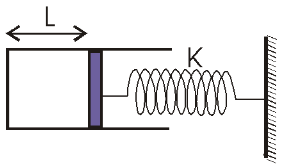
competishun — Power of real gurus	**CAPS-20**
PHYSICS	**KTG & Thermodynamic**
TARGET : JEE- Advanced 2021	

SCQ (Single Correct Type) :

1. A fixed container is fitted with a piston which is attached to a spring of spring constant k. The other end of the spring is attached to a rigid wall. Initially the spring is in its natural length and the length of container between the piston and its side wall is L.Now an ideal diatomic gas is slowly filled in the container so that the piston moves quasistatically. It pushed the piston by x so that the spring now is compressed by x. The total rotational kinetic energy of the gas molecules in terms of the displacement x of the piston is (there is vacuum outside the container)



(A) k x L (B) 4k x L (C) kx(x+L) (D) $\frac{2kx^2}{L}$

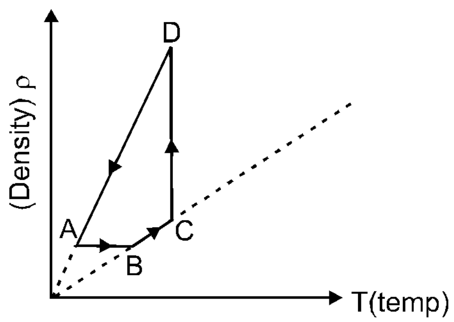
Ans. **(C)**

Sol. Rotational K.E. = Rotational degree of freedom x $\frac{1}{2}$nRT

$= 2 \times \frac{1}{2}nRT \quad = nRT \quad = PV$

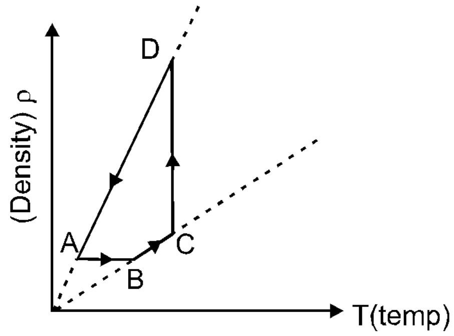
$= PA \cdot \frac{V}{A}$ = force on piston $(L + x)$ $= kx\,(L + x)$

2. Consider the following cyclic process for mono–atomic ideal gas. This process consist of four process AB,BC,CD and DA. The number of processes out of these four processes in which heat is supplied to the gas is :



(A) 1 (B) 2 (C) 3 (D) 4

Ans. **(B)**

Sol. 

AB :

V is constant so W = 0

ΔT is positive so $\Delta U > 0$

So, Q > 0

BC: $\frac{1/V}{T}$ is constant

So VT is constant

So PV^2 = constant

$W = \frac{nR\Delta T}{1-2} = -nR\Delta T$

$\Delta U = \frac{3}{2} nR\Delta T$

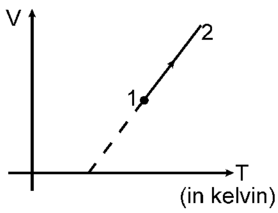
$Q = \frac{1}{2} nR\Delta T$

CD: W < 0 $\Delta U = 0$

So, Q < 0.

DA: $Q = \frac{1}{2} nR\Delta T < 0.$

3. V–T diagram for a process of a given mass of ideal gas is as shown in the figure. During the process pressure of gas.



(A) first increases then decreases (B) continuously decreases

(C) continuously increases (D) first decreases then increases.

Ans. **(B)**

Sol. V = KT + C

$P = \frac{nRT}{V} \Rightarrow P = \frac{nRT}{KT + C}$

$$\frac{dP}{dT} = \frac{nRC}{(KT+C)^2}$$

As C < 0 by diagram

$\Rightarrow \quad \frac{dP}{dT} < 0$ for all T $\Rightarrow$ P continuously decreases.

4. An ideal gas at temperature 'T' follows the law $P^2 = V$ where P is pressure and V is volume of gas, Then the volume expansion co-efficient of gas is.

(A) $\frac{1}{T}$ (B) $\frac{2}{T}$ (C) $\frac{3}{T}$ (D) $\frac{2}{3T}$

Ans. (D)

Sol. as $\left.\begin{matrix} P^2 = v \\ P = \sqrt{v} \end{matrix}\right\}$

Pv = nRT

$\sqrt{v}.v = nRT$

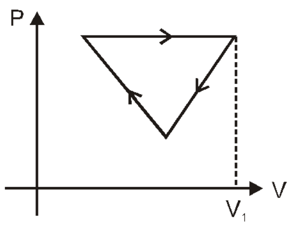
$v^{3/2} = nRT$

$\frac{3}{2}\ell nv = \ell n(nR) + \ell nT$

$\frac{3}{2}\frac{dv}{v} = \frac{dT}{T}$

Volume expansion coefficient $= \frac{dv}{vdT} = \frac{2}{3T}$

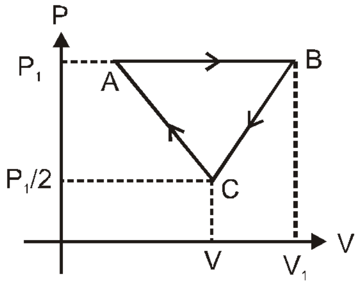
5. The diagram shown an equilateral cyclic process on P-V diagram. The ratio of maximum pressure to minimum pressure is 2. The maximum temperature in the process is T_1. The work done by the two moles of ideal gas in the cyclic process is equal to (R is universal gas constant)-



(A) $\frac{R^2T_1^2}{4\sqrt{3}V_1^2}$ (B) $\frac{4R^2T_1^2}{\sqrt{3}V_1^2}$ (C) $\frac{R^2T_1^2}{\sqrt{3}V_1^2}$ (D) $\frac{2}{\sqrt{3}}\frac{R^2T_1^2}{V_1^2}$

Ans. (C)

Sol.



In ΔABC

$P_A = 2P_C = P_1$

$\tan 60° = \frac{P_1/2}{(V_1 - V)}$

$(V_1 - V) = \frac{P_1}{2\sqrt{3}}$

Work done = Area of triangle = $\frac{1}{2} \times 2(V_1 - V) \times \frac{P_1}{2} = \frac{P_1^2}{4\sqrt{3}} = \frac{(nRT_1)^2}{4\sqrt{3}V_1^2} = \frac{R^2T_1^2}{\sqrt{3}V_1^2}$

6. 56 gm of N_2 gas is passed through a reversible process in which internal energy changes with pressure (P) according to $U \propto P^2$. Initial temperature of the gas is T_0. During the process, if volume of the gas is doubled, then heat given to the gas will be :

(A) $3RT_0$ (B) $16T_0$ (C) $9RT_0$ (D) $18RT_0$

Ans. **(D)**

Sol. $U \propto P^2 \Rightarrow T \propto P^2 \Rightarrow T \propto (T/V)^2 \Rightarrow T \propto V^2$

So, if volume is doubled, temperature will be four time $(T_0 \rightarrow 4T_0)$

$T \propto P^2 \Rightarrow PV \rightarrow P^2 \Rightarrow PV^{-1}$ = cont. $\longleftrightarrow PV^x$ = const.

So x = – 1

Molar heat capacity $C = C_V + \frac{R}{1-x} = \frac{5}{2}R + \frac{R}{1-(-1)} = 3R$

Heat given $= nC\Delta T = (2)\ (3R)\ (4T_0 - T_0) = 18RT_0$

MCQ (One or more than one correct) :

7. A certain thermodynamic cycle is given by the equation

$$2\left(\frac{P-P_0}{P_0}\right)^2+2\left(\frac{V-V_0}{V_0}\right)^2=1$$

where P and V represent the pressure and the volume of the ideal gas involved. If P_0 and V_0 are both expressed in SI units then their values are equal. The cycle is clockwise when represented on the P-V diagram and the number of moles of the gas is n. Now choose the correct statements:

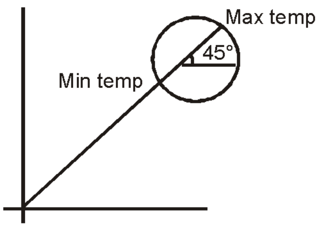
(A) The maximum temperature obtained in the cycle is $\frac{9V_0P_0}{4nR}$

(B) The minimum temperature obtained in the cycle is $\frac{V_0P_0}{4nR}$

(C) The total work done by the gas in one cycle is $\frac{\pi V_0P_0}{2}$

(D) The maximum pressure obtained in the cycle is $P_0\left[1+\frac{1}{\sqrt{2}}\right]$

Ans. **(ABCD)**



Sol.

The cycle is a circle with semi-major axis = $\frac{P_0}{\sqrt{2}}$ and semi-minor axis = $\frac{V_0}{\sqrt{2}}$ and centre is P_0V_0

maximum pressure $P_0+\frac{P_0}{\sqrt{2}}$

At maximum temperature, P = $P_0+\frac{P_0}{\sqrt{2}}\cos45° = \frac{3}{2}P_0$

and V = $V_0+\frac{V}{\sqrt{2}}\sin 45° = \frac{3}{2}V_0$

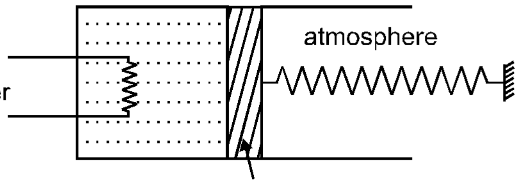
PV = nRT

$\Rightarrow T_{max} = \frac{9}{4}\frac{P_0V_0}{nR}$, similar for $T_{min.}$,

Total work = area of ellipse = πab

$= \pi\frac{P_0}{\sqrt{2}}\times\frac{V_0}{\sqrt{2}} = \frac{\pi P_0V_0}{2}$

8. An ideal gas is filled in a cylinder as shown in figure. The initial temperature, pressure and volume of the gas are T_0, P_0 and V_0 respectively where P_0 = atmospheric pressure. A light and smooth piston of area A is connected to a spring of spring constant K, which is initially in natural length. Now the gas is heated slowly for some time, due to which the piston moves out slowly by a distance 'x'. Then :



(A) Final pressure of the gas is $P_0 + \frac{Kx}{A}$

(B) Final temperature of the gas is $\left(1+\frac{Kx}{P_0A}\right)\left(1+\frac{Ax}{V_0}\right)T_0$

(C) The gas is undergoing constant pressure process

(D) Work done by the gas is $\frac{1}{2}Kx^2$

Ans. **(AB)**

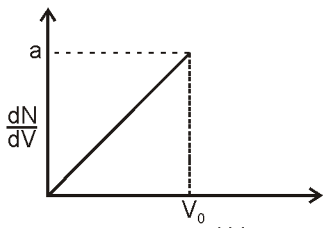
Sol. $PA = P_0A + kx$ (equilibrium)

$$\frac{T_2}{P_2V_2} = \frac{T_1}{P_1V_1}$$

$$T_2 = \frac{T_0}{V_0}(V_0 + Ax)\frac{P_2}{P_1}$$

$$= T_0\left(1+\frac{Ax}{V_0}\right)\left(\frac{P_0+kx/A}{P_0}\right) = \left(1+\frac{Kx}{P_0A}\right).\left(1+\frac{Ax}{V_0}\right)T_0$$

9. Graph shows a hypothetical speed distribution for a sample of N gas particle (for $V > V_0$; $\frac{dN}{dV} = 0$), $\frac{dN}{dV}$ is rate of change of number of particles with change veloicty



(A) The value of aV_0 is 2N.

(B) The ratio V_{avg}/V_0 is equal to 2/3.

(C) The ratio V_{rms}/V_0 is equal to $1/\sqrt{2}$.

(D) Three fourth of the total particle has a speed between 0.5 V_0 and V_0.

Ans. **(ABCD)**

Sol. Area under the curve is equal to number of molecules of the gas sample. Hence

$$N = \frac{1}{2} . a . V_0 \Rightarrow aV_0 = 2N$$

$$V_{avg} = \int_0^\infty v \quad N(V) \quad dV = \frac{1}{N}\int_0^{V_0} C \quad . \quad \left(\frac{a}{v_0}.V\right) dV = \frac{2}{3}V_0 \Rightarrow \frac{V_{avg}}{V_0} = \frac{2}{3}$$

$$v_{rms}^2 = \frac{1}{N}\int_0^\infty V^2 \quad N(V) \quad dV = \frac{1}{N}\int_0^{V_0} V^2\left(\frac{a}{V_0}.V\right) dV = \frac{V_0^2}{2} \Rightarrow \frac{V_{rms}}{V_0} = \frac{1}{\sqrt{2}}$$

Area under the curve from 0.5 V_0 to V_0 is $\frac{3}{4}$ of total area.

10. One mole of an ideal gas undergoes a process such that $P \alpha \frac{1}{\sqrt{T}}$. The molar heat capacity of this process is 4R, R = universal gas constant.

(A) The work done by the gas is 1.5 $R\Delta T$ where ΔT is the change in temperature

(B) Degree of freedom of the gas is 5.

(C) On increase of temperature, volume increases

(D) On increase of temperature, volume decreases

Ans. **(ABC)**

Sol. $P \alpha \frac{1}{\sqrt{T}} \quad \Rightarrow \quad PV^{1/3} = \text{constant}$

$$C = \frac{f}{2}R - \frac{R}{\frac{1}{3}-1} \quad \Rightarrow \quad 4R = \frac{f}{2}R + \frac{3}{2}R \quad \Rightarrow \quad f = 5$$

$$W = -\frac{R}{a-1}\Delta T = -\frac{R\Delta T}{\frac{1}{3}-1} = 1.5\ R\Delta T$$

$\Rightarrow$ W > 0, when $\Delta T > 0$. Hence $\Delta V > 0$.

11. An ideal diatomic gas undergoes a process in which its internal energy (U) relates to the volume (V) as $U = \alpha\sqrt{V}$, here α is a constant. The internal energy of gas is increased by 100 J.

(A) The work performanced by gas is 80 J.

(B) The amount of heat to be transferred to gas is 180 J.

(C) The molar specific heat of gas is $\frac{9R}{2}$

(D) The molar specific heat of gas is $\frac{5R}{2}$

Ans. **(ABC)**

Sol. Internal energy

$U = \alpha\sqrt{V}$

since $U \propto T$, and $T = \frac{PV}{nR}$

$\therefore$ $PV^{1/2}$ = constant

polytropic process with

$\mu = \frac{1}{2}$

work done : $w = \frac{\Delta U \quad (\gamma - 1)}{\left(\frac{1}{2}\right)} = \frac{100 \times \left(\frac{7}{5} - 1\right)}{\frac{1}{2}}$

$w = 80$ J

amount of heat transferred

$Q = \Delta U \left[1 + \frac{\gamma - 1}{\frac{1}{2}}\right] = 100\left[1 + \left(\frac{7}{5} - 1\right) 2\right]$

Q = 180 J

molar heat capacity

$C = \frac{R}{\left(\frac{7}{5} - 1\right)} + \frac{R}{1/2}$

$C = \frac{9R}{2}$

12. One mole of an ideal monatomic gas is taken from temperature T_0 to $2T_0$ by the process T^4P^{-1} = constant.

(A) The molar specific heat of gas is $-\frac{3R}{2}$ (B) The molar specific heat of gas is $\frac{3R}{2}$

(C) Work done by gas is $-3RT_0$ (D) Work done by gas is $3R_0T_0$

Ans. **(AC)**

Sol. T^4P^{-1} = constant

$PV^{4/3}$ = constant, polytropic process with $\mu = \frac{4}{3}$

molar specific heat

$c = \frac{R}{\gamma - 1} - \frac{R}{\mu - 1} = \frac{3R}{2} - \frac{3R}{1} = -\frac{3R}{2}$

Work done by gas

$\therefore w = \frac{nR(T_1 - T_2)}{(4/3 - 1)}$

$w = -3RT_0$

13. An ideal gas is taken from the state A (pressure P, volume V) to the state B (pressure P/2, volume 2V) along a straight line path in the P-V diagram. Select the correct statement(s) from the following :

(A) The work done by the gas in the process A to B exceeds the work that would be done by it if the system were taken from A to B along the isotherm.

(B) In the T-V diagram, the path AB becomes a part of the parabola.

(C) In the P-T diagram, the path AB becomes a part of hyperbola.

(D) In going from A to B, the temperature T of the gas first increases to a maximum value and then decreases.

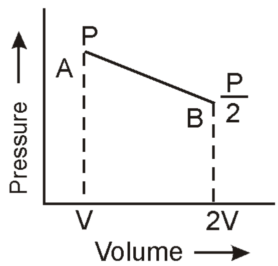
Ans. **(ABD)**

Sol. The situation is shown in figure.

Workdone from A to B = area of P-V diagram

$$= \left(P + \frac{P}{2}\right) V = \frac{3PV}{2} \quad = \frac{3RT}{2}$$

In case of isothermal process, the workdone is given by



$$= RT \log_e \frac{V_2}{V_1} \quad = RT \times 2.303 \log_{10}{}^{2}$$

$$= RT \times 2.303 \times 0.3010 = 0.6903\ RT$$

Hence (a) is correct.

The equation of straight line is given by

$$\frac{P}{P_0} + \frac{V}{V_0} = 1$$

Here P_0, V_0 are the intercepts on P and V-axes respectively.

In T-V graph,

$P = RT/V$ (assuming one moe of gas)

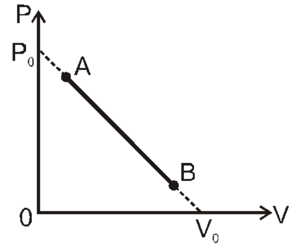
$$\therefore \frac{RT}{P_0 V} + \frac{V}{V_0} = 1 \qquad \text{or} \qquad \frac{RT}{P_0 V} = \left(1 - \frac{V}{V_0}\right)$$

$$T = \frac{P_0 V}{R}\left(1 - \frac{V}{V_0}\right) = \frac{P_0 V}{R} - \frac{P_0 V^2}{R V_0}$$

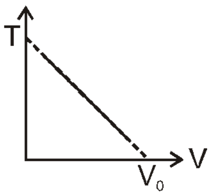
This represents a parabola. So (B) is correct.

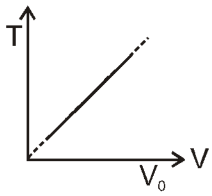
Comprehension Type Question:

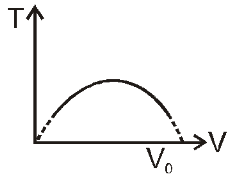
One mole of an ideal monatomic gas undergoes a linear process from A to B, in which is pressure P and its volume V change as shown in figure

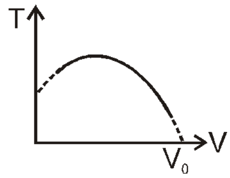


14. The absolute temperature T versus volume V for the given process is

(A) 

(B) 

(C) 

(D) 

Ans. **(C)**

15. The maximum temperature of the gas during this process is

(A) $\frac{P_0V_0}{2\ R}$ (B) $\frac{P_0V_0}{4\ R}$ (C) $\frac{3\ P_0V_0}{4\ R}$ (D) $\frac{3\ P_0V_0}{2\ R}$

Ans. **(B)**

16. As the volume of the gas is increased, in some range of volume the gas expands with absorbing the heat (the endothermic process) ; in the other range the gas emits the heat (the exothermic process). Then the volume after which if the volume of gas is further increased the given process switches from endothermic to exothermic is

(A) $\frac{2V_0}{8}$ (B) $\frac{3V_0}{8}$ (C) $\frac{5V_0}{8}$ (D) none of these

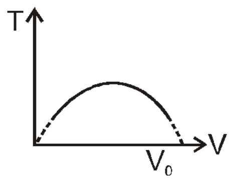
Ans. **(C)**

Sol. From The P–V graph, the relation between P and V is $P = -\frac{P_o}{V_o}V + P_0$ .... (1)

Also the ideal gas state equation for one mole is PV =RT .... (2)

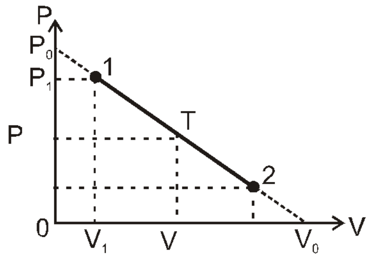
From equation (1) and (2) is $T = \frac{P_o}{R}V \left(1 - \frac{V}{V_o}\right)$

Hence the graph of T vs. V is a parabola given by



Obviously T is maximum at V=. $\frac{V_o}{2}$ There maximum value of T is $\frac{P_0V_0}{4\ R}$

$Q = \Delta U + W$



where ΔU is the change in the internal energy of the gas; and W is work, done by the gas. For one mole of the monatomic ideal gas ΔU = 3/2R ΔT. Work equals the area under the graph P vs. V

Therefore, for the process from the initial state with $P_1V_1 = 3/2\ RT_1$ to the state with P,V,T the heat given to system is

$$Q = (3/2)\ R\ (T - T_1) + (1/2)\ (P + P_1)(V - V_1)$$

$$= \frac{3}{2}\ (PV - P_1V_1) + \frac{1}{2}\ (PV + P_1V - PV_1 - P_1V_1) \qquad .... (3)$$

$$= 2PV + \frac{1}{2}P_1V - \frac{1}{2}PV_1 - 2P_1V_1$$

from equation 1 and 3 we get

$$Q = -\ 2\left(\frac{P_0}{V_0}\right)V^2 + \frac{5}{2}P_0V - 2P_0V_1\ \left(\frac{5}{4} - \frac{V_1}{V_0}\right)$$

The process switches from endothermic to exothermic as $\frac{dQ}{dV}$ changes from positive to negative, that is at $\frac{dQ}{dV} = 0$. Solving we get $V = \frac{5}{8}V_0$

Numerical based Questions :

17. Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is m_A and that in the B is m_B. The gas in each cylinder is now allowed to expand isothermally to the same final volume 2V. The changes in the pressure in A and B are found to be ΔP and 1.5ΔP respectively. Then find $\frac{14m_B}{m_A}$.

Ans. **21**

Sol. Process is isothermal. Therefore, T = constant. $\left(P \propto \frac{1}{V}\right)$ volume is increasing therefore, pressure will decrease.

In chamber A

$$\Delta P = (P_A)_i - (P_A)_f = \frac{n_A RT}{V} - \frac{n_A RT}{2V} = \frac{n_A RT}{2V} \qquad ......(1)$$

In chamber B

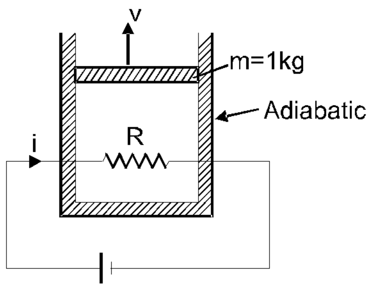
$$1.5\ \Delta P = (P_B)_i - (P_B)_f = \frac{n_B RT}{V} - \frac{n_B RT}{2V} = \frac{n_B RT}{2V} \qquad ......(2)$$

From (1) and (2)

$$\frac{n_A}{n_B} = \frac{1}{1.5} = \frac{2}{3} \qquad \text{or} \qquad \frac{m_A/M}{m_B/M} = \frac{2}{3}$$

or $\frac{m_A}{m_B} = \frac{2}{3}$ or $3m_A = 2m_B$

18. Current i = 2A flows through the resistance R = 10Ω. With what constant speed v (in m/s), must the piston move in upward direction so that temperature of ideal gas may remain unchanged. (g = 10 m/s²)



Ans. **v = 4 m/s**

Sol. Energy supply by resistance

$= i^2.R$

By first law of thermodynamics

$\Delta Q = \Delta U + \Delta W$

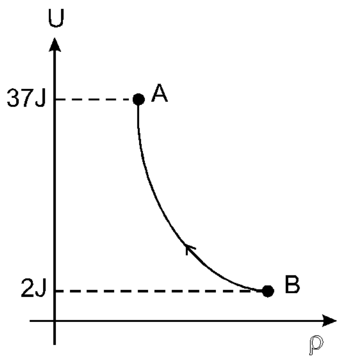
$\Delta U = 0$

$\therefore \quad i^2 R = mg \left(\frac{dx}{dt}\right)$

$v = \frac{I^2R}{mg} = \frac{2\times2\times10}{1\times10}$

v = 4 m/s.

19. Figure shows the variation of internal energy "U" with the density""of one mole of ideal diatomic gas. Process BA is a part of rectangular hyperbola. If the work done by gas in the process BA is W joules. Find W ?



Ans. **14**

Sol. For rectangular hyperbola

XY = constant

$U\rho$ = constant

$n\, C_V T\rho$= constant

$$n\frac{5}{2}R\ T\frac{M}{V} = \text{constant}$$

$$\frac{5}{2}P\frac{VM}{V} = \text{constant} \quad \Rightarrow \quad P = \text{constant (isobaric process)}$$

$\Delta U_{B \to A} = 37 - 2 = 35\ J = \frac{5}{2}nR\Delta T$

$14\ J = nR\Delta T$

$\therefore \quad W = 14\ J$

Matrix Match Type :

20. An ideal monoatomic gas undergoes different types of processes which are described in column-I. Match the corresponding effects in column-II. The letters have usual meaning.

Column-I	**Column-II**
(A) $P = 2V^2$	(p) If volume increases then temperature will also increase.
(B) PV^2 = constant	(q) If volume increases then temperature will decrease.
(C) $C = C_V + 2R$	(r) For expansion, heat will have to be supplied to the gas.
(D) $C = C_V - 2R$	(s) If temperature increases then work done by gas is positive.

Ans. (A) p,r,s (B) q (C) p,r,s (D) q,r

Sol. (A) If $P = 2V^2$, from ideal gas equation we get

$2V^3 = nRT$

$\therefore$ with increase in volume

(i) Temperature increases implies dU = +ve

(ii) dW = +ve

Hence dQ = dU + dW = +ve

(B) If PV^2 = constant, from ideal gas equation we get VT = K (constant)

Hence with increase in volume, temperature decreases

Now $dQ = dU + PdV = nC_v dT - \frac{PK}{T^2} dT \quad [\because dV = -\frac{K}{T^2} dT]$

$$= nC_v dT - \frac{PV}{T} dT = n(C_v - R)\, dT$$

$\therefore$ with increase in temperature dT = +ve and since $C_v > R$ for monoatomic gas. Hence dQ = +ve as temperature is increased

(C) $dQ = nC\,dT = nC_v\,dT + PdV$

$\Rightarrow n(C_v + 2R)\,dT = nC_v dT + PdV$

$\therefore 2nRdT = PdV \quad \therefore \frac{dV}{dT} = +ve$

Hence with increase in temperature volume increases and vice versa.

$\therefore dQ = dU + dW = +ve$

(D) $dQ = nC\,dT = nC_v\,dT + PdV$

or $\quad n(C_v - 2R)dT = nC_r dT + PdV$

or $-2nRdT = PdV \quad \therefore \frac{dV}{dT} = -ve$

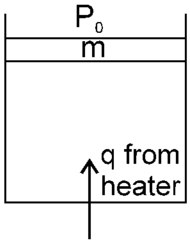
$\therefore$ with increase in volume temperature decreases.

Also $dQ = n(C_v - 2R)dT$

with increase in temperature dT = +ve but $C_v < 2R$ for monoatomic gas. Therefore dQ = –ve with increase in temperature.

Subjective Type Questions :

21. Two moles of an ideal monoatomic gas are contained in a vertical cylinder of cross sectional area A as shown in the figure. The piston is frictionless and has a mass m. At a certain instant a heater starts supplying heat to the gas at a constant rate q J/s. Find the steady velocity of the piston under isobaric condition. All the boundaries are thermally insulated.



Ans. $\frac{2q}{5(mg + P_0A)}$

Sol. $dQ_p = nC_p dT$

Let heat be supplied for time dt

$$\therefore \quad qdt = 2.\frac{5R}{2}.\frac{\left[\frac{mg}{A} + P_0\right]Adx}{2R}$$

where $\left[dT = \frac{Pdv}{nR}\right]$

$\therefore \quad qdt = \frac{5}{2}(mg + P_0A)dx \qquad \therefore \quad \frac{dx}{dt} = \frac{2q}{5(mg + P_0A)}$.

22. A piston can freely move inside a horizontal cylinder closed from both ends. Initially, the piston separates the inside space of the cylinder into two equal parts each of volume V_0, in which an ideal gas is contained under the same pressure p_0 and at the same temperature. What work has to be performed in order to increase isothermally the volume of one part of gas η times compared to that of the other by slowly moving the piston?

Ans. $W = p_0V_0 \ln [(\eta +1)^2 /4\eta$

Sol. Final volume of left part = η x final volume of right – part

$V_0 + V = \eta(V_0 - V)$

$V = \frac{(\eta - 1)V_0}{\eta + 1}$

in isothermal process, work done = $p_0V_0\ell n \left(\frac{V_2}{V_1}\right)$

work done by left part of gas on piston is

$W_1 = p_0V_0 \ell n \left(\frac{V_0 + V}{V_0}\right)$

$= p_0V_0 \ell n \left(1 + \frac{V}{V_0}\right)$

$= p_0V_0 \ell n \left(1 + \frac{\eta - 1}{\eta + 1}\right)$

$= p_0V_0 \ell n \left(\frac{2\eta}{\eta + 1}\right)$

similarity, work done by right part of gas on piston is

$W_2 = p_0V_0 \ell n \left(\frac{V_0 - V}{V_0}\right)$

$= p_0V_0 \ell n \left(1 - \frac{\eta - 1}{\eta + 1}\right)$

$= p_0V_0 \ell n \left(\frac{2}{\eta + 1}\right)$

let work done by applied force in W_{ext} then

$W_1 + W_2 + W_{ext} = 0$

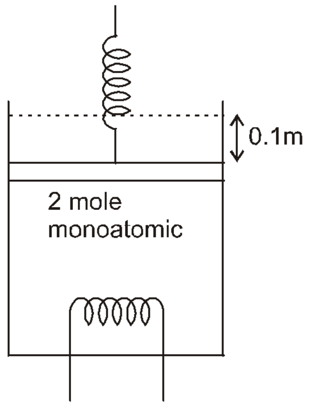
$W_{ext} = - W_1 - W_2$

$W_{ext} = p_0V_0 \ell n \frac{(\eta + 1)^2}{4\eta}$

23. Two moles of an ideal monoatomic gas are confined within a cylinder by a massless and frictionless spring loaded piston of cross-sectional area 4×10^{-3} m^2. The spring is, initially in its relaxed state. Now the gas is heated by an electric heater, placed inside the cylinder, for some time. During this time, the gas expands and does 50 J of work in moving the piston through a distance 0.10 m. The temperature of the gas increases by 50 K. Calculate the spring constant and the heat supplied by the heater. $P_{atm} = 1 \times 10^5$ N/m^2.R = 8.314 J/mol-K

Ans. K = 2000 N/m, Q = 1297 Joules approx.

Sol. $W_{gas} = 50$ J



$\Delta T = 50$ K

$P_{atm} = 1 \times 10^5$ N/m^2

From work energy theorem

$\Rightarrow \quad W_{spring} + W_{atm} + W_{gas} = \Delta KE.$

$\Rightarrow \quad -\frac{1}{2}K(0.1)^2 - 10^5 \times 4 \times 10^{-3} \times 0.1 + 50 = 0$

$\Rightarrow \quad K = 2000$ N/m

$\Delta Q = \Delta U + W$

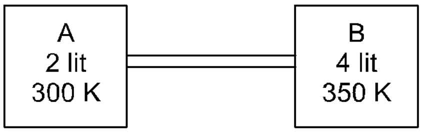
$= 2 \times \frac{3}{2} R \times 50 + 50$

$= 50 + 150$ R Joules. $\simeq 1297$ J

24. Two vessels A and B, thermally insulated, contain an ideal monoatomic gas. A small tube fitted with a valve connects these vessels. Initially the vessel A has 2 liters of gas at 300 K and 2×10^5 N m^{-2} pressure while vessel B has 4 liters of gas at 350 K and 4×10^5 Nm^{-2} pressure. The valve is now opened and the system reaches equilibrium in pressure and temperature. Calculate the new pressure and temperature. (R = $\frac{25}{3}$ J/mol-K)

Ans. P = $\frac{10}{3} \times 10^5$ N/m^2, T = $\frac{10500}{31}$ K $\simeq 338.71$ K

Sol.



2×10^5 N/m^2 $\quad$ 4×10^5 N/m^2

From $\quad PV = nRT$

$$n = \frac{PV}{RT} \qquad \Rightarrow n_A = \frac{2\times10^5\times2\times10^{-3}}{\frac{25}{3}\times300} = \frac{4}{25},\ n_B = \frac{4\times10^5\times4\times10^{-3}}{\frac{25}{3}\times350} = \frac{96}{175}$$

$$U_1 + U_2 = U_{mix}$$

$$\Rightarrow \quad n_A C_V T_1 + n_B C_V T_2 = (n_A + n_B) C_V T \qquad \Rightarrow \qquad T = \frac{n_A T_1 + n_B T_2}{n_A + n_B}$$

Putting values

$$T = \frac{10500}{31} \text{ K} = 338.71 \text{ K} \qquad \Rightarrow \qquad P = \frac{(n_A + n_B)RT}{V_A + V_B}$$

Putting values

$$P = \frac{10}{3} \times 10^5 \text{ N/m}^2$$

25. An ideal gas ($C_p/C_v = \gamma$) having initial pressure P_0 and volume V_0. (a) The gas is taken isothermally to a pressure $2P_0$ and then adiabatically to a pressure $4P_0$. Find the final volume. (b) The gas is brought back to its initial state. It is adiabatically taken to a pressure $2P_0$ and then isothermally to a pressure $4P_0$. Find the final volume.

Ans. $\frac{V_0}{2^{\frac{\gamma+1}{\gamma}}}$ **in each cases**

Sol. Initial pressure of an ideal gas = P_0

initial volume of an ideal gas = V_0

(a) For isothermal process $\quad P_2 V_2 = P_1 V_1$

$$\Rightarrow \quad V_2 = \frac{P_1V_1}{P_2} = \frac{V_0}{2}$$

For adiabatic process $P_3 V_3^{\gamma} = P_2 V_2^{\gamma}$

$$\Rightarrow \quad V_3 = \left(\frac{P_2}{P_3}\right)^{1/\gamma} V_2 \qquad \Rightarrow \qquad V_3 = \frac{V_0}{2^{\frac{\gamma+1}{\gamma}}}$$

(b) For adiabatic process

$$P_2 V_2^{\gamma} = P_1 V_1^{\gamma} = V_2 = \left(\frac{P_1}{P_2}\right)^{1/\gamma} V_1 \qquad \Rightarrow \qquad V_2 = \frac{V_0}{2^{\frac{1}{\gamma}}}$$

For isothermal process $\qquad P_3 V_3 = P_2 V_2 \qquad \Rightarrow \qquad V_3 = \frac{P_2V_2}{P_3} \qquad \Rightarrow V_3 = \frac{V_0}{2^{\frac{\gamma+1}{\gamma}}}$