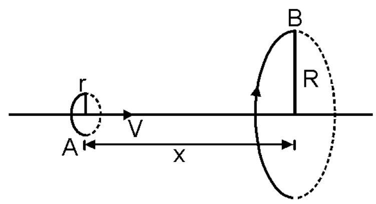
competishun PHYSICS TARGET : JEE- Advanced 2021	CAPS-15 EMI + AC

SCQ (Single Correct Type) :

1. Loop A of radius r(r << R) moves towards loop B with a constant velocity V in such a way that their planes are always parallel. What is the distance between the two loops (x) when the induced emf in loop A is maximum



(A) R (B) $\frac{R}{\sqrt{2}}$ (C) $\frac{R}{2}$ (D) $R\left(1-\frac{1}{\sqrt{2}}\right)$

Ans. **(C)**

Sol. $\phi_A = \frac{\mu_0 i \pi R^2}{2\pi(R^2+x^2)^{3/2}}.\pi r^2$

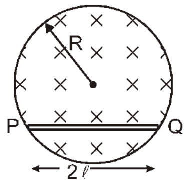
$E_A = -\frac{d\phi}{dt} = \frac{\mu_0 i\pi}{2} R^2 r^2 \ (-3/2)\ (R^2 + x^2)^{-5/2} \ . \ 2x\ (v)$

E_A is maximum when $\frac{dE_A}{dx} = 0 \quad \Rightarrow \quad \frac{d}{dx}\frac{x}{(R^2+x^2)^{5/2}} = 0$

or $(R^2 + x^2)^{5/2} - \frac{5x}{2}(R^2 + x^2)^{3/2}\, 2x = 0$

or $R^2 + x^2 - 5x^2 = 0$ or $x = \frac{R}{2}$ **Ans.**

2. A uniform magnetic field, $B = B_0 t$ (where B_0 is a positive constant), fills a cylindrical volume of radius R, then the potential difference in the conducting rod PQ due to electrostatic field is :



(A) $B_0\ell\ \sqrt{R^2+\ell^2}$ (B) $B_0\ell\ \sqrt{R^2-\frac{\ell^2}{4}}$ (C) $B_0\ell\ \sqrt{R^2-\ell^2}$ (D) $B_0R\ \sqrt{R^2-\ell^2}$

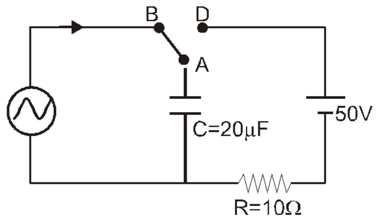
Ans. **(C)**

Sol. $\int E d\ell = \varepsilon$, $E = \frac{r}{2}\frac{dB}{dt}$, $E\cos\theta = \frac{r\cos\theta}{2} B_0 = \frac{h}{2} B_0$

$V_Q - V_P = \left(\frac{h}{2}B_0\right)2\ell = B_0\ \ell\ \sqrt{R^2-\ell^2}$

MCQ (One or more than one correct) :

3. At time t = 0, terminal A in the circuit shown in the figure is connected to B by a key and alternating current $I(t) = I_0\cos(\omega t,)$, with I_0 = 1A and ω = 500 rad s^{-1} starts flowing in it with the initial direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If C = 20μ, R = 10Ω and the battery is ideal with emf of 50V, identify the correct statement (s)



(A) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is 1×10^{-3} C.

(B) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise

(C) Immediately after A is connected to D. the current in R is 10A.

(D) $Q = 2 \times 10^{-3}$ C.

Ans. **(CD)**

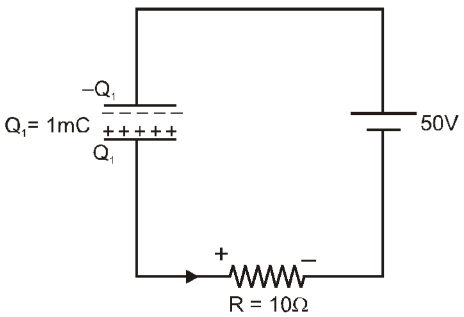
Sol. Charge on capacitor will be maximum at $t = \frac{\pi}{2\omega}$

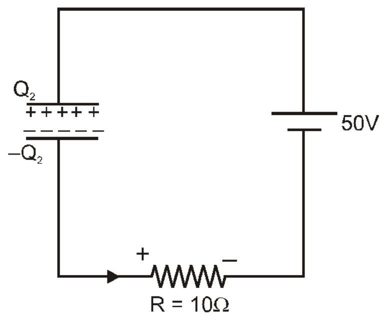
$Q_{max} = 2 \times 10^{-3}$ C

(A) charge supplied by source from t = 0 to $t = \frac{7\pi}{6\omega}$

$$Q = \int_0^{\frac{7\pi}{6\omega}} \cos(500t)dt = \left[\frac{\sin 500t}{500}\right]_0^{\frac{7\pi}{6\omega}} = \frac{\sin\frac{7\pi}{6}}{500} = -1\text{mC}$$

Just after switching **In steady state**





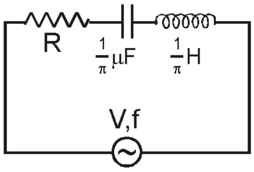
Apply KVL just after switching

$$50 + \frac{Q_1}{C} - IR = 0 \Rightarrow \quad I = 10\text{ A}$$

In steady state Q_2 = 1mC

net charge flown from battery = 2mC

4. In the AC circuit shown below, the supply voltage has constant rms value V but variable frequency f. At resonance, the circuit :



(A) has a current I given by $I = \frac{V}{R}$

(B) has a resonance frequency 500 Hz

(C) has a voltage across the capacitor which is 180^0 out of phase with that across the inductor

(D) has a current given by $I = \frac{V}{\sqrt{R^2 + \left(\frac{1}{\pi} + \frac{1}{\pi}\right)^2}}$

Ans. **(ABC)**

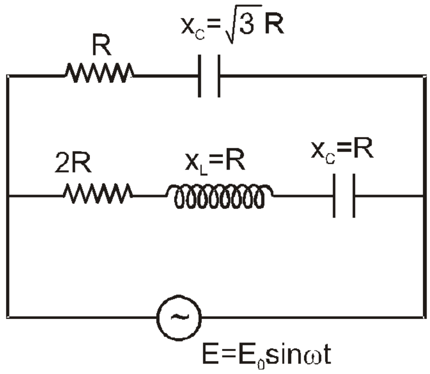
Sol. Resonance frequency $f = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$ = 500 Hz

At resonance

$Z = R$ & $I = \frac{V}{z} = \frac{V}{R}$

L & C are in out of phase.

5. Choose the correct options for the given arrangement (Symbols have their usual meaning)



(A) Current from the AC source is $\frac{\pi}{6}$ phase ahead than voltage.

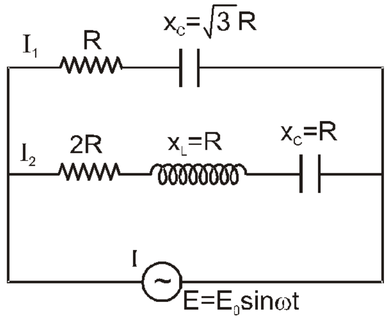
(B) Impendence of the circuit is $\frac{2R}{\sqrt{3}}$

(C) Current from resistor R as function of time is $\frac{E_0}{2R}\sin\left(\omega t + \frac{\pi}{3}\right)$

(D) Current from resistor 2R as function of time is $\frac{E_0}{2R}\sin\omega t$

Ans. **(ABCD)**

Sol.

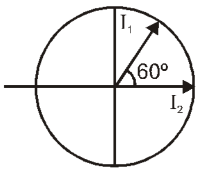


$I_1 = \frac{E_0}{2R}\sin\left(\omega t + \frac{\pi}{3}\right)$

$I_2 = \frac{E_0}{2R}\sin\omega t$

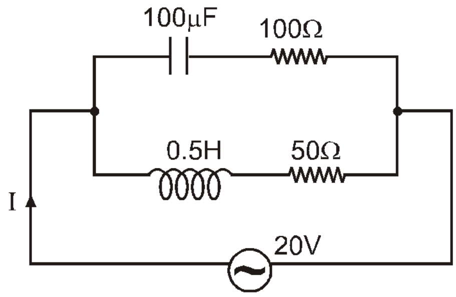
$I = I_1 + I_2$

$= 2\left(\frac{E_0}{2R}\cos 30^0\right)\sin\left(\omega t + \frac{\pi}{6}\right)$



$\frac{\sqrt{3}E_0}{2R}\sin\left(\omega t + \frac{\pi}{6}\right)$

6. In the given circuit, the AC source has ω = 100 rad/s. considering the inductor and capacitor to be ideal, the correct choice (s) is(are)



(A) The current through the circuit, I is approximately 0.3 A

(B) The current through the circuit, I is $0.3\sqrt{2}$ A.

(C) The voltage across 100Ω resistor = $10\sqrt{2}$ V

(D) The voltage across 50Ω resistor = 10V

Ans. **(AC)**

Since $I_{rms} = \frac{1}{\sqrt{10}} \approx$ 0.3 A so A may or may not be correct.

Sol. C = 100 μF, $\frac{1}{\omega C} = \frac{1}{(100)\ \ (100\times10^{-6})}$

X_C = 100 Ω, X_L = ωL = (100) (.5) = 50 Ω

$Z_1 = \sqrt{x_C^2 + 100^2} = 100\sqrt{2}\Omega$

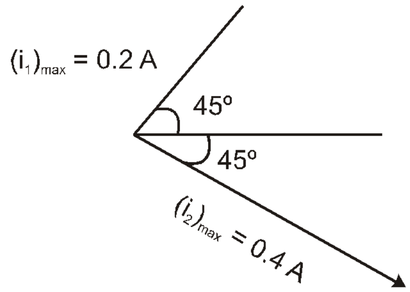
$Z_2 = \sqrt{x_L^2 + 50^2} = \sqrt{50^2 + 50^2} = 50\sqrt{2}$

$\varepsilon = 20\sqrt{2}\sin\omega t$

$i_1 = \frac{20\sqrt{2}}{100\sqrt{2}}\sin(\omega t + \pi/4)$

$i_1 = \frac{1}{5}\sin(\omega t + \pi/4)$

$I_2 = \frac{20\sqrt{2}}{50\sqrt{2}}\sin(\omega t - \pi/4)$



$I = \sqrt{(.2)^2 + (.4)^2}$

$= (.2)\ \sqrt{1+4}$

$= \frac{1}{5}\sqrt{5} = \frac{1}{\sqrt{5}}$

$(I)_{rms} = \frac{1}{\sqrt{2}\sqrt{5}} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

$\approx 0.3A$

$(V_{100\Omega})_{rms} = (I_1)_{rms}) \times 100 = \left(\frac{0.2}{\sqrt{2}}\right) \times 100 = \frac{20}{\sqrt{2}} = 10\sqrt{2}$ V

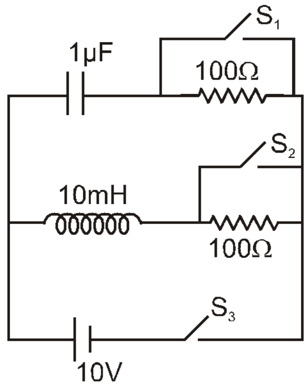
$V_{50\Omega})_{rms} = \left(\frac{0.4}{\sqrt{2}}\right) \times 50 = \frac{20}{\sqrt{2}} = 10\sqrt{2}$ V

Since $I_{rms} = \frac{1}{\sqrt{10}} \approx 0.3$ A so A may or may not be correct.

Comprehension Type Question:

Comprehension # 1

Switches S_1 and S_2 are open while S_3 remains closed for long time such that capacitor becomes fully charged and current in inductor coil becomes maximum. Now switches S_1 and S_2 are closed and at the same time S_3 is opened at t = 0. Assume that battery and inductor coil are ideal.



7. Charge (in μC) on capacitor as a function of time is :

(A) $q = 10\sqrt{2}\ \sin\left(10^4 t(\text{sec.}^{-1}) + \frac{\pi}{4}\right)$ (B) $q = 10 \sin\left(10^4 t(\text{sec.}^{-1}) + \frac{\pi}{2}\right)$

(C) $q = 10 \sin\left(10^4 t(\text{sec.}^{-1}) + \frac{\pi}{4}\right)$ (D) $q = 10\sqrt{2} \sin\left(10^2 t(\text{sec.}^{-1}) + \frac{\pi}{4}\right)$

Ans. (A)

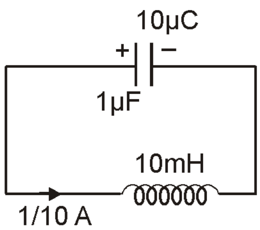
8. Maximum current in the inductor coil at any time t > 0 is :

(A) $\frac{1}{10}$ A (B) $\frac{1}{5\sqrt{2}}$ A (C) $\frac{1}{5}$ A (D) 1 A

Ans. (B)

Sol. (7 to 8)

After closing S_1, S_2 and S_3 :



At t = 0

At any Time t :

$$\frac{d^2q}{dt^2} = -\omega^2 q$$

$\Rightarrow$ $q = Q_{max} \sin(\omega t + \phi)$

$$\frac{1}{2}(10\text{ mH})\left(\frac{1}{10}\right)^2 + \frac{(10\mu C)^2}{2.(1\mu F)} = \frac{Q_{max}^2}{2.(1\mu F)} = \frac{1}{2}(10\text{mH})\, I_{max}{}^2$$

$\Rightarrow$ $Q_{max} = 10\sqrt{2}\,\mu C$

$I_{max} = \frac{1}{5\sqrt{2}} A$

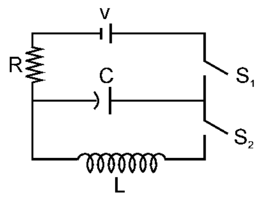
At t = 0, q = 10 μC so $\phi = \frac{\pi}{4}$

Also $\frac{di}{dt} = q$

so $\frac{di}{dt}$ will become maximum when charge becomes maximum.

Comprehension # 2

The capacitor of capacitance C can be charged (with the help of a resistance R) by a voltage source V, by closing switch S_1 while keeping switch S_2 open. The capacitor can be connected in series with an inductor 'L' by closing switch S_2 and opening S_1.



9. Initially, the capacitor was uncharged. Now, switch S_1 is closed and S_2 is kept open. If time constant of this circuit is τ, then

(A) after time interval τ, charge on the capacitor is CV/2

(B) after time interval 2τ, charge on the capacitor is $CV(1 - e^{-2})$

(C) the work done by the voltage source will be half of the heat dissipated when the capacitor is fully charged

(D) after time interval 2τ, charge on the capacitor is $CV(1 - e^{-1})$

Ans. **(B)**

Sol. $Q = Q_0(1 - e^{-t/\tau})$

$Q = CV(1 - e^{-t/\tau})$ after time interval 2τ.

10. After the capacitor gets fully charged, S_1 is opened and S_2 is closed so that the inductor is connected in series with the capacitor. Then,

(A) at t = 0, energy stored in the circuit is purely in the form of magnetic energy

(B) at any time t > 0, current in the circuit is in the same direction

(C) at t > 0, there is no exchange of energy between the inductor and capacitor

(D) at any time t > 0, instantaneous current in the circuit will have maximum value $V\sqrt{\frac{C}{L}}$, where C is the capacitance and L is the inductance.

Ans. **(D)**

Sol. $q = Q_0 \cos \omega t$

$i = -\frac{dq}{dt} = Q_0 \omega \sin \omega t \quad \Rightarrow \quad i_{max} = C\omega V = V\sqrt{\frac{C}{L}}$

11. If the total charge stored in the LC circuit is Q_0, then for $t \geq 0$

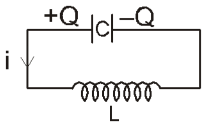
(A) the charge on the capacitor is $Q = Q_0 \cos\left(\frac{\pi}{2} + \frac{t}{\sqrt{LC}}\right)$

(B) the charge on the capacitor is $Q = Q_0 \cos\left(\frac{\pi}{2} - \frac{t}{\sqrt{LC}}\right)$

(C) the charge on the capacitor is $Q = -LC\frac{d^2Q}{dt^2}$

(D) the charge on the capacitor is $Q = -\frac{1}{\sqrt{LC}}\frac{d^2Q}{dt^2}$

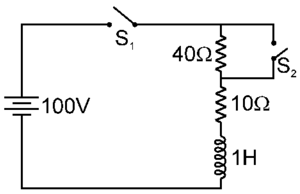
Ans. **(C)**

Sol. 

$L\frac{di}{dt} - \frac{Q}{C} = 0, \quad -L\frac{d^2Q}{dt^2} - \frac{Q}{C} = 0, \quad Q = -LC\frac{d^2Q}{dt^2}$

Numerical based Questions :

12. In the circuit diagram shown in the figure the switches S_1 and S_2 are closed at time t = 0. After time t = (0.1) ℓn 2 sec, switch S_2 is opened. The current in the circuit at time, t = (0.2) l n 2 sec is equal to $\frac{x}{32}$ amp. Findout value of x.



Ans. 67

Sol. $i = i_0 [1 - e^{-tR/\ell}]$

when S_1 and S_2 are closed then at t = 0.1 ℓn 2

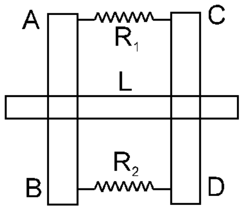
$i = \frac{100}{10}\left[1 - e^{-\frac{10\times0.1\ell n2}{1}}\right] \quad \Rightarrow \quad i = 10\left[1 - \frac{1}{2}\right] = 5A.$

when S_2 is opened after that at t = 0.2ℓn2 – 0.1ℓn2

$i' = \frac{100}{50}\left[1 - e^{-\frac{50\times0.1}{1}\ell n2}\right] + 5e^{\frac{-50\times0.1\ \ell n\ 2}{1}}$

$i' = 2 \times \left[1 - \frac{1}{32}\right] + \frac{5}{32} \quad \Rightarrow \quad i' = 2\times\frac{31}{32} + \frac{5}{32} = \frac{67}{32}A$

13. Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at the two ends by resistance R_1 and R_2 as shown in the figure. A horizontal metallic bar L of mass 0.2 kg slides without friction, vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6T perpendicular to the plane of the rails. It is observed that when the terminal velocity is attained, the power dissipated in R_1 and R_2 are 0.76 W and 1.2 W respectively. If the terminal velocity of bar L is x m/s and R_1 is y Ω and R_2 is ZΩ then find the value of x + 76y + 10z. (g = 9.8 m/s²)



Ans. 40

Sol. For terminal velocity,

Mg = ILB

here $I = \frac{\varepsilon}{R_{eq}} = \frac{BV_0L}{R_{eq}}$

$$Mg = \frac{B^2L^2V_0}{R_1R_2/R_1+R_2}$$

$$V_0 = Mg.\ \frac{R_1R_2}{R_1+R_2}\ \frac{1}{B^2L^2} \qquad \text{............(i)}$$

Given that

$I_1^2\ R_1 = 0.76$ ......(ii)

& $I_2^2\ R_2^2 = 1.2$ ......(iiii)

where $I_1 = \frac{\varepsilon}{R_1}$ and $I_2 = \frac{\varepsilon}{R_2}$

Solve (i), (ii) and (iii)

method II (Better sol.)

power of gravitational force = $P_1 + P_2$

mg V_T = 0.76 + 1.20

So, V_T = 1 m/s

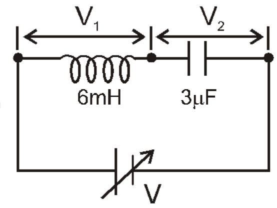
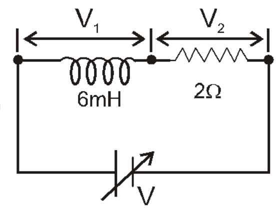
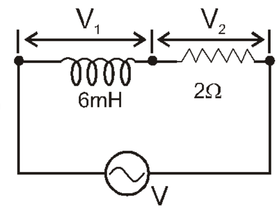
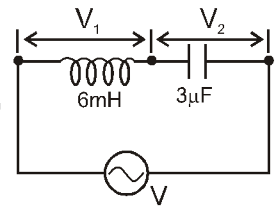
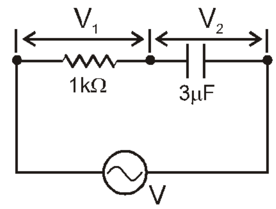
$\varepsilon = BV_T\ell = 0.6$ volt $\quad P_1 = \frac{\varepsilon^2}{R_1} \quad \therefore R_1 = \frac{\varepsilon^2}{P_1} = \frac{(0.6)^2}{0.76}$

$R_1 = \frac{36}{76}\ \Omega \quad$ similarly $R_2 = \frac{0.36}{1.20} = \frac{3}{10}\Omega$

$$\frac{R_1}{R_2} = \frac{30}{19}$$

Matrix Match Type :

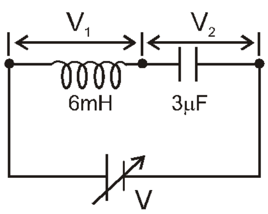
14. You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in **Column II.** When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2. (indicated in circuits) are related as shown in **Column I.** Match the two column.

Column I		**Column II**
(A) $I \neq 0, V_1$ is proportional to I	(p)	
(B) $I \neq 0, V_2 > V_1$	(q)	
(C) $V_1 = 0$, $V_2 = V$	(r)	
(D) $I \neq 0, V_2$ is proportional to I	(s)	
	(t)	

Ans. **(A) – r,s,t ; (B) – q,r,s,t ; (C) – p,q ; (D) – q,r,s,t**

As per given conditions, there will be no steady state in circuit ‘p’, so it should not be considered in options of ‘c’.

Sol. (p)



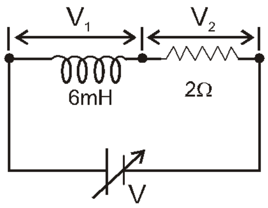
As I is steady state current

$V_1 = 0$; $I = 0$

Hence, $V_2 = V$

So , answer of P $\Rightarrow$ C

(q)



In the steady state ;

$V_1 = 0$ as $\frac{dI}{dt} = 0$

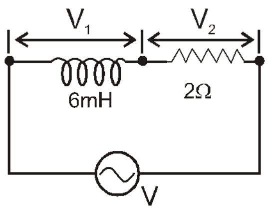
$\therefore$ $V_2 = V = IR$

or $V_2 \propto I$

and $V_2 > V_1$

So , answer of q $\Rightarrow$ B, C, D

(r)



Inductive reactance $X_L = \omega L$

$X_L = 6\pi \times 10^{-1}\ \Omega$

and resistance $= R = 2\Omega$

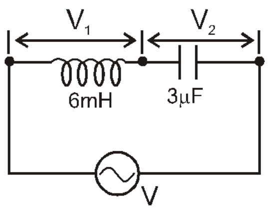
So, $V_1 = IX_L$

and $V_2 = IR$

Hence, $V_2 > V_1$

So, Answer of r $\Rightarrow$ A,B,D

(s)



Here, $V_1 = IX_L$, where, $X_L = 6\pi \times 10^{-1}\ \Omega$

Also, $V_2 = IX_C$, where, $X_C = \frac{10^4}{3\pi}$

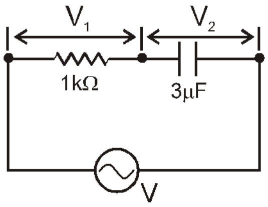
So, $V_2 > V_1$

$V_1 \propto I$

$V_2 \propto I$

So, answer of s $\Rightarrow$ A,B,D

(t)



Here, $V_1 = IR$, where, R = 1000 Ω , $X_C = \frac{10^4}{3\pi}\ \Omega$

$V_2 = IX_C$, where, $X_C = \frac{10^4}{3\pi}\Omega$

So, $V_2 > V_1$

and $V_1 \propto I$

$V_2 \propto I$

So, answer of t $\Rightarrow$ A,B,D

Ans. (A) – r,s,t ; (B) – q,r,s,t ; (C) – p,q ; (D) – q,r,s,t

Note : For circuit 'p' :

$V - \frac{Ldi}{dt} - \frac{q}{C} = 0$ or $CV = CL\frac{di}{dt} + q$ or $0 = LC\frac{d^2i}{dt^2} + \frac{dq}{dt}$ or $\frac{d^2i}{dt^2} = -\frac{1}{LC}\frac{dq}{dt}$

So, $i = i_0 \sin\left(\frac{1}{\sqrt{LC}}t + \phi_0\right)$

As per given conditions, there will be no steady state in circuit 'p'. So it should not be considered in options

Subjective Type Questions :

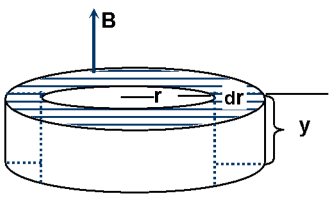
15. A metal disc of radius R, resistivity ρ, thickness y is placed in a vertical magnetic field of induction $B = B_0 \sin \omega t$ where $\omega = 2\pi f$. Find the total eddy current loss in the metal disc.

Ans. $P = \frac{\pi^3 B_0^2 f^2 R^4 y}{4\rho}$

Sol. The changing magnetic field induces an emf in each segment of the disc. Consequently the induced current in the annular ring is given as

$I = \frac{E}{R_0}$; E = induced emf and

R_0 = resistance of the ring.



$\Rightarrow$ The power loss, due to eddy (Induced) current in the ring

$dP = i^2 R_0$

$\Rightarrow \quad dP = \frac{E^2}{R_0}$; where $E = \frac{-d\ [BA]}{dt} = -A\frac{d}{dt}[B_0 \sin \omega t]$

$-\pi r^2 B_0 \omega \cos \omega t$

$\Rightarrow \quad dP = \frac{\pi^2 r^4 B_0^2 \omega^2 \cos^2 \omega t}{R_0}$; Putting $R_0 = \left(\frac{\rho 2\pi r}{y\ dr}\right)$

We obtain $dP = \frac{\pi^2 r^4 B_0^2 \omega^2 \cos^2 \omega t}{(2\rho\pi r / y\, dr)} = \frac{\pi y B_0^2 \omega^2 r^3 dr\ \cos^2 \omega t}{2\rho}$

$\Rightarrow$ The total power loss in the disc

$P = \frac{\pi y B_0^2 \omega^2 \cos^2 \omega t}{2\rho} \int_0^R r^3 dr$

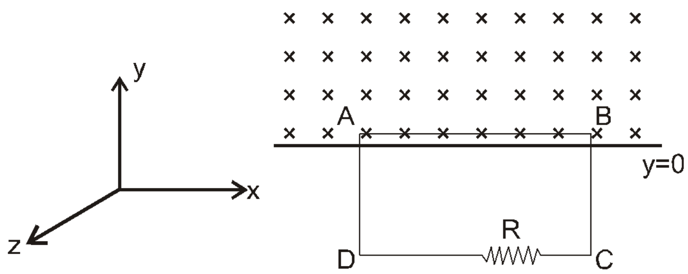
$\Rightarrow \quad P = \frac{\pi y B_0^2 \omega^2 R^4}{8\rho} \cos^2 \omega t$

Since $\frac{1}{T}\int_0^T \cos^2 \omega t.dt = \frac{1}{2}$

where $T = \frac{2\pi}{\omega}$, $P = \frac{\pi y B_0^2 \omega^2 R^4}{16\rho}$

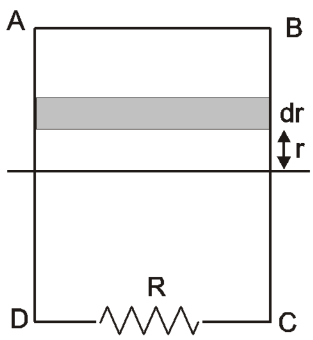
Putting $\omega = 2\pi f$, $P = \frac{\pi^3 B_0^2 f^2 R^4 y}{4\rho}$

16. A square loop ABCD of side ℓ is moving in xy plane with velocity $\vec{v} = \beta t\ \hat{j}$. There exists a non-uniform magnetic field $\vec{B} = -B_0(1 + \alpha y^2)\ \hat{k}$ $(y > 0)$, where B_0 and α are positive constants. Initially, the upper wire of the loop is at y = 0. Find the induced voltage across the resistance R as a function of time. Neglect the magnetic force due to induced current.



Ans. $\varepsilon = -B_0 I\beta\left(t + \frac{\alpha \times \beta^2 t^5}{4}\right)$

Sol. Flux inside the strip = B. l. dr = $B_0(1 + \alpha r^2)$. ldr



Total flux in the loop = $\int_0^y B_0(1+\alpha r^2)\ \ l dr$ $\qquad \phi = B_0 l.\left(y+\frac{\alpha y^3}{3}\right)$

Induced emf $\varepsilon = \frac{-d\phi}{dt} = -B_0 l \left(1+\frac{3\alpha y^2}{3}\right)\frac{dy}{dt} = -\ B_0 l\ \left(1+\alpha y^2\right)\frac{dy}{dt}$

Given $v = \beta t$

$$\frac{dy}{dt} = \beta t \qquad \Rightarrow \qquad \int_0^y dy = \int_0^t \beta t dt \qquad \Rightarrow \qquad y = \frac{\beta t^2}{2}$$

$$\therefore \qquad \varepsilon = -\ B_0 l \left(1+\frac{\alpha \times \beta^2 t^4}{4}\right)\beta t \qquad \Rightarrow \qquad \varepsilon = -\ B_0 l \beta \left(t+\frac{\alpha\times\beta^2 t^4}{4}\right)$$

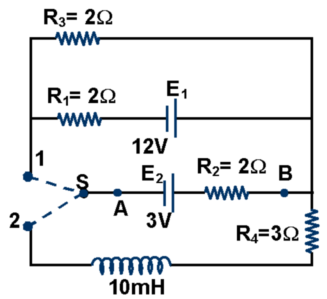
17. A circuit containing a two position switch S is shown below.

(a) The switch S is in position 1. Find the potential difference $V_A - V_B$ and rate of production of heat in R_1.

(b) If now the switch S is put in position 2 at t = 0, find

(i) steady current in R_4.

(ii) The time when the current in R_4 is half the steady value. Also, calculate the energy stored in the inductor L at that time.



Sol. (a) When the switch is in position 1, the equivalent circuit is shown in the figure. No current flows through the inductor.

Using Kirchoff's 2nd law to the mesh CDFEC,

$2I_1 + 2(I_1 + I_2) = 12$

$2I_1 + I_2 = 6$ . . . (1)

For the mesh EFGHE

$2I_2 + 2(I_1 + I_2) = 12 - 3 = 9$

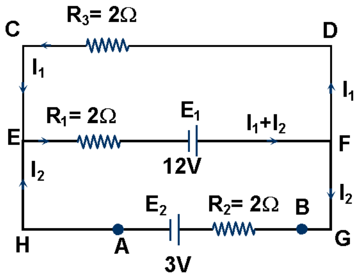
$\Rightarrow \quad 2I_1 + 4I_2 = 9$ . . . (2)

From equation (1) and (2), we get,

$I_1 = 2.5$ A, $I_2 = 1.0$ A

$V_B - V_A = 2 \times 1 + 3 = 5$ V

$\Rightarrow \quad V_A - V_B = -5$ V



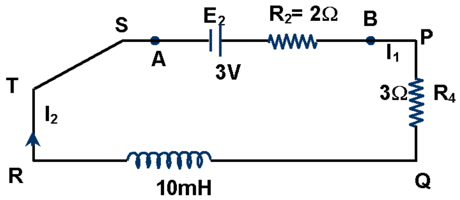
Rate of heat produced in $R_1 = (I_1 + I_2)^2 R_1$

$= (3.5)^2 \times 2 = 24.5$ W

(b) When the switch S is in position 2, the equivalent circuit is shown in the figure.

(i) Steady current in R_4,

$E_2/(R_2 + R_4) = 3/2+3 = 0.6$ A



(ii) Growth of current in an L-R circuit is given by

$I = I_o(I - e^{-Rt./L})$

When current reaches half of its steady value,

$I/I_o = ½ = 1 - e^{-Rt/L}$

$\Rightarrow \quad e^{-\frac{Rt}{L}} = \frac{1}{2}$

$\Rightarrow \quad \frac{Rt}{L} = 2.303 \log 2 = 0.639$

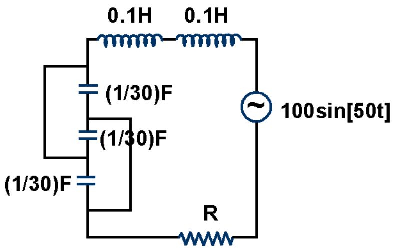
$\therefore \quad t = 0.639 \frac{L}{R} = \frac{0.639 \times 10 \times 10^{-3}}{5}$ s

$\therefore \quad t = 1.386 \times 10^{-3}$ s

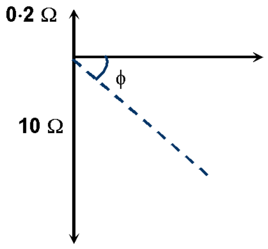
Thus, current increases to one half of its final steady value in 1.386×10^{-3} s. The energy stored when the current in the circuit attains the value $I_o/2$ (0.3A) is given by

$$E = \frac{1}{2} LI^2 = \frac{1}{2} \times 10 \times 10^{-3} \times (0.3)^2 J = 4.5 \times 10^{-4} J$$

18. Find the value of the resistance R so that the power factor of the given circuit is $\frac{1}{\sqrt{2}}$. Also find the peak current in this case.



Sol. C′ = 0.1F capacitor are in parallel.



$$\frac{1}{\omega C'} = \frac{1}{50 \times 0 \cdot 1} = 0.2\Omega$$

$$\omega L' = 50 \times 0 \cdot 2 = 10\Omega$$

$\Rightarrow \quad \cos\phi = \frac{1}{\sqrt{2}}$

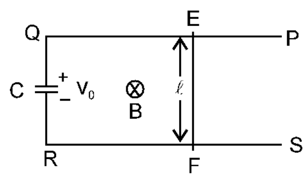
$\Rightarrow \quad \phi = 45°$

$\therefore \quad R = 10\Omega - 0.2\Omega = 9.8\ \Omega$

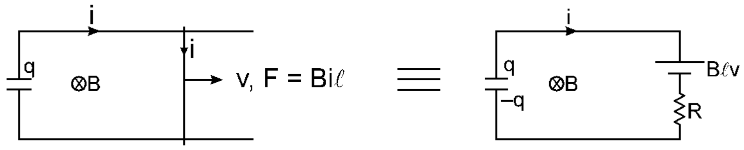
$\Rightarrow \quad Z = \sqrt{9 \cdot 8^2 + 9 \cdot 8^2} = 9 \cdot 8\sqrt{2}\Omega$.

$\therefore \quad$ Peak current $= \frac{100}{9 \cdot 8 \times \sqrt{2}} = 7 \cdot 22$ Amperes.

19. In the figure shown 'PQRS' is a fixed resistanceless conducting frame in a uniform and constant magnetic field of strength B. A rod 'EF' of mass 'm', length 'ℓ' and resistance R can smoothly move on this frame. A capacitor charged to a potential difference 'V_0' initially is connected as shown in the figure. Find the velocity of the rod as function of time 't' if it is released at t = 0 from rest.



Sol. Due to charged capacitor current will flow in the rod in downward direction. Hence the rod will experience magnetic force towards right. Then an emf (motional) will be induced in the rod. Let the charge on capacitor and speed of rod at any time t be q and v respectively.



Applying loop law we get

$$\frac{q}{C} - iR - B\ell v = 0 \qquad ...(1)$$

The force on rod is

$$F = m\frac{dv}{dt} = Bi\ell \qquad ...(2)$$

differentiating equation (1) w.r.t. time t we get

$$-\frac{i}{C} + R\,\frac{di}{dt} - B\ell\frac{dv}{dt} = 0 \qquad ...(3)$$

from equation (2) and (3) we get

$$\frac{di}{dt} = \left[\frac{B^2\ell^2}{mR} + \frac{1}{RC}\right] i \qquad \Rightarrow \qquad = \frac{di}{i} - Kdt \qquad ....(4)$$

where $K = -\frac{B^2\ell^2C + m}{mRC}$

at t = 0 sec, q = CV_0 and v = 0

$\therefore$ from equation (1) the current at t = 0 is $i_0 = \frac{V_0}{R}$.

integrating equation (4)

$$\int_{V_0/R}^{i} \frac{di}{i} = -K\int_0^t dt$$

we get

$$i = \frac{V_0}{R}\, e^{-Kt}$$

from equation (2)

$$dv = \frac{B\ell}{m}\, i\,dt \qquad \text{or} \qquad dv = \frac{B\ell V_0}{mR}\, e^{-Kt}\,dt$$

integrating the equation

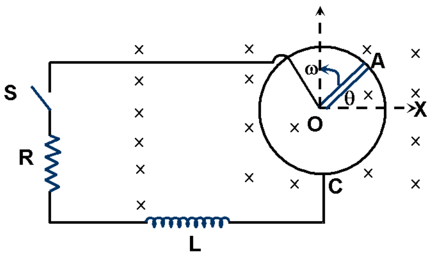
$$\int_0^v dv = \int_0^t \frac{B\ell}{m}\frac{V_0}{R}\, e^{-Kt}\,dt$$

$$\Rightarrow \quad v = \frac{B\ell V_0}{mR}\left(-\frac{1}{K}\right)[e^{-Kt} - e^0] = \frac{B\ell V_0}{mRK}[1 - e^{-Kt}]$$

By substituting 'K' we get

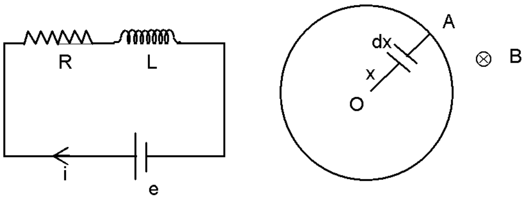
$$v = \frac{B\ell C\;V_0}{m + B^2\ell^2C}\left(1 - e^{-\left(\frac{B^2\ell^2}{mR} + \frac{1}{RC}\right)t}\right)$$

20. A metal rod OA of mass m and length r is kept rotating with a constant angular speed ω in a vertical plane about a horizontal axis at the end O. The free end A is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform and constant magnetic induction $\vec{B}$ is applied perpendicular and into the plane of rotation as shown in figure. An inductor L and an external resistance R are connected through a switch S between the point O and a point C on the ring to form an electrical circuit. Neglect the resistance of the ring and the rod. Initially, the switch is open. What is the induced emf across the terminals of the switch?



Sol. (a) de = B (xω) dx

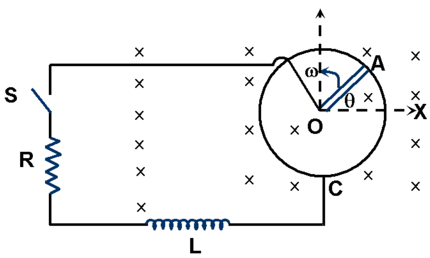
$\Rightarrow$ e = $\dfrac{B\omega r^2}{2}$



21. In the previous question the switch S is closed at time t = 0.

(i) Obtain an expression for the current as a function of time.

(ii) In the steady state, obtain the time dependence of the torque required to maintain the constant angular speed. Given that the rod OA was along the positive X–axis at t = 0.



Sol. (b) (i) $i = i_0 (1 - e^{-Rt/L})$

$\Rightarrow$ i_0 = e/R

$\Rightarrow$ $i = \dfrac{B\omega r^2}{2R}\left(1 - e^{-Rt/L}\right)$

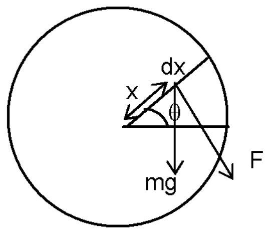
(ii) In steady state, $i = i_0$

Net torque τ = mg (r/2) cos θ + $\int$ Fx

$\tau = \dfrac{mgr\cos\theta}{2} + \int_0^r i_0 B.xdx$

$$= \frac{mgr\cos\theta}{2} + \int_0^r \frac{B\omega r^2}{2R}.Bxdx$$

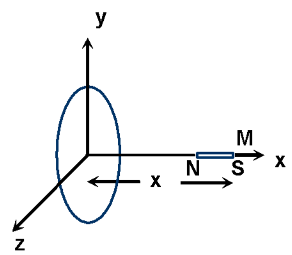
$$\tau = \frac{mgr\cos\omega t}{2} + \frac{1}{4R}B^2\omega r^4$$



22. An infinitesimal bar magnet of dipole moment M is pointing and moving with speed v in the x-direction. A closed circular conducting loop of radius a and negligible self inductance lies in the Y-Z plane with its center at x = 0 and its axis coinciding with x-axis. Find the force opposing the motion of the magnet, if the resistance of the loop is R. Assuming $x \gg R$.

Sol. A bar magnet can be thought of as a current carrying coil of dipole moment M. The magnetic field of a current carrying coil at a distance x on its axis ($x \gg$ radius of the coil) can be given as

$$B = \frac{\mu_0 (2M)}{4\pi x^3} = \frac{\mu_0 M}{2\pi x^3}$$



⇒ The flux linked with the given loop = ϕ = B.(area of the loop)

⇒ $$\phi = B(\pi a^2) = \frac{\mu_0 M a^2}{2x^3}$$

Emf induced in the loop

$$E = -\frac{d\phi}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 M a^2}{2x^3}\right]$$

⇒ $$E = \frac{3\mu_0 M a^2}{2x^4}\frac{dx}{dt} = \frac{3\mu_0 M a^2}{2x^4}v$$

The induced current in the loop

$$I = \frac{E}{R} = \frac{3\mu_0 M a^2 v}{2Rx^4}$$

The induced dipole moment of the loop = $M' = I(\pi a^2)$

⇒ $$M' = \left(\frac{3\mu_0 M a^2 v}{2Rx^4}\right)(\pi a^2)$$

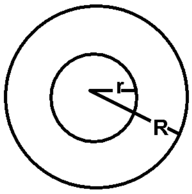
The force of interaction between two dipoles of dipole moment pointing along the line joining their centres can be written as

$$F = \frac{3\mu_0 MM'}{2\pi x^4}$$; where x = separation between them.

Putting the value of M′ we obtain

$$F = \frac{9\mu_0^2 M^2 a^4}{4x^8 R} v$$

23. Find the magnetic energy stored in the system of two concentric loops of radii r and R (R>>r). Find the mutual inductance of the system. If the loops have currents of same magnitude i_0, find the magnetic energy stored. Ignore the effect of self inductance

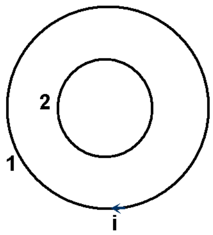


Sol. Let a current i be set up in the loop 1. It develops a magnetic field of induction $B_1 = \frac{\mu_0 i}{2R}$ at its center. The flux linked by the (smaller) loop 2 = $\phi_{12} = B_1(\pi r^2)$

$\Rightarrow \quad \phi_{12} = \pi B_1 r^2$

Putting the value of $B_1 = \frac{\mu_0 i}{2R}$ we obtain mutual inductance

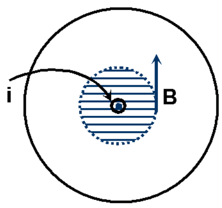
$$M = \frac{\phi_{12}}{i} = \frac{\pi\mu_0 r^2}{2R}$$



Ignoring the self inductance of the loop, the magnetic energy stored, $U = MI_1I_2$

$$U = \frac{\pi\mu_0 r^2 i_0^2}{2R} \qquad (\because i_1 = i_2 = i_0)$$

24. A long solid cylindrical straight conductor carries a current i_0. Find the magnetic energy stored per meter inside it.



Sol. The current enclosed within the shaded circle

$$I = \frac{i_0}{\pi R^2}\pi r^2 = \frac{i_0 r^2}{R^2}$$

Applying Ampere's law for the dotted loop we obtain

$$\oint B \cdot d\ell = \mu_0 I$$

$\Rightarrow \quad B(2\pi r) = \mu_0 \left(\frac{i_0 r^2}{R^2}\right) \quad \Rightarrow \quad B = \frac{\mu_0 i_0 r}{2\pi R^2}$

Energy stored in the elementary volume dV (= $2\pi r\ell dr$) is given as,

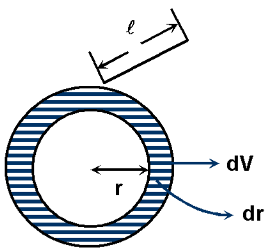
$$dU = \frac{B^2}{2\mu_0} dV$$

$\Rightarrow$ The total energy stored in the conductor

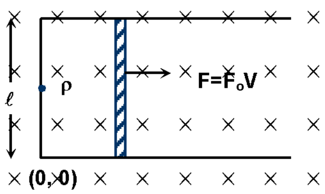
$$U = \int \frac{B^2}{2\mu_0} dV = \frac{1}{2\mu_0}\int_0^R \left(\frac{\mu_0 i_0 r}{2\pi R^2}\right)^2 (2\pi r\ell dr)$$

$$\Rightarrow \quad U = \frac{\mu_0 i_0^2 \ell}{4\pi \; R^4}\int_0^R r^3 dr$$

$$\Rightarrow \quad U = \frac{\mu_0 i_0^2 \ell}{4\pi} \times \frac{1}{4} = \frac{\mu_0 i_0^2}{16\pi} \quad (\because \ell = 1)$$



25. Two infinite parallel wires, having the cross sectional area a and resistivity k are connected at a junction point P (as shown in the figure). A slide wire of negligible resistance and having mass 'm' and length 'ℓ' can slide between the parallel wires, without any frictional resistance.



If the system of wires is introduced to a magnetic field of intensity B (into the plane of paper) and the slide wire is pulled with a force which varies with the velocity of the slide wire as F = F_0V, then find the velocity of the slide wire as a function of the distance travelled. (The slide wire is initially at origin and has a velocity v_0)

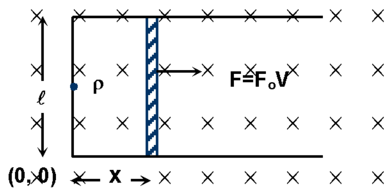
Sol. At any given instance of time the slide wire is at distance x from origin, then the resistance of the circuit is R = $\frac{k(2x+\ell)}{a}$. If the velocity of slide wire is V, then the emf generated is $B\ell V$ so we have

$$B\ell V - \frac{k}{a}(2x + \ell)I = 0$$

or, $\quad I = \frac{B\ell a}{k}\left(\frac{V}{2x+\ell}\right)$

This current exerts magnetic force in the wire given by F′

So, $F' = I\ell B = \frac{B\ell a}{k}\left(\frac{V}{2x+\ell}\right)\ell B = \frac{B^2\ell^2 a}{k}\left(\frac{V}{2x+\ell}\right)$



Since, $F - F' = \frac{mdV}{dt}$

So $$F_0 V - \frac{B^2\ell^2 a}{k}\left(\frac{V}{2x+\ell}\right) = mV\frac{dV}{dx}$$

$$\frac{F_0}{m} - \frac{B^2\ell^2 a}{km}\left(\frac{1}{2x+\ell}\right) = \frac{dV}{dx}$$

$$\int_0^x \left[\frac{F_0}{m} - \frac{B^2\ell^2 a}{km}\left(\frac{1}{2x+\ell}\right)\right]dx = \int_{v_0}^{v} dv$$

$$\frac{F_0}{m}x - \frac{B^2\ell^2 a}{2km}\left\{\ln\left(\frac{2x+\ell}{\ell}\right)\right\} + v_o = V$$