competishun — Power of real gurus **PHYSICS** TARGET : JEE- Advanced 2021	**CAPS-22** **WAVE OPTICS**

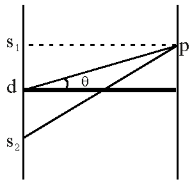
SCQ (Single Correct Type) :

1. In a Young's double slit experiment, the separation between the slits is d, distance between the slit and screen is D (D >> d). In the interference pattern, there is a maxima exactly in front for each slit. Then the possible wavelength(s) used in the experiment are

(A) d^2/D, $d^2/2D$, $d^2/3D$
(B) d^2/D, $d^2/3D$, $d^2/5D$
(C) $d^2/2D$, $d^2/4D$, $d^2/6D$
(D) none of these

Ans. (C)

Sol.



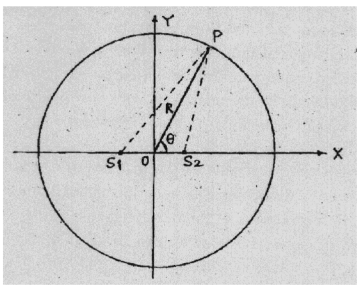
$$s_2p - s_1p = \frac{d \times y}{D} = \frac{d \times (d/2)}{D} = \frac{d^2}{2D}$$

$$\frac{d^2}{2D} = n\lambda$$

$$\lambda = \frac{d^2}{2nD}, \; n = 1, 2, \ldots\ldots\ldots\ldots.$$

$$\lambda = \frac{d^2}{2D}, \frac{d^2}{4D}, \frac{d^2}{6D}$$

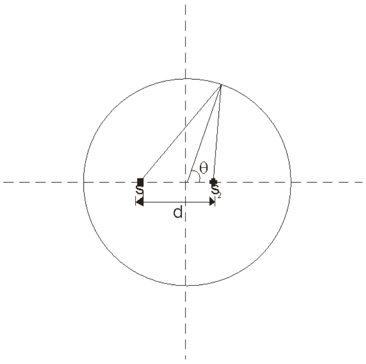
2. Two coherent sources of light S_1 and S_2, equidistant from the origin, are separated by a distance 2λ as shown. They emit light of wavelength λ. Interference is observed on a screen placed along the circle of large radius R. Point is seen to be a point of constructive interference. Then angle θ (other than 0° and 90°) is



(A) 45°
(B) 30°
(C) 60°
(D) Not possible in the first quandrant

Ans. (C)

Sol.



$\Delta x = d\cos\theta = n\lambda$

$d = 2\lambda$

$\because \quad \cos\theta = \frac{n}{2}$

$n = 1$

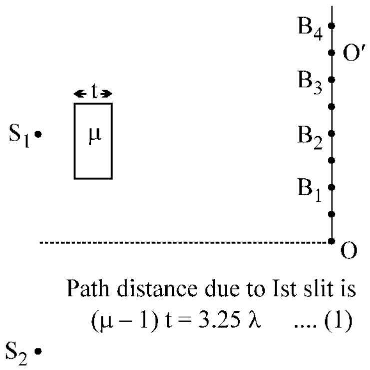
$\cos\theta = \frac{1}{2}$

$\theta = 60°$

3. In a regular YDSE, when thin film of refractive index μ is placed in front of the upper slit then it is observed that the intensity at the central point becomes half of the original intensity. It is also observed that the initial 3^{rd} maxima is now below the central point and the initial 4^{th} minima is above the central point. Now, a film of refractive index μ_1 and thickness same as the above film, is put in the front of the lower slit also. It is observed that whole fringe pattern shifts by one fringe width. What is the value of μ_1?

(A) $(4\mu+9)/12$ (B) $(4\mu+9)/13$ (C) $(4\mu+9)/11$ (D) None

Ans. **(B)**



Sol.

$\Delta\phi = \frac{2\pi}{\lambda}\, t\,(\mu - 1)$ ... (1)

$6\pi < \Delta\phi < 7\pi$ ... (2)

$I = I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$

$I = I_0 / 2$

$\Rightarrow \quad \cos\left(\frac{\Delta\phi}{2}\right) = \pm \frac{1}{\sqrt{2}} \quad \Rightarrow \quad \left(\frac{\Delta\phi}{2}\right) = 2n\pi \pm \frac{\pi}{4}$

$\Delta\phi = 4n\pi \pm \pi / 2 \quad$ ... (3)

Solving equation (2) & (3) we get

$\Delta\phi = 6.5\,\pi$

from (1)

$6.5\,\pi = \frac{2\pi}{\lambda}\, t\,(\mu - 1) \quad$ ... (4)

Introducing record film shifted the fringe pattern by one fringe with means.

$t\,(\mu_1 - 1) = \lambda \quad$ ... (5)

Solving 4 & 5

$\Rightarrow \quad \mu_1 = \frac{4\mu + 9}{13}$

4. Large number (N) of coherent waves superimpose at a point in the medium represented by-

$y_1 = a \sin (kx - \omega t)$

$y_2 = a \sin (kx - \omega t + \phi)$

$y_3 = a \sin (kx - \omega t + 2\phi)$

....

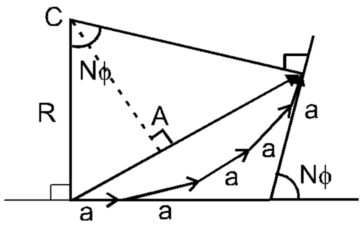
$y_N = a \sin(kx - \omega t + N\,\phi)$

where ϕ is very small. Then amplitude of resultant wave is-

(A) zero (B) Na (C) $(a/\phi) \sin(N\phi/2)$ (D) $2(a/\phi) \sin(N\phi/2)$

Ans. **(D)**

Sol.



$2R \sin\left(\frac{N\phi}{2}\right) = A$

$N\,\phi = \frac{Na}{R}$

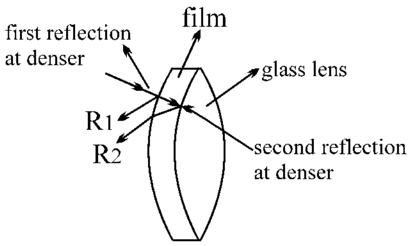
$\Rightarrow \quad A = 2\left(\frac{a}{\phi}\right) \sin\left(\frac{N\phi}{2}\right)$

5. Many people's glasses appear to be a blue-green colour when viewed under reflected light. A thin film of index of refraction n = 1.35 is applied to the outside surface of the glass so that the film/glass interface does not reflect any red light incident near normal of wavelength λ = 630 nm. What thickness must the film layer be in order to achieve this? Take the index of refractions of air and glass to be 1.0 and 1.6 respectively.

(A) 157.5 nm (B) 315.0 nm (C) 233.3 nm (D) 116.7 nm

Ans. **(D)**

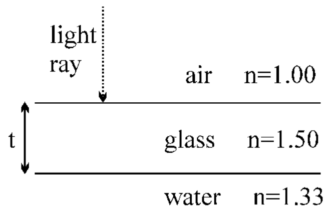
Sol.



$\Delta X_{net} = 2\mu t = \lambda/2$

$t = \lambda/4\mu = \frac{(630 \times 10^{-9})}{4 \times 1.35} = 116.7$ nm

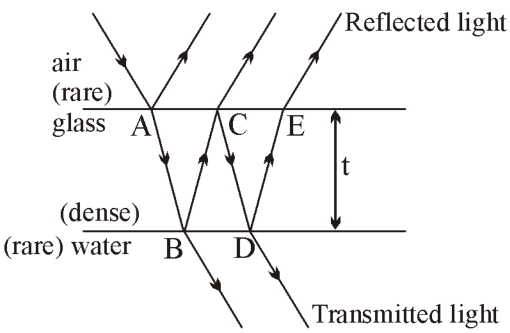
6. A light ray is incident normal to a thin layer of glass. Given the figure, what is the minimum thickness of the glass that gives the reflected light an orangish color (λ_{air}= 600 nm)?



(A) 50 nm (B) 100 nm (C) 150 nm (D) 200 nm

Ans. **(B)**

Sol. For reflected light to have orangish color, rays from A, C, E must be out of phase for l = 600 nm



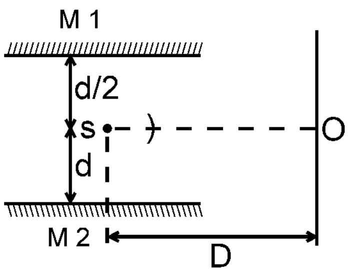
or $\delta = (2n + 1)\pi$

or $2\mu_g t\ (2n + 1)\ \frac{\lambda}{2}$

i.e. $t = (2n+1)\frac{\lambda}{4\mu_g}$

or $t_{min} = \frac{\lambda}{4\mu_g} = 100$ nm

7. M_1 and M_2 are plane mirrors and kept parallel to each other. At point O there will be a maxima for wavelength. Light from monochromatic source S of wavelength λ is not reaching directly on the screen. Then λ is : [$D >> d$, $d >> \lambda$]



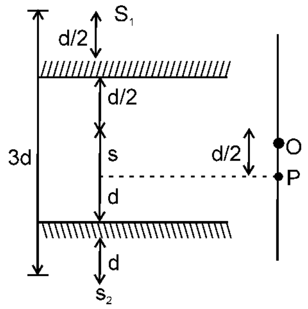
(A) $\frac{3d^2}{D}$ (B) $\frac{3d^2}{2D}$ (C) $\frac{d^2}{D}$ (D) $\frac{2d^2}{D}$

Ans. **(B)**

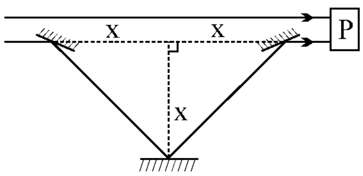
Sol. The situation can be taken as if there are two soureces S_1 and S_2 as shown in figure. Due to these S_1 and S_2 , the central maxima will be at P at a distance d / 2 from O.

for 'O' to be a maxima :

Path difference = $\frac{3d.d}{2D} = n\lambda \Rightarrow \lambda = \frac{3d^2}{2nD}$ ie. $\lambda = \frac{3d^2}{2d}, \frac{3d^2}{4d}$



8. Microwaves of wavelength λ = 5 cm and intensity I_0 are split and recombined by the metallic mirror system as shown in the figure. The intensity of the microwave at the detector P is minimum if x is



(A) 0.88 cm (B) 3.54 cm (C) 3.02 cm (D) 6.04 cm

Ans. **(D)**

Sol. Due to reflection, $\Delta x = \lambda/2$

Due to geometry, $\Delta x = \sqrt{2}x \times 2 - 2x = 2(\sqrt{2}-1)x$

Total $\Delta x = \frac{\lambda}{2} + 2(\sqrt{2}-1)x = \left(n+\frac{1}{2}\right)\lambda$

$x = \frac{n\lambda}{2(\sqrt{2}-1)} = \frac{n \times 5}{2 \times 0.41} = n \times 6.04$

9. An YDSE is performed with bi-chromatic light (5500Å and 6000Å) for d = 2 mm and D = 1m. Distance of first complete maxima from the central maxima on the screen, is

(A) 1.1 mm (B) 2.2 mm (C) 3.3 mm (D) 4.4 mm

Ans. **(C)**

Sol. $y = \frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$

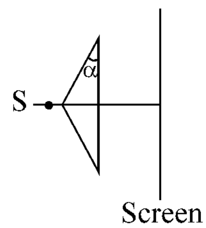
$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{12}{11}$

$n_1 = 12$

$y = \frac{12 \times 5500 \times 10 \times 1}{2 \times 10^{-3}} = 330 \times 10^{-5} = 3.3$ mm]

MCQ (One or more than one correct) :

10. For the biprism experiment shown in the figure, the fringe width increases when



(A) biprism is moved towards the slit

(B) entire apparatus is submerged in a liquid having R.I. less than that of prism

(C) a biprism having smaller angle α is used

(D) the slit width is reduced

Ans. **(ABC)**

Sol. $d = 2a(\mu - 1)\alpha$

$\beta = \frac{D\lambda}{d} \Rightarrow \frac{D\lambda}{2a(\mu-1)\alpha} = \beta$

(A) a is decreased so $\beta \uparrow$

(B) $\beta' = \frac{D(\lambda/\mu_m)}{2a\alpha\left(\frac{\mu}{\mu_m}-1\right)} = \frac{D\lambda}{2a\alpha}\frac{1}{(\mu-\mu_m)}$

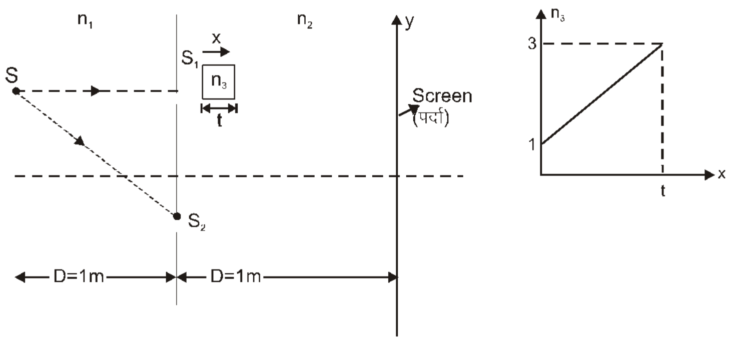
Now, $(\mu - \mu_m) < (\mu - 1)$

so, $\beta \uparrow$

(C) $\alpha \downarrow \Rightarrow \beta \uparrow$

Comprehension Type Question:

In YDSE arrangement as shown in figure, fringes are seen on screen using monochromatic source S having wavelength 3000 Å (in air). S_1 and S_2 are two slits seperated by d = 1 mm and D = 1m. Left of slits S_1 and S_2 medium of refractive index $n_1 = 2$ is present and to the right of S_1 and S_2 medium of $n_2 = 3/2$, is present. A thin slab of thickness 't' is placed in front of S_1. The refractive index of n_3 of the slab varies with distance from it's starting face as shown in figure.



11. In order to get central maxima at the centre of screen, the thickness of slab required is :

(A) 1 μm (B) 2 μm (C) 0.5 μm (D) 1.5 μm

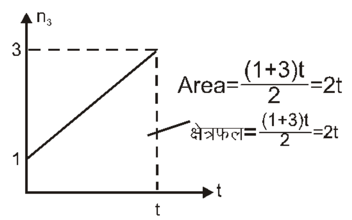
Ans. **(B)**

Sol. Path difference,

$$\Delta x = n_1 SS_2 + n_2S_2P - \left[(n_1SS_1 + n_2S_1P) - \int_0^t (n_3 - n_2)dx\right]$$

$$= n_1 (SS_2 - SS_1) + n_2(S_2P - S_1P) - \int_0^t n_3 dx + n_2 t$$

In order to get central maxima at centre of screen –



$$O = \frac{2\times(1\times10^{-3})^2}{2\times1} + 0 - 2t + \frac{3t}{2}$$

$0.5\,t = 1\ \mu m.$

$t = 2\ \mu m.$

12. If thickness of the slab is selected 1 μm, then position of central maxima will be : (y–coordinate)

(A) $\frac{1}{3}$mm (B) $-\frac{1}{3}$mm (C) $\frac{1}{6}$mm (D) $-\frac{1}{6}$mm

Ans. (B)

Sol. From previous equation :

$$0 = 1\ \mu m + \frac{3yd}{2D} - 0.5\ t$$

$$\frac{3}{2}\frac{yd}{D} = -0.5\ \mu m$$

$$y = -\left(\frac{10^{-6}}{3}\right)\left(\frac{1m}{1\times10^{-3}}\right)$$

$$= \frac{10^{-3}}{3} = \frac{-1}{3}\text{mm, below centre.}$$

13. Fringe width on the screen is :

(A) 0.4 mm (B) 0.1 mm (C) 0.2 mm (D) 0.3 mm

Ans. (C)

Sol. $\beta = \frac{\lambda D}{n_2 d} = \frac{3000\times10^{-10}\times1\times2}{3\times1\times10^{-3}}$

$= 2 \times 10^{-4}$ m = 0.2 mm.

Numerical based Questions :

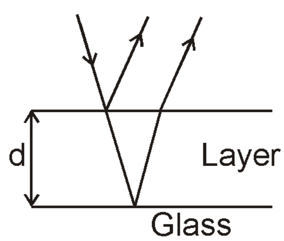
14. To reduce the light reflected by the glass surface of a camera lens, the surface is coated with a thin layer of another material which has an index of refraction (μ = 7/4) smaller than that of glass. The least thickness of the layer, to ensure that light falling perpendicularly on the surface and having wavelengths, λ_1 = 700nm and λ_2 = 420nm will be weekly reflected for both wavelengths is x × 10^{-8} m. Find x ?

Ans. 30

Sol. $2\mu d = (2n-1)\frac{\lambda}{2}$ (n = 1, 2, 3,.....)

$\Rightarrow \quad d = \frac{(2n-1)\lambda}{4\mu}$

For light of wavelength $\lambda_1 = 700\text{nm}$



$$d_1 = \frac{700}{4\mu}, \frac{2100}{4\mu}, \frac{3500}{4\mu}$$

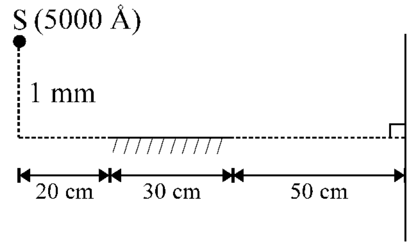
for light of wavelength $\lambda_2 = 420\text{nm}$

$$d_2 = \frac{420}{4\mu}, \frac{1260}{4\mu}, \frac{2100}{4\mu} \quad \Rightarrow \quad d_{min} = \frac{2100}{4\mu}$$

$$= \frac{2100}{4 \times \frac{7}{4}}$$

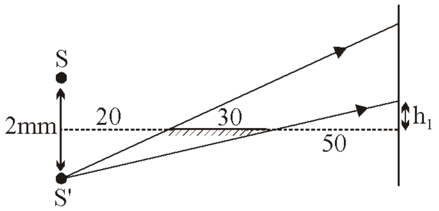
$d_{min} = 300\text{nm} = 3 \times 10^{-7}\text{m}$ = 30×10^{-8} m

15. Find the total number of fringes formed on screen in the Lloyd's mirror arrangement shown.



Ans. **12**

Sol.



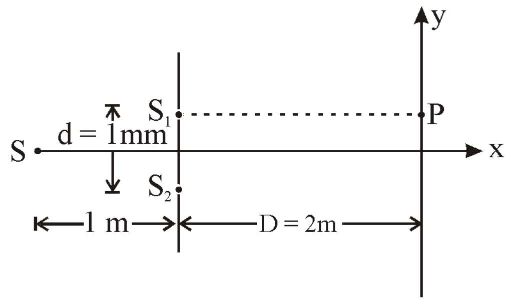
$\frac{0.1}{h_1} = \frac{50}{50} \Rightarrow h_1$ = 1mm

$\frac{L + h_1}{0.1} = \frac{80}{20} \Rightarrow L + h_1$ = 4 mm $\qquad \Rightarrow$ L = 3 mm

$B = \frac{\lambda D}{d} = \frac{5 \times 10^{-7} \times 1}{2 \times 10^{-3}} = 2.5 \times 10^{-4}$ m

$N = \frac{L}{B} = \frac{3 \times 10^{-3}}{2.5 \times 10^{-4}} = \frac{300}{25} = 12$

16. In a modified YDSE the source S of wavelength 5000Å oscillates about axis of setup according to the equation $y = 0.5 \sin\left(\frac{\pi}{6}\right) t$, where y is in millimeter and t in second. At what time t will the intensity at P, a point exactly in front of slit S_1, be maximum for the first time ?



Ans. 1

Sol. The path difference at point P,

$$\Delta x = (SS_2 - SS_1) + (S_2P - S_1P)$$

$$= \frac{dy}{D_1} + \frac{d(d/2)}{D_2}$$

For constructive interference,

$$\Delta x = \frac{dy}{D_1} + \frac{d^2}{2D_2} = n\lambda$$

$$\frac{(10^{-3})(0.5 \sin \pi\ t) \times 10^{-3}}{1} + \frac{(10^{-3})^2}{2 \times 2} = n\lambda$$

$$(0.5 \sin\left(\frac{\pi}{6}\right)t) \times 10^{-6} + 0.25 \times 10^{-6}$$

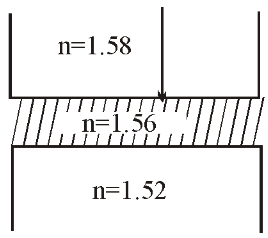
$$= (5000 \times 10^{-10})n = 0.5 \times 10^{-6}n$$

$$\sin\left(\frac{\pi}{6}\right)t = \frac{0.5n - 0.25}{0.5}$$

For the minimum value of t, n = 1.

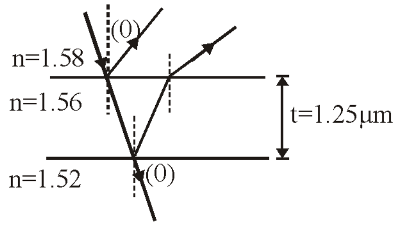
$$\sin\left(\frac{\pi}{6}\right)t = \frac{1}{2} \Rightarrow \quad \left(\frac{\pi}{6}\right)t = \frac{\pi}{6} \quad \text{or} \quad t = 1 \text{ sec.}$$]

17. A thin film of plastic (n = 1.56) is 0.25 μm thick. It is sandwiched between two glass slabs with refractive indice of 1.58 and 1.52, respectively. White light (400 – 700 nm) is first incident normally on the slab for which n = 1.58. Which visible wavelength (in nm) is missing in the reflected light ?



Ans. 520 nm

Sol. $\therefore\ 2t\mu = \left(n + \frac{1}{2}\right)\lambda$ (for destructive interference)



$\Rightarrow 2t\mu = \left(n + \frac{1}{2}\right)\lambda$

$\Rightarrow 2t(1.56) = \left(n + \frac{1}{2}\right)\lambda$

$\Rightarrow 2 \times 0.25 \times 10^{-6} \times 1.56 = \left(n + \frac{1}{2}\right)\lambda$

$\therefore \left(n + \frac{1}{2}\right)\lambda = 7.8 \times 10^{-7} = 780 \text{ nm}$

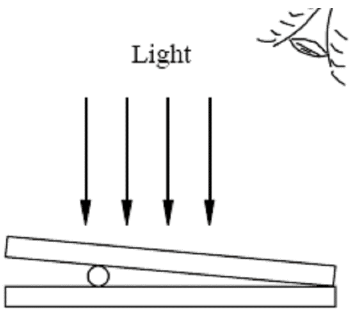
$\lambda = \frac{780}{n + \frac{1}{2}}$

$n = 0, \lambda = 1560 \text{ nm}$

$n = 1, \lambda = \frac{1560}{3} = 520 \text{ nm}$

$n = 2, \lambda = 780 \times \frac{2}{5} = 312 \text{ nm}$ (not possible)

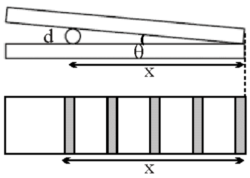
18. A very fine hair-like fiber is stuck between two microscope slides. As a result of this, there is a wedge of air between them. When the slides are illuminated normally (from above) with light of wavelength 550 nm, bright and dark interference bands are formed. The fiber is seen to lie at the position of fifth dark band counting from the common edge. What is the diameter (in nm) of the fiber?



Ans. **1100**

Sol. $d = x\theta$

$x = \frac{n\lambda}{2\theta}$ = condition of dark fringe



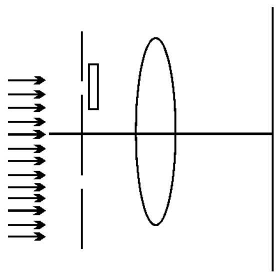
n = 0, first dark fringe at join of plates

n = 4, fifth dark fringe at fiber

$d = x\theta = \frac{4\lambda}{2} = 2\lambda$

$d = 2 \times 550$ nm $\Rightarrow 1100$ nm

19. The intensity received at the focus of the lens is I when no glass slab has been placed in of the slit. Both the slits are of the same dimension and the plane wavefront inc perpendicularly on them, has wavelength l. On placing the glass slab, the intensity reduc 3I/4 at the focus. Find out the minimum thickness of the glass slab (in Å) if its refractive is 3/2. Given l = 6933Å, μ = 1.5.



Ans. **2311**

Sol. $I = 4\,I_0 \cos^2\frac{\phi}{2}$

Case - 1, $\phi = 0 \Rightarrow I = 4I_0$

Case - 2, $I = \frac{3I}{4} = 4I_0 \cos^2\frac{\phi}{2}$

$\Rightarrow \cos^2\frac{\phi}{2} = \frac{3}{4}$

$\cos\frac{\phi}{2} = \frac{\pm\sqrt{3}}{2} \Rightarrow \frac{\phi}{2} = \frac{\pi}{6}$ $\qquad \phi = \frac{\pi}{3}$

Now, $\phi = \frac{(\mu-1)t \times 2\pi}{\lambda}$

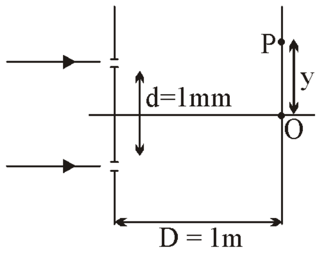
$\frac{\pi}{3} = \frac{(\mu-1)t - 2\pi}{\lambda}$

$t = \frac{\lambda}{6(\mu-1)}$

$t = \frac{6933}{3} = 2311$ Å

Matrix Match Type :

20. A parallel beam of visible light consisting of wavelengths λ_1 & λ_2 is incident on a standard YDSE apparatus with d = 1mm, D = 1m. P is a point on the screen at a distance y from center of screen O. y = y_1 is the nearest point above O where the two maxima coincide. y = y_2 is the nearest point above O where the two minima coincide. β_1 & β_2 are fringe width corresponding to wave length λ_1 & λ_2.



	Column-I		**Column-II**
(A)	β_1 = 0.3 mm, β_2 = 0.5 mm	(P)	The 2nd nearest point above O where two maxima coincide is y = $2y_1$
(B)	β_1 = 0.3 mm, β_2 = 0.4 mm	(Q)	y_2 has no finite value
(C)	β_1 = 0.2 mm, β_2 = 0.4 mm	(R)	The 2nd nearest point above O where the two minima coincide is y = $3y_2$.
(D)	β_1 = 0.2 mm, β_2 = 0.6 mm	(S)	y_1 = LCM of β_1 & β_2
		(T)	The 2nd nearest point above O where two minima coincide is y = $2y_2$.

Ans. (A) P, R,S (B) P,Q,S (C) P,Q,S (D) P,R,S

Sol. (1) $y_1 = n_1\beta_1 = n_2\beta_2$ = LCM of β_1 and β_2

$2y_1 = 2n_1\beta_1 = 2n_2\beta_2$

Hence at this point both maxima again coincide.

$$y_2 = \left(n_1 - \frac{1}{2}\right)\beta_1 = \left(n_2 - \frac{1}{2}\right)\beta_2$$

$$\frac{\beta_1}{\beta_2} = \frac{n_2 - \frac{1}{2}}{n_1 - \frac{1}{2}}$$

$\Rightarrow$ $\frac{\beta_1}{\beta_2} = \frac{2n_2 - 1}{2n_1 - 1}$ which will have a solution. Iff $\frac{\beta_1}{\beta_2}$ expressed as a proper fraction will be of form $\frac{odd}{odd}$.

Option B & C : $\frac{\beta_1}{\beta_2}$ is of form $\frac{odd}{even}$. Hence no solution i.e. the two minima will never coincide.

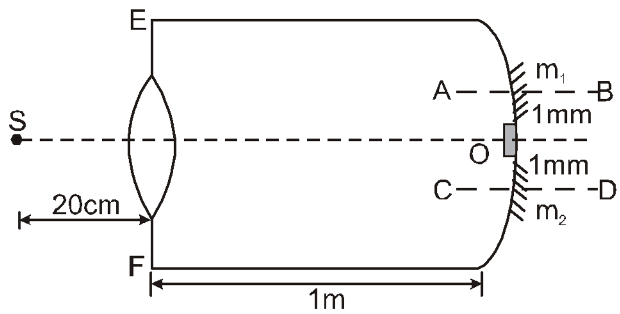
For option A& D : $\frac{\beta_1}{\beta_2}$ is of form $\frac{\text{odd}}{\text{even}}$. Hence at some finite y_2 the two minima will coincide.

At $2y_2$ the two maxima (and not minima) will coincide.

$\therefore$ $y = 3y_2$ is the next nearest point where minima coincide.

Subjective Type Questions :

21. An equi convex lens of focal length 10 cm (in air) and R.I. 3/2 is put at a small opening on a tube of length 1 m fully filled with liquid of R.I. 4/3. A concave mirror of radius of curvature 20 cm is cut into two halves m_1 and m_2 and placed at the end of the tube. m_1 & m_2 are placed such that their principal axis AB and CD respectively are separated by 1 mm each from the principal axis of the lens. A slit S placed in air illuminates the lens with light of frequency 7.5×10^{14} Hz. The light reflected from m_1 and m_2 forms interference pattern on the left end EF of the tube. O is an opaque substance to cover the hole left by m_1 & m_2. Find :



(a) the position of the image formed by lens water combination.

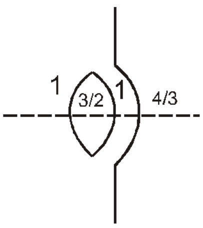
(b) the distance between the images formed by m_1 & m_2.

(c) width of the fringes on EF.

Ans. (a) 80 cm behind the lens (b) 4 mm (c) $\beta = 60\ \mu m$

Sol. Lets find out the radius of curvature of equi. convex lens.

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) \Rightarrow \frac{1}{10} = \left(\frac{3}{2} - 1\right)\left(\frac{2}{R}\right) \Rightarrow R = 10 \text{ cm.}$$



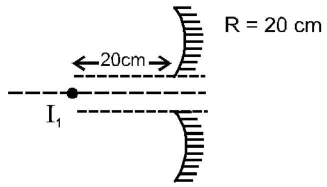
Now

for lens :

$$\frac{1}{V} - \frac{1}{-20} = \frac{1}{10} \Rightarrow \frac{1}{V} = \frac{1}{20}$$

$\Rightarrow$ for surface of tube (of R = 10 cm.)

$$\frac{\mu_2}{V} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{4/3}{V} - \frac{1}{+20} = \frac{4/3 - 1}{-10} \Rightarrow V = +80 \text{ cm.}$$



(b) Now for mirrors.

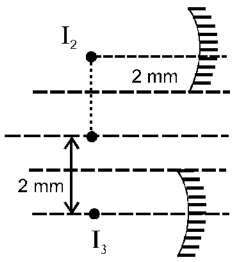
As the object for the mirrors is at 20 cm so the image will be at 20 cm only

$u = -2f \quad \Rightarrow \; v = -2f$ also.

$\Rightarrow \quad$ magnification $= m = \dfrac{y_I}{y_0} = \dfrac{-v}{u}$

$\Rightarrow \quad \dfrac{y_I}{-(1\ \text{mm})} = -\left(\dfrac{-20}{-20}\right) \qquad \Rightarrow \qquad y_I = +(1\ \text{mm})$

so the final images are like.



so the distance between the images is 4 mm.

(c) Now, these I_2 and I_4 behave as the 2 sources for fringe pattern.

$$\Rightarrow \quad \beta = \frac{\lambda D}{d} = \frac{vD}{fd} = \frac{(c/\mu)\ D}{fd}$$

$$= \left(\frac{3\times10^8}{\frac{4}{3}}\right) \times \frac{0.8}{\frac{3}{4}\times10^{15}\times(4\times10^{-3})} \qquad = 60\ \mu\text{m}.$$

22. In a YDSE experiment, the distance between the slits & the screen is 100 cm. For a certain distance between the slits, an interference pattern is observed on the screen with the fringe width 0.25 mm. When the distance between the slits is increased by $\Delta d = 1.2$ mm, the fringe width decreased to $n = 2/3$ of the original value. In the final position, a thin glass plate of refractive index 1.5 is kept in front of one of the slits & the shift of central maximum is observed to be 20 fringe width. Find the thickness of the plate & wavelength of the incident light.

Ans. $\lambda = 600$ nm , $t = 24\ \mu$m

Sol Clearly $\beta_{initial} = \dfrac{\lambda D}{d_i}$

$\Rightarrow \quad 0.25 \times 10^{-3} = \dfrac{\lambda}{d} \times 1$ m

$\Rightarrow \quad \dfrac{\lambda}{d} = 2.5 \times 10^{-4} \qquad$ ....(1)

Afterwards:

$$\frac{2}{3}\ \beta_i = \frac{\lambda D}{(d+\Delta d)}$$

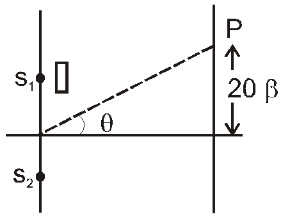
$$\Rightarrow \qquad \frac{\lambda}{d+\Delta d} = \frac{2}{3}\times\frac{2.5\times10^{-4}}{1} \qquad ....(2)$$

Dividing (1) and (2)

$$\frac{d+\Delta d}{d} = \frac{3}{2} \quad \Rightarrow \quad d = 2\,(\Delta d) \quad = 2.4 \text{ mm.}$$

& $\quad \lambda = 2.4 \times 2.5 \times 10^{-7}$ m $\qquad = 600$ nm.

Now



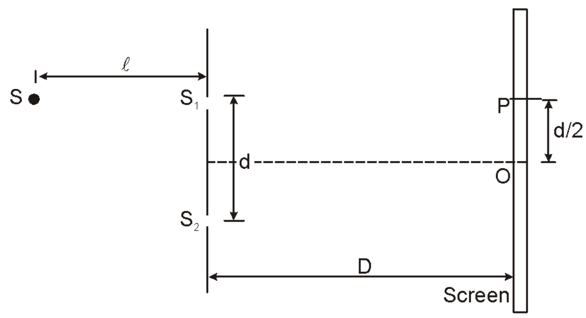
Now P becomes central maxima.

$\Rightarrow$ for point P: $d \sin\theta = (\mu - 1)\, t \;\Rightarrow\; d.\ \frac{20\beta}{D}\tan\theta \approx (\mu - 1)t$

$\Rightarrow$ $d = (\mu - 1)t \qquad \Rightarrow \qquad \frac{d}{D}\times\frac{20\times\lambda D}{d} = (1.5 - 1)t$

$$\Rightarrow \qquad t = \frac{20\ \lambda}{0.5} = \frac{20\times600\times10^{-9}}{0.5} = 24\ \mu\text{m.}$$

23. A source S is kept directly behind the slit S_1 in a double–slit apparatus. Find the phase difference at a point O which is equidistant from S_1 & S_2. What will be the phase difference at P if a liquid of refraction index μ is filled; (wavelength of light in air is λ due to the source). Assume same intensity due to S_1 and S_2 on screen and position at liquid. ($\lambda << d$, $d << D$, $\ell >> d$)

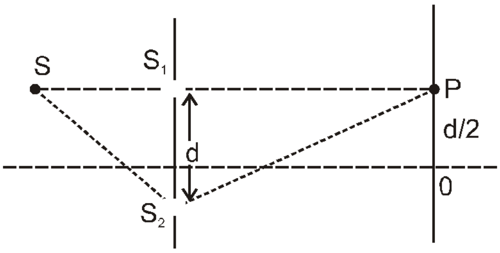


(a) between the screen and the slits.

(b) between the slits & the source S. In this case find the minimum distance between the points on the screen where the intensity is half the maximum intensity on the screen.

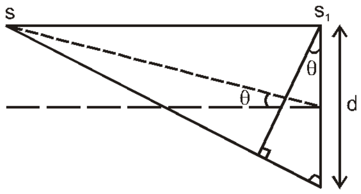
Ans. **(a)** $\Delta\phi = \left(\frac{1}{\ell}+\frac{\mu}{D}\right)\frac{\pi d^2}{\lambda}$ $\qquad$ **(b)** $\Delta\phi = \left(\frac{\mu}{\ell}+\frac{1}{D}\right)\frac{\pi d^2}{\lambda}$; $\qquad D_{min} = \frac{\beta}{2} = \frac{\lambda D}{2d}$

Sol.



at O; path difference

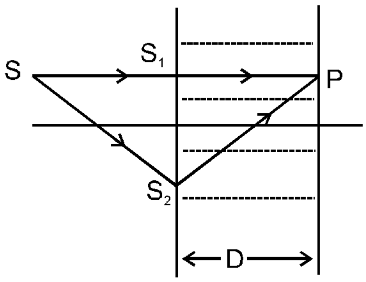
= $d\sin\theta$ where θ is the angle as shown: (as $\ell >> d$)



so $\Delta P = d\sin\theta \cong d\tan\theta \approx \frac{d.d/2}{\ell} = \frac{d^2}{2\ell}$

$\Rightarrow \quad \Delta\phi = \frac{2\pi}{\lambda} \times \frac{d^2}{2\ell} = \frac{\pi d^2}{\lambda \ell}$.

(a) Now if a liquid is filled as shown.



path difference = $(SS_2 - SS_1) + S_2P - S_1P$

$$= \frac{d^2}{2\ell} + \frac{d^2}{2D}$$

$\Rightarrow \quad$ phase difference = $\frac{d^2}{2\ell} \times \frac{2\pi}{\lambda} + \frac{d^2}{2D} \times \frac{2\pi}{(\lambda/\mu)}$

$$= \frac{d^2\pi}{\lambda}\left(\frac{1}{\ell} + \frac{\mu}{D}\right)$$

(b) (i) if liquid is filled on the left side.

$$\Delta\phi = \frac{d^2}{2\ell} \times \frac{2\pi}{\lambda/\mu} + \frac{d^2}{2D} \times \left(\frac{2\pi}{\lambda}\right)$$

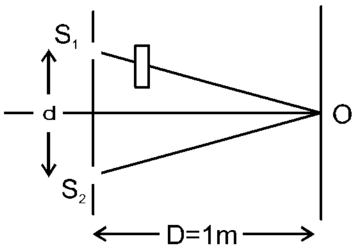
$$= \frac{\pi d^2}{\lambda}\left(\frac{\mu}{\ell} + \frac{1}{D}\right)$$

(ii) Clearly it is asking about $\beta/2$ { $\lambda << d$}.

$\Rightarrow \quad D_{min} = \frac{\beta}{2} = \frac{\lambda D}{2d}$

24. A monochromatic light of λ = 5000 Å is incident on two slits separated by a distance of 5×10^{-4} m. The interference pattern is seen on a screen placed at a distance of 1 m from the slits. A thin glass plate of thickness 1.5×10^{-6} m & refractive index μ = 1.5 is placed between one of the slits & the screen. Find the intensity at the centre of the screen, if the intensity there is I_0 in the absence of the plate. Also find the lateral shift of the central maximum.

Ans. 0, 1.5 mm



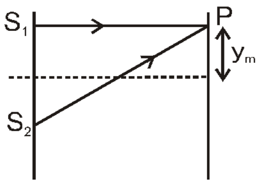
Sol.

After the thin plate is placed between one of the slits and the screen, an additional path difference [$\Delta P = (\mu - 1)\, t$] is created between the rays emerging out of S_1 and S_2. Thus

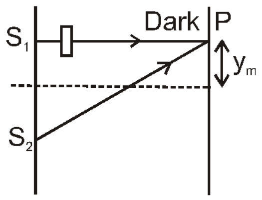
$S_1O - S_2O = (\mu - 1)\, t = 0.75 \times 10^{-6}$ m

$$\frac{(\mu - 1)t}{\lambda} = \frac{3}{2} \quad \Rightarrow \quad (\mu - 1)t = \frac{3}{2}\lambda$$

Point O would be an interference minimum. Thus intensity at O is zero.



$$S_2P - S_2P = \frac{dy_m}{D}$$



$$S_2P - S_1P = \frac{dy_m}{D} - (\mu - 1)t$$

For central maxima, the path difference between the interfering beams is zero.

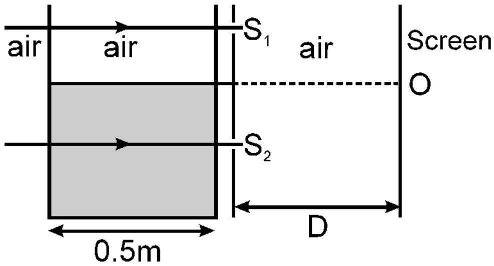
i.e. $S_2P - S_1P = 0$

$$\Rightarrow \quad \frac{dy_m}{D} = (\mu - 1)t$$

$$\Rightarrow \quad y_m = \frac{D}{d}(\mu - 1)t = \frac{1}{5 \times 10^{-4}} \times 0.75 \times 10^{-6} = 1.5 \text{ mm}$$

Thus, the effect of putting a thin film in front of one of the slits is to shift the central maximum towards that slit.

25. In a YDSE, radiowaves of wavelength λ = 50 mm (in air) are used. Two parallel beams are first passed through a glass vessel of base length 0.5 m as shown, filled upto some height (shaded region) with a liquid whose refractive index is varying with distance ' x ' (in meters) from the left face as $\mu(x) = \mu_0(a + x)$ where $\mu_0 = 4/3\ m^{-1}$ and a = 1 m. If intensity due to each slit at O (symmetrical with respect to S_1 and S_2) is I. Find the intensity at point O on the screen.



Ans I

Sol. Optical path length in medium

$$= \int \mu dx = \int_0^{0.5} \frac{4}{3}(1+x)dx = \frac{5}{6}$$

$\therefore$ Path difference $= \frac{5}{6} - \frac{1}{2} = \frac{5-3}{6} = \frac{5}{6}$

Phase difference $= \frac{1/3 \times 2\pi}{50 \times 10^{-3}} = \frac{40\pi}{3} = 12\pi + \frac{4\pi}{3}$

Resultant intensity = I + I + 2 $\sqrt{I}\sqrt{I}\cos\frac{4\pi}{3}$ = I