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PHYSICS

TARGET : JEE- Advanced 2021

CAPS-13

CAPACITANCE

SCQ (Single Correct Type) :

1. One plate of a parallel plate capacitor(5 μF) has a fixed charge 10 μC. The charge q(in μC)on the other plate is varied with time t(in seconds) as q = 2t. The potential difference (in volts) between the plates will vary as

(A) $|1-0.2t|$ (B) $|1+0.2t|$ (C) 0.5t (D) 0.2t

Ans. **(A)**

Sol. Let the charge on the other plate be q

Electric field between the plates

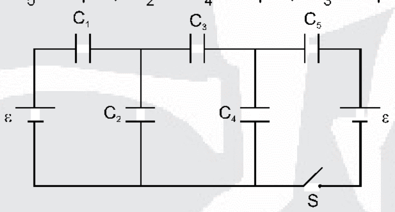
$= \left|\dfrac{q-10}{2\varepsilon_o A}\right|$ where A = area of plate

Potential difference between the plates

$= \left|\dfrac{q-10}{2\varepsilon_o A}.d\right|$ where d = distance between the plates

$= \left|\dfrac{q-10}{2\ C}\right| = \left|\dfrac{2t-10}{2\ (5)}\right| = |0.2\ t-1|$

2. In the circuit shown $C_1 = C_5 = 5\mu F$, $C_2 = C_4 = 3\mu F$, $C_3 = 6\mu F$, $\varepsilon = 20$ v.



Choose the **incorrect** option :

(A) Charge on C_1 is equal to charge on C_5 when switch S is closed and each is equal to 37.5 μC

(B) Charge on C_1 is equal to 50μC when switch S is open

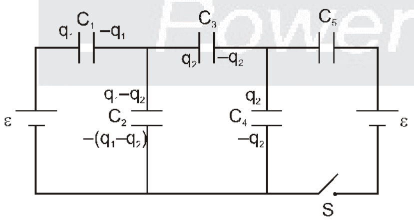
(C) Charge on C_4 is equal to 30μC when switch S is open

(D) Charge on C_3 is zero when switch S is closed

Ans. **(C)**

Sol. S open :

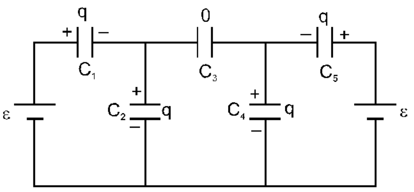
$$\frac{q_1}{C_1}+\frac{q_1-q_2}{C_2}=\varepsilon$$



$$\frac{q_2}{C_3}+\frac{q_2}{C_4}-\frac{(q_1-q_2)}{C_2}=0$$

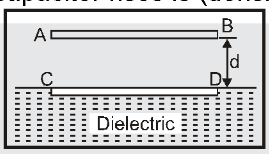
Solving q_1 = 50 μC, q_2 = 20 μC

S closed :



$$\frac{q}{C_1}+\frac{q}{C_2}=\varepsilon \qquad \Rightarrow \qquad q = 37.5\ \mu C.$$

3. AB and CD are two plates each of area 'a' such that the plate CD is just submerged into a liquid whose dielectric constant is k. If the plates are charged using a battery to charge density σ, the height to which the level of liquid in the capacitor rises is (density of liquid is ρ).



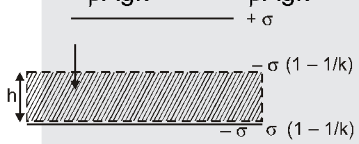
(A) $\frac{\sigma^2(k^2+1)}{2\varepsilon_0\rho g k^2}$ (B) $\frac{\sigma^2}{2\varepsilon_0\rho g k}$ (C) $\frac{\sigma^2(k-1)}{2\varepsilon_0\rho g k}$ (D) $\frac{\sigma^2(k^2-1)}{2\varepsilon_0\rho g k^2}$

Ans. **(D)**

Sol. The weight of risen column of liquid = electric force

$\rho Agh = \sigma\left[1-\frac{1}{k}\right]AE$

$h = \frac{\sigma[k-1]AE}{\rho Agk} = \frac{\sigma[k-1]A}{\rho Agk}.E$

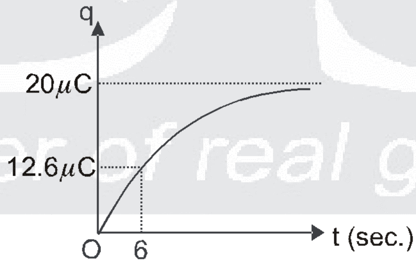


$E = \frac{\sigma}{2\varepsilon_0}+\frac{\sigma}{2k\varepsilon_0}$

$\frac{\sigma}{2\varepsilon_0}\left(\frac{k+1}{k}\right)$

$h = \frac{\sigma}{\rho g k}[k-1]\ \frac{\sigma}{2\varepsilon_0}\ \frac{[k-1]}{k} = \frac{\sigma^2[k^2-1]}{2\varepsilon_0\rho g k^2}$

4. Charge q versus t graph for a capacitor, initally uncharged is charged with a battery of emf 5V as shown in figure. The resistance R of the circuit is



(A) 2MΩ (B) 6MΩ (C) 3MΩ (D) 1.5MΩ

Ans. **(D)**

Sol. Time constant = t = 6 sec

$\Rightarrow$ RC = 6 .... (i)

Q_{max} = CE = 20 .... (ii)

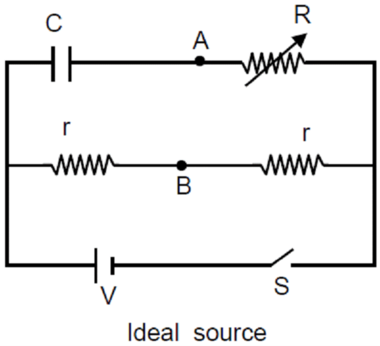
C × 5 = 20μc

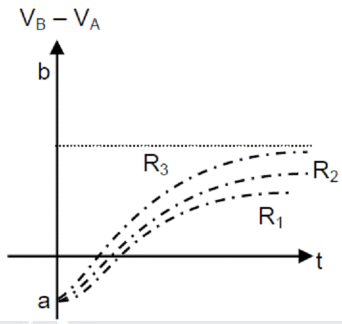
$\Rightarrow$ C = 4μF

Since RC = 6

$\Rightarrow = R = \frac{6}{4\times10^{-6}} = 1.5\ M\Omega$

5. An uncharged capacitor C and a variable resistance R are connected to an ideal source and tworesistors with the help of a switch at t = 0 as shown in the figure. Initially capacitor is uncharged and switch S is closed at t = 0 sec. The graph between V_B . V_A for variable resistor R for its three different value R_1, R_2, and R_3 versus time is shown in the figure II. Choose the correct statement.



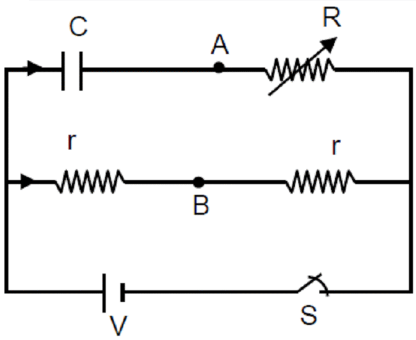


(A) $R_1 < R_2 < R_3$ (B) $R_1 > R_2 > R_3$ (C) $R_1 < R_3 < R_2$ (D) |a| < |b| (E) |a| > |b|

Ans. **(B)**

Sol. Consider circuit at any time t after closing the switch :

$I_1 = \frac{V}{2r}$ and $I_2 = \frac{V}{R}e^{-\frac{t}{RC}}$



$\Rightarrow V_A - V_B = RI_2 - rI_1 = R \times \frac{V}{R}e^{-\frac{t}{RC}} - r \times \frac{V}{2r}$

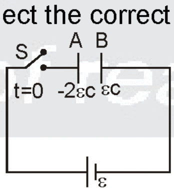
$\Rightarrow V_A - V_B = \frac{-V}{2}\left(1 - 2e^{-\frac{t}{RC}}\right)$

$\Rightarrow a = \frac{-V}{2}$, and $b = \frac{+V}{2}$

When $V_B - V_A = 0 \Rightarrow t_0 = RC\,\ell n(2)$, Hence greater the R, greater the t_0

MCQ (One or more than one correct) :

6. A parallel plate capacitor of capacitance 'C' has charges on its plates initially as shown in the figure. Now at t = 0, the switch 'S' is closed. Select the correct alternative(s) for this circuit diagram.



(A) In steady state the charges on the outer surfaces of plates 'A' and 'B' will be same in magnitude and sign.

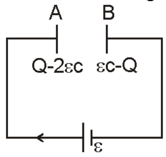
(B) In steady state the charges on the outer surfaces of plates 'A' and 'B' will be same in magnitude and opposite in sign.

(C) In steady state the charges on the inner surfaces of the plates 'A' and 'B' will be same in magnitude and opposite in sign.

(D) The work done by the cell by the time steady state is reached is $\frac{5\varepsilon^2 C}{2}$.

Ans. **(ACD)**

Sol. Suppose charge flown through the battery is Q, then charge distribution will be as :



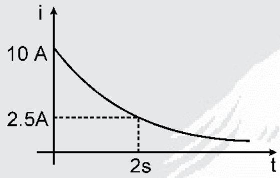
The electric field in the region between A and B is = $\frac{Q-2\varepsilon C}{2A\in_0} - \frac{\varepsilon C-Q}{2A\in_0} = \frac{2Q-3\varepsilon C}{2A\in_0}$

$\therefore$ P.D. between the plates, $\frac{2Q-3\varepsilon C}{2A\in_0}.d = \varepsilon \Rightarrow \frac{2Q-3\varepsilon C}{2}\frac{1}{C} = \varepsilon$

$\Rightarrow 2Q = 5\varepsilon C \quad \Rightarrow Q = \frac{5\varepsilon C}{2}$

$\therefore$ w.d. by battery = $\varepsilon Q = \frac{5\varepsilon^2 C}{2}$

7. The figure shows, a graph of the current in a discharging circuit of a capacitor through a resistor of resistance 10 Ω.



(A) The initial potential difference across the capacitor is 100 volt.

(B) The capacitance of the capacitor is $\frac{1}{10\,\ell n2}$ F.

(C) The total heat produced in the circuit will be $\frac{500}{\ell n2}$ joules.

(D) The thermal power in the resistor will decrease with a time constant $\frac{1}{2\ell n2}$ second.

Ans. **(ABCD)**

Sol. $V_0 = I_0R = 10 \times 10 = 100$ volts (since, I_0 = 10amp from figure)

Also : $I = I_0e^{-t/RC}$

Taking log ; $\log\left(\frac{I_0}{I}\right) = \frac{t}{RC}$

$\Rightarrow \quad C = \frac{t}{R\log(I_0/I)}$

At ; t = 2 sec, $I = 2.5$ A

$C = \frac{2}{10\log\left(\frac{10}{2.5}\right)}$

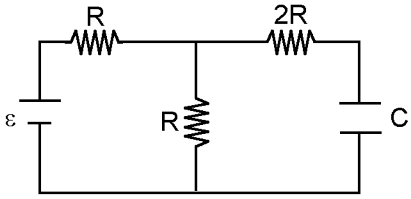
$C = \frac{2}{10\log 4} = \frac{2}{10\times 2\log 2} = \frac{1}{10\ell n2}$

$C = \frac{1}{10\ell n2}$.

Heat produced = $\frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{1}{10\ell n2}\right)(100^2) = \frac{500}{\ell n2}$ joules.

Hence (C) is correct

8. Circuit shown in the figure is in steady state. Now the capacitor is suddenly filled with medium of dielectric constant K = 2.



(A) Current through '2R' just after this moment is $\frac{\varepsilon}{10R}$

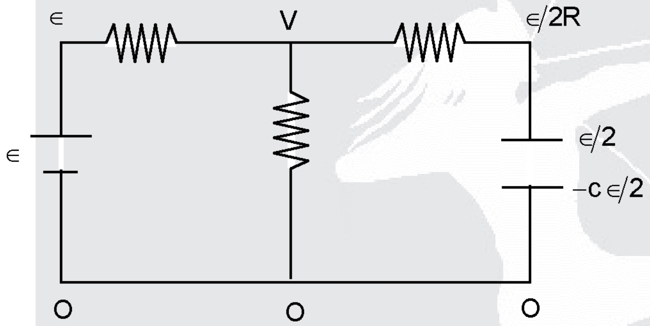
(B) Current through '2R' just after this moment is $\frac{\varepsilon}{15R}$

(C) Current through battery just after this moment is $\frac{11\varepsilon}{20R}$

(D) Potential difference across capacitor just after this moment is $\frac{\varepsilon}{4}$

Ans. **(ACD)**

Sol.



$$\frac{V}{R}+\frac{V-\varepsilon}{R}+\frac{V-\frac{\varepsilon}{2k}}{2R}=0$$

2v + 2v - $2\varepsilon + V - \frac{\varepsilon}{2k} = 0$

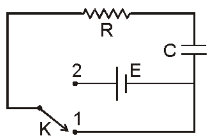
5V = $2\varepsilon + \frac{\varepsilon}{2k}$

$$\frac{2\varepsilon+\frac{\varepsilon}{2k}}{5}=\frac{9\varepsilon}{20}$$

Comprehension Type Question:

Comprehension # 1

In the shown circuit involving a resistor of resistance R Ω, capacitor of capacitance C farad and an ideal cell of emf E volts, the capacitor is initially uncharged and the key is in position 1. At t = 0 second the key is pushed to position 2 for t_0 = RC seconds and then key is pushed back to position 1 for t_0 = RC seconds. This process is repeated again and again. Assume the time taken to push key from position 1 to 2 and vice versa to be negligible.



9. The charge on capacitor at t = 2RC second is

(A) CE (B) CE$\left(1-\frac{1}{e}\right)$ (C) CE$\left(\frac{1}{e}-\frac{1}{e^2}\right)$ (D) CE$\left(1-\frac{1}{e}+\frac{1}{e^2}\right)$

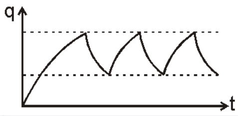
Ans. **(C)**

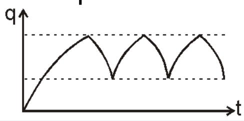
10. The current through the resistance at t = 1.5 RC seconds is

(A) $\frac{E}{e^2R}(1-\frac{1}{e})$ (B) $\frac{E}{eR}(1-\frac{1}{e})$ (C) $\frac{E}{R}(1-\frac{1}{e})$ (D) $\frac{E}{\sqrt{e}R}(1-\frac{1}{e})$

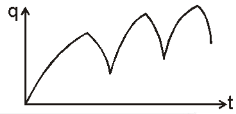
Ans. **(D)**

11. Then the variation of charge on capacitor with time is best represented by

(A) 

(B) 

(C) 

(D) 

Ans. **(C)**

Sol. **(9 to 11)**

For t = 0 to t_o = RC seconds, the circuit is of charging type. The charging equation for this time is

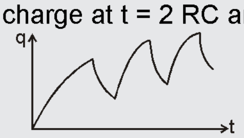
$$q = CE(1 - e^{-\frac{t}{RC}})$$

Therefore the charge on capacitor at time t_0 = RC is $q_o = CE(1 - \frac{1}{e})$

For t = RC to t = 2RC seconds, the circuit is of dicharging type. The charge and current equation for this time are

$$q = q_o e^{-\frac{t-t_o}{RC}} \quad \text{and} \quad i = \frac{q_o}{RC} e^{-\frac{t-t_o}{RC}}$$

Hence charge at t = 2 RC and current at t = 1.5 RC are



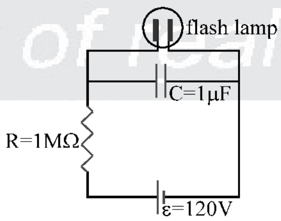
$$q = q_o e^{-\frac{2RC-RC}{RC}} = \frac{q_o}{e} = \frac{1}{e}CE(1 - \frac{1}{e})$$

and $i = \frac{q_o}{RC} e^{-\frac{1.5RC-RC}{RC}} = \frac{q_o}{\sqrt{e}RC} = \frac{E}{\sqrt{e}R}(1 - \frac{1}{e})$ respectively

Since the capacitor gets more charged up from t = 2RC to t= 3RC than in the interval t=0 to t=RC, the graph representing the charge variation is as shown in figure

Comprehension # 2

A highway emergency flasher uses a 120 volt battery, a 1 MΩ resistor, a 1 mF capacitor and a neon flash lamp in the circuit shown in the figure. The flash lamp has a resistance more than 10^{10} Ω when the voltage across it is less than 110V. Above 110 V, the neon gas ionizes, the lamp's resistance drops to 10 Ω, and the capacior discharges completely. Until the capacitor voltage reaches the breackdown voltage V_b = 110 V, the large resistance of the flash lamp ensures that it draws a negligible current.



The capacitor charges as if the lamp were absent. At V_b, however, the lamp resistance quickly becomes negligible, and the capacitor discharges through the lamp as if the battery and the series resistor were absent. The time between the flashes is the time for the capacitor to charge to V_b. The flash duration is roughly the time for the capacitor to discharge through the lamp, or about 3 time constant of the capacitor–lamp circuit. The flash energy is the stored energy in the capacitor at 110 volt.

12. The flash interval is found by solving for the time when the capacitor voltage is V_b = 110 V. $V_b = \varepsilon(1 - e^{-t/CR})$, ℓn 12 = 2.5). Flash interval is

(A) 2 s (B) 2/5 s (C) 5/2 s (D) 1 s

Ans. **(C)**

Sol. $V_b = \varepsilon_0 (1 - e^{-t/RC})$

$\Rightarrow 110 = 120 (1 - e^{-t/RC})$

$\Rightarrow e^{-t/RC} = 1/12$

$\Rightarrow t/RC = \ell n/12 = 2.5$

$\Rightarrow t = RC \times 2.5 = 10^6 \times 10^{-6} \times 2.5 = 5/2$ sec

13. Time constant (τ_c) of the capacitor–lamp circuit is-

(A) 20 μs (B) 15 μs (C) 30 μs (D) 10 μs

Ans. **(D)**

Sol. $\tau_0 = 10^{-6} \times 10 = 10\ \mu s$

14. Flash duration is

(A) 10 μs (B) 20 μs (C) 30 μs (D) 5 μs

Ans. **(C)**

Sol. Flash duration = $3\tau_0 = 30\ \mu s$

15. The energy in the flash is

(A) 6.1 mJ (B) 6.1 J (C) 3 mJ (D) 12.2 mJ

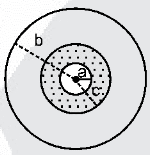
Ans. **(A)**

Sol. Energy in flash

$= \frac{1}{2}CV^2 = \frac{1}{2} \times 1 \times 10^{-6} \times 110 \times 110 = 6.1$ mJ

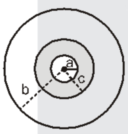
Numerical based Questions :

16. A spherical capacitor is made of two conducting spherical shells of radii a and b = 3a. The space between the shells is filled with a dielectric of dielectric constant K = 3 upto a radius c = 2a as shown. If the capacitance of given arrangement is n times the capacitance of an isolated spherical conducting shell of radius a. Then find value of n.



Ans. n = 3

Sol.



$\int d\ V = \int E.\ dr$ $\quad V = \int_a^c \frac{q}{4\pi\ \epsilon_0} \frac{dr}{kr^2} + \int_c^b \frac{q}{4\pi\ \epsilon_0} \frac{dr}{r^2} = \frac{q}{4\pi \epsilon_0} \frac{1}{k}\left[-\frac{1}{r}\right]_a^c + \frac{q}{4\pi \epsilon_0}\left[-\frac{1}{r}\right]_c^b$

$= \frac{q}{4\pi \epsilon_0}\left[\frac{1}{k}\left(\frac{1}{a}-\frac{1}{c}\right)-\left(\frac{1}{c}-\frac{1}{b}\right)\right] = \frac{q}{4\pi \epsilon_0}\left[\frac{bc-ab-kab+kac}{kabc}\right]$

$V = \frac{q}{4\pi \epsilon_0}\left[\frac{ba(b-c)+b(c-a)}{kabc}\right]$; $C_{eq} = \frac{q}{V}$

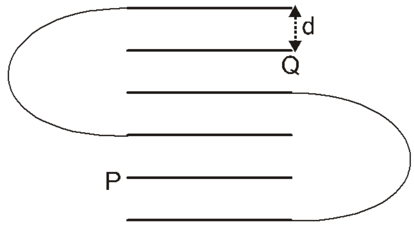
$C_{eq} = \frac{4\pi\ \epsilon_0\ kabc}{ka(b-c)+b(c-a)} = \frac{4\pi\ \epsilon_0\ (3)\ (a)\ (3a)\ (2a)}{3a\ \ (3a-2a)+3a(2a-a)}$

$C_{eq} = 3\ (4\pi\epsilon_0 a\ )$

= 3 (capacitance of capacitor of isolated conducting shell of radius a)

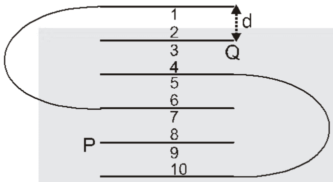
$\Rightarrow$ n = 3

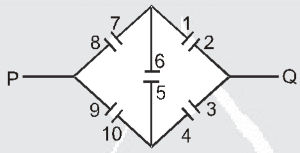
17. Six identical parallel metallic large plates are located in air at equal distances d to neighbouring plates. The area of each plate is A. Some of the plates are connected by conducting wires to each other. The capacitance of the system of plates between two points P and Q in pF is :
(Take A = 0.05 m^2, d = 17.7 mm, ε_0 = 8.85 $\times 10^{-12}$ F/m).



Ans. **25**

Sol.

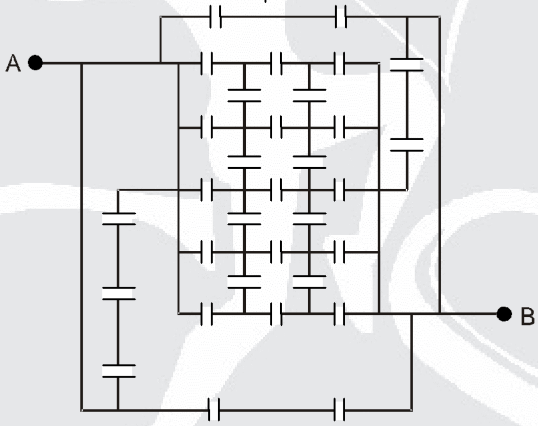




The equivalent capacitance between points PQ is capacitance between two neighboring plates by wheat stone bridge.

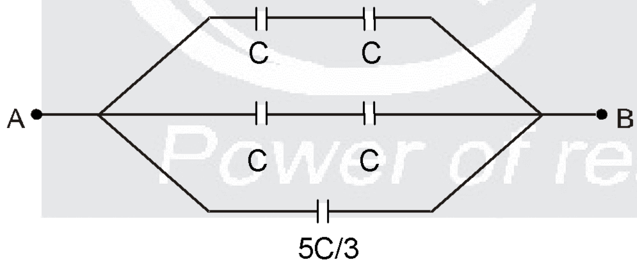
$$C_{eq} = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 0.05}{17.7 \times 10^{-3}} \text{ F} = 25 \text{ pF}.$$

18. 32 capacitors are connected in a circuit as shown in the figure. The capacitance of each capacitor is 3μF. Find equivalent capacitance across AB in μF.



Ans. **8**

Sol. C_{eq} = 8C/3 = 8μF



19. A parallel plate capacitor is to be designed which is to be connected across 1 kV potential difference. The dielectric material which is to be filled between the plates has dielectric constant K = 6π and dielectric strength 10^7 V/m.
For safely the electric field is never to exceed 10% of the dielectric strength. With such specifications, if we want a capacitor of capacitance 50 pF, what minimum area (in mm^2) of plates is required for safe working? (use $\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ in MKS)

Ans. 300

Sol. $E < 10^6 \Rightarrow \frac{10^3}{d} < 10^6$

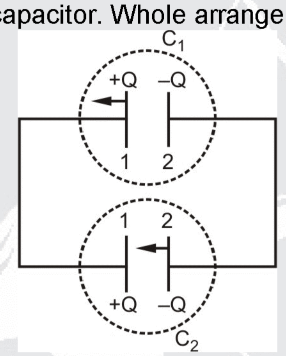
$d > 10^{-3}\ m^2 \Rightarrow C = \frac{k\varepsilon_0 A}{d}$

$d = \frac{k\varepsilon_0 A}{C} > 10^{-3}$

$A > \frac{10^{-3} \times C}{k\varepsilon_0} \Rightarrow A > \frac{10^{-3} \times 50 \times 10^{-12}}{(6\pi) \times \left(\frac{1}{36\pi} \times 10^{-9}\right)} = 300\ mm^2$

Matrix Match Type :

20. Identical capacitors A and B, each plate area S and separation between the plates d, are connected in parallel. Charge Q is given to each capacitor. Whole arrangement is shown in figure.



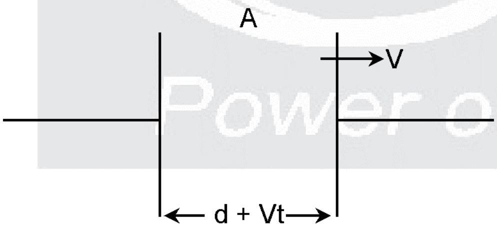
Now Keeping one plate of capacitor B fixed another plate is moved towards the first plate with very small constant velocity V from t = 0 and keeping one plate of A fixed another plate is moved away from the first plate with small constant velocity V from t = 0.

Match the proper entries from column-II to column-I for above described system.

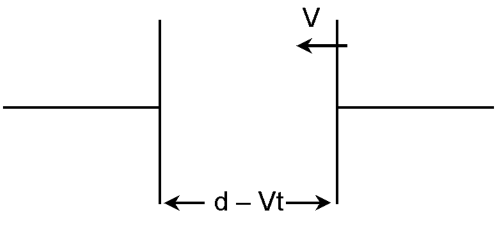
Column-I	Column-II
(A) Charge on capacitor A as function of time	(P) $\frac{(d-Vt)Q}{d}$
(B) Charge on capacitor B as function of time	(Q) $\frac{(d+Vt)Q}{d}$
(C) Current in the circuit as function of time	(R) decreases
(D) Ratio of electrostatic potential energy Stored between the plates of capacitor A and between the plates of capacitor B as function of time.	(S) $\frac{QV}{d}$

Ans. **(A) – P, R ; (B) – Q ; (C) – S ; (D) – R**

Sol.



$C_A = \frac{\varepsilon_0 S}{d + Vt}$



$C_B = \frac{\varepsilon_0 S}{d - Vt}$

$$\frac{C_A}{C_B} = \frac{d - Vt}{d + Vt} \; ; \; \frac{Q_A}{Q_B} = \frac{C_A}{C_B} = \frac{d - Vt}{d + Vt} \; ; Q_A + Q_B = 2Q$$

$$Q_A = \frac{(d - Vt)Q}{d}$$

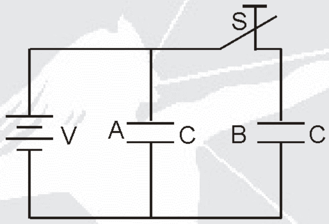
$$Q_B = \frac{(d + Vt)Q}{d}$$

$$i = \frac{dQ_B}{dt} = -\frac{dQ_A}{dt} = \frac{QV}{d}$$

$$\frac{U_A}{U_B} = \frac{C_A}{C_B} = \frac{d - Vt}{d + Vt}$$

Subjective Type Questions :

21. The figure shows two identical parallel plate capacitors connected to a battery with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant (or relative permittivity) 3. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric.



Ans. $\frac{3}{5}$

Sol. Before opening the switch potential difference across both the capcitors is V, as they are in paralle. Hence, energy stored in them is,

$U_A = U_B = \frac{1}{2}CV^2$ $\quad \therefore U_{Total} = CV^2 = U_i$ .... (1)

After opening the switch, potential difference across it is V and its capacity is 3C

$\therefore \quad U_A = \frac{1}{2}(3C)V^2 = \frac{3}{2}CV^2$

In case of capacitor B, charge strored in it is q = CV and its capacity is also 3C. Therefore,

$$U_B = \frac{q^2}{2(3C)} = \frac{CV^2}{6}$$

$\therefore U_{Total} = \frac{3CV^2}{2} + \frac{CV^2}{6} = \frac{10}{6}CV^2 = \frac{5CV^2}{3} = U_f$ .... (2)

From Eqs.(1) and (2) $\quad \frac{U_i}{U_f} = \frac{3}{5}$

22. Calculate the capacitance of a parallel plate condenser, with plate area A and distance between plates d, when filled with a dielectric whose dielectric constant varies as;

$K(x) = 1 + \frac{\beta x}{\epsilon_0} \quad 0 < x < \frac{d}{2}$; $\quad K(x) = 1 + \frac{\beta}{\epsilon_0}(d - x) \quad \frac{d}{2} < x < d.$

For what value of β would the capacity of the condenser be twice that when it is without any dielectric?

Ans. $C = \dfrac{A\beta}{2\ell n\left(1 + \frac{\beta d}{2\epsilon_0}\right)}$, $\quad \beta d = 4\epsilon_0 \ell n\left(1 + \frac{\beta d}{2\epsilon_0}\right)$

Solution of this equation gives required value of β .

Sol. $\frac{1}{dC} = \int \frac{dx}{K(x) \;.\; A\epsilon_0} = \int_0^{d/2} \frac{dx}{\left[1 + \frac{\beta x}{\epsilon_0}\right] A\epsilon_0} + \int_{\frac{d}{2}}^{d} \frac{dx}{\left[1 + \frac{\beta x}{\epsilon_0}(d - x)\right] A\epsilon_0}$

$$\int \frac{1}{dC} = \frac{\frac{1}{A\in_0}\ \ell n\left[1+\frac{\beta x}{\in_0}\right]_0^{d/2}}{\frac{\beta}{\in_0}} + \frac{\frac{1}{A\in_0}\ \ell n\left[1+\frac{\beta}{\in_0}(d-x)\right]_{d/2}^{d}}{-\frac{\beta}{\in_0}} \ ..$$

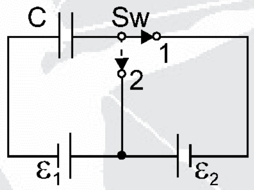
$$\int \frac{1}{dC} = \frac{1}{A\beta}\ell n\left[1+\frac{\beta d}{2\in_0}\right] + \frac{1}{A\beta}\ \ell n\left[1+\frac{\beta d}{2\in_0}\right]$$

$$C_{eq} = \frac{A\beta}{2\ell n\left[1+\frac{\beta d}{2\in_0}\right]}$$

Now $C_{eq} = 2C_0$ (C_0 = capacitance when it is without any dielectric)

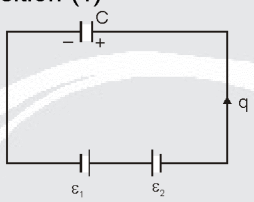
$$= \frac{A\beta}{2\ell n\left[1+\frac{\beta d}{2\in_0}\right]} = \frac{2\in_0 A}{d} \qquad \beta d = 4\in_0 \ell n\left(1+\frac{\beta d}{2\in_0}\right)$$

23. What amount of heat will be generated in the circuit shown in the figure, after the switch Sw is shifted from position 1 to position 2?



Ans. $Q = \frac{1}{2}C\varepsilon_2^2$. It is remarkable that the result obtained is independent of ε_1.

Sol. When S is at position (1)



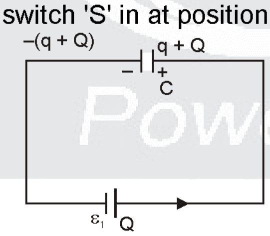
By kVL

$\Rightarrow \qquad \varepsilon_1 - \varepsilon_2 - \frac{q}{C} = 0$

$q = (\varepsilon_1 - \varepsilon_2)C$ ....(1)

$\therefore \qquad$ Energy stored $= \frac{1}{2}C(\varepsilon_1 - \varepsilon_2)^2 = \frac{q^2}{2C}$

When switch 'S' in at position (2)



By kVL ;

$\varepsilon_1 = \frac{q+Q}{C}$ Put q from (1)

$\varepsilon_1 = \frac{(\varepsilon_1 - \varepsilon_2)C + Q}{C}$

$Q = \varepsilon_2 C$ ....(2)

Energy stored $= \frac{(Q+q)^2}{2C}$

$\therefore \qquad$ Work done by battery ε_1

$W = \varepsilon_1 Q = \varepsilon_1\varepsilon_2 C$

$\therefore$ Heat produced ;

$H = W - \Delta U$

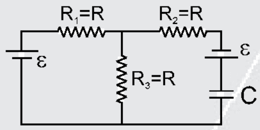
$\therefore$ $H = \varepsilon_1\varepsilon_2 C - (U_f - U_i)$

$$H = \varepsilon_1\varepsilon_2 C - \left[\frac{(Q+q)^2}{2C} - \frac{q^2}{2C}\right]$$

Put Q and q from (1) & (2)

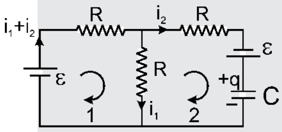
$\therefore$ $H = \frac{1}{2} C\varepsilon_2^{\ 2}$

24. In the figure shown the capacitor is initially uncharged. Find the current in R_3 (= R) at time 't'.



Ans. $i = \frac{\varepsilon}{2R}\left(1 - e^{-\frac{2t}{3RC}}\right)$

Sol.



Applying Kirchoff's law in
Loop1

$$\varepsilon - (i_1 + i_2) R - i_1 R = 0 \quad ...(1)$$

Loop 2

$$-i_2 R + \varepsilon - \frac{q}{C} + i_1 R = 0 ....(2)$$

eliminating i_1 from (1) and (2)

$$\varepsilon - \frac{q}{C} - i_2 R + \frac{\varepsilon - i_2 R}{2} = 0 \quad \text{or} \quad \frac{3\varepsilon}{2} - \frac{q}{C} - \frac{3}{2} i_2 R = 0$$

$$i_2 = \frac{dq}{dt}$$

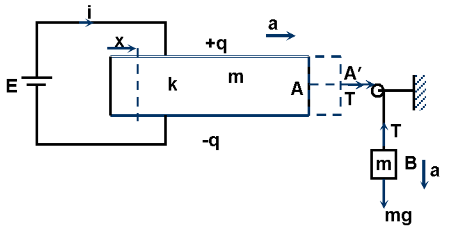
$$\Rightarrow \quad \frac{3C\varepsilon - 2q}{2C} = \frac{3}{2} R \frac{dq}{dt} \quad \text{or} \quad \int_0^q \frac{dq}{3C\varepsilon - 2q} = \int_0^t \frac{dt}{3RC}$$

$$\text{or} \quad -\frac{1}{2}\ln\left(\frac{3C\varepsilon - 2q}{3C\varepsilon}\right) = \frac{t}{3RC} \quad \text{or} \quad 1 - \frac{2q}{3C\varepsilon} = e^{-\frac{2t}{3RC}}$$

$$\Rightarrow \quad q = \frac{3C\varepsilon}{2}\left(1 - e^{-\frac{2t}{3RC}}\right) \qquad i_2 = \frac{dq}{dt} = \left(\frac{\varepsilon}{R}\right) e^{-\frac{2t}{3RC}}$$

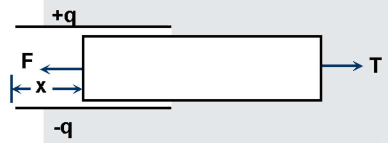
from (1), $\quad i_1 = \frac{\varepsilon - i_2 R}{2R} = \frac{\varepsilon}{2R}\left(1 - e^{-\frac{2t}{3RC}}\right)$

25. A smooth dielectric slab A of mass m and dielectric constant k is placed between the plates of a parallel plate capacitor and connected to a block B of equal mass m through a string and pulley arrangement as shown in the figure. The capacitor plates with separation δ and width b are connected to a battery of emf E as shown in the figure.



The pulley and string are massless and $E = \sqrt{\dfrac{mg\delta}{b(k-1)\varepsilon_0}}$. Find the current supplied by the battery as a function of time t.

Sol. Let the length of the plate be ℓ, width be b and plate separation be δ. The capacitance, C as a function of x is given by



$$C(x) = \frac{\varepsilon_0 bx}{\delta} + \frac{k\varepsilon_0 b(\ell - x)}{\delta}$$

The force on the dielectric is,

$$F_x = \frac{1}{2}E^2\frac{dC}{dx} = -\frac{\varepsilon_0}{2}(k-1)E^2\frac{b}{\delta}$$

For the two blocks, we can write,

$$-T + mg = ma \qquad \text{. . . (1)}$$

$$\text{and } T - \frac{\varepsilon_0}{2}(k-1)E^2\frac{b}{\delta} = ma \qquad \text{. . . (2)}$$

$$\therefore \quad \frac{d^2x}{dt^2} = a$$

$$= \frac{1}{2m}\left[mg - \frac{\varepsilon_0}{2}(k-1)\frac{E^2 b}{\delta}\right] = \frac{g}{4}$$

or, $x = \dfrac{1}{2}\left(\dfrac{g}{4}\right)t^2$ (Since the block starts from rest.)

Now, q = CE

$$\text{and } i = \frac{dq}{dt} = \frac{d}{dt}C(x)E$$

$$= \frac{dC}{dx}E.\frac{dx}{dt} = -\frac{\varepsilon_0 b}{\delta}(k-1)E\left(\frac{g}{4}t\right)$$

$$= -\frac{(k-1)E}{4}\frac{\varepsilon_0 b}{\delta}gt = -\frac{g}{4}t \times \frac{mg}{E}$$

$$i(t) = -\frac{mg^2 t}{4E}.$$