal velocity $\vec{V}_P$ and Q alone in L_1 has terminal velocity $\vec{V}_Q$, then



(A) $\dfrac{|\vec{V}_P|}{|\vec{V}_Q|} = \dfrac{\eta_1}{\eta_2}$ (B) $\dfrac{|\vec{V}_P|}{|\vec{V}_Q|} = \dfrac{\eta_2}{\eta_1}$ (C) $\vec{V}_P.\vec{V}_Q > 0$ (D) $\vec{V}_P.\vec{V}_Q < 0$

Ans. **(AD)**



Sol.

For floating

$(\rho_1 + \rho_2)V = (\sigma_1 + \sigma_2)V$

$\rho_1 + \rho_2 = \sigma_1 + \sigma_2$

since strings in taut so

$\rho_1<\sigma_1$ $\rho_2>\sigma_2$

$$V_P = \frac{2}{9}\frac{r^2(\sigma_2 - \rho_1)g}{\eta_2}$$

$$V_Q = \frac{2}{9}\frac{(\sigma_1 - \rho_2)g}{\eta_1}$$

since$\sigma_2 - \rho_1 = -(\sigma_1 - \rho_2)$

$$\left|\frac{V_P}{V_Q}\right| = \frac{\eta_1}{\eta_2}$$

$\vec{V}_P \; . \; \vec{V}_Q < 0$ because V_P and V_Q are opposite

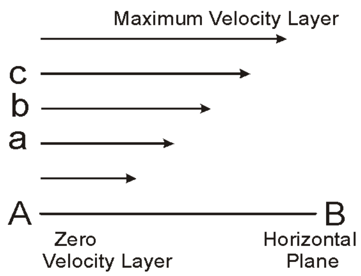
Comprehension Type Question:

When a solid body slides over another solid body, a frictional-force begins to act between them. This force opposes the relative motion of the bodies. Similarly, when a layer of a liquid slides over another layer of the same liquid, a frictional-force acts between them which opposes the relative motion between the layers. This force is called 'internal frictional-force'.

Suppose a liquid is flowing in streamlined motion on a fixed horizontal surface AB (Fig.). The layer of the liquid which is in contact with the surface is at rest due to adhesive forces between the liquid and the surface, while the velocity of other layers increases with distance from the fixed surface. In the Fig., the lengths of the arrows represent the increasing velocity of the layers. Thus there is a relative motion between adjacent layers of the liquid. Let us consider three parallel layers a, b and c. Their velocities are in the increasing order. The layer a tends to retard the layer b, while b tends to retard c.

Thus each layer tends to decrease the velocity of the layer above it. Similarly, each layer tends to increase the velocity of the layer below it. This means that in between any two layers of the liquid, internal tangential forces act which try to destroy the relative motion between the layers. These forces are called 'viscous forces'. If the flow of the liquid is to be maintained, an external force must be applied to overcome the dragging viscous forces. In the absence of the external force, the viscous forces would soon bring the liquid to rest. **The property of the liquid by virtue of which it opposes the relative motion between its adjacent layers is known as 'viscosity'.**

The property of viscosity is seen in the following examples :



(i) A stirred liquid, when left, comes to rest on account of viscosity. Thicker liquids like honey, coaltar, glycerine, etc. have a larger viscosity than thinner ones like water. If we pour coaltar and water on a table, the coaltar will stop soon while the water will flow upto quite a large distance.

(ii) If we pour water and honey in separate funnels, water comes out readily from the hole in the funnel while honey takes enough time to do so. This is because honey is much more viscous than water. As honey tends to flow down under gravity, the relative motion between its layers is opposed strongly.

(iii) We can walk fast in air, but not in water. The reason is again viscosity which is very small for air but comparatively much larger for water.

Viscosity comes into play only when there is a relative motion between the layers of the same material. This is why it does not act in solids.

VELOCITY GRADIENT AND COEFFICIENT OF VISCOSITY

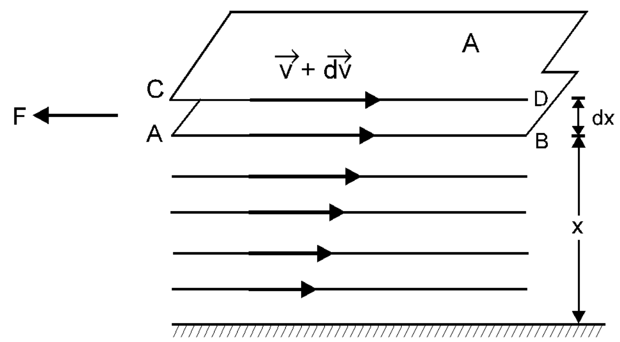
The property of a liquid by virtue of which an opposing force (internal friction) comes into play whenever there is a relative motion between the different layers of the liquid is called viscosity. Consider a flow of a liquid over the horizontal solid surface as shown in fig. Let us consider two layers AB and CD moving with velocities $\vec{v}$ and $\vec{v} + d\vec{v}$ at a distance x and (x + dx) respectively from the fixed solid surface.

According to Newton, the viscous drag or back ward force (F) between these layers depends.

(i) directly proportional to the area (A) of the layer and (ii) directly proportional to the velocity gradient $\left(\frac{dv}{dx}\right)$ between the layers.

i.e. $F \propto A \dfrac{dv}{dx}$ or $\boxed{F = -\eta A \dfrac{dv}{dx}}$ ...(1)

η **is called Coefficient of viscosity**. Negative sign shows that the direction of viscous drag (F) is just opposite to the direction of the motion of the liquid.



SIMILARITIES AND DIFFERENCES BETWEEN VISCOSITY AND SOLID FRICTION

Similarities

Viscosity and solid friction are similar as

1. Both oppose relative motion. Whereas viscosity opposes the relative motion between two adjacent liquid layers, solid friction opposes the relative motion between two solid layers.

2. Both come into play, whenever there is relative motion between layers of liquid or solid surfaces as the case may be.

3. Both are due to molecular attractions.

Differences between them →

	Viscosity	Solid Friction
(i)	Viscosity (or viscous drag) between layers of liquid is directly proportional to the area of the liquid layers.	(i) Friction between two solids is independent of the area of solid surfaces in contact.
(ii)	Viscous drag is proportional to the relative velocity between two layers of liquid.	(ii) Friction is independent of the relative velocity between two surfaces.
(iii)	Viscous drag is independent of normal reaction between two layers of liquid.	(iii) Friction is directly proportional to the normal reaction between two surfaces in contact.

EFFECT OF TEMPERATURE ON THE VISCOSITY

The viscosity of liquids decrease with increase in temperature and increase with the decrease in temperature. That is, $\eta \propto \frac{1}{\sqrt{T}}$. On the other hand, the value of viscosity of gases increases with the increase in temperature and vice-versa. That is, $\eta \propto \sqrt{T}$ (This takes into account the diffusion of the gases).

SOME APPLICATIONS OF VISCOSITY

Knowledge of viscosity of various liquids and gases have been put to use in daily life. Some applications of its knowledge are discussed as under →

1. As the viscosity of liquids vary with temperature, proper choice of lubricant is made depending upon season. Coaltar used for making of roads is heated to reduce its viscosity so that it can be easily laid on the road.

2. Liquids of high viscosity are used in shock absorbers and buffers at railway stations.

3. The phenomenon of viscosity of air and liquid is used to damp the motion of some instruments like galvanometer

UNITS OF COEFFICIENT OF VISCOSITY

From the above formula, we have $\eta = \frac{F}{A(\Delta v_x / \Delta z)}$

$$\therefore \quad \text{dimensions of } \eta = \frac{[MLT^{-2}]}{[L^2][LT^{-1}/L]} = \frac{[MLT^{-2}]}{[L^2T^{-1}]} = [ML^{-1}T^{-1}]$$

Its unit is kg/(meter-second)

In C.G.S. system, the unit of coefficient of viscosity is dyne s cm^{-2} and is called poise. In SI the unit of coefficient of viscosity is N sm^{-2} and is called decapoise.

1 decapoise = 1 N sm^{-2} = (10^5 dyne) × s × (10^2 cm)$^{-2}$ = 10 dyne s cm^{-2} = 10 poise

3. A man is rowing a boat of mass m with a constant velocity 'v_0' in a river the contact area of boat is 'A' and coefficient of viscosity is η. The depth of river is 'D'. Find the force required to row the boat. Assume that velocity gradient is constant.

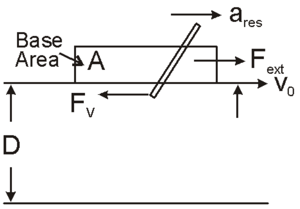
Ans. $\frac{\eta A v_0}{D}$

Sol. $F_{ext} - F_V = m\, a_{res}$

As boat moves with constant velocity $a_{res} = 0$

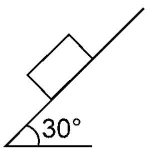
$$F_{ext} = F_V$$

But $F_V = \eta A \frac{dv}{dz}$, but $\frac{dv}{dz} = \frac{v_0 - 0}{D} = \frac{v_0}{D}$



then $F_{ext} = F_V = \frac{\eta A v_0}{D}$

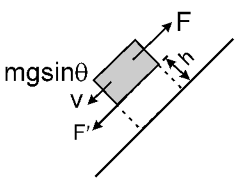
4. A cubical block (of side 2m) of mass20 kg slides on inclined plane lubricated with the oil of viscosity $\eta = 10^{-1}$ poise with constant velocity of 10 m/sec. ($g = 10$ m/sec^2) find out the thickness of layer of liquid. (10^{-1} poise $= 10^{-2}$ Nsm^2)



Ans. 4 mm

Sol. $F = F' = \eta A \frac{dv}{dz} = mg \sin\theta \quad \frac{dv}{dz} = \frac{v}{h}$

$20 \times 10 \times \sin 30° = \eta \times 4 \times \frac{10}{h}$



$h = \frac{40 \times 10^{-2}}{100}$ – [$\eta = 10^{-1}$ poise $= 10^{-2}$ N-sec-m^{-2}]

$= 4 \times 10^{-3}$ m $= 4$ mm

Now Answer the questions below :

5. A metal square plate of 10 cm side rests on a 2 mm thick caster oil layer. Calculate the horizontal force needed to move the plate with speed 3 cm s^{-1} : (Coefficient of viscosity of caster oil is 15 poise.)

(A) 2.25×10^{-2} N (B) 2.25×10^{-1}N (C) 2.25×10^{-3} N (D) 2.25×10^{-4} N

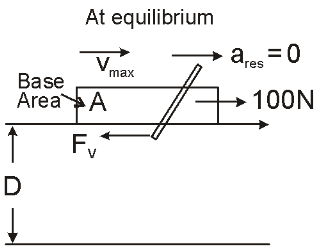
Ans. **(B)**

6. A man starts rowing his stationary cuboidal boat of base area A = 10m². The driving force on the boat due to rowing is 100 N in the direction of motion. Find the maximum velocity that the boat can achieve. Also find the time in which he will attain half of this maximum velocity. [Take coefficient of viscosity of water = 15 poise] The depth of the lake is 10 m and the combined mass of man and the boat to be 150 kg. (u = 0, velocity gradient uniform)

Sol. At equilbrium

$F_{ext} = F_V$

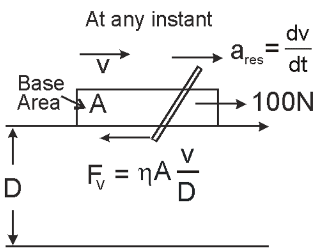
$F_{ext} = 100$ N and $F_V = \eta A \dfrac{V_{max}}{D} = 1.5 \times 10 \times \dfrac{V_{max}}{D}$



$\therefore 100 = 1.5\, V_{max}$ $\Rightarrow V_{max} = \dfrac{200}{3}$ m/s

At any instant

$F_{ext} - F_V = m\dfrac{dv}{dt}$ $\Rightarrow 100 - \eta A \dfrac{v}{D} = m \dfrac{dv}{dt}$



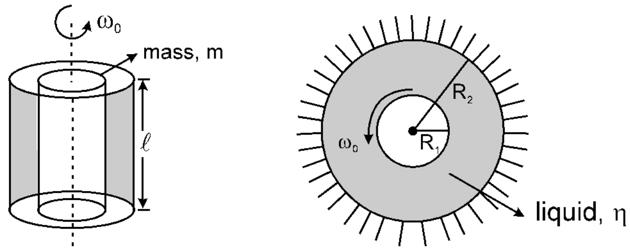
$\therefore 100 - 1.5\, v = 150 \dfrac{dv}{dt}$

from (1) $\displaystyle\int_0^t dt = \int_0^{\frac{V_{max}}{2}} 150 \frac{dv}{100 - 1.5v}$

$\Rightarrow$ t = $150 \displaystyle\int_0^{100/3} \frac{\ell n(100 - 1.5v)}{-1.5}$

$\Rightarrow$ t = 100 ℓn2 seconds

7. As per the shown figure the central solid cylinder starts with initial angular velocity ω_0. Find out the time after which the angular velocity becomes half. (Velocity gradient uniform)



Sol. $F = \eta A \dfrac{dv}{dz}$, where $\dfrac{dv}{dz} = \dfrac{\omega R_1 - 0}{R_2 - R_1}$

$$F = \eta \frac{2\pi R_1 \ell \omega R_1}{R_2 - R_1}$$

and $\tau = FR_1 = \dfrac{2\pi\eta R_1^3 \omega \ell}{R_2 - R_1}$

$$I\alpha = \frac{2\pi\eta R_1^3 \omega \ell}{R_2 - R_1} \Rightarrow \frac{MR_1^2}{2}\left(-\frac{d\omega}{dt}\right) = \frac{2\pi\eta R_1^3 \omega \ell}{R_2 - R_1}$$

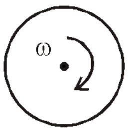
or $-\displaystyle\int_{\omega_0}^{\omega_0/2} \frac{d\omega}{\omega} = \frac{4\pi\eta R_1 \ell}{m(R_2 - R_1)} \Rightarrow t = \frac{m(R_2 - R_1)\ell n2}{4\pi\eta\ell R_1}$

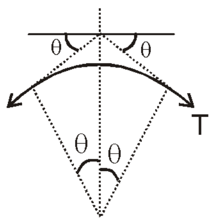
Numerical based Questions :

8. A thin ring of radius R is made of a material of density ρ and Young's modulus Y. If the ring is rotated about its centre in its own plane with angular velocity ω, if the small increase in its radius is $\dfrac{2\rho\omega^2 R^3}{\lambda Y}$ then find λ.

Ans. 2

Sol.





$$2T\sin\theta = \left(\frac{m}{2\pi R}R.2\theta\right)\omega^2 R'$$

For small 'θ' -

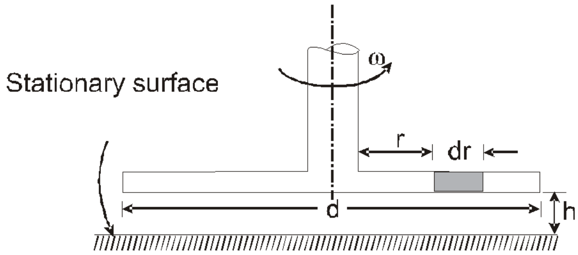
$$T = \frac{m\omega^2 R'}{2\pi}$$

$$\frac{T}{A} = \left(\frac{2\pi R' - 2\pi R}{2\pi R}\right)Y$$

$$\frac{m\omega^2 R'}{2\pi A} = \frac{\Delta R}{R}Y \text{ , If } R' \simeq R$$

$$\Delta R = \frac{m\omega^2 R^2}{2\pi AY} \text{ but } m = A\,2\pi R\,\rho \Rightarrow \Delta R = \frac{A2\pi R\rho\omega^2 R^2}{2\pi AY} = \frac{\rho\ \ \omega^2 R^3}{Y}$$

9. A circular disc of a diameter 'd' is slowly rotated in a liquid of large viscosity 'η' at a small distance 'h' from a fixed surface as shown in figure. If an expression for torque 'τ' necessary to maintain an angular velocity 'ω' is $\frac{\pi\eta\omega d^4}{\lambda h}$ then find λ.



Ans. **32**

Sol. $d\tau = r\eta\left(2\pi r dr\frac{\omega r}{h}\right)$

$$\tau = \int_0^r \frac{\eta}{h}\,2\pi\omega r^3 dr$$

$$\tau = \int_0^{d/2}\frac{\eta\omega}{h}2\pi\ \ r^3 dr$$

$$\tau = \left[\frac{2\pi\eta\omega}{h}\ \ \frac{r^4}{4}\right]_0^{d/2}$$

$$\tau = \frac{2\pi\eta\omega}{h}\ \ \frac{d^4}{4\ \times\ 16}$$

$$\tau = \left(\frac{\pi\eta\omega d^4}{32h}\right)$$

10. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity η = 3 poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity η = 2 poiseulles. The ratio of the terminal velocities of P and Q is :

Ans. **3**

Sol. $6\pi\eta rv + \rho_L Vg = \rho_0 Vg$

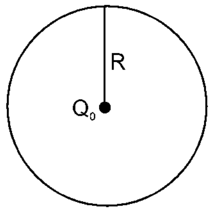
$$\frac{v_P}{v_Q} = \frac{(\rho_P V_P - \rho_L V_p)g}{6\pi\eta_p r_p} \times \frac{6\pi\eta_Q r_Q}{(\rho_Q V_Q - \rho_L V_Q)}$$

$$= \frac{r_p^3(8-0.8)}{\eta_p . r_p(8-1.6)} \times \frac{r_Q . \eta_Q}{r_Q^3}$$

$$= \left(\frac{r_p}{r_Q}\right)^2 \times \left(\frac{\eta_Q}{\eta_p}\right) \times \left(\frac{7.2}{6.4}\right) = 4 \times \frac{7.2}{6.4} \times \frac{2}{3} = 3$$

Subjective Type Questions :

11. In a ring having linear charge density λ, made up of wire of cross-section area A, young modulus y, a charge Q_0 is placed at it's centre. If initial radius is 'R', then find out change in radius



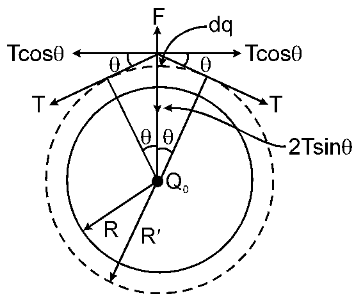
Ans. $\Delta R = \dfrac{k\lambda Q_0}{AY}$

Sol. Considering an element of angular width 2θ -

$dq = \lambda R' \cdot 2\theta \qquad \Rightarrow \qquad F = 2T \sin\theta$

$\dfrac{kdq.\ Q_0}{R'^2} = 2T\sin\theta \qquad \Rightarrow \qquad \dfrac{k\lambda R' \times 2\theta.\ Q_0}{R'^2} = 2T\sin\theta$

if θ is small, then $\sin\theta = \theta$



further $\dfrac{k\lambda Q_0}{R'} = T$

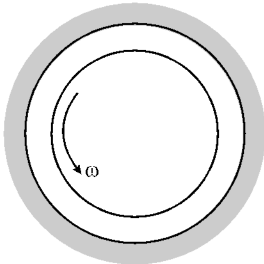
But $\quad Y = \dfrac{\text{stress}}{\text{strain}}, \quad \text{strain} = \dfrac{2\pi(R'-R)}{2\pi R} = \dfrac{R'-R}{R}$

$Y = \dfrac{T}{a(R'-R)} \qquad \Rightarrow \qquad R' - R = \dfrac{k\lambda Q_0}{R'AY}$

$Y = \dfrac{T}{A\dfrac{\Delta R}{R}}$

$\Rightarrow \qquad \Delta R = \dfrac{TR}{AY} = \dfrac{k\lambda Q_0 R}{R'AY} = (R \simeq R')$

12. A cylinder of 150 mm radius rotates concentrically inside a fixed cylinder of 155 mm radius. Both cylinders are 300 mm long. Determine the viscosity of the liquid which fills the space between the cylinders if a torque of 0.98 N-m is required to maintain an angular velocity of 60 r.p.m.



Ans. η = 0.77 N-sec/m^2.

Sol. $\tau = Fr$

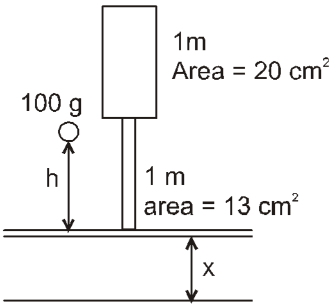
$$F = \frac{\eta(2\pi r_i) \times \omega r_i \times \ell}{r_0 - r_i} \Rightarrow \tau = 2\pi\eta r_i^3 \times \frac{\omega \times \ell}{(r_0 - r_i)}$$

on solving and putting values

η = 0.77 N-s/m^2

13. A vertical rod 2 m long , fixed at the upper end , is 13 cm^2 in area for '1 m' and 20 cm^2 in area for 1 m. A collar is attached to the free end. Through what height can a load of 100 kg fall on to collar to cause maximum stress of 50 N/mm^2. Y = 200000 N/mm^2. (g = 9.8 m/s^2)

Ans. 1.33 cm



Sol.

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}, \; k_1 = \frac{A_1 Y}{\ell}, \; k_2 = \frac{A_2 Y}{\ell}, \; \ell = 1 \text{ m}$$

$$k = \frac{k_1 k_2}{k_1 + k_2} = \frac{A_1 A_2}{(A_1 + A_2)} \frac{Y}{\ell}$$

$$k = \frac{52}{33} \times 10^8$$

$F = 50 \times 10^6 \times 13 \times 10^{-4} = 650 \times 10^2$ N

$F = kx$

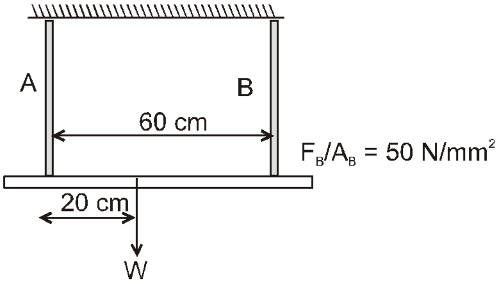
$$x = \frac{165}{4} \times 10^{-5} \text{ m} \qquad mg(h + x) = \frac{1}{2}kx^2 = \frac{F^2}{2k}$$

on solving h = 1.33 cm

14. Two rods 'A' & 'B' of equal free length hang vertically 60 cm apart and support a rigid bar horizontally. The bar remains horizontal when carrying a load of 5000 kg at 20 cm from 'A'. If the stress in 'B' is 50 N/mm², find the stress in 'A' and the areas of 'A' and 'B'.

Given $Y_B = 9 \times 10^4$ N/mm², $Y_A = 2 \times 10^5$ N/mm², g = 10 m/sec²

Ans. $\frac{1000}{9}$ N/mm², 300 mm², $\frac{1000}{3}$ mm²



Sol.

$Y_A = 2 \times 10^5$ N/mm²

$Y_B = 9 \times 10^4$ N/mm²

$F_A \times 20 = F_B \times 40$

$F_A = 2F_B$

$F_A + F_B = 5000\ g$

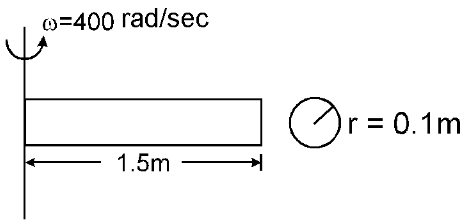
$F_B = \frac{5000\ g}{3}$, $F_A = \frac{10000\ g}{3}$

$\sigma_B = 50 = \frac{F_B}{A_B}$ $A_B = \frac{50 \times 10^3}{3 \times 50} = \frac{1000}{3}$ mm²

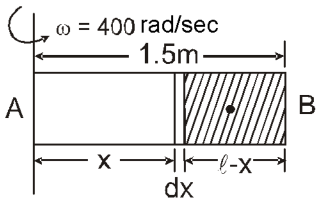
$\Delta\ell = \frac{F_B \ell}{A_B Y_B} = \frac{F_A \ell}{A_A Y_A}$, $\sigma_A = \frac{F_A}{A_A} = \frac{Y_A}{Y_B}\sigma_B = \frac{20}{9} \times 50 = \frac{1000}{9}$ N/mm²

$\sigma_A = \frac{1000}{9} = \frac{10^5}{3\ A_A}$ $A_A = 300$ mm²

15. Given Y = 2×10^{11} N/m², $\rho = 10^4$ kg/m³. Find out elongation in rod.



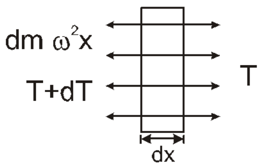
Sol.

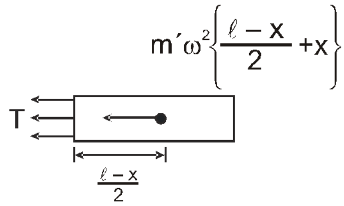


mass of shaded portion

$m' = \frac{m}{\ell}(\ell - x)$ [where m = total mass = $\rho A\lambda$]

$T = m'\omega^2\left[\frac{\ell - x}{2} + x\right] \Rightarrow T = \frac{m}{\ell}(\ell - x)\omega^2\left(\frac{\ell + x}{2}\right) \qquad T = \frac{m\omega^2}{2\ell}(\ell^2 - x^2)$





this tension will be maximum at A $\left(\frac{m\omega^2\ell}{2}\right)$ and minimum at 'B' (zero), elongation in element

of width 'dx' = $\frac{Tdx}{AY}$

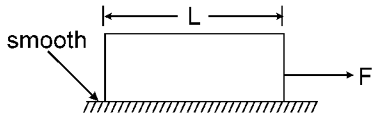
Total elongation

$$\delta = \int \frac{Tdx}{AY} = \int_0^{\ell} \frac{m\omega^2(\ell^2 - x^2)}{2\ell \quad AY} \, dx$$

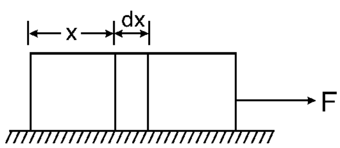
$$\delta = \frac{m\omega^2}{2\ell \quad AY}\left[\ell^2 x - \frac{x^3}{3}\right]_0^{\ell} = \frac{m\omega^2 \times 2\ell^3}{2\ell AY \times 3} = \frac{m\omega^2\ell^2}{3AY} = \frac{\rho A\ell\omega^2\ell^2}{3AY}$$

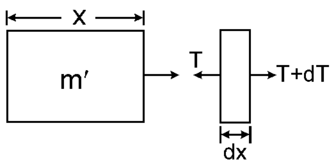
$$\delta = \frac{\rho\omega^2\ell^3}{3Y} = \frac{10^4 \times (400) \times (1.5)^3}{3 \times 2 \times 10^{11}} = 9 \times 10^{-3} \text{ m} = 9\text{mm}$$

16. Find out the elongation in block. If mass, area of cross-section and young modulus of block are m, A and Y respectively.



Sol.





Acceleration, $a = \frac{F}{m}$ then $T = m'a$ where $\Rightarrow$ $m' = \frac{m}{\ell}x$

$T = \frac{m}{\ell}x\,\frac{F}{m} = \frac{F\ x}{\ell}$

Elongation in element 'dx' = $\frac{Tdx}{AY}$

total elongation, $\delta = \int_0^{\ell} \frac{Tdx}{AY}$ $\quad d = \int_0^{\ell} \frac{Fxdx}{A\ell Y} = \frac{F\ell}{2AY}$