competishun — Power of real gurus

PHYSICS

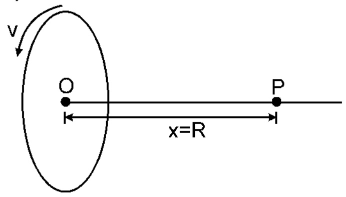
TARGET : JEE- Advanced 2021

CAPS-14

EMF

SCQ (Single Correct Type) :

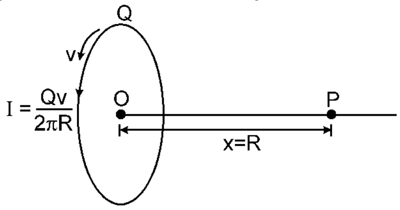
1. A uniformly charged ring of radius R is rotated about its axis with constant linear speed v of each of its particles. The ratio of electric field to magnetic field at a point P on the axis of the ring distant x = R from centre of ring is (c is speed of light)



(A) $\frac{c^2}{v}$ (B) $\frac{v^2}{c}$ (C) $\frac{c}{v}$ (D) $\frac{v}{c}$

Ans. **(A)**

Sol. Let Q be the charge on the ring. The electric field at point P is



$$\therefore \quad E = \frac{1}{4\pi \epsilon_0} \frac{Qx}{(x^2 + R^2)^{3/2}} = \frac{1}{4\pi \epsilon_0} \frac{QR}{(2R^2)^{3/2}}$$

The rotating charged (Q) ring is equivalent to a ring in which current I flows, such that

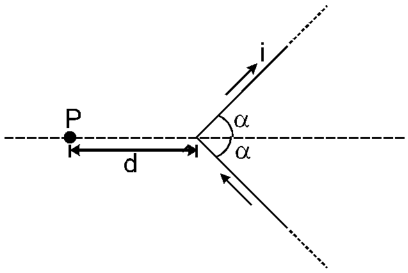
$$I = \frac{Qv}{2\pi R}$$

The magnetic field at point P is

$$B = \frac{\mu_0}{4\pi} \frac{2\pi I R^2}{(x^2 + R^2)^{3/2}} = \frac{\mu_0}{4\pi} \frac{QvR}{(2R^2)^{3/2}}$$

$$\therefore \quad \frac{E}{B} = \frac{1}{\mu_0 \epsilon_0 v} = \frac{c^2}{v}$$

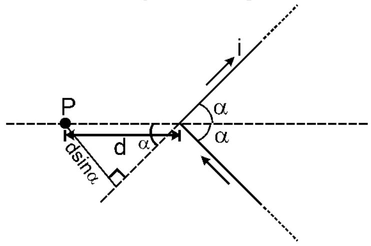
2. If the magnetic field at 'P' can be written as K tan $\left(\frac{\alpha}{2}\right)$ then K is :



(A) $\frac{\mu_0 I}{4\pi d}$ (B) $\frac{\mu_0 I}{2\pi d}$ (C) $\frac{\mu_0 I}{\pi d}$ (D) $\frac{2\mu_0 I}{\pi d}$

Ans. **(B)**

Sol. Let us compute the magnetic field due to any one segment :



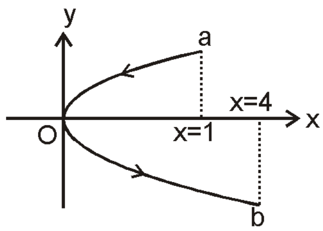
$B = \frac{\mu_0 i}{4\pi(d\sin\alpha)}(\cos 0^0 + \cos(180 - \alpha))$

$= \frac{\mu_0 i}{4\pi(d\sin\alpha)}(1 - \cos\alpha) = \frac{\mu_0 i}{4\pi d}\tan\frac{\alpha}{2}$

Resultant field will be :

$B_{net} = 2B = \frac{\mu_0 i}{2\pi d}\tan\frac{\alpha}{2} \quad \Rightarrow \quad k = \frac{\mu_0 i}{2\pi d}$

3. A conducting wire is bent in the form of a parabola $y^2 = x$ carrying a current i = 1 A as shown in the Figure. This wire is placed in a magnetic field $\vec{B} = -2\hat{k}$ tesla. The unit vector in the direction of force (on the given portion a to b) is :



(A) $\frac{3\hat{i} + 4\hat{j}}{5}$ (B) $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$ (C) $\frac{\hat{i} + 2\hat{j}}{\sqrt{5}}$ (D) $\frac{\hat{i} - 2\hat{j}}{\sqrt{5}}$

Ans. **(B)**

Sol. Coordinates of point a is (1, 1) and of b is (4, –2).

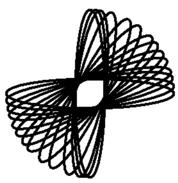
$\vec{F} = (\vec{L} \times \vec{B})\, i,\ \vec{L} = 3\hat{i} - 3\hat{j}$

$\vec{F} = \left((3\hat{i} - 3\hat{j}) \times (-2\hat{k})\right) i$

$\vec{F} = 6\hat{i} + 6\hat{j}$

Unit vector along force is $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

4. The current in coil (as shown in figure) is I (centers of all the circular loops lie at same point) and angular spread of coil is 90º, n is number of turns per unit radian and R is radius of each turn. (Assume that turns are very close)



(A) B at common centre will be $\frac{\mu_0 nI}{\sqrt{2}R}$

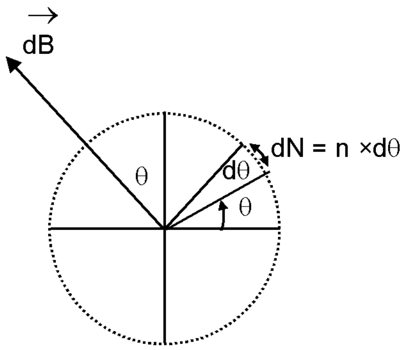
(B) B at common centre will be $\frac{\sqrt{2}\mu_0 nI}{R}$

(C) B at common centre will be $\frac{\sqrt{3}\mu_0 nI}{R}$

(D) B at common centre will be $\frac{\sqrt{5}\mu_0 nI}{R}$

Ans. **(A)**

Sol.



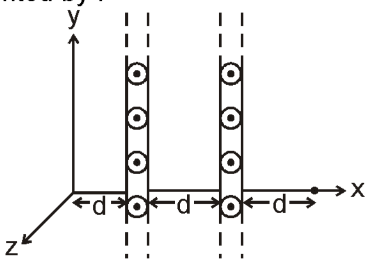
$dB = \frac{\mu_0 (dN) \times I}{2R}$

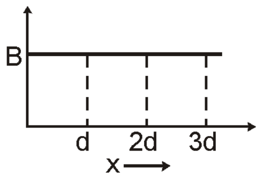
$dB_y = dB\sin\theta$

$\int dB_y = \frac{\mu_0 nI}{2R}\int_0^{\pi/2} \sin\theta \, d\theta \Rightarrow B_y = \frac{\mu_0 nI}{2R} = B_x$

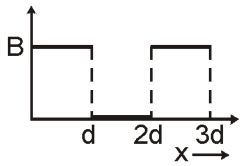
$B_{centre} = \frac{\mu_0 nI}{\sqrt{2}R}$

5. Two large conducting planes carrying current perpendicular to x-axis are placed at (d, 0) and (2d, 0) as shown in figure. Current per unit width in both the planes is same and current is flowing in the outward direction. The variation of magnetic induction (taken as positive if it is in positive y-direction) as function of 'x' ($0 \leq x \leq 3d$) is best represented by :

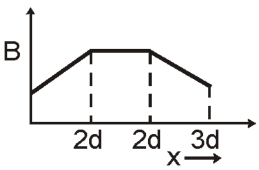




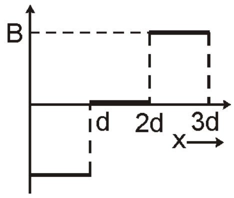
(A)



(B)



(C)



(D)

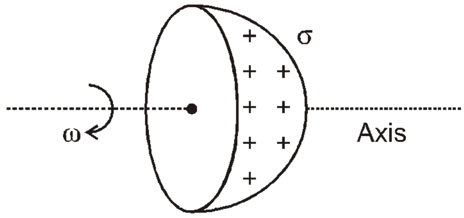
Ans. **(D)**

Sol. For $0 \leq x \leq d$; magnetic induction is negative.

For $d \leq x \leq 2d$; magnetic fields due to the two planes cancel each other, hence becomes zero.

For $2d \leq x \leq 3d$; magnetic field B is positive.

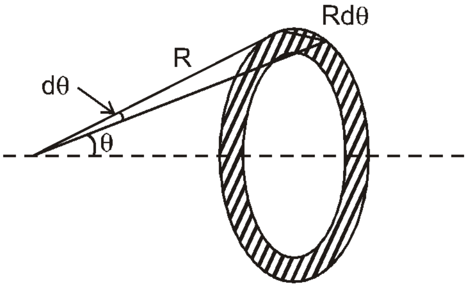
6. A hemispherical shell of radius R, having uniform charge density σ rotated about its axis of symmetry with constant angular velocity ω, then magnetic field strength at its centre is (μ_0 = magnetic permeability of free space)



(A) $\frac{1}{3}\omega\sigma\mu_0 R$ (B) zero (C) $\frac{2}{3}\omega\sigma\mu_0 R$ (D) $\omega\sigma\mu_0 R$

Ans. **(A)**

Sol. $dB = \frac{\mu_0}{4\pi}\frac{2\pi dI}{R^3}(R\sin\theta)^2$ $dI = \frac{dq}{2\pi}.\omega = \omega\sigma R^2\sin\theta.d\theta$

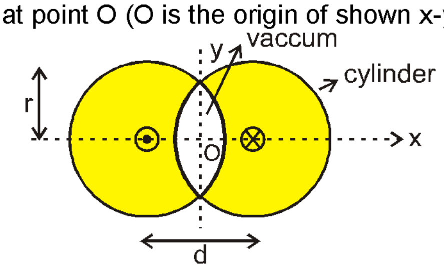


$B = \int \frac{\mu_0}{4\pi}.\frac{2\pi}{R}\sin^3\theta\,\omega\sigma R^2.d\theta$

$R = \frac{\mu_0\omega\sigma R}{2}\int_0^{\pi/2}\sin\theta(1-\cos^2\theta)d\theta$

$= \frac{1}{3}\omega\sigma\mu_0 R$

7. Two long cylinders (with axis parallel) are arranged as shown to form overlapping cylinders, each of radius r, whose centers are separated by a distance d. Current of density J (Current per unit area) flows into the plane of page along the right shaded part of one cylinder and an equal current flows out of the plane of the page along the left shaded part of the other, as shown. The magnitude and direction of magnetic field at point O (O is the origin of shown x-y axes) are :

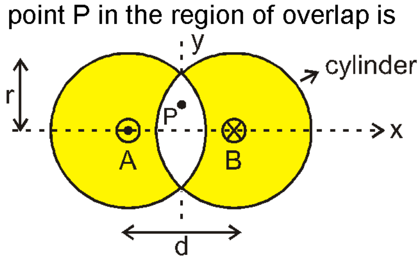


(A) $\frac{\mu_0}{2\pi}\pi Jd$, in the + y-direction (B) $\frac{\mu_0}{2\pi}d^2\frac{J}{r}$, in the + y-direction

(C) zero (D) none of these

Ans. **(A)**

Sol. Let the current density in complete left cylinder is $\vec{J}$, then current density in complete right cylinder is $-\vec{J}$. Then magnetic field at any point P in the region of overlap is



$$\vec{B} = \frac{\mu_0}{2}\vec{J}\times\overrightarrow{AP} + \frac{\mu_0}{2}(-\vec{J}\times\overrightarrow{BP})$$

$$= \frac{\mu_0}{2}\vec{J}\times(\overrightarrow{AP}+\overrightarrow{PB}) = \frac{\mu_0}{2}(\vec{J}\times\overrightarrow{AB})$$

Therefore magnitude of field at any point in region of overlap is $= \frac{\mu_0}{2}$ Jd and its direction is along positive y-direction at any point P in overlap region.

8. An infinite straight wire of radius 'R' is carrying a constant current 'i' flowing uniformly in it's cross-section along it's length then magnetic energy stored per unit length of this wire is [inside the wire].

(A) $\frac{\mu_0 i^2}{2}$ (B) $\frac{\mu_0 i^2}{16\pi}$ (C) $\frac{\mu_0 i^2}{8}$ (D) $\frac{\mu_0 i^2}{2\pi^2}$

Ans. **(B)**

Sol. (2) $\frac{dE}{dv} = \frac{\left(\frac{\mu_0 i}{2\pi R^2}\cdot r\right)^2}{2\mu_0}$

$$\frac{E}{\ell} = \frac{\mu_0 I^2}{8\pi^2 R^4}\cdot\int_0^R r^2 \times 2\pi r dr$$

$$\frac{E}{\ell} = \frac{\mu_0 I^2}{16\pi}$$

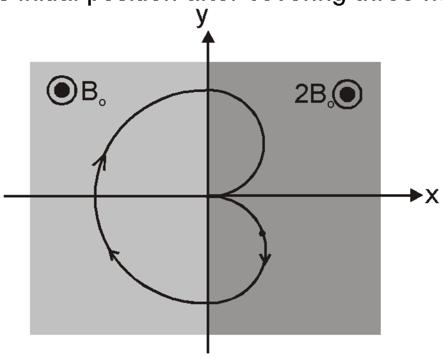
9. In region x > 0, a uniform and constant magnetic field $\vec{B}_1 = 2B_0\hat{k}$ exists. Another uniform and constant magnetic field $\vec{B}_2 = B_0\hat{k}$ exists in region x < 0. A positively charged particle of mass m and charge q is crossing origin at time t = 0 with a velocity $\vec{u} = u_0\hat{i}$. The particle comes back to its initial position after a time : (B_0, u_0 are positive constants)

(A) $\frac{3}{2}\frac{\pi m}{qB_0}$ (B) $\frac{2\pi m}{qB_0}$ (C) $\frac{3\pi m}{qB_0}$

(D) Particle does not come back to its initial position.

Ans. **(B)**

Sol. The motion of positively charged particle in the given combination of magnetic field will be as shown. The particle comes back to its initial position after covering three half circles.



Hence net time taken to come back to initial position is

$$T = \frac{\pi m}{2qB_0} + \frac{\pi m}{qB_0} + \frac{\pi m}{2qB_0} = \frac{2\pi m}{qB_0}$$

10. A charged particle having charge +q and mass m enters in a region where magnetic field varies with x-coordinate as:

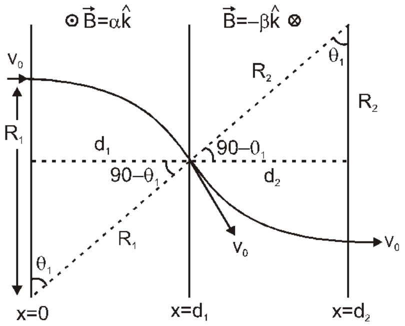
$$\vec{B} = \begin{cases} 0 & x < 0 \\ \alpha\hat{k} & 0 < x < d_1 \\ -\beta\vec{k} & d_2 > x > d_1 \\ 0 & x > d_2 \end{cases}$$

where α and β are positive constants with appropriate dimension. If in the region $x < 0$ and $x > d_2$ charge particle has velocity $\vec{v} = v_0\hat{i}$ choose the proper relation.

(A) $\frac{\beta}{\alpha} = \frac{d_1}{d_2 - d_1}$ (B) $\frac{\beta}{\alpha} = \frac{d_2}{d_1}$ (C) $\frac{\beta}{\alpha} = \frac{2d_1}{d_2}$ (D) $\frac{\beta}{\alpha} = \frac{2d_2}{d_1}$

Ans. **(A)**

Sol.



$R_1 \sin\theta_1 = d_1$

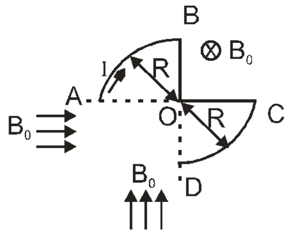
$R_2 \sin\theta_1 = d_2 - d_1$

$\frac{R_1}{R_2} = \frac{d_1}{d_2 - d_1}$

$\frac{mV_0}{\alpha q} \frac{\beta q}{mV_0} = \frac{d_1}{d_2 - d_1}$

$\frac{\beta}{\alpha} = \frac{d_1}{d_2 - d_1}$

11. Wire bent as ABOCD as shown, carries current I entering at A and leaving at D. Three uniform magnetic fields each B_0 exist in the region as shown. The force on the wire is



(A) $\sqrt{3}\,IRB_0$ (B) $\sqrt{5}\,IRB_0$ (C) $\sqrt{8}\,IRB_0$ (D) $\sqrt{6}\,IRB_0$

Ans. **(D)**

Sol. $\vec{F} = \vec{F} = I\vec{\ell} \times \vec{B}$

$\vec{\ell} = \overrightarrow{AD} = R(\hat{i} - \hat{j})$

$\vec{B} = B_0\,(\hat{i} + \hat{j} - \hat{k})$

$\therefore \quad \vec{F} = IRB_0\;(\hat{i} - \hat{j}) \times (\hat{i} + \hat{j} - \hat{k})$

$$= IRB_0 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 1 & -1 \end{vmatrix}$$

$= IRB_0\;(\hat{i} + \hat{j} + 2\hat{k})$

$F = IRB_0\sqrt{6}$

12. A thin circular disc of mass m, uniform surface charge density σ and having radius R is placed on a thin film of viscous liquid of thickness t and coefficient of viscosity η. The film has circular cross-section of same area as that of the disc. A uniform magnetic field exists perpendicular to the plane of the disc and in same area. If the magnetic field starts increasing at a constant rate β, what angular velocity (about its axis) must be given to the disc so that it continues to rotate with the same angular velocity.

(A) $\frac{\sigma\beta t}{2\eta}$ (B) $\frac{2\sigma\beta t}{3\eta}$ (C) $\frac{3\sigma\beta t}{2\eta}$ (D) $\frac{3\sigma\beta t}{4\eta}$

Ans. **(A)**

Sol. (A) Consider the disc to be made up of large number of elementary rings. Consider one such ring of radius x & thickness dx, accelerating torque on the elementary ring

$$d\tau_1 = dqEx = \sigma(2\pi x dx)\left(\frac{1}{2}\beta x\right)x$$

$\therefore$ Net accelerating torque

$$\tau_1 = \pi\sigma\beta\int_0^R x^3 dx = \frac{\pi\sigma\beta R^4}{4}$$

Similarly, retarding torque

$$d\tau_2 = \eta(2\pi x dx)\left(\frac{\omega x}{t}\right)x$$

$$d\tau_2 = \frac{2\pi\eta\omega}{t}x^3 dx \qquad \Rightarrow \tau_2 = \frac{2\pi\eta\omega}{t}\int_0^R x^3 dx = \frac{\pi\eta\omega R^4}{2t}$$

For constant angular velocity

$$\tau_1 = \tau_2$$

$$\frac{\pi\sigma\beta R^4}{4} = \frac{\pi\eta\omega R^4}{2t}$$

$$\Rightarrow \omega = \frac{\sigma\beta t}{2\eta}$$

MCQ (One or more than one correct) :

13. A proton is fired from origin with velocity $\vec{v} = v_0\hat{j} + v_0\hat{k}$ in a uniform magnetic field $\vec{B} = B_0\hat{j}$

In the subsequent motion of the proton

(A) its z-coordinate can never be negative

(B) its x-coordinate can never be positive

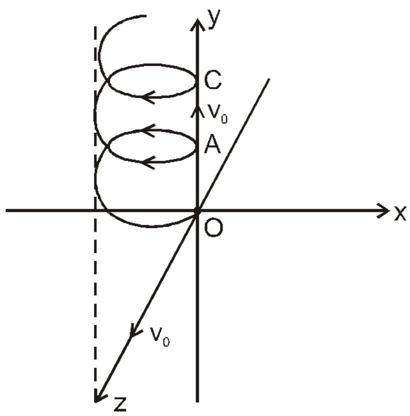
(C) its x-and z-coordinates cannot be zero at the same time

(D) its y-coordinate will be proportional to its time of flight

Ans. **(BD)**

Sol. The path of the particle will be helix as shown in figure. Clearly x-coordinate is always negative. z-coordinate can be negative and positive both. x and z coordinate will be zero at the same time at points A, C etc.

$y = v_0 t \Rightarrow y \propto t$



14. A coil of radius R carries a current I. Another concentric coil of radius r (r << R) carries current $\frac{I}{2}$. Initially planes of the two coils are mutually perpendicular and both the coils are free to rotate about common diameter. They are released from rest from this position. The masses of the coils are M and m respectively (m < M). During the subsequent motion let K_1 and K_2 be the maximum kinetic energies of the two coils respectively and let U be the magnitude of maximum potential energy of magnetic interaction of the system of the coils. Choose the correct options.

(A) $\frac{K_1}{K_2} = \frac{M}{m}\left(\frac{R}{r}\right)^2$

(B) $K_1 = \frac{Umr^2}{mr^2 + MR^2}$ $K_2 = \frac{UMR^2}{mr^2 + MR^2}$

(C) $U = \frac{\mu_0 \pi I^2 r^2}{4R}$

(D) $K_2 >> K_1$

Ans. **(BCD)**

Sol. $\tau_1 = \tau_2$

$$\frac{\int \tau dt}{I} = \omega$$

$$\omega \propto \frac{1}{I}$$

$$K.E. = \frac{1}{2}I\omega^2 \propto \frac{1}{I}$$

$$\frac{K_1}{K_2} = \frac{\frac{mr^2}{2}}{\frac{MR^2}{2}}$$

$$\frac{K_1}{K_2} = \frac{m}{M}\left(\frac{r}{R}\right)^2$$

$$K_1 + K_2 = U$$

$$K_2 \frac{m}{M}\left(\frac{r}{R}\right)^2 + K_2 = U$$

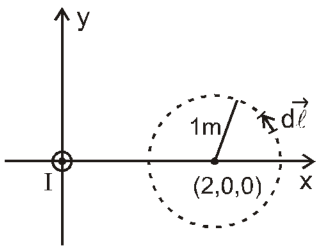
$$K_2 = \frac{UMR^2}{mr^2 + MR^2}$$

$$K_1 = \frac{Umr^2}{mr^2 + MR^2}$$

$$U = \frac{I}{2}\pi r^2 \frac{\mu_0 I}{2R} = \frac{\mu_0 I^2 \pi r^2}{4R}$$

Comprehension Type Question:

An infinitely long wire lying along z-axis carries a current I, flowing towards positive z-direction. There is no other current, consider a circle in x-y plane with centre at (2 meter, 0, 0) and radius 1 meter. Divide the circle in small segments and let $d\vec{\ell}$ denote the length of a small segment in anticlockwise direction, as shown.



15. The path integral $\oint \vec{B} \cdot d\vec{\ell}$ of the total magnetic field $\vec{B}$ along the perimeter of the given circle is,

(A) $\frac{\mu_0 I}{8}$ (B) $\frac{\mu_0 I}{2}$ (C) $\mu_0 I$ (D) 0

Ans. **(D)**

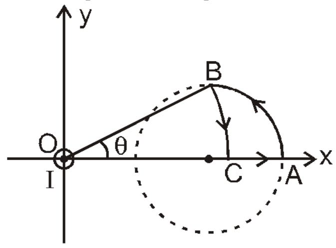
Sol. Since there is no current passing through circular path, the integral $\oint \vec{B}\cdot d\vec{\ell}$ along the dotted circle is zero.

16. Consider two points A(3,0,0) and B(2,1,0) on the given circle. The path integral $\int_A^B \vec{B}\cdot d\vec{\ell}$ of the total magnetic field $\vec{B}$ along the perimeter of the given circle from A to B is,

(A) $\frac{\mu_0 I}{\pi}\tan^{-1}\frac{1}{2}$ (B) $\frac{\mu_0 I}{2\pi}\tan^{-1}\frac{1}{2}$ (C) $\frac{\mu_0 I}{2\pi}\sin^{-1}\frac{1}{2}$ (D) 0

Ans. **(B)**

Sol. Let segment OB =OC and arc BC is a circular arc with centre at origin. Since the shown closed path ABCA encloses no current, the path integral of magnetic field over this path is zero.



Hence $\int_A^B \vec{B}\cdot d\vec{\ell} + \int_B^C \vec{B}\cdot d\vec{\ell} + \int_C^A \vec{B}\cdot d\vec{\ell} = 0.$

Because is perpendicular to segment AC at all points , therefore $\int_C^A \vec{B}\cdot d\vec{\ell} = 0.$

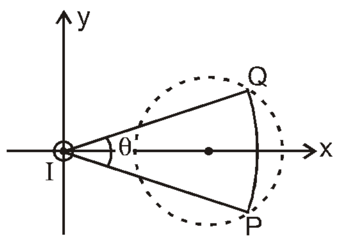
Hence $\int_A^B \vec{B}\cdot d\vec{\ell} = \int_C^B \vec{B}\cdot d\vec{\ell} = \frac{\mu_o I}{2\pi}\frac{OB(\theta)}{OB} = \frac{\mu_o I}{2\pi}\tan^{-1}\frac{1}{2}$

17. The maximum value of path integral $\int \vec{B}\cdot d\vec{\ell}$ of the total magnetic field $\vec{B}$ along the perimeter of the given circle between any two points on the circle is

(A) $\frac{\mu_0 I}{12}$ (B) $\frac{\mu_0 I}{8}$ (C) $\frac{\mu_0 I}{6}$ (D) 0

Ans. **(C)**

Sol. Consider two points P and Q lying on dotted circle and equidistant from origin O. We draw a circular arc QP with centre at origin O. The path integral of magnetic field, that is $\int \vec{B}\cdot d\vec{\ell}$, along the dotted circle between two points P and Q is also is equal to path integral $\int \vec{B}\cdot d\vec{\ell}$ along the arc QP whose centre is at origin.



Therefore the path integral of magnetic field $\int \vec{B}\cdot d\vec{\ell}$ along the dotted circle between two points P and Q

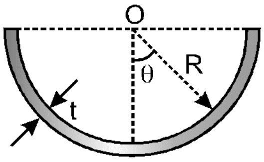
$= \frac{\mu_o I}{2\pi}\frac{OP(\theta)}{OP} = \frac{\mu_o I}{2\pi}\theta.$

The value of θ will be maximum when chord OQ and chord OP will be tangent to the dotted circle, that is, $\theta = \frac{\pi}{3}$.

Hence the required maximum value = $\frac{\mu_0 I}{6}$.

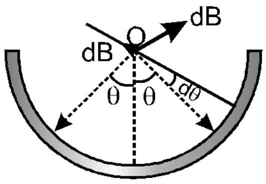
Numerical based Questions :

18. Conductor of length ℓ has shape of a semi cylinder of radius R (<< ℓ). Cross section of the conductor is shown in the figure. Thickness of the conductor is t (<< R) and conductivity of its material varies with angle θ according to the law $\sigma = \sigma_0 \cos\theta$ where σ_0 is a constant. If a battery of emf ε is connected across its end faces (across the semi–circular cross-sections), the magnetic induction at the mid point O of the axis of the semi-cylinder is found to be $B = \frac{2\mu_0\sigma_0\varepsilon t}{x\ell}$. What is the value of x.



Ans. 8

Sol.



$$dB = \frac{\mu_0 di}{2\pi R} = \frac{\mu_0 \times \frac{\varepsilon}{\ell}(\sigma_0 \cos\theta)Rd\theta \times t}{2\pi R}$$

$$B = \int_0^{\pi/2} 2dB\cos\theta = \frac{2\mu_0\sigma_0\varepsilon t}{8\ell}$$

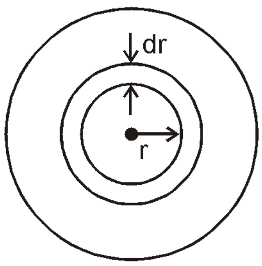
19. The current density $\vec{J}$ inside a long, solid, cylindrical wire of radius a = 12 mm is in the direction of the central axis, and its magnitude varies linearly with radial distance r from the axis according to $J = \frac{J_0 r}{a}$, where $J_0 = \frac{10^5}{4\pi}$ A/m^2. Find the magnitude of the magnetic field at $r = \frac{a}{2}$ in μT.

Ans. 10

Sol. Current in the element = J(2πr . dr) = J(2πr . dr)

Current enclosed by Amperian loop of radius $\frac{a}{2}$

$$I = \int_0^{a/2} \frac{J_0 r}{a} \cdot 2\pi r \cdot dr = \frac{2\pi J_0}{3a}\left(\frac{a}{2}\right)^3 = \frac{\pi J_0 a^2}{12}$$



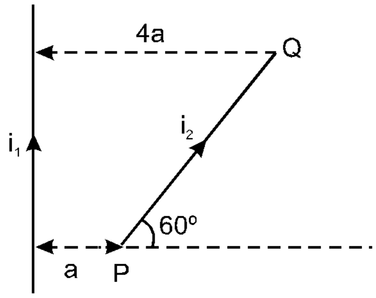
Applying Ampere's law

$$B \cdot 2\pi \cdot \frac{a}{2} = \mu_0 \cdot \frac{\pi J_0 a^2}{12} \Rightarrow B = \frac{\mu_0 J_0 a}{12}$$

On putting values

B = 10 μT

20. A long straight conductor carries current 'i_1'. A wire PQ carrying current 'i_2' is placed as shown. The net force on PQ is $\frac{2\mu_0 i_1 i_2}{\pi}\ell nx$, then write the value of 'x'.

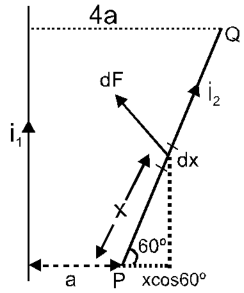


Ans. 2

Sol. $dF = B\,i_2\,dx$

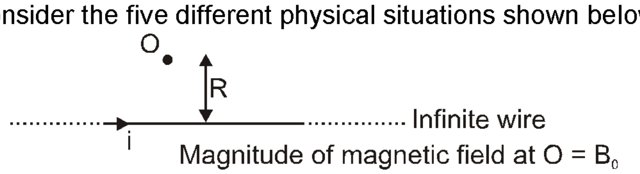
$$= \frac{\mu_0 i_1 i_2}{2\pi(a + x\cos 60^\circ)}dx$$

$$F = \int_0^{6a} dF = \frac{\mu_0 i_1 i_2}{\pi}\ln\left(\frac{4a}{a}\right) = \frac{2\mu_0 i_1 i_2}{\pi}\ \ell n\ 2$$

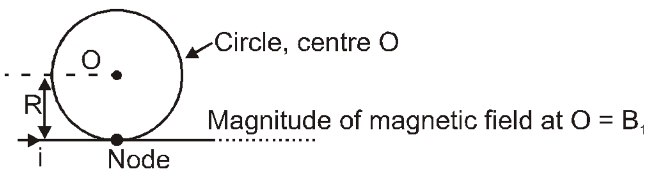


Matrix Match Type :

21. Consider the five different physical situations shown below. All the symbols have their usual meaning.

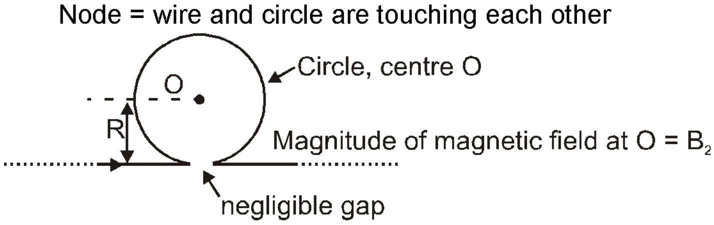


(a)

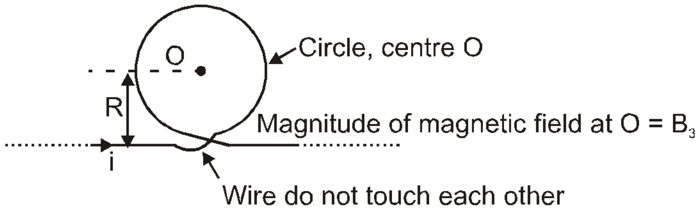


(b)

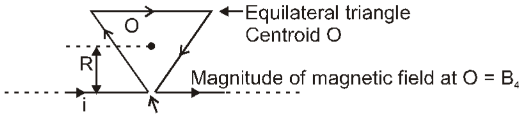
Node = wire and circle are touching each other



(c)



(d)



(e)

Then match the following :

Column – I	**Column – II**
(A) $\frac{B_1}{B_0}$	(p) Less than 1.
(B) $\frac{B_2}{B_1}$	(q) More than 2 but less than 3.
(C) $\frac{B_3}{B_2}$	(r) 1
(D) $\frac{B_3}{B_4}$	(s) More than 1

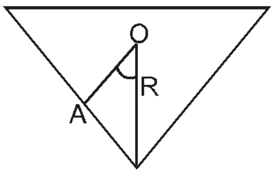
Ans. **(A) → (r) ; (B) → (q), (s) ; (C) → (s) ; (D) → (p)**

Sol. $B_0 = \frac{\mu_0 i}{2\pi R}$

$B_1 = \frac{\mu_0 i}{2\pi R} = B_0$ ($\because$ no current through the circle)

$B_2 = \frac{\mu_0 i}{2\pi R} - \frac{\mu_0 i}{2R} = B_0 - B_0\pi = B_0 (1 - \pi) = -2.14\ B_0$

$B_3 = \frac{\mu_0 i}{2\pi R} + \frac{\mu_0 i}{2R} = B_0 + B_0\pi = B_0 (1 + \pi) = 4.14\ B_0$



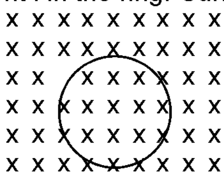
$B_4 = \frac{\mu_0 i}{2\pi R} - 3 \times \frac{\mu_0 i}{4\pi(R\cos 60°)}$ (2 sin 60°)

($\therefore$ OA = R cos 60°)

$= \frac{\mu_0 i}{2\pi R} - \frac{\mu_0 i}{2\pi R} \cdot 3\sqrt{3} = B_0\,(1 - 3\sqrt{3}) = -4.20\ B_0$

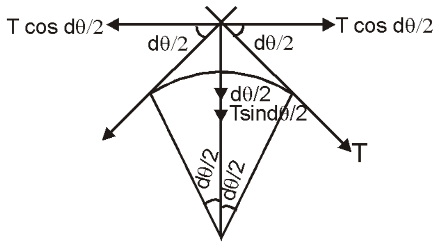
Subjective Type Questions :

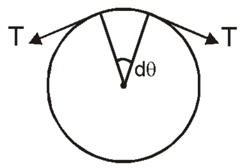
22. A ring of mass m and radius r is rotated in uniform magnetic field B which is perpendicular to the plane of the loop with constant angular velocity ω_0. Find the net ampere force on the ring and the tension developed in the ring if there is a current i in the ring. Current and rotation both are clockwise.

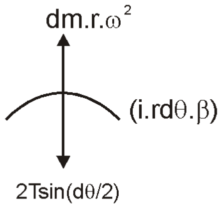


Ans. 0, $\frac{r}{2\pi}(m\omega_0^2 + 2\pi iB)$

Sol. Net ampere force acting on a closed loop in uniform magnetic field is zero.





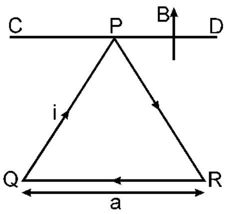


$2T(\sin\frac{d\theta}{2}) = (dm.r.\omega^2) + r.d\theta.iB$

$T.d\theta = \frac{m}{2\pi r}.rd\theta.r.\omega^2 + r.d\theta.i.B$

$T = \frac{r}{2\pi}\ (m\omega_0^2 + 2\pi i.B).$

23. A loop PQR formed by three identical uniform conducting rods each of length ' a ' is suspended from one of its vertices (P) so that it can rotate about horizontal fixed smooth axis CD. Initially plane of loop is in vertical plane. A constant current ' i ' is flowing in the loop. Total mass of the loop is ' m '. At t = 0, a uniform magnetic field of strength B directed vertically upwards is switched on. Acceleration due to gravity is ' g '. then Find
the minimum value of B so that the plane of the loop becomes horizontal (even for an instant) during its subsequent motion.



Ans. $\frac{4mg}{3ia}$

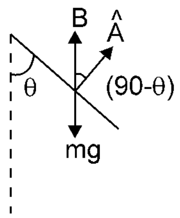
Sol. Applying Energy conservation, initially, kinetic energy = 0
gravitational P.E. = 0 (say) & Magnetic P.E. = μB

where, μ = magnetic moment of the loop = i. $\left(\frac{\sqrt{3}a^2}{4}\right)$

Finally when the loop becomes horizontal, Kinetic energy = 0

gravitational P.E.= $\left(\frac{a}{\sqrt{3}}\right)$mg(because mg acts on the centre of mass)

magnetic P.E. = 0



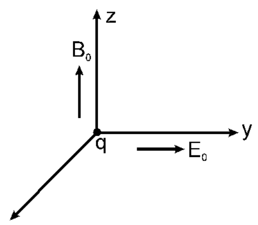
$\Rightarrow \qquad 0 + 0 + \mu B = 0 + \frac{mga}{\sqrt{3}} + 0 \Rightarrow \qquad B = \frac{mga}{\sqrt{3}\ \mu} = \frac{4mg}{3ia}$

24. A positive charge particle of charge ' q ' & mass ' m ' is released at origin. There are uniform magnetic field and electric field in the space given by $\vec{E} = E_0\hat{j}$ & $\vec{B} = B_0\hat{k}$, where E_0 & B_0 are constants. Find the ' y ' co-ordinate of the particle at time ' t '.

Ans. $y = \frac{E_0 m}{qB_0^2}\left[1 - \cos\frac{qB_0}{m}t\right]$

Sol. $m\frac{dv}{dt} = qE_0\hat{j} + q\left[v_x\hat{i} + v_y\hat{j}\right]$ ´ $B_0\hat{k}$

$$m\frac{dv_y}{dt}\hat{j} + m\frac{dv_x}{dt}\hat{i} = [qE_0 - qv_xB_0]\hat{j} + qv_yB_0\hat{i}$$



$m\frac{dv_y}{dt} = [qE_0 - qv_xB_0]$ .....(1)

$m\frac{dv_x}{dt} = qv_yB_0$ .....(2)

From (1) $v_x = \left[qE_0 - m\frac{dv_y}{dt}\right]\frac{1}{qB_0}$

From (2) $\frac{m}{qB_0}\frac{d}{dt}\left[qE_0 - m\frac{dv_y}{dt}\right] = qv_yB_0$

$-\frac{d^2v_y}{dt^2} = \frac{q^2v_yB_0^2}{m^2}$ or $\frac{d^2v_y}{dt^2} + \frac{q^2v_yB_0^2}{m^2} = 0$

Solution of above equation :

$v_y = A\sin(\omega t + \phi)$ .....(3)

where $\omega = \frac{qB_0}{m}$ at $t = 0$, $v_y = 0$, $\phi = 0$ $v_y = A\sin\omega t$

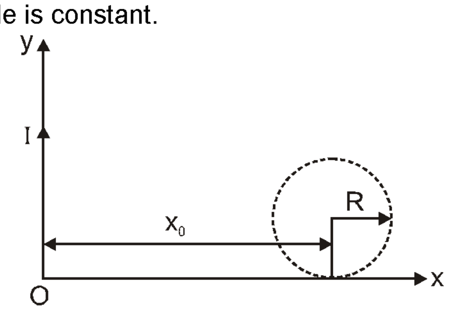
at $t = 0$, $a = \frac{qE_0}{m}$ $a = \frac{dv_y}{dt} = A\omega\cos\omega t$ $\frac{qE_0}{m} = A\times\frac{qB_0}{m} \Rightarrow A = \frac{E_0}{B_0}$

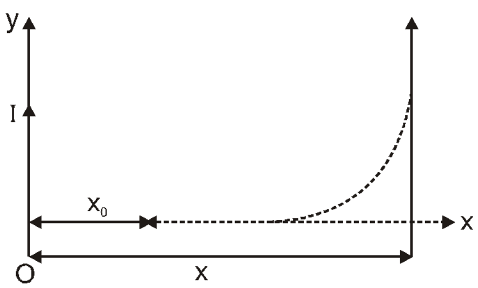
This equation (3) $v_y = \frac{E_0}{B_0}\sin\omega t\,\frac{dy}{dt} = \frac{E_0}{B_0}\sin\omega t \Rightarrow y = \left[-\frac{E_0}{B_0}\cos\omega t\right]_0^t\times\frac{1}{\omega}$

$y = \frac{E_0 m}{B_0\times qB_0}[1 - \cos\omega t]$ $\Rightarrow y = \frac{E_0 m}{qB_0^2}\left[1 - \cos\frac{qB_0}{m}t\right]$

25. A positive point charge q of mass m, kept at a distance x_0 (in the same plane) from a fixed very long straight current is projected normally away from it with speed v. Find the maximum separation between the wire and the particle.

Sol. Since the magnetic field is not uniform, the particle doesn't follows circular path but the speed (v) of the particle is constant.





Here the magnetic field set-up by the straight current is along the negative z-axis, the initial velocity of the particle is along x-axis and the force F is in the x-y plane.
The force at time t after starting from point P is
F = q(v × B)

or $F = q\left[(v_x\hat{i} + v_y\hat{j}) \times \left(\frac{\mu_0 I}{2\pi x}(-k)\right)\right]$

$= \frac{\mu_0 q I}{2\pi x}(-v_y\hat{i} + v_x\hat{j})$

So, $F_x = \frac{-\mu_0}{2\pi}\frac{qIv_y}{x}$ $\therefore$ $a_x = \frac{-\mu_0 qIv_y}{2\pi m x}$

or $\frac{v_x dv_x}{dx} = \frac{\mu_0 qIv_y}{2\pi m x}$ ......(i)

But $v_x^2 + v_y^2 = v^2$

$\therefore$ $2v_x dv_x + 2v_y dv_y = 0$

or $v_x dv_x = - v_y dv_y$ ......(ii)

From Eqs. (i) and (ii)

$\frac{2\pi m}{\mu_0 qI} \times dv_y = \frac{dx}{x}$

or $\frac{2\pi m}{\mu_0 qI}\int_0^v dv_y = \int_{x_0}^{x}\frac{dx}{x}$ or $\frac{2\pi m}{\mu_0 qI}v = \ell n\frac{x}{x_0}$

$\therefore$ $x = x_0 e^{2\pi m v/\mu_0 qI}$