PHYSICS

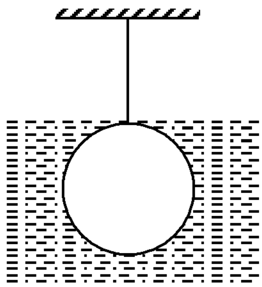
TARGET : JEE- Advanced 2021

CAPS-23

FLUID MECHANICS + SURFACE TENSION

SCQ (Single Correct Type) :

1. A sphere of radius R and relative density 4 is hanging with the help of a string such that it remains just inside water. The ratio of force exerted by the liquid on upper and lower half of the sphere is

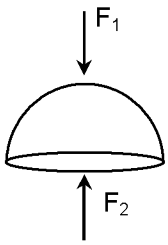


(A) 1 : 5 (B) 1 : 4 (C) 2 : 5 (D) 1 : 3

Ans. **(A)**

Sol. Consider the upper half sphere only, let F_1 & F_2 be forces exerted by the water on curved and flat surface respectively

$F_2 - F_1 = F_B$



$$\rho Rg(\pi R^2) - F_1 = \frac{2}{3}\pi \quad \rho \quad R^3 g$$

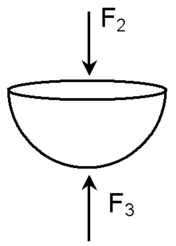
$$F_1 = \frac{1}{3}\pi \quad \rho \quad R^3 g$$

Similarly for lower half

$F_3 - F_2 = F_B$

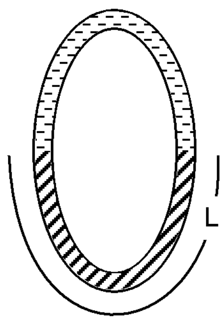
$$F_3 - \rho Rg(\pi R^2) = \frac{2}{3}\pi\rho R^3 g$$

$$F_3 = \frac{5}{3}\pi \quad \delta R^3 g$$



$\therefore \frac{F_1}{F_3} = \frac{1}{5}$

2. A tube of given shape has cross-sectional area S and total length 2L. its bottom and upper halves are filled with two non-viscous, non-compressible liquids of densities 3ρ, ρ respectively. If the liquid interface is slightly displaced then find its angular frequency.



(A) It is not SHM (B) $\sqrt{\frac{3g}{2L}}$ (C) $\sqrt{\frac{2g}{L}}$ (D) $\sqrt{\frac{g}{L}}$

Ans. **(D)**

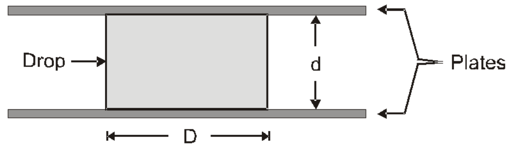
Sol. Restoring force

$F = 2s\,(3\rho - \rho)\,gx = 4s\rho gx$

$m = Ls\rho + Ls3\rho = 4\,Ls\rho$

$\omega = \sqrt{\frac{4s\rho g}{4L\rho s}} = \sqrt{\frac{g}{L}}$

3. A drop of liquid of surface tension σ is in between the two smooth parallel glass plates held at a distance d apart from each other in zero gravity. The liquid wets the plate so that the drop is a cylinder of diameter D with its curved surface at right angles to both the plates. Determine the force acting on each of the plates from drops under the given considerations.



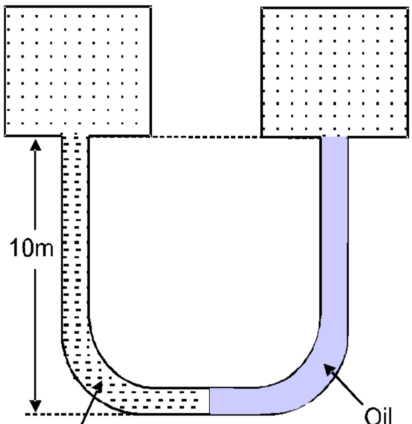
(A) $\frac{\sigma\pi D}{2}$ (B) $\frac{\sigma^2\pi D}{2}$ (C) zero (D) None of these

Ans. **(A)**

Sol. $F_{net} = \sigma\pi D - P_{ex} \times \frac{\pi D^2}{4}$ $\qquad$ $\left[P_{ex.} = \left(\frac{\sigma}{r_1} + \frac{\sigma}{r_2}\right) = \frac{\sigma}{r_1} \quad (r_2 = \infty)\right]$

$F_{net} = \sigma\pi D - \frac{2\sigma}{D} \times \frac{\pi D^2}{4} = \frac{\sigma\pi D}{2}$

4. One side of vertical U–tube contains water and other contains oil as shown in figure. The ends of the tube are connected to the two vessels of same volume. Vessel A is filled with air at atmospheric pressure. The absolute temperature inside vessel B is one fourth that of vessel A and number of moles inside vessel B is 5 times that of vessel A. The relative density of the oil is: (atmospheric pressure $=10^5$ Pa, g = 10 m/s^2)



(A) $\frac{2}{3}$ (B) $\frac{3}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

Ans. **(B)**

Sol. $P_A + \rho_w g \times 10 = P_B + \rho_{Oil}\, g \times 10$

$\frac{n_A R T_A}{V} + 10\rho_w g = \frac{n_B R T_B}{V} + 10\rho_{oil} g$

$\frac{n_A R T_A}{V} + 10\rho_w g = \frac{5 n_A R}{V}\frac{T_A}{4} + 10\rho_{oil} g$

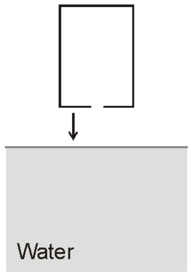
$P_0 + 10\rho_w g = \frac{5}{4}P_0 + 10\rho_{oil} g$

$\frac{P_0}{4} = 10g\,(\rho_w - \rho_{oil})$

$\rho_{oil} = 750$

$\frac{\rho_{oil}}{\rho_W} = \frac{3}{4}$.

5. An empty container has a circular hole of radius r at its bottom. The container is pushed into water very slowly as shown. To what depth the lower surface of container (from surface of water) can be pushed into water such that water does not flow into the container ?



(Surface tension of water = T, density of water = ρ)

(A) $\frac{2T}{\rho gr}$ (B) $\frac{4T}{\rho gr}$ (C) $\frac{T}{\rho gr}$ (D) $\frac{T}{2\rho gr}$

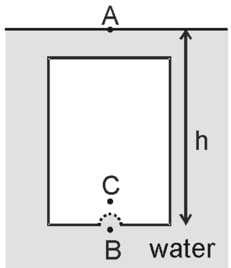
Ans. (A)

Sol. Let the container is dipped to depth h, so the contact angle becomes θ

$P_A = P_0$

$P_B = P_0 + \rho gh$

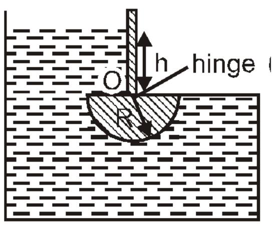
$P_C = P_0 + \rho gh - \frac{2T}{r}\cos\theta = P_0$



$\cos\theta = \frac{\rho ghr}{2T} \leq 1$

$\boxed{h \leq \frac{2T}{\rho gr}} \Rightarrow \boxed{h_{max} = \frac{2T}{\rho gr}}$

6. Suppose there is a tank having weightless gate having a cylindrical bottom as shown in diagram. It can rotate about a hinge O. Water is filled in the tank so that the gate just opens i.e., rotates clockwise about the hinge when the height of water level is h above the flat portion of the gate as shown. Obtain minimum value of h.

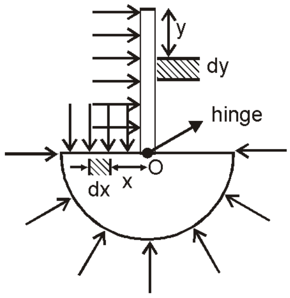


(A) R (B) $\sqrt{2}$R (C) R/2 (D) $\sqrt{3}$R

Ans. **(D)**

Sol. Anticlock wise torque about hinge 'O' {for horizontal portion}

$$\tau_1 = \int_0^R \rho gh(b\ \ dx)x = \frac{\rho ghbR^2}{2}, \quad \text{\{b is the width of gate\}}$$



For vertical portion : $\{\tau_2$ clockwise $\}$

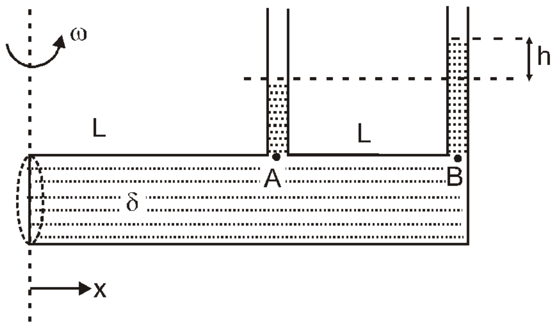
$$\tau_2 = \int_0^h \rho gy(b\ \ dy)(h-y) = \frac{\rho gbh^3}{6}$$

The gate just open :

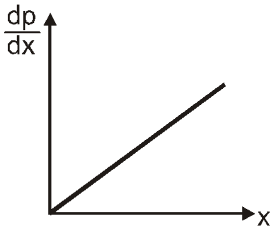
$$\tau_1 = \tau_2 \Rightarrow \frac{\rho ghbR^2}{2} = \frac{\rho gbh^3}{6} \Rightarrow \boxed{h \geq \sqrt{3}R}$$

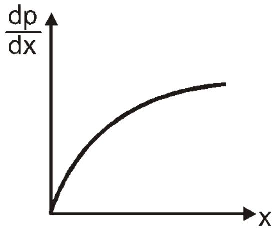
Comprehension Type Question:

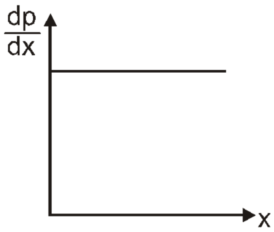
A horizontal thin tube of length 2L completely filled with a liquid of density δ rotates about a vertical axis passing through one of its end and along the diameter of its side face with an constant angular velocity ω. Two vertical thin long tubes are fitted with the horizontal tube to measure the pressure difference between points A and B which are at same horizontal level as shown. Neglect viscous forces and surface tension of liquid.



7. The graph of the pressure gradient $\left(\frac{dP}{dx}\right)$ with the distance x measured from the axis of rotation is :

(A) 

(B) 

(C) 

(D) None of these

Ans. **(A)**

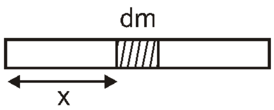
8. The difference in the heights (h) of the liquid column in the two vertical thin tubes is :

(A) $\frac{\omega^2 L^2}{2g}$ (B) $\frac{3\omega^2 L^2}{2g}$ (C) $\frac{5\omega^2 L^2}{2g}$ (D) None of these

Ans. **(B)**

Sol. $dm = \delta A dx$

$dPA = (\delta A dx)a$



$\frac{dP}{dx} = \delta\omega^2 x$

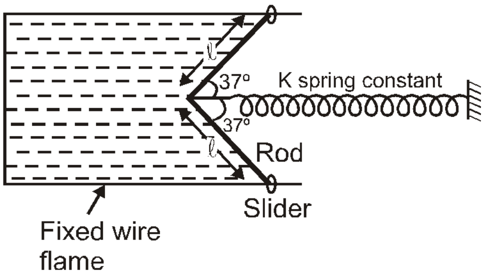
The difference in heights

$h = y_2 - y_1 = \frac{\omega^2 (2L)^2}{2g} - \frac{\omega^2 L^2}{2g}$

$= \frac{3\omega^2 L^2}{2g}$

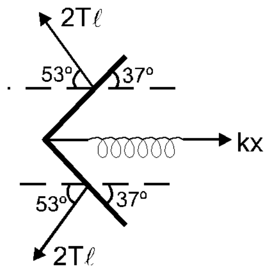
Numerical based Questions :

9. A rigid bent light rod of total length 2ℓ can slide on fixed wire frame with the help of frictionless sliders. There is thin liquid film (surface tension T) between bent rod and wire frame. In equilibrium the elongation in spring is given by $\left(\frac{4T\ell\alpha}{5K}\right)$. Then find the value of α.



Ans. 3

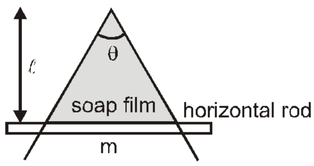
Sol.



$2(2T\ell)\cos 53° = Kx$

$\frac{4T\ell 3}{5K} = x.$

10. A wire is bend at an angle θ. A rod of mass m can slide along the bended wire without friction as shown in figure. If a soap film is maintained in the frame and frame is kept in a vertical position and rod is in equilibrium. If rod is displaced slightly in vertical direction. The time period of small oscillation is $a\pi\sqrt{\frac{b\ell}{g}}$ sec. where a and b are constant number then a+b is :

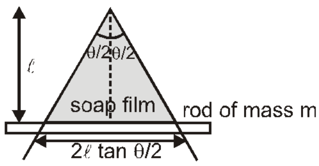


Ans. 3

Sol. $2T\left(2\ell\tan\frac{\theta}{2}\right) = mg\ ,\ T = \frac{mg}{4\ell\tan\frac{\theta}{2}}$

if x be the displacement in vertical direction of the rod from equilibrium position

$F_{res} = -2x\tan\frac{\theta}{2}T\times 2\ ,\ a = \frac{-4T\tan\frac{\theta}{2}}{m}x\ ,$



$$T = 2\pi\sqrt{\frac{m}{4T\tan\frac{\theta}{2}}} = 2\pi\sqrt{\frac{m}{4\frac{mg}{4\ell}}} = 2\pi\sqrt{\frac{\ell}{g}}$$

$\therefore$ a + b = 3 **Ans. 3**

11. Due to the excess pressure of surface tension find the change in the radius (in Å) of a liquid drop of radius 1 mm, surface tension 0.075 N/m and Bulk modulus 1.25 × 10^{+8} N/m^2.

Ans. 4

Sol. $p = \frac{2s}{r} = \frac{BdV}{V}$

$V = \frac{4}{3}\pi r^3$

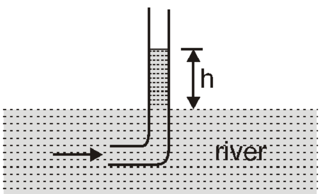
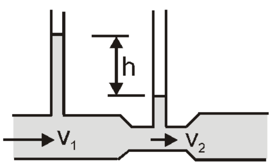
$dV = 4\pi r^2 dr$

$\frac{2s}{r} = B \times \frac{4\pi r^2 dr}{4/3\pi r^3}$

$dr = \frac{2s}{3B} = \frac{2\times 0.075}{3\times 1.25\times 10^8} = \frac{0.15}{3\times 1.25} \times 10^{-8} = 4Å$

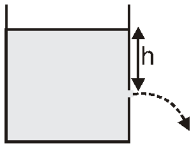
Matrix Match Type :

12. In **Column** I, position of a water level are shown in certain cases and certain things are given and certain quantities are asked.Correctly match the asked quantity with the quantities given in column II.

Column–I	**Column–II**
(A) 	(p) 2gh
Pitot tube is used to find the speed v of the flow of the river water. The expression for v^2 is :	
(B) 	(q) $\frac{2gh}{3}$

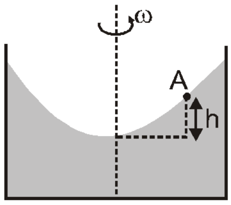
Area of cross section of the narrower part of the venturi tube is 1/2 of that of the wider part. If the velocity in the wider part is v_1 and that in the narrower part is v_2 then

v_1^2 is :

(C) 

(r) 4gh

If the velocity of the efflux from a wide stationary tank and small orifice is v then v^2 is :

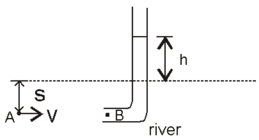
(D) 

(s) 6gh

A cylindrical beaker containing some liquid is rotated with constant angular speed about its axis. If the speed of the point A is v then v^2 is :

Ans. **(A) –p ; (B) – q ; (C) – p ; (D) – p**

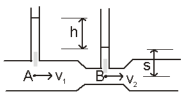
Sol. (A) Applying Bernoulli's equation between A and B



$$\frac{p_a + s\rho g}{\rho} + \frac{v^2}{2} = \frac{p_a + s\rho g + h\rho g}{\rho}$$

$v^2 = 2gh$

(B) Applying Bernoulli's equation between A and B



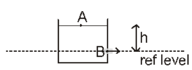
$$\frac{p_a + s\rho g + h\rho g}{\rho} + \frac{v_1^2}{2} = \frac{p_a + s\rho g}{\rho} + \frac{v_2^2}{2}$$

$\therefore \quad v_2^2 - v_1^2 = 2gh$

Also from continuity eq, $v_2 = 2v_1$

$\therefore \quad v_1^2 = \frac{2gh}{3}$

(C) (A) Applying Bernoulli's equation between A and B



$$\frac{p_a}{\rho} + gh \ \frac{p_a}{\rho} = + \frac{v^2}{2}$$

$v^2 = 2gh$

(D) $h = \frac{\omega^2 r^2}{2g}$

$\therefore$ $v^2 = \omega^2 r^2 = 2gh$.

Subjective Type Questions :

13. Two identical cylindrical vessels with their bases at the same level each contain a liquid of density ρ as shown in figure. The height of the liquid in one vessel is h_2 and other vessels h_1, the area of either base is A. Find the work done by gravity in equalizing the levels when the two vessels are connected.

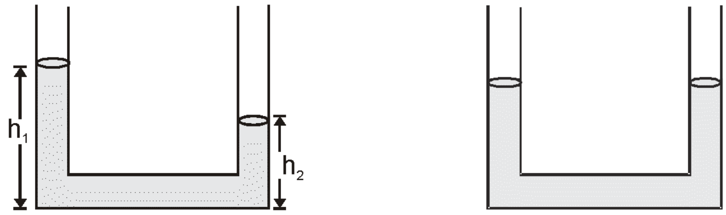
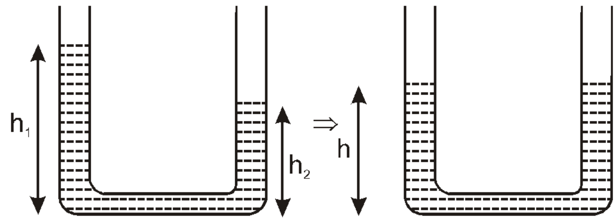


Figure (1) Figure (2)

Ans. $\frac{gA\rho}{4}(h_1 - h_2)^2$

Sol. Let h be level in equilibrium. Equating the volumes, we have



$Ah_1 + Ah_2 = 2Ah$

$\therefore$ $h = \left(\frac{h_1 + h_2}{2}\right)$

Work done by gravity = $U_i - U_f$

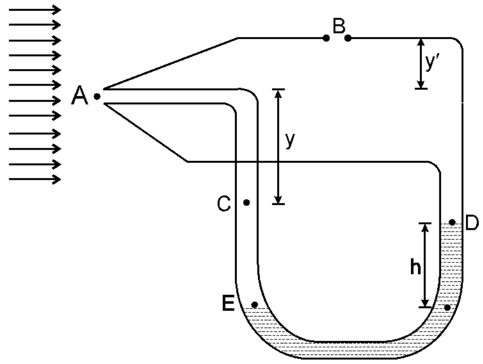
$W = \left(m_1 g \frac{h_1}{2} + m_2 g \frac{h_2}{2}\right) - (m_1 + m_2) g \frac{h}{2}$

$= \frac{Ah_1 \rho g h_1}{2} + \frac{Ah_2 \rho g h_2}{2} - [Ah_1 \rho + Ah_2 \rho] g \left(\frac{h_1 + h_2}{4}\right)$

Simplifying this, we get

$W = \frac{\rho A g}{4} (h_1 - h_2)^2$

14. A Pitot tube is shown in figure. Wind blows in the direction shown. Air at inlet A is brought to rest, whereas its speed just outside of opening B is unchanged. The U tube contains mercury of density ρ_m. Find the speed of wind with respect to Pitot tube. Neglect the height difference between A and B and take the density of air as ρ_a.



Ans. $v = \sqrt{\dfrac{2(\rho_m - \rho_a)gh}{\rho_a}}$

Sol. Bernoulli's equation between A and B gives

$$\frac{p_A}{\rho_a} = \frac{p_B}{\rho_a} + \frac{v^2}{2} \qquad \Rightarrow \qquad v^2 = 2\left[\frac{p_A - p_B}{\rho_a}\right]$$

Also equating pressures at horizontal level of E

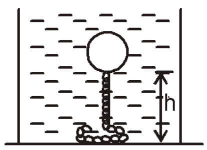
$$p_A + \rho_a gy + \rho_a gh = p_B + \rho_a gy' + \rho_a gy + \rho_m gh.$$

$\Rightarrow$ $p_A + \rho_a gh = p_B + \rho_m gh$ $\qquad [\because y'\ 0]$

$$p_A - p_B = (\rho_m - \rho_a)\, gh.$$

$$v^2 = \frac{2(\rho_m - \rho_a)gh}{\rho_a} \qquad v = \sqrt{\frac{2(\rho_m - \rho_a)gh}{\rho_a}}$$ **Ans.**

15. One end of a long iron chain of linear mass density λ is fixed to a sphere of mass m and specific density 1/3 while the other end is free. The sphere along with the chain is immersed in a deep lake. If specific density of iron is 7, the height h above the bed of the lake at which the sphere will float in equilibrium is (Assume that the part of the chain lying on the bottom of the lake exerts negligible force on the upper part of the chain.) :



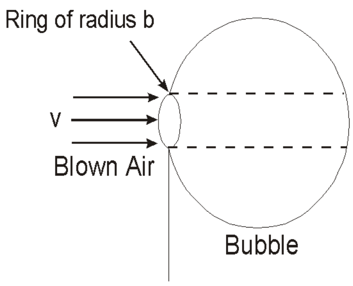
Ans $\dfrac{7m}{3\lambda}$

Sol. Weight of sphere + chain = $(m + \lambda h)g$

Buoyant force = $\left(3m + \dfrac{\lambda h}{7}\right) g$

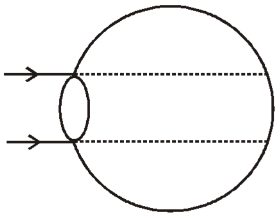
for equilibrium, weight = Buoyant force $\qquad$ or , $m + \lambda h = 3m + \dfrac{\lambda h}{7}$ $\qquad$ or $h = \dfrac{7m}{3\lambda}$

16. Bubbles are made by dipping a circular ring of radius b in a soap solution and then blowing air on the film formed on the ring. Assume that the blown air is in the form of a cylinder of radius b. It has speed v and stops after striking the surface of the bubble being formed. The bubble grows spherically. Let the radius R of the bubble (>> b), so that the air strikes the bubble surface perpendicularly. The surface tension of the solution is T and air density is ρ. Obtain the radius of the bubble when it separates from the ring in terms of the given parameters (neglect the mass of the bubble).



Ans. $\frac{4T}{\rho v^2}$

Sol.



We know that force = $\frac{(dm)\times V}{dt}$

where $\frac{dm}{dt}$ = rate of mass transferred.

Now $\frac{dm}{dt}$ is also eqval to $\frac{dm}{dt} = \frac{\rho\times(\pi b^2)\times(Vdt)}{dt} = \rho\pi b^2 V$.

so force = $\rho V^2 \pi b^2$

= pressure exerted by air on walls = $\frac{\rho V^2\pi b^2}{\pi b^2} = \rho V^2$.

when the thrust of this pressure becomes equal to the excess pressure

$\Rightarrow \quad \rho V^2 = \frac{4T}{r} \Rightarrow r_{final} = \frac{4T}{\rho V^2}$

Alternate

When force due to surface tension on bubbles is equal to the Force due to blowing air bubble leave contact with ring (separate from ring)

$F = 2 \times (2\pi bT)\sin\theta \qquad (\sin\theta = \frac{b}{R})$

$F = 4\pi Tb \left(\frac{b}{R}\right) = \rho\pi b^2 v^2$

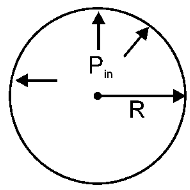
$R = \frac{4T}{\rho v^2}$

17. A soap bubble of radius R and surface tension T is formed in vacuum. It is slowly charged so that it slowly expands. It is stopped charging when the radius becomes 2R. Find the amount of charge given to the bubble. (T = constant)

Ans. $Q = \sqrt{768\pi^2 R^3 \in_0 T}$

Sol. we have initially $P_{in} = \frac{4T}{R}$ ....(1)

Now when extra charge is given, for equilibrium we have



$P'_{in} + \frac{\sigma^2}{2\varepsilon_0} = \left(\frac{4T}{2R}\right)$ ....(2)

Now we have $P'_{in} = \frac{(P_{in})v_i}{v_f}$ [isothermal change]

where v_i - initial volume of bubble

v_f - final volume of bubble

so, $P'_{in} = \frac{(P_{in})}{8}$

so, (1) equation will give

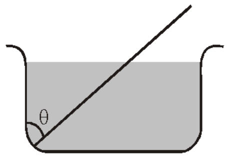
$$\frac{4T}{8R} + \frac{\sigma^2}{2\varepsilon_0} = \frac{4T}{2R} \quad \Rightarrow \quad \frac{\sigma^2}{2\varepsilon_0} = \frac{4T}{2R}\left(1 - \frac{1}{4}\right) = \frac{12T}{8R}$$

so, $\sigma^2 = \frac{12T\varepsilon_0}{4R} = \left(\frac{3T\varepsilon_0}{R}\right)$ so, $\sigma = \sqrt{\frac{3T\varepsilon_0}{R}}$

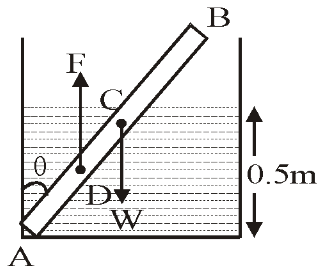
now charge $Q = \sigma \,.\, 4\pi\,(2R)^2 = 4\pi.(4R^2)\sqrt{\frac{3T\varepsilon_0}{R}} = \sqrt{256 \times 3T \;\; \varepsilon_0 R^2 \pi^2}$

$Q = \sqrt{768\pi^2 R^3 T\varepsilon_0}$

18. A wooden plank of length 1m and uniform cross–section is hinged at one end to the bottom of a tank as shown in figure. The tank is filled with water upto a height 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ that the plank makes with the vertical in the equilibrium position. (exclude the cosθ = 0°).



Sol. Let cross-section area of the plank is A then weight of plank W= (1 × A) 0.5 × g length of plank inside the water = $\frac{0.5}{\cos\theta}$



So upthrust on the plank

$\left(\frac{0.5}{\cos\theta}\right)$ = A × 1 × g

torque about point A

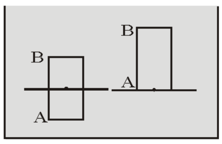
W × AC sin θ = Th × AD sinθ

(1 × A) × 0.5 × g × 0.5 sinθ

= $\left(\frac{0.5}{\cos\theta}\right)$A × 1 × g × $\left[\left(\frac{1}{2}\right)\times\frac{0.5}{\cos\theta}\right]$ sinθ

$\Rightarrow 1 = \frac{1}{2\cos^2\theta} \Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$

19. A cylindrical rod of length ℓ = 2m & density $\frac{\rho}{2}$ floats vertically in a liquid of density ρ as shown in figure.



(i) Show that it performs SHM when pulled slightly up & released & find its time period. Neglect change in liquid level.

(ii) Find the time taken by the rod to completely immerse when released from position shown in (b). Assume that it remains vertical throughout its motion. (take g=π^2 m/s^2)

Sol. Intially : $mg = f_B \Rightarrow mg = Vd_Lg = Ahd_Lg$

when pulled slightly up by x then

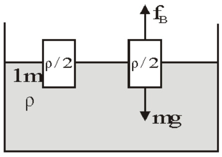
$f_{net} = mg - f_B = mg - A(h-x)d_Lg$

$= mg - Ahd_Lg + Axd_Lg$

$f_{net} = Axd_Lg$

force directry propotional to x therefore if will perform S.H.M.

(ii) $ma = (mg - Vd_L(g))$



$a = \left(g - \frac{Ad_Lxg}{A(H)d_m}\right)$

$a = g - \frac{2gx}{2}$

$\frac{d^2x}{dt^2} + gx - g = 0$ can be compered with

$\frac{d^2x}{dt^2} + \omega^2 x - g = 0 \Rightarrow w = \sqrt{g}$

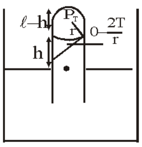
$T = \frac{2\pi}{\omega} \Rightarrow$ and time required is = T/2, t = 1 sec

20. When a vertical capillary of length ℓ with the sealed upper end was brought in contact with the surface of a liquid, the level of this liquid rise to the height h. The liquid density is ρ, the inside diameter of the capillary is d, the contact angle is θ, the atmospheric pressure is P_0. Find the surface tension of the liquid.

Ans. $T = \left(\frac{P_0 h}{\ell - h} + \rho g h\right)\frac{d}{4\cos\theta}$

Sol. From diagram $r\cos\theta = \frac{d}{2} \Rightarrow r = \frac{d}{2\cos\theta}$

$P_0\ell A = P_T(A)(\ell - h)$



$P_T = \frac{P_0\ell}{\ell - h}$; $P_A = \left(\frac{P_0\ell}{\ell - h} - \frac{2T}{r}\right)$

$$P_B = \frac{P_0 \ell}{\ell - h} - \frac{2T}{r} + \rho g h = P_0$$

$$= \left(\frac{P_0 h}{\ell - h} + \rho g h \right) = \frac{2T}{d}(2\cos\theta)$$

$$T = \left(\frac{P_0 h}{\ell - h} + \rho g h \right) \frac{d}{4\cos\theta}$$