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PHYSICS

TARGET : JEE- Advanced 2021

CAPS-17

Nuclear Physics

SCQ (Single Correct Type) :

1. Nuclei of radioactive element A are produced at rate ' t^2 ' at any time t. The element A has decay constant λ. Let N be the number of nuclei of element A at any time t. At time $t = t_0$, $\frac{dN}{dt}$ is minimum. Then the number of nuclei of element A at time $t = t_0$ is

(A) $\frac{\lambda t_0^2 - 2t_0}{\lambda^2}$ (B) $\frac{t_0 - \lambda t_0^2}{\lambda^2}$ (C) $\frac{2t_0 - \lambda t_0^2}{\lambda}$ (D) None of these

Ans. **(A)**

Sol. $\frac{dN}{dt} = t^2 - \lambda N$

for $\frac{dN}{dt}$ to be minimum; $\frac{d^2N}{dt^2} = 0$

$\Rightarrow \frac{d^2N}{dt^2} = 2t - \lambda \frac{dN}{dt} = 2t - \lambda (t^2 - \lambda N) = 0$ or $N = \frac{\lambda t_0^2 - 2t_0}{\lambda^2}$

2. A radioactive sample decays by β-emission. In first two seconds 'n' β-particles are emitted and in next 2 seconds, '0.25n' β-particles are emitted. The half life of radioactive nuclei is

(A) 2 sec (B) 4 sec (C) 1 sec (D) none

Ans. **(C)**

Sol. Let initially No $\rightarrow$ atoms were present

$\therefore n = N_0 - N_0 e^{-\lambda(2)} \longrightarrow$ (i)

and $1.25\, n = N_0 - N_0 e^{-\lambda(2+2)} \longrightarrow$ (ii) [Total β particle emitted in 4 seconds]

Dividing (ii) by (i)

$1.25 = \frac{1 - e^{-4\lambda}}{1 - e^{-2\lambda}}$

On solving $e^{-2\lambda} = \frac{1}{4}$

$\therefore -2\lambda = -\ln 4$

$\therefore \frac{-2\ln 2}{T_{1/2}} = -2 \ln 2$

$\therefore T_{1/2} = 1$ second]

3. A factory produces a radionuclide at a constant rate R. Initially there were no nuclide present. Then activity of the sample produced by factory just after an average life of nuclide passes away is :

(A) R (B) Re^{-1} (C) $R(1 - e^{-1})$ (D) $\frac{R}{2}(1 - e^{-1})$

Ans. **(C)**

Sol. Between t = t & t = t + dt

No. of radionuclide formed = Rdt

If average life = $t_0\left(=\frac{1}{\lambda}\right)$

$\Rightarrow$ It will decay for time interval $(t_0 - t)$

Then activity after time $(t_0 - t)$ will become

$dA = \lambda R dt e^{-\lambda(t0 - t)}$

$\Rightarrow A = \int_0^{t_0} \lambda R e^{-\lambda(t_0 - t)} dt$

$= R(1 - e^{-\lambda t_0}) = R(1 - e^{-1})$ $\left(\because\ t_0 = \frac{1}{\lambda}\right)$

4. A radionuclide with decay constant λ is produced in a reactor at a constant rate α nuclei per second. During each decay energy E_0 MeV is released. If the production of radio nuclides started at t = 0, total energy released upto time t is given by equation

(A) $E_0\alpha(1 - e^{-\lambda t})$ (B) $E_0\frac{\alpha}{\lambda}(1 - e^{-\lambda t})$ (C) $E_0(\alpha t - \frac{\alpha}{\lambda}e^{-\lambda t})$ (D) $E_0\left(\alpha t - \frac{\alpha}{\lambda}(1 - e^{-\lambda t})\right)$

Ans. **(D)**

Sol. $\frac{dN}{dt} = \alpha - \lambda N$

$\Rightarrow \int_0^N \frac{dN}{\alpha - \lambda N} = \int_0^t dt$

$\Rightarrow \frac{1}{-\lambda}\ell n\left(\frac{\alpha - \lambda N}{\alpha}\right) = t$

$\Rightarrow N = \frac{\alpha}{\lambda}(1 - e^{-\lambda t})$: No. of undecayed nucleii

$\therefore$ No. of decayed nucleii in time t = $\alpha t - \frac{\alpha}{\lambda}(1 - e^{\lambda t})$

Hence, energy released $= E_0\left(\alpha t - \frac{\alpha}{\lambda}(1 - e^{\lambda t})\right)$

5. The count rate observed from a radioactive source at 't' second was N_0 and at 4t second it was $\frac{N_0}{16}$. The count rate observed, at $\left(\frac{11}{2}\right)t$ second will be

(A) $\frac{N_0}{128}$ (B) $\frac{N_0}{64}$ (C) $\frac{N_0}{32}$ (D) None of these

Ans. **(B)**

Sol. Let initially substance have N_i nuclei then

$N= N_i \; e^{-\lambda t}$; $\frac{dN}{dt} = -\lambda N_i e^{-\lambda t}$

it is given that,

At t=t, $\frac{dN}{dt} = -\lambda N_i e^{-\lambda t} = N_0$

And at t= 4t, $\frac{dN}{dt} = -\lambda N_i e^{-4\lambda t} = \frac{N_0}{16}$

Dividing both, we get $e^{3\lambda t} = 16$

At t= 11t/2, $\frac{dN}{dt} = -\lambda N_i e^{-\frac{11}{2}\lambda t} \quad = -\lambda N_i e^{-\frac{8}{2}\lambda t} \times e^{-\frac{3}{2}\lambda t} = \frac{N_0}{64}$

6. The count rate from 100 cm^3 of a radio-active liquid is C. Some of this liquid is now discarded. The count rate of the remaining liquid is found to be C/10 after three half-lives. The volume of the remaining liquid in cm^3 is

(A) 20 (B) 40 (C) 60 (D) 80

Ans. **(D)**

Sol. Intial count rate of $1\,cm^3$ of liquid $= \frac{C}{100}$

After 3 half-lives, count rate of the liquid $= \frac{1}{8} \times \frac{C}{100}$

Let V be the volume of the remaining liquid . The CR of this liquid $= \frac{VC}{800}$

$$\frac{C}{10} = \frac{VC}{800} \Rightarrow V = 80$$

7. An isolated nucleus which was initially at rest, disintegrates into two nuclei due to internal nuclear forces and no γ rays are produced. If the ratio of their kinetic energy is found to be $\frac{64}{27}$ then :

(A) Ratio of their de–broglie wavelength is $\frac{\sqrt{64}}{\sqrt{27}}$ respectively

(B) Ratio of their speed is $\frac{64}{37}$ respectively

(C) Ratio of their nuclear radius is $\frac{3}{4}$ respectively

(D) Ratio of their nuclear radius is $\frac{4}{3}$ respectively

Ans. (C)

Sol. $P_1 = P_2 = P$

$m_1v_1 = m_2v_2$

$$\left(\frac{P^2}{2m_1}\right)\Big/\left(\frac{P^2}{2m_2}\right) = \frac{64}{27} \quad \Rightarrow \quad \frac{m_2}{m_1} = \frac{64}{27} = \frac{v_1}{v_2}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{h/P_1}{h/P_2} = 1 : 1$$

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{27}{64}\right)^{1/3} = \frac{3}{4}$$

MCQ (One or more than one correct) :

8. Two radioactive nuclei A and B are present in equal numbers to begin with. Three day later, number of A nuclei are 3 times number of B nuclei. Choose the correct statement.

(A) $\lambda_B - \lambda_A = \frac{\ell n3}{3 \text{ days}}$

(B) $\lambda_A - \lambda_B = \frac{\ell n3}{3 \text{ days}}$

(C) the ratio of activity rate of A and B after 3 days is $\frac{3}{1}$

(D) the ratio of activity rate of A and B after 3 days is less than $\frac{3}{1}$.

Ans. (AD)

Sol. $N_A = N_0 e^{-3\lambda_A} \quad \Rightarrow \quad N_B = N_0 e^{-3\lambda_B}$

$N_A = 3N_B \quad \Rightarrow \quad e^{-3\lambda_A} = 3e^{-3\lambda_B}$

$e^{3(\lambda_B - \lambda_A)} = 3$

$R_A = \lambda_A \times 3N_0 \quad \Rightarrow \quad R_B = \lambda_B \times N_0 \quad \Rightarrow \quad \frac{R_A}{R_B} = \frac{3\lambda_A}{\lambda_B}$

9. $^{64}_{29}Cu$ can decay by β^- or β^+ emission, or electron capture. It is known that $^{64}_{29}Cu$ has a half life of 12.8 hrs with 40% probability of β^- decay, 20% probability of β^+ decay and 40% probability of electron capture. The mass of $^{64}_{29}Cu$ is 63.92977 amu while $^{64}_{30}Zn$ is 63.92914 amu and $^{64}_{28}Ni$ is 63.92796 amu. What is the half life for electron capture?

(A) 5.12 Hrs. (B) 32 Hrs. (C) 2.56 Hrs. (D) 16 Hrs.

Ans. **(B)**

Sol. $^{64}_{29}Cu \rightarrow ^{64}_{30}Zn + ^{0}_{-1}e + \bar{\nu}$

$^{64}_{29}Cu \rightarrow ^{64}_{28}Ni + ^{0}_{+1}e + \nu$

$^{64}_{29}Cu + ^{0}_{-1}e \rightarrow ^{64}_{28}Ni + \nu$

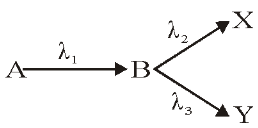
$\lambda = \lambda_1 + \lambda_2 + \lambda_3$ ($\lambda_1 = 0.4\ \lambda$, $\lambda_2 = 0.2\ \lambda$, $\lambda_3 = 0.4\ \lambda$)

$T_{1/2}$ e– capture = $\frac{T_{1/2}}{0.4} = \frac{12.8}{0.4}$ = 32 hrs.

Comprehension Type Question:

Comprehension#1

The rate at which a particular decay process occurs in a radio active sample, is proportional to the number of radio active nuclei present. If N is the number of radio active nuclei present at some instant, the rate of change of N is $\frac{dN}{dt} = -\lambda N$.



Consider radioactive decay of A to B which may further decay either to X or to Y, λ_1, λ_2 and λ_3 are decay constants for A to B decay, B to X decay and B of Y decay respectively. If at t = 0 number of nuclei of A, B, X and Y are N_0, N_0, zero and zero respectively and N_1, N_2, N_3, N_4 are number of nuclei A,B,X and Y at any instant.

10. Rate of accumulation of B of any instant will be –

(A) $N_1\lambda_1 + N_2\lambda_2 + N_3\lambda_3$ (B) $N_1\lambda_1 - N_3\lambda_2 - N_4\lambda_3$

(C) $N_1\lambda_1 - N_2\lambda_2 - N_2\lambda_3$ (D) $N_1\lambda_1 + N_2\lambda_2 - N_3\lambda_3$

Ans. **(C)**

Sol. Rate of production of B depends on the decaying rate (A) :

$$\frac{d_{N_A}}{dt} = -\lambda_1 N_A = \lambda_1 N_1$$

B is decaying simultaneously with two rates

$$\frac{d_{N_B}}{dt} = -\lambda_2 N_B = -\lambda_2 N_2 \qquad \frac{d_{N_B}}{dt} = -\lambda_3 N_B = -\lambda_3 N_2$$

Number of nuclei of 'B' is = $\lambda_1 N_1 - \lambda_2 N_2 - \lambda_3 N_2$

11. The number of nuclei of B will first increase then after a maximum value, it will decreases, if–

(A) $\lambda_1 > \lambda_2 + \lambda_3$ (B) $\lambda_1 = \lambda_2 = \lambda_3$

(C) $\lambda_1 = \lambda_2 + \lambda_3$ (D) For any values of λ_1, λ_2 and λ_3

Ans. **(A)**

Sol. B will increase when $\lambda N_1 > (\lambda_2 + \lambda_3)N_2$

as initially $N_1 = N_2 = N_0 \Rightarrow \lambda_1 > \lambda_2 + \lambda_3$

12. At t = ∞, which of the following is incorrect ?

(A) $N_2 = 0$ (B) $N_3 = \frac{N_0\lambda_2}{\lambda_2+\lambda_3}$ (C) $N_4 = \frac{2N_0\lambda_3}{\lambda_2+\lambda_3}$ (D) $N_3+N_4+N_1+N_2=2N_0$

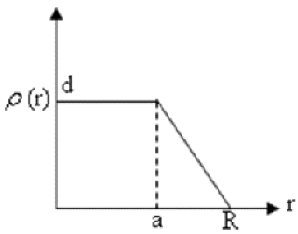
Ans. **(B)**

Sol. $N_2 = 0$, $N_1=0$ as both will decay completely :

$N_3 = \frac{2N_0\lambda_2}{\lambda_2+\lambda_3}$ therefore B is incorrect

Comprehension # 2

The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density ρ (r) (charge per unit volume) is dependent only on the radial distance r from the centre of the nucleus, as shown. The electric field is only along radial direction



13. The electric field at r = R is :

(A) independent of a (B) directly proportional to a

(C) directly proportional to a^2 (D) inversely proportional to a

Ans. **(A)**

Sol. $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}; Q = \text{total charge} = Ze$

$$\Rightarrow E = \frac{Ze}{4\pi\epsilon_0 R^2}$$

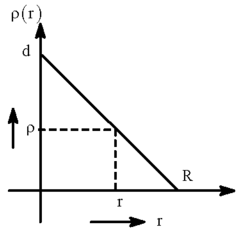
⇒ E at r = R is independent of a

14. For a = 0, the value of d(maximum value of ρ as shown in the figure) is :

(A) $\frac{3Ze}{4\pi R^3}$ (B) $\frac{3Ze}{\pi R^3}$ (C) $\frac{4Ze}{3\pi R^3}$ (D) $\frac{Ze}{3\pi R^3}$

Ans. **(B)**

Sol. ρ versus r graph for a = 0 will be as shown



$$\frac{d}{R} = \frac{\rho}{R-r} \Rightarrow \rho = \frac{d(R-r)}{R} \text{ for } r \le R$$

$$Q = Ze = \int_0^R \rho\left(4\pi x^2\right)dx = \frac{\pi d R^3}{3} \text{ or } d = \frac{3Ze}{\pi R^3}$$

Numerical based Questions :

15. A radioactive sample contains two radio nucleoids A and B having decay constant λ hr^{-1} and 2λ hr^{-1}. Initially 25% of total decay comes from A. How long (in hr) will it take before 75% of total decay comes from A. [Take λ = ln 3]

Ans. **2**

Sol. Initially let no. of molecules of A = N_A and that of B = N_B

$$\frac{\lambda N_A}{\lambda N_A + 2\lambda N_B} = \frac{25}{100}$$

$\therefore$ $100\ \lambda N_A = 25\ \lambda N_A + 50\ \lambda N_B$ $\Rightarrow$ $N_A = \frac{2}{3} N_B$

Now, $N_A' = N_A\ e^{-\lambda t} = \frac{2}{3} N_B e^{-\lambda t}$

$N_B' = N_B\ e^{-2\lambda t}$

$$\therefore \quad \frac{\frac{2}{3}\lambda N_B e^{-\lambda t}}{\frac{2}{3}\lambda N_B e^{-\lambda t} + 2\lambda N_B e^{-2\lambda t}} = \frac{75}{100} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{1}{1+3e^{-\lambda t}} = \frac{3}{4} \quad \Rightarrow \quad 1 = 9e^{-\lambda t} \quad \Rightarrow \quad e^{\lambda t} = 9$$

So, $t = \frac{\ln(9)}{\lambda}$ hr $= \frac{2\ln(3)}{\ln(3)}$ hr = 2 hr **Ans.**]

16. The ratio of the components in a mixture, which consists of two elements, is to be determined. The atomic number of the elements are big, their atomic mass numbers are the same and the amount of the sample is only 58 mg. We know that both elements β decay when they are bombarded by neutrons. They behave similarly when absorbing neutrons. The half-life of element A is half an hour and the half-life of element B is an hour. Right after the neutron irradiation the β emission is measured. At this time 80 particles are measured in 2 seconds, and after an hour only 29 particles are measured during 2 seconds. Determine the mass of the element A (in mg) in the sample. Assume that spontaneous decay statistical law is obeyed by nuclei of both the elements.

Ans. **22 mg**

Sol. $\frac{dN_1}{dt} = \frac{400}{10} = \lambda_1 N_1 + \lambda_2 N_2$

$40 = \frac{\ell n2}{1800} N_1 + \frac{\ell n2}{3600} N_2$

$14.5 = \frac{\ell n2}{1800} \times \frac{N_1}{4} + \frac{\ell n2}{3600} \times \frac{N_2}{2}$

$40 = x_1 + x_2$

$$\begin{array}{l} 29 = \frac{x_1}{2} + \frac{x_2}{2} \\ \hline 11 = \frac{x_1}{2} \end{array}$$

$x_1 = 22 = \frac{\ell n2N_1}{1800}$

$40 = 22 + x_2$

$x_2 = 18 = \frac{\ell n2N_2}{3600}$

$1 + \frac{N_1}{N_2} = \frac{22 \times 18}{18 \times 36} = \frac{N_2}{N_1 + N_2} = \frac{18}{29}$

$\frac{N_1}{N_1 + N_2} = \frac{11}{29}$

$m_1 = \frac{N_1}{N_1 + N_2} \times M \qquad = 22 \text{ mg}$

17. In a sample initially there are equal number of atoms of two radioactive isotopes A and B. 3 days later the number of atoms of A is twice that of B. Half life of B is 1.5 days. What is half life of isotope A ? (in days)

Ans. **3**

Sol. | | A | B |
|---|---|---|
| t = 0 | N_0 | N_0 |
| t_0 = 3 days | 2N | N |

$2N = N_0\,(0.5)\; t_0/\tau_1$

$N = N_0\,(0.5) t_0/\tau_2$

$2 = (0.5)\; t_0 \left(\frac{1}{\tau_1} - \frac{1}{\tau_2}\right)$

$\Rightarrow 0.5^{-1} = (0.5)\left(\frac{3}{\tau_1} - 2\right)$ $\qquad \Rightarrow -1 = \frac{3}{\tau_1} - 2 \therefore\ \tau_1 = 3$ days

Subjective Type Questions :

18. The nucleus of $^{230}_{90}Th$ is unstable against α-decay with a half-life of 7.6×10^3 years. Write down the equation of the decay and estimate the kinetic energy of the emitted α-particle from the following data : $m\left(^{230}_{90}Th\right)$=230.0381 amu, m $\left(^{226}_{88}Ra\right)$ = 226.0254 amu and m $\left(^{4}_{2}He\right)$ = 4.0026 amu.

Ans. $^{230}_{90}Th \rightarrow {}^{226}_{88}Ra + {}^{4}_{2}He + Q$; 9.25 MeV.

Sol. The equation of the decay is

$$^{230}_{90}Th \rightarrow {}^{226}_{88}Ra + {}^{4}_{2}He + Q$$

The energy Q is given by

$$Q = [m\,(Th) - m\,(Ra) - m(He)]c^2$$

Using the given data and c^2 = 931.5 MeV/amu, we get Q = 9.41 MeV. This energy is shared by Ra and He. If the original nucleus Th is at rest, i.e. if the momentum of the system before α-decay is zero, the total momentum after the decay will also be zero. Thus Ra and He will have equal and opposite linear momenta.

$\therefore \qquad m_{He}\, v_{He} = -\, m_{Ra}\, v_{Ra}$

or $\qquad m_{He}\, v_{He} = m^2{}_{Ra}\, v^2{}_{ra} \qquad$ (i)

Now $\qquad \frac{K.E.(HE)}{K.E.(Ra)} = \frac{1/2 m_{He} v_{He}^2}{1/2 m_{Ra} v_{Ra}^2}$

$$= \left(\frac{m_{He} v_{He}}{m_{Ra} v_{Ra}}\right) \times \left(\frac{m_{Ra}}{m_{He}}\right)$$

$= \frac{m_{Ra}}{m_{He}} \qquad$ [use Eq. (i)]

$\approx \frac{226}{4} \approx 56.6 \qquad$ (ii)

i.e. the kinertic energy of He is 56.5 times that of Ra, the total energy being

K.E. (He) + K.E.(Ra) = 9.41 MeV $\qquad$ (iii)

From eqs (ii) and (iii) We have K.E. (Ra) = 0.16 MeV and K.E. (He) 9.25 MeV.

19. Radium being a member of the uranium series occurs in uranium ores. If the half lives of uranium and radium are respectively 4.5×10^9 and 1620 years calculate the $\frac{N_{radiun}}{N_{Uranium}}$ in Uranium ore at equlibrium.

Ans. $1/2.78 \times 10^6$

Sol. In the series decay $A \rightarrow B \rightarrow C$, if $\lambda_A << \lambda_B$ the transient equilibrium occurs when

$$\frac{N_B}{N_A} = \frac{\lambda_A}{\lambda_B}$$

Here A = Uranium and B = Radium

$\lambda_A = \frac{1}{\tau_A} = 0.693 / 4.5 \times 109 \text{ year}^{-1}$ $\lambda_B = \frac{1}{\tau_B} = 0.693 / 1.620 \text{ year}^{-1}$

N (Rad) / N (U) $\approx 1.620 / 4.5 \times 10^9 = 1/2.78 \times 10^6$

20. Find the binding energy of a nucleus consisting of equal numbers of protons and neutrons and having the radius one and a half time smaller than that of Al^{27} nucleus. [atomic mass of $_4Be^8$ = 8.0053 u, $_1H^1$ = 1.007826u, $_0n^1$= 1.008665 u]

Ans. Be^8, E_b = 56.5 MeV.

Sol. $\therefore$ $R = R_0 A^{1/3}$

$R_{Al} = R_0(27)^{1/3}$

$\therefore$ $R = R_0A^{1/3}$

But $R = \frac{R_{Al}}{\frac{3}{2}} = \frac{2}{3}R_{Al}$

or $R_0A^{1/3} = \frac{2}{3}R_0(27)^{1/3}$

$\therefore$ $A = \left(\frac{2}{3}\right)^3 27 = \frac{8}{27} \times 27 = 8$

$\Delta m = (4m_H + 4m_n - M_N)$amu

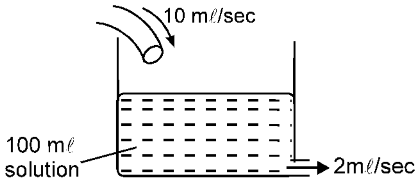
$= (4 \times 1.007826 + 4 \times 1.008665 - 8.00531)$ amu

$= 0.060664$ amu

$E_b = \Delta mc^2 = 0.060664 \times 931.5$ MeV

$= 56.50$ MeV $= 56.5$ MeV

21. A 100 mℓ solution having activity 50 dps is kept in a beaker. It is now constantly diluted by adding water at a constant rate of 10 mℓ/sec and 2 mℓ/sec of solution is constantly being taken out. Find the activity of 10 mℓ solution which is taken out, assuming half life to be effectively very large.



Sol. The volume of liquid in beaker at any instant of time t is

$$V = 100 + 8t$$

The volume of liquid ejected in t seconds is 2t

$\therefore$ Number of active atoms being taken out is

$$-dN = \frac{N}{V}\,2dt \qquad \text{or} \qquad -\frac{dN}{dt} = \frac{2N}{V} = \frac{2N}{100+8t}$$

multiplying both sides with disintegration constant.

$$-\lambda\, dN = \lambda N \frac{2dt}{V} \qquad \text{or} \qquad -dA = A.\frac{2dt}{V}$$

where A is activity of the solution. The time taken for 10 ml solution to come out is 5 second.

or $$\int_{A_0}^{A} \frac{dA}{A} = \int_0^5 \frac{-2t}{100+8t}\,dt$$

or $$A = A_0 \left(\frac{5}{7}\right)^{1/4}$$

$\therefore$ required activity of the solution is $A - A_0 = A_0 \left[1-\left(\frac{5}{7}\right)^{1/4}\right]$

Ans. $A_0 \left[1-\left(\frac{5}{7}\right)^{1/4}\right]$ **where A_0 = 50 dps**

22. A charged capacitor of capacity C is discharged through a resistance R. A radioactive sample decays with an average life J. If the ratio of electrostatic field energy stored in the capacitor to the activity of the radioactive sample remains constant with time then $R = (xJ/C)$. Where x is ______

Ans. 2

Sol. $\frac{E}{A} = \frac{\frac{1}{2C}\left(Q_0 e^{-t/RC}\right)^2}{A_0 e^{-\lambda t}}$

For this ratio to be constant,

with time; $\frac{2t}{RC} = \lambda t$ or $\lambda = \frac{2}{CR}; \frac{1}{J} = \frac{2}{CR}$

23. A radio nuclide with half-life T days emits $\beta-$ particles of average kinetic energy E J. The radionuclide is used as a source in a machine which generates electric eneergy with efficiency 25%. The number of moles of the nuclide required to generate electrial energy at an initial rate P is $n = \frac{yTP}{EN\ln(2)}$ where 'y' is. (Here N is Number of active nuclei in 1 mole)

Ans. **4**

Sol. Let 'n' be the number of moles of the ratio nuclide number of nuclei in the nuclide = nN; $\lambda = \frac{\ln(2)}{T}$;

Rate of decay $A = \lambda(nN) = \frac{\lambda n \ln(2)}{T}$

Rate of release of energy = AE

Rate of generate of electrical energy P = ηAE

$= \frac{25}{100}\frac{nNE\ln(2)}{T} \qquad \therefore n = \frac{4TP}{NE\ln(2)}$

24. A radioactive isotope is being produced at a constant rate dN/dt = R in an experiment .The isotope has a half-life $t_{1/2}$. Show that after a time t >> $t_{1/2}$, the number of active nuclei will become constant. Find the value of this constant. Suppose the production of the radioactive isotope starts at t = 0. Find the number of active nuclei at time t.

Ans. $\frac{Rt_{1/2}}{\ell n2}$; $\frac{R}{\lambda}(1 - e^{-\lambda t})$

Sol. Let N = no. of isotope at any instant

$$\frac{dN}{dt} = R - \lambda N \qquad \Rightarrow \qquad \frac{dN}{R - \lambda N} = dt$$

On integrating with initial condition t = 0, N = 0, we get

$$\int_0^N \frac{dN}{R - \lambda N} = \int_0^t dt \Rightarrow \frac{1}{-\lambda}\ln\frac{R - \lambda N}{R} = t \Rightarrow N = \frac{R}{\lambda}(l - e^{-\lambda t})$$

when t >> $t_{½}$

$N \approx \frac{R}{\lambda} = \frac{Rt_{½}}{\ell n2}$

25. Nuclei of radioactive element A are being produced at a constant rate α. The element has a decay constant λ. At time t = 0, there are N_0 nuclei of the element.

(a) Calculate the number N of nuclei of A at time t.

(b) If $\alpha = 2N_0\lambda$, calculate the number of nuclei of A after one half-life of A and also the limiting value of N as $t \to \infty$.

Ans. (a) $N = \frac{1}{\lambda}\,[\alpha - (a - \lambda N_0)\, e^{-\lambda t}$; (b) $\frac{3N_0}{2}$, $2N_0$]