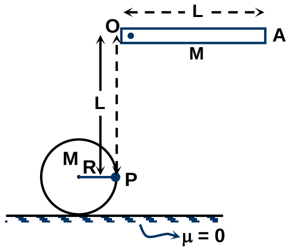
competishun (Power of real gurus) **PHYSICS** TARGET : JEE- Advanced 2023	**CAPS-8** **ROTATIONAL DYNAMICS-1**

SCQ (Single Correct Type) :

1. Moment of inertia of a uniform solid cone about an axis passing through its centre of gravity and parallel to its base is (m is mass of cone, its height is h and its radius R = h)

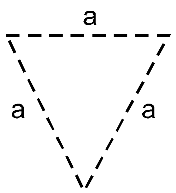
(A) $\frac{10}{3}MR^2$ (B) $\frac{3}{5}MR^2$ (C) $\frac{3}{10}MR^2$ (D) $\frac{3}{16}MR^2$

2. A slender rod of mass M and length L hinged at O is kept horizontal and then released. The other end of the rod strikes a solid sphere of mass M and radius R (at point P) kept on a smooth horizontal surface. The points O and P are on the same vertical line. After the collision, the rod comes to rest. The angular speed of the sphere after the collision is



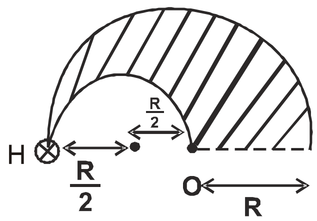
(A) $\sqrt{\frac{3g}{L}}$ (B) $\frac{R}{L}\sqrt{\frac{3g}{L}}$ (C) $\frac{L}{R}\sqrt{\frac{3g}{L}}$ (D) zero

3. All sides of an equilateral triangle are diameter of three identical uniform semicircular rings each of mass m. Plane of each ring is perpendicular to the plane of paper. Then moment of inertia of this system of three semicircular rings about an axis through centroid of triangle and perpendicular to plane of paper is :



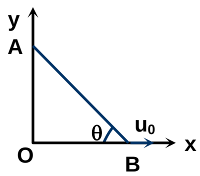
(A) $\frac{5ma^2}{24}$ (B) $\frac{5ma^2}{16}$ (C) $\frac{5ma^2}{8}$ (D) $\frac{5ma^2}{6}$

4. A uniform circular plate of radius R, from which a semicircular portion of radius $\frac{R}{2}$ is removed, shown in figure. The moment of inertia about an axis passing through its end 'H' and perpendicular to the plane of plate is [Assume mass of remaining portion of the plate is m]



(A) $\frac{13}{8}mR^2$ (B) $\frac{15}{8}mR^2$ (C) $\frac{3}{8}mR^2$ (D) $\frac{mR^2}{8}$

5. The end B of the rod AB which makes angle θ with the floor is being pulled with a constant velocity u_0 as shown. The length of the rod is ℓ. Which of the following is/are correct?



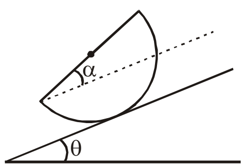
(A) Velocity of end A is $\frac{7}{3}u_0$ downwards when $\theta = 37°$.

(B) Angular velocity of the rod is $\frac{5}{3}\frac{u_0}{\ell}$ when $\theta = 37°$.

(C) Angular velocity of the rod is constant.

(D) Velocity of end A is constant.

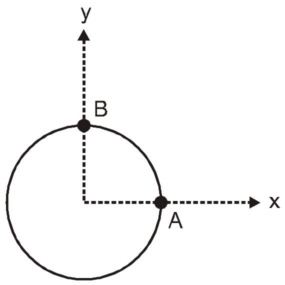
6. A uniform thin hemispherical shell is kept static on an inclined plane of angle θ = 30º as shown. If the surface of the inclined plane is sufficiently rough to prevent sliding then the angle α made by the plane of hemisphere with inclined plane is :



(A) value of μ is needed (B) 30º

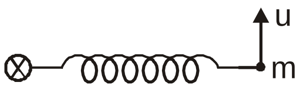
(C) 45º (D) 60º

7. A non-uniform disc of mass m and radius R, hinged at some point and performing pure rotation with respect to hinge, in horizontal plane with an angular velocity ω. At certain instant center of the disc is at origin in the mentioned co-ordinate system and velocity of particle A is $\vec{V}_A = -\frac{\omega R}{4}\left(3\hat{i} - 4\hat{j}\right)$ m/s. Velocity of particle B at the given instant is –



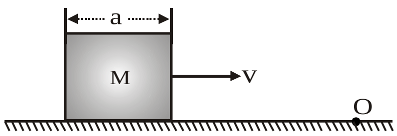
(A) $\frac{4}{5}\omega R\left(\hat{i} - \hat{j}\right)$ (B) $\frac{4}{5}\omega R\left(\hat{j} - \hat{i}\right)$ (C) $-\frac{7\omega R}{4}\hat{i}$ (D) $-\frac{4\omega R}{5}\hat{i}$

8. In the figure shown a particle of mass 'm' which is tied to an end of massless spring of natural length ℓ is given a velocity 'u' parpendicular to the spring. Initially the spring is undeformed. The maximum extension of the spring is 'ℓ'. The motion is on a horizontal smooth surface. The spring constant is



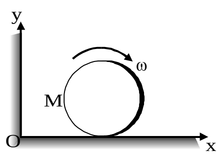
(A) $\frac{mu^2}{\ell^2}$ (B) $\frac{mu^2}{8\ell^2}$ (C) $\frac{mu^2}{4\ell^2}$ (D) $\frac{3mu^2}{4\ell^2}$

9. A cubical block of side 'a' moving with velocity v on a horizontal smooth plane as shown. It hits a ridge at point O. The angular speed of the block after it hits O is :-



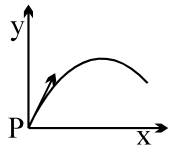
(A) $\frac{3v}{4a}$ (B) $\frac{3v}{2a}$ (C) $\frac{\sqrt{3}}{\sqrt{2}a}$ (D) zero

10. A disc of mass M and radius R is rolling (pure) with is angular speed ω on a horizontal plane as shown. The magnitude of angular momentum of the disc about the origin O is :-



(A) $\left(\frac{1}{2}\right)MR^2\omega$ (B) $MR^2\omega$ (C) $\left(\frac{3}{2}\right)MR^2\omega$ (D) $2MR^2\omega$

11. At time t = 0, a ball of mass 'm' is thrown vertically from a point P (x = 0, y = 0) with initial velocity $(v_x\hat{i}+v_y\hat{j})$. The acceleration due to gravity is $\vec{g}=-g\hat{j}$. The angular momentum vector $\vec{L}(t)$ of the ball about the point P at time t is



(A) $-\frac{mv_x gt^2}{2}\hat{k}$

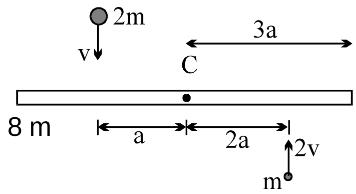
(B) $2mv_x v_y t\hat{k}$

(C) $(-2mv_x v_y t+\frac{v_x gt^2}{2})\hat{k}$

(D) $-2mv_x v_y t\hat{i}+\frac{mv_x gt^2}{2}\hat{j}$

MCQ (One or more than one correct) :

12. A uniform bar of length 6a and mass 8m lies on a smooth horizontal table. Two point masses m and 2m moving in the same horizontal plane with speeds 2v, and v, respectively, strike the bar (as shown in figure) and stick to the bar after collision. Denoting angular velocity, total energy and velocity of centre of mass by ω, E and V_c respectively, we have after collision



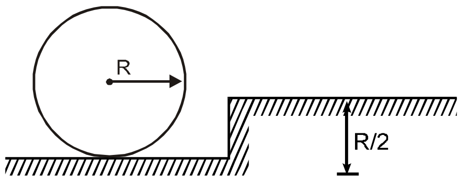
(A) $V_c = 0$

(B) $\omega = \frac{3v}{5a}$

(C) $\omega = \frac{v}{5a}$

(D) $E = \frac{3mv^2}{5}$

13. A wheel (to be considered as a ring) of mass m and radius R rolls without sliding on a horizontal surface with constant velocity v. It encounters a step of height R/2 at which it ascends without sliding.



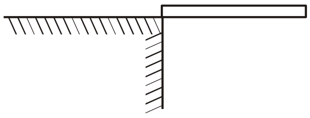
(A) the angular velocity of the ring just after it comes in contact with the step is 3v/4R

(B) the normal reaction due to the step on the wheel just after the impact is $\frac{mg}{2}+\frac{9\,mv^2}{16\,R}$

(C) the normal reaction due to the step on the wheel increases as the wheel ascends

(D) the friction will be absent during the ascent.

14. A uniform rod of length *l* is falling down with a velocity v_0 when one of it's end hits a fixed edge as shown.



(A) If the collision is elastic, the centre of mass may have upwards velocity just after the collision

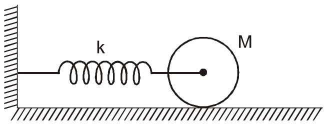
(B) If the collision is partially elastic, the centre of mass may come to rest just after the collision

(C) If the collision is perfectly inelastic, the centre of mass has a downward velocity just after the collision

(D) If the collision is elastic, the centre of mass has a downward velocity just after the collision

Comprehension Type Question:

A solid cylinder attached with a horizontal massless spring (see figure) can roll without slipping along a horizontal surface. If the system is released from rest at a position in which spring is stretched by 0.3 m.



15. Period of oscillation of c.o.m of cylinder is

(A) $2\pi\sqrt{\frac{M}{k}}$ (B) $2\pi\sqrt{\frac{3M}{2k}}$ (C) $2\pi\sqrt{\frac{2M}{3k}}$ (D) $2\pi\sqrt{\frac{M}{2k}}$

16. If k = 20 N/m, Rotational K.E. of cylinder as it passes through mean position is

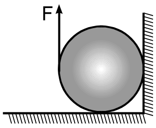
(A) 0.3 J (B) 0.45 J (C) 0.6 J (D) 0.9 J

17. Translation K.E. of cylinder as it passes through mean position is

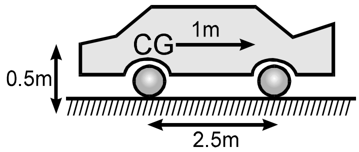
(A) 0.3 J (B) 0.45 J (C) 0.6 J (D) 0.9 J

Numerical based Questions :

18. Figure shows a vertical force F that is applied tangentially to a uniform cylinder of weight W. The coefficient of static friction between the cylinder & all surfaces is 0.5. Find in terms of W. the maximum force that can be applied without causing the cylinder to rotate.

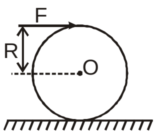
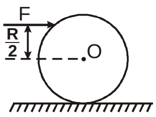
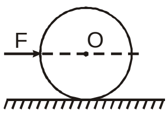
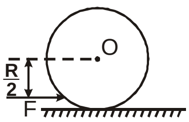


19. A 2000 kg car is driven by an engine which provides torque to two rear driving wheels. The wheel base of the car is 2.5 m. The centre of the mass of the car is located 1 m from the front wheel at a height of 0.5 m from the ground. If μ = 0.7 is the friction with the road, calculate the minimum time taken to accelerate the car to 60 kmph. Neglect friction on front wheels.



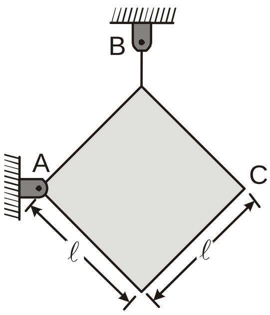
Matrix Match Type :

20. A uniform solid cylinder of mass m and radius R is placed on a rough horizontal surface where friction is sufficient to provide pure rolling. A horizontal force of magnitude F is applied on cylinder at different positions with respect to its centre O in each of four situations of column-I, due to which magnitude of acceleration of centre of mass of cylinder is a. Match the appropriate results in column-II for conditions of column-I.

Column-I	**Column-II**
(A) 	(p) Friction force on cylinder will not be zero.
(B) 	(q) $a = \frac{F}{m}$
(C) 	(r) $a \neq \frac{F}{m}$
(D) 	(s) the direction of friction force acting on cylinder is towards left

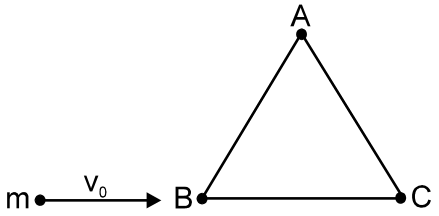
Subjective Type Questions :

21. A uniform square plate of mass m is supported as shown. If the cable suddenly breaks, determine just after that moment. The angular acceleration of the plate.



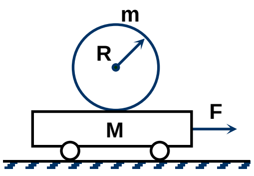
22. The angular momentum of a particle relative to a certain' point O varies with time as $\vec{M} = \vec{a} + \vec{b}t^2$, where $\vec{a}$ and $\vec{b}$ are constant vectors, with $\vec{a} \perp \vec{b}$. Find the force moment N relative to the point O acting on the particle when the angle between the vectors N and M equals 45°.

23. Three particles A, B, C of mass m each are joined to each other by massless rigid rods to form an equilateral triangle of side a. Another particle of mass m hits B with a velocity v_0 directed along BC as shown. The colliding particle stops immediately after impact.



Calculate the time required by the triangle ABC to complete half revolution in its subsequent motion.

24. Determine the maximum horizontal force F that can be applied to the plank of mass M for which the solid sphere does not slip as it begins to roll on the plank. The sphere has mass m and radius R. The coefficient of static and kinetic friction between the sphere and the plank are μ_s and μ_k respectively.



25. A right circular uniform solid cone of mass m and semi vertical angle α is placed over a rough horizontal surface. Assuming friction to be sufficient to prevent slipping, find the minimum force that can be applied at the tip of the cone so that it just topples.