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PHYSICS

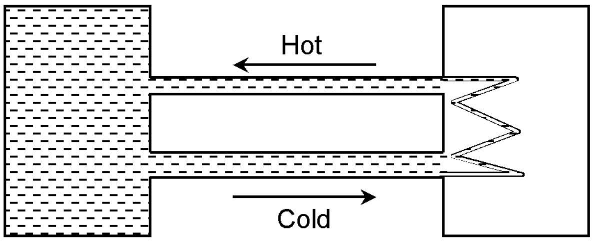
TARGET : JEE- Advanced 2021

CAPS-21

CALORIMETRY + THERMAL EXPANSION + HEAT TRANSFER

SCQ (Single Correct Type) :

1. A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is :



(Specific heat of water is 4.2 kJ kg^{-1} K^{-1} and the density of water is 1000 kg m^{-3})

(A) 1600 (B) 2067 (C) 2533 (D) 3933

Ans. **(B)**

Sol. Heat generated in device in 3 hours = $3 \times 3600 \times 3 \times 10^3 = 324 \times 10^5$J

Heat used to heat water = $ms\Delta\theta = 120 \times 1 \times 4.2 \times 10^3 \times 20$ J

Heat absorbed by coollant = Pt = $324 \times 10^5 - 120 \times 1 \times 4.2 \times 10^3 \times 20$ J

Pt = $(325 - 100.8) \times 10^5$ J

$$P = \frac{223.2 \times 10^5}{3600} = 2067W$$

2. If specific heat capacity of a substance in solid and liquid state is proportional to temperature of the substance, then if heat is supplied to the solid initially at – 20°C (having melting point 0°C) at constant rate. Then the temperature dependence of solid with time will be best represented by :

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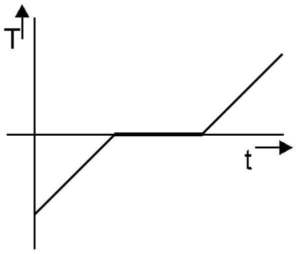

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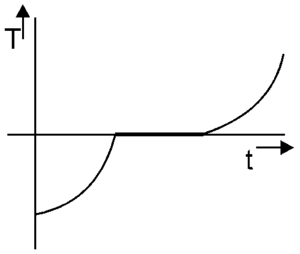

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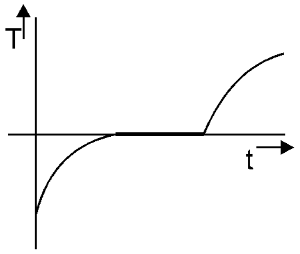

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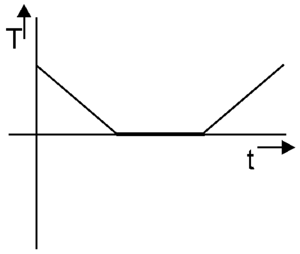


Ans. **(C)**

Sol. As dQ = msdT

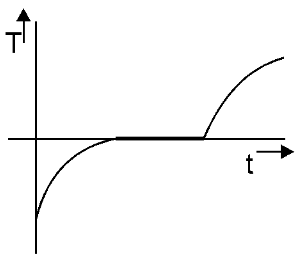
$$\frac{dQ}{dt} = ms\frac{dT}{dt}$$

From question : $S \propto T$

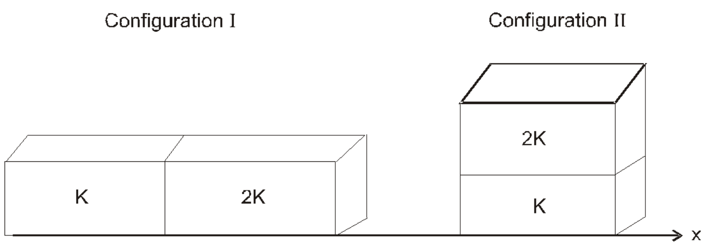
or $S = K_1T$. (K_1 being propertionality constant)

Also, $\frac{d\theta}{dt}$ = constant = K_2 (say) $\Rightarrow ms\frac{dT}{dt} = K_2 \Rightarrow m(K_1T)\frac{dT}{dt} = K_2$

$\Rightarrow \left(m\frac{K_1}{K_2}\right)\frac{T^2}{2} = t \Rightarrow T \propto \sqrt{t}$ Hence, the graph will be



3. Two rectangular blocks, having indentical dimensions, can be arranged either in configuration I or in configuration II as shown in the figure, On of the blocks has thermal conductivity k and the other 2k. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9s to transport a certain amount of heat from the hot end to the cold end in the configuration I. The time to transport the same amount of heat in the configuration II is :



(A) 2.0 s (B) 3.0 s (C) 4.5 s (D) 6.0 s

Ans. (A)

Sol. In configuration 1 equivalent thermal resistance is $\frac{3R}{2}$

In configuration 2 equivalent thermal resistance is $\frac{R}{3}$

Thermal Resistance $\propto$ time taken by heat flow from high temperature to low temperature

4. A resistor has initial resistance 'R_0' at 0ºC. Now, it is connected to an ideal battery of constant emf = 'v'. If the temperature co–efficient of resistance is α, then after how much time, will its temperature be 'TºC' (mass of the wire = m, specific heat capacity of the wire = S). Assume the resistance varies linearly with temperature neglect heat loss to the surrounding.

(A) $\frac{msR_0T}{v^2}$ (B) $\frac{m_0SR_0}{v^2}$ (T/2) (C) $\frac{mSR_0}{v^2}$ $(T + \frac{\alpha T^2}{2})$ (D) $\frac{mSR_0}{v^2}$ $T(1 + \alpha T)$

Ans. (C)

Sol. Rate of heat produced

$$\frac{d\theta}{dt} = \frac{v^2}{R} = \frac{v^2}{R_0(1+\alpha(T-0))} = \frac{v^2}{R_0(1+\alpha T)} \text{ and } \frac{d\theta}{dt} = \frac{dT}{dt}ms$$

$$\Rightarrow \quad ms\frac{dT}{dt} = \frac{v^2}{R_0(1+\alpha T)}$$

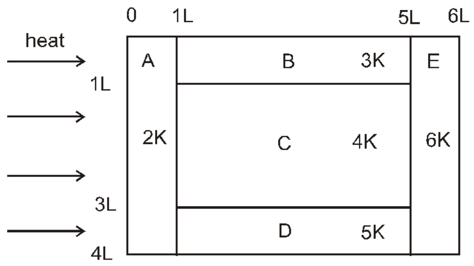
$$\int_{T=0}^{T=T}(1+\alpha T)dT = \frac{v^2}{R_0 ms}\int_{t=0}^{t=t}dt$$

$$T + \frac{\alpha T^2}{2} = \frac{v^2}{R_0 ms}t$$

$$t = \frac{R_0 ms}{v^2}\left(T + \frac{\alpha T^2}{2}p\right).$$

MCQ (One or more than one correct) :

5. A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat 'Q' flows only from left to right through the blocks. Then in steady state



(A) heat flow through A and E slabs are same

(B) heat flow through slab E is maximum

(C) temperature difference across slab E is smallest

(D) heat flow through C = heat flow through B + heat flow through D.

Ans. **(ACD)**

Sol. **A :** At steady state, heat flow through A and E are same.

C : $\Delta T = i \times R$

'i' is same for A and E but R is smallest for E.

D : $i_B = \dfrac{\Delta T}{R_B}$

$i_C = \dfrac{\Delta T}{R_C}$

$i_D = \dfrac{\Delta T}{R_D}$

if $i_c = i_B + i_D$

Hence $\dfrac{1}{R_C} = \dfrac{1}{R_B} + \dfrac{1}{R_D}$ $\quad\Rightarrow \dfrac{8KA}{\ell} = \dfrac{3KA}{\ell} + \dfrac{5kA}{\ell}$

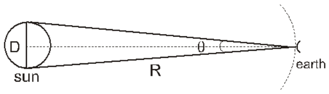
6. The solar constant is the amount of heat energy received per second per unit area of a perfectly black surface placed at a mean distance of the Earth from the Sun, in the absence of Earth's atmosphere, the surface being held perpendicular to the direction of Sun's rays. Its value is 1388 W/m^2 .

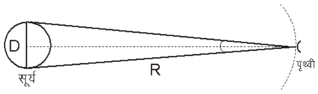
If the solar constant for the earth is 's'. The surface temperature of the sun is TK, D is the diameter of the Sun, R is the mean distance of the Earth from the Sun . The sun subtends a small angle 'θ' at the earth. Then correct options is/are :–

(A) $s = \sigma T^4 \left(\dfrac{D}{R}\right)^2$ (B) $s = \dfrac{\sigma T^4}{4}\left(\dfrac{D}{R}\right)^2$ (C) $s = \dfrac{\sigma T^4}{4}\theta^2$ (D) $s = \dfrac{\sigma T^4}{4}\left(\dfrac{R}{D}\right)^2$

Ans. **(BC)**

Sol.





Let the diameter of the sun be D and its distance from the earth be R.

$$\frac{D}{R} = \theta$$

The radiation emitted by the surface of the sun per unit time is

$$4\pi\left(\frac{D}{2}\right)^2 \sigma T^4 = \pi D^2 \sigma T^4.$$

At distance R, this radiation falls on an area $4\pi R^2$ in unit time. The radiation received at the earth's surface per unit time per unit area is, therefore.

$$s = \frac{\pi D^2 \sigma T^4}{4\pi R^2} = \frac{\sigma T^4}{4}\left(\frac{D}{R}\right)^2.$$

Thus, $s \propto T^4$ and $s \propto \theta^2$

Comprehension Type Question:

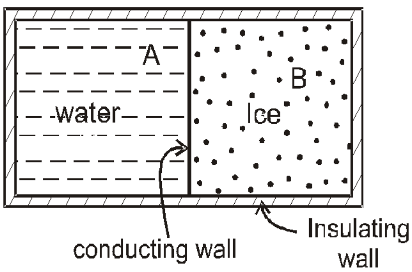
Comprehension # 1

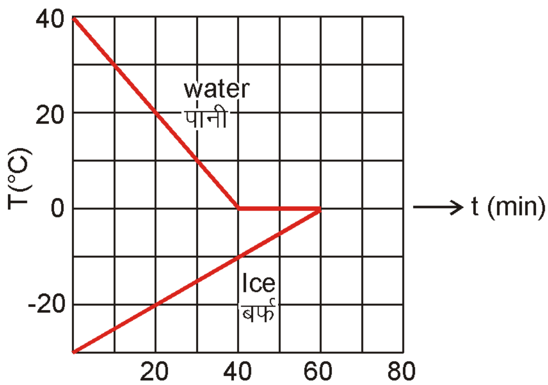
A 0.60 kg sample of water and a sample of ice are placed in two compartments A and B that are separated by a conducting wall, in a thermally insulated container. The rate of heat transfer from the water to the ice though the conducting wall is constant P, until thermal equilibrium is reached. The temperature T of the liquid water and the ice are given in graph as functions of time t. Temperature of the compartments remain homogeneous during whole heat transfer process.

Given specific heat of ice = 2100 J/kg-K

Given specific heat of water = 4200 J/kg-K

Latent heat of fusion of ice = 3.3×10^5 J/kg





7. The value of rate P is

(A) 42.0 W (B) 36.0 W (C) 21.0 W (D) 18.0 W

Ans. **(A)**

Sol. In 40 min. temperature of water has come down by 40°C.

Therefore rate $P = \frac{mS\Delta T}{t} = \frac{0.60 \times 4200 \times 40}{40 \times 60} = 42.0$ W

8. The initial mass of the ice in the container is equal to

(A) 0.36 kg (B) 1.2 kg (C) 2.4 kg (D) 3.6 kg

Ans. **(C)**

Sol. Sample of ice has been receiving heat at constant rate P from water. Its temperature has increased by 30°C in time 60 min.

Therefore $\frac{m_i s_i \Delta T_i}{P} = 60$ min. $\Rightarrow$ $m = \frac{(60 \times 60s) \times (42 \quad W)}{(2100J/kg) \ . \ (30°C)} = 2.4$ kg

9. The mass of the ice formed due to conversion from the water till thermal equilibrium is reached, is equal to

(A) 0.12 kg (B) 0.15 kg (C) 0.25 kg (D) 0.40 kg

Ans. **(B)**

Sol. Thermal equilibrium reaches after 60 min. Ice conversion takes place for 20 min. During this time water at 0°C continues to give heat at rate P.

$m \times L_f = P \times (20 \times 60s) \Rightarrow m = \frac{42 \times 20 \times 60}{3.3 \times 10^5}$ kg = 0.15 kg

Comprehension # 2

A body cools in a surrounding of constant temperature 30 ºC. Its heat capacity is 2J/ºC. Initial temperature of the body is 40ºC. Assume Newton's law of cooling is valid. The body cools to 38ºC in 10 minutes.

10. In further 10 minutes it will cool from 38ºC to :

(A) 36ºC (B) 36.4ºC (C) 37ºC (D) 37.5ºC

Ans. **(B)**

Sol. We have $\theta - \theta_s = (\theta_0 - \theta_s)\, e^{-kt}$

where θ_0 = Initial temperature of body = 40°C

θ = temperature of body after time t.

Since body cools from 40 to 38 in 10min, we have

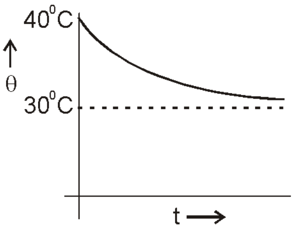
$38 - 30 = (40 - 30)\, e^{-k\,10}$ .... (1)

Let after 10 min, The body temp. be θ

$\theta - 30 = (38 - 30)\, e^{-k\,10}$ .... (2)

$\frac{(1)}{(2)}$ gives $\frac{8}{\theta - 30} = \frac{10}{8}$, $\theta - 30 = 6.4$ $\Rightarrow$ $\theta = 36.4$ °C

11. The temperature of the body in ºC denoted by θ the variation of θ versus time t is best denoted as

(A)


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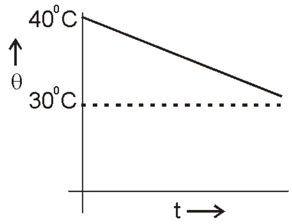
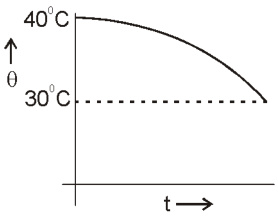


(C)


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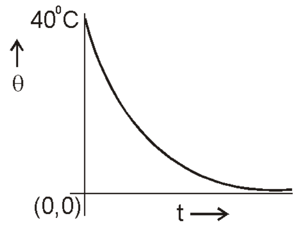


Ans. **(A)**

Sol. Temperature decreases exponentially.

12. When the body temperature has reached 38 ºC, it is heated again so that it reaches to 40ºC in 10 minutes.The total heat required from a heater by the body is:

(A) 3.6J (B) 7J (C) 8 J (D) 4 J

Ans. **(C)**

Sol. During heating process from 38 to 40 in 10 min. The body will lose heat in the surrounding which will be exactly equal to the heat lost when it is cooled from 40 to 38 in 10 min, which is equal to ms $\Delta\theta = 2 \times 2 = 4$ J.

During heating process heat required by the body = m s $\Delta\theta$ = 4 J.

$\therefore$ Total heat required = 8 J.

Numerical based Questions :

13. In an insulated vessel, 0.05 kg steam at 373 K and 0.45 kg of ice at 253 K are mixed. Find the final temperature of the mixture (in Kelvin).

Given, L_{fusion} = 80 cal/gm = 336 J/gm, $L_{vaporization}$ = 540 cal/gm = 2268 J/gm,

S_{ice} = 2100 J/kg K = 0.5 cal/gm K and S_{water} = 4200 J/kg K = 1 cal/gmK

Ans. 273 K.

Sol. $\Sigma\Delta Q = 0$

Heat lost by steam to convert into 0ºC water

$H_L = 0.05 \times 540 + 0.05 \times 100 \times 1$

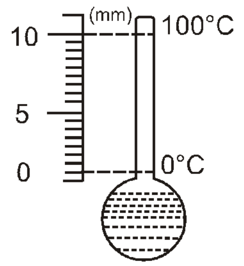
$= 27 + 5 = 32$ kcal

Heat required by ice to change into 0º C water

$H_g = 0.45 \times \frac{1}{2} \times 20 + 0.45 \times 80 = 4.5 + 36.00 = 40.5$ kcal

Thus, final temperature of mixture is 0ºC = 273 K

14. Level of a certain liquid at 0°C and 100° C are 0 and 10 mm on a given fixed scale (as shown in fig.) coefficient of volume expansion this liquid varies with temperature as $\gamma = \gamma_0\left(1+\frac{T}{100}\right)$ (where T in °C). Find the level (in mm) of liquid at 48°C



Ans. **4**

Sol. dv = Adx

$v_0\,\gamma_0\left(\frac{T}{100}+1\right)dT = Adx$

$\Rightarrow v_0\,\gamma_0\int_0^{100}\left(\frac{T}{100}+1\right)dT = A\int_0^{10}dx \Rightarrow v_0\,\gamma_0\times\left(\frac{100^2}{2\times100}+100\right) = A\times10$

$\Rightarrow v_0\,\gamma_0\times150 = A\times10$.....(I)

II case

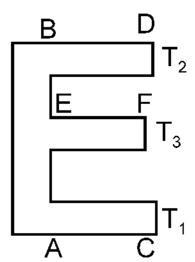
$\Rightarrow v_0\,\gamma_0\int_0^{48}\left(\frac{T}{100}+1\right)dT = A\int_0^{x}dx$

$v_0\,\gamma_0\times\left(\frac{48^2}{2\times100}+48\right) = A\times x$ .....(II)

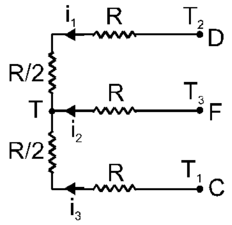
$\frac{1}{15}\times48\,(0.24+1) = x$

x = 1.24 × 3.2 = 3.968 = 4 mm

15. Four thin identical rods AB, AC, BD and EF made of the same material are joined as shown. The free-ends C, D and F are maintained at temperatures T_1, T_2 and T_3 respectively. Assuming that there is no loss of heat to the surroundings, the temperature at joint E when the steady state is attained is $\frac{1}{K}(2T_1+2T_2+3T_3)$. Find K (E is mid point of AB)



Ans. 7

Sol. 

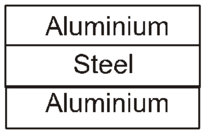
by KCL at junction we can find T.

$i_1 + i_2 + i_3 = 0$

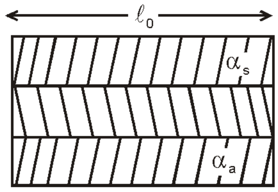
$$\frac{T_2 - T}{R + \frac{R}{2}} + \frac{T_3 - T}{R} + \frac{T_1 - T}{R + \frac{R}{2}} = 0 .$$

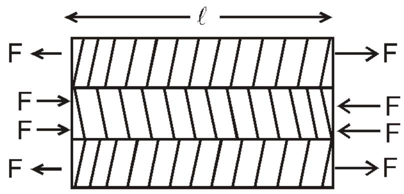
Subjective Type Questions :

16. Two Aluminium rods and a steel rod of equal cross-sectional area and equal length ℓ_0 are joined rigidly side by side as shown in figure. Initially the rods are at 0ºC. Find the length of the rod at the temperature θ if young's modulus of elasticity of the aluminium and steel are Y_a and Y_s respectively and coefficient of linear expansion of aluminium and steel are α_a and α_s respectively.



Ans. $\ell_0 \left[1 + \frac{2Y_a\alpha_a + Y_s\alpha_s}{2Y_a + Y_s} \theta \right]$

Sol. at 0 ºC 

at θ ºC 

If rods are free to expand.

$\ell_s = \ell_0 (1 + \alpha_s\theta)$

$\ell_a = \ell_0 (1 + \alpha_a\theta)$

$Y = \frac{F/A}{x/\ell}$

$x = \frac{F \times \ell}{AY}$ if $\alpha_s > \alpha_a$

for steel (compression) $x = \frac{2F \times \ell_0}{AY_s} = \ell_0 (1 + \alpha_s \Delta\theta) - \ell$ .....(i)

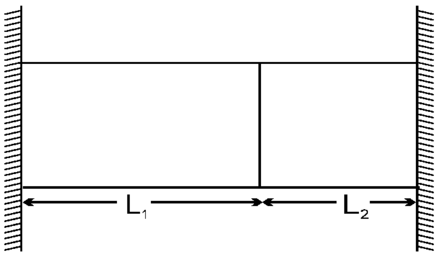
for aluminium (expansion) $x = \frac{F \times \ell_0}{AY_a} = \ell - \ell_0 (1 + \alpha_a \Delta\theta)$ .....(ii)

by solving (i) and (ii)

we get

then $\ell = \ell_0 \left[1 + \frac{2Y_a\alpha_a + Y_s\alpha_s}{2Y_a + Y_s} \theta\right]$ **Ans.**

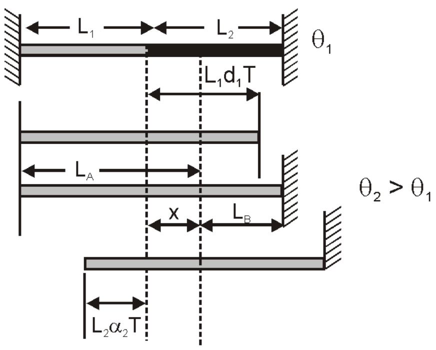
17. Two rods of different metals having same area of cross section A are placed end to end between two massive platforms, as shown in the figure. The first rod has a length L_1, coefficient of linear expansion α_1 and Young's modulus Y_1. The corresponding quantities for the second rod are L_2, α_2, and Y_2. The temperature of both the rods is now increased by T° C. Find the force with which the rods act on each other (at the higher temperature) in terms of given quantities. Also find the lengths of the rods at the higher temperature. Assume that there is no change in the cross sectional area of the rods and that the rods do not bend. There is no deformation of the walls.



Ans. $F = \frac{AT(L_1\alpha_1 + L_2\alpha_2)Y_1Y_2}{L_1Y_2 + L_2Y_1}$, Length of the first rod = $L_1 + \frac{L_1L_2T(Y_1\alpha_1 - Y_2\alpha_2)}{L_1Y_2 + L_2Y_1}$,

Length of the second rod = $L_2 + \frac{L_1L_2T(Y_2\alpha_2 - Y_1\alpha_1)}{L_1Y_2 + L_2Y_1}$]

Sol. $L_A = L_1 + x$



$L_B = L_2 - x$

$Y = \frac{F \times \ell}{Ae}$

$e = \frac{F \times \ell}{YA}$ = extension or compression due to force

$e = L_1\alpha_1 T - x = \frac{FL_1}{Y_1 A}$ ....(1)

$e = L_2\alpha_2 T + x = \frac{FL_2}{Y_2 A}$ ....(2)

By adding equation (1) and equation (2)

We get $F = \frac{AT(L_1\alpha_1 + L_2\alpha_2)Y_1Y_2}{L_1Y_2 + L_2Y_1}$ **Ans.**

By dividing (1) and (2)

We get $x = \frac{L_1L_2T(Y_1\alpha_1 - Y_2\alpha_2)}{L_1Y_2 + L_2Y_1}$

So, $L_A = L_1 + x$; $L_B = L_2 - x$ **Ans.**

18. A clock with an iron pendulum keeps correct time at 20º C. How much will it lose or gain in a day if the temperature changes to 40º C? (Coefficient of cubical expansion of iron = 0.000036/º C)

Ans. 10.368 s

Sol. Gain or loss in time due to thermal expansion

$$\Delta t = \frac{1}{2} \times \Delta\theta \times t \times \alpha \left(\alpha = \frac{0.000036}{3} /^\circ C\right)$$

t = duration time

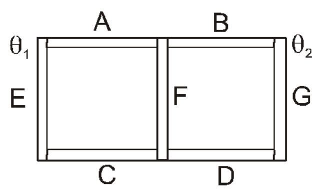
1 day = 24 × 3600 sec.

$$\therefore \quad \Delta t = \frac{1}{2} \times \frac{0.000036}{3} \times 20 \times 24 \times 3600$$

Δt = 10.368 sec.

Note : If we increase temperature then time period increases and watch becomes slow.

19. Seven rods A, B, C, D, E, F and G are joined as shown in figure. All the rods have equal cross-sectional area A and length l. The thermal conductivities of the rods are $K_A = 2K_C = 3K_B = 6K_D = K_0$. The rod E is kept at a constant temperature θ_1 and the rod G is kept at a constant temperature $\theta_2(\theta_2 > \theta_1)$. (a) Show that the rod F has a uniform temperature $\theta = (3\theta_1 + \theta_2)/4$. (b) Find the rate of heat flow from the source which maintains the temperature θ_2.



Ans. $\frac{3K_0A(\theta_2 - \theta_1)}{8l}$

Sol. $\frac{R_A}{R_C} = \frac{k_C}{k_A} = \frac{1}{2}$

& $\frac{R_B}{R_D} = \frac{k_D}{k_B} = \frac{1}{2}$

$\because$ $\frac{R_A}{R_C} = \frac{R_B}{R_D}$ $\Rightarrow$ Balanced W. S. B.

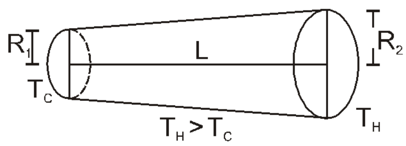
$\Rightarrow$ $\frac{(\theta_2 - \theta)}{R_B} = \frac{\theta_2 - \theta_1}{R_A + R_B}$ $\Rightarrow$ $\theta = \frac{3\theta_1 + \theta_2}{4}$

$\Rightarrow$ Rate of heat flow from the source

$$= \frac{(\theta_2 - \theta_1)}{\left(\frac{(R_A + R_B)(R_C + R_D)}{R_A + R_B + R_C + R_D}\right)}$$

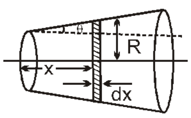
$$= \frac{(\theta_2 - \theta_1)}{\left(\frac{\left(\frac{1}{k_A} + \frac{1}{k_B}\right)\left(\frac{1}{k_C} + \frac{1}{k_D}\right)\frac{\ell}{A}}{\frac{1}{k_A} + \frac{1}{k_B} + \frac{1}{k_C} + \frac{1}{k_D}}\right)} = \frac{3k_0 A(\theta_2 - \theta_1)}{8\ell}$$

20. Find the rate of heat flow through a cross-section of the rod shown in figure ($T_H > T_C$). Thermal conductivity of the material of the rod is K.



Ans. $\frac{K\pi R_1 \ \ R_2(T_H - T_C)}{L}$

Sol.



$i_H = \frac{T_H - T_C}{R_{eq}}$

$dR = \frac{dx}{K\pi R^2}$ $\tan\theta = \frac{R - R_1}{x} = \frac{R_2 - R_1}{L}$

$\Rightarrow$ $R = R_1 + x\left(\frac{R_2 - R_1}{L}\right)$

$$R_{eq} = \int dR = \int \frac{dx}{k\pi\left[R_1 + \frac{x(R_2 - R_1)}{L}\right]^2}$$

$\Rightarrow \qquad R_{eq} = \dfrac{L}{k\pi R_1 R_2}$

$i_H = \dfrac{T_H - T_C}{R_{eq}}$. $= \dfrac{K\pi R_1 \quad R_2(T_H - T_C)}{L}$

21. Two chunks of metal with heat capacities C_1 and C_2, are interconnected by a rod of length ℓ and cross-sectional area S and fairly low heat conductivity K. The whole system is thermally insulated from the environment. At a moment t = 0 the temperature difference between the two chunks of metal equals $(\Delta T)_0$. Assuming the heat capacity of the rod to be negligible, find the temperature difference between the chunks as a function of time.

An. $\Delta T = (\Delta T)_0 \, e^{-\alpha t}$, where $\alpha = (1/C_1 + 1/C_2)\, SK/\ell$

Sol. $\dfrac{dQ}{dt} = \dfrac{Ks}{\ell}(T_1 - T_2)$

where T_1 and T_2 are temperatures of two chunks as function of time 't'.

$-C_1\dfrac{dT_1}{dt} = \dfrac{Ks}{\ell}(T_1 - T_2)$

$C_1\dfrac{dT_2}{dt} = \dfrac{Ks}{\ell}(T_1 - T_2)$

or $\qquad -\dfrac{dT_1}{dt} = \dfrac{Ks}{\ell C_1}(T_1 - T_2)$

or $\qquad \dfrac{dT_2}{dt} = \dfrac{Ks}{\ell C_2}(T_1 - T_2)$

or $\qquad \dfrac{-d(T_1 - T_2)}{dt} = \dfrac{Ks}{\ell}\left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right)$

or $\qquad -\displaystyle\int_{\Delta T_0}^{\Delta T} \dfrac{d(T_1 - T_2)}{(T_1 - T_2)} = \dfrac{Ks}{\ell}\left[\dfrac{1}{C_1} + \dfrac{1}{C_2}\right] \int_0^t dt$

Ans. $\Delta T = \Delta T_0 \, e^{-\alpha t}$

where $\alpha = (1/C_1 + 1/C_2)\, SK/\ell$

22. A liquid at 30° C is poured very slowly into a Calorimeter that is at temperature of 110°C. The boiling temperature of the liquid is 80°C. It is found that the first 5 gm of the liquid completely evaporates. After pouring another 80 gm of the liquid the equilibrium temperature is found to be 50°C. The ratio of the Latent heat of the liquid to its specific heat will be _____ ºC. [Neglect the heat exchange with surrounding]

Ans. **270.00, 120.00**

Sol. **CASE-I: If calorimeter is open and after evaporation liquid escapes**

$5 \times S \times 50 + 5L = W \times 30$ ......(1)

$80 \times S \times 20 = W \times 30$ ......(2)

$80 \times S \times 20 = 5 \times S \times 50 + 5L$

$5L = 1350\ S \Rightarrow \frac{L}{S} = 270$

CASE-II: If calorimeter is closed (vapour not allowed to escape)

Heat gain = Heat loss

5S (80–30) + 5L = W (110–80)

S = Specific heat of liquid

L = Latent heat of liquid

W = Water equivalent of calorimeter

$250S + 5L = W \times 30$ ...... (1)

Now 80gm liquid is poured

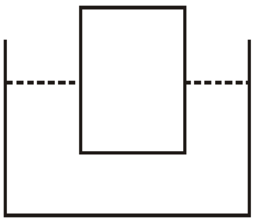
Heat gain = Heat loss

Here final temperature = 50ºC

$80 \times S \times 20 = 5L + 5S \times 30 + W \times 30$ ..... (2)

From (1) & (2) $\Rightarrow \frac{L}{S} = 120$ **Ans**

23. A cylindrical isotropic solid of coefficient of linear expansion α and density ρ floats in a liquid of coefficient of volume expansion γ and density d as shown in the diagram



Column I	**Column II**
(A) volume of cylinder inside the liquid remains constant	(p) $\gamma = 0$
(B) volume of cylinder outside the liquid remains constant	(q) $\gamma = 2\alpha$
(C) Height of cylinder outside the liquid remains constant	(r) $\gamma = 3\alpha \frac{d}{\rho}$
(D) Height of cylinder inside the liquid remain constant	(s) $\gamma = (2\alpha + \alpha \frac{d}{\rho})$

Ans. (A) – (p) ; (B) – (r) ; (C) – (s) ; (D) – (q)

Sol. (A) Buoyant force = Mg = constant = $V_{sub}dg \Rightarrow V_{sub} = \frac{Mg}{dg}$.

volume of displace fluid = constant

$\therefore$ density of fluid must be constant.

(B) $(V_{solid} - V_{sub})$ = constant

$$\Rightarrow \left(V_{solid} - \frac{M_{solid}}{\rho_{liquid}}\right) = \text{constant} \qquad V \times 3\alpha \times \Delta T = \frac{M\gamma\Delta T}{d} \quad \Rightarrow \quad \gamma = 3\alpha\frac{d}{\rho}$$

(C) $Ah_{in}\, d_{liquid} = A(h_{in} + h_{out})\, \rho_{solid}$ = M (mass of solid)

$$h_{out} = \frac{M}{A\rho_{solid}} - h_{in} = \frac{M}{A\rho_{solid}} - \frac{M}{Ad_{liquid}} = \text{constant}$$

$$\frac{M(1+3\alpha\Delta T)}{A(1+2\alpha\Delta T)\ \rho} - \frac{M\ \ (1+\gamma\Delta T)}{A(1+2\alpha\Delta T)\ \ d} = \frac{M}{A\rho} - \frac{M}{Ad}$$

$$\gamma = 2\alpha + \alpha\frac{d}{\rho}$$

(D) $Ah_{in}\, d_{liquid}\, g$ = Buoyant force = constant = Mg

$$A_0\,(1 + 2\,\alpha\,\Delta T)\, h_{in}\ \frac{d}{1+\gamma\Delta T} = \text{constant}$$

$$h_{in} = \frac{M}{A_0 d}\ (1 + (\gamma - 2\alpha)\,\Delta T)$$

$$\gamma = 2\alpha$$