

PHYSICS

TARGET : JEE- Advanced 2023

CAPS-25

GRAVITATION

SCQ (Single Correct Type) :

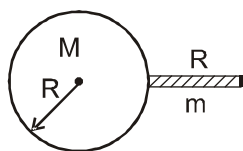
1. A cavity of radius $R/2$ is made inside a solid sphere of radius R . The centre of the cavity is located at a distance $R/2$ from the centre of the sphere. The gravitational force on a particle of mass ' m ' at a distance $R/2$ from the centre of the sphere on the line joining both the centres of sphere and cavity is (opposite to the centre of cavity). [Here $g = GM/R^2$, where M is the mass of the solid sphere]

(A) $\frac{mg}{2}$ (B) $\frac{3mg}{8}$ (C) $\frac{mg}{16}$ (D) none of these

2. Maximum height reached by a rocket fired with a speed equal to 50% of the escape velocity from earth's surface is given by (R is radius of earth) :

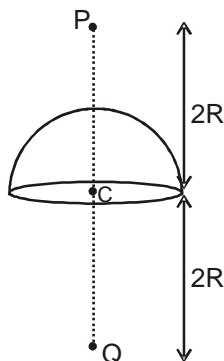
(A) $\frac{R}{3}$ (B) $3R$ (C) $\frac{2R}{3}$ (D) R

3. A uniform thin rod of mass m and length R is placed normally on surface of earth as shown. The mass of earth is M and its radius is R . Then the magnitude of gravitational force exerted by earth on the rod is



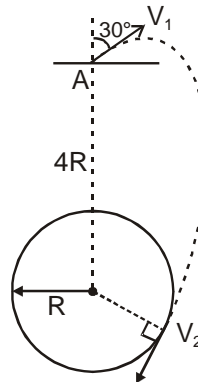
(A) $\frac{GMm}{2R^2}$ (B) $\frac{GMm}{4R^2}$ (C) $\frac{4GMm}{9R^2}$ (D) $\frac{GMm}{8R^2}$

4. If Gravitational field due to uniform thin hemispherical shell at point P is I , then the magnitude of gravitational field at Q is (Mass of hemisphere is M , radius R) -



(A) $\frac{GM}{2R^2} - I$ (B) $\frac{GM}{2R^2} + I$ (C) $\frac{GM}{4R} - I$ (D) $2I - \frac{GM}{2R^2}$

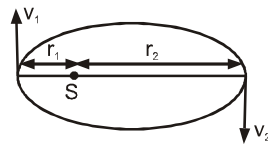
5. Altitude at which acceleration due to gravity decreases by 0.1% approximately: (Radius of earth = 6400 km)
 (A) 3.2 km (B) 6.4 km (C) 2.4 km (D) 1.6 km
6. A particle is projected from point A, that is at a distance $4R$ from the centre of the earth, with speed V_1 in a direction making 30° with the line joining the centre of the earth and point A, as shown. Find the speed V_1 if particle passes grazing the surface of the earth. Consider gravitational interaction only between these two. (use $\frac{GM}{R} = 6.4 \times 10^7 \text{ m}^2/\text{s}^2$)



- (A) $4\sqrt{2}$ km/s (B) $3\sqrt{2}$ km/s (C) $6\sqrt{2}$ km/s (D) $5\sqrt{2}$ km/s

MCQ (One or more than one correct) :

7. A planet is revolving in an elliptical orbit around the sun as shown in figure. The areal velocity (area swapped by the radius vector with respect to sun in unit time) is :

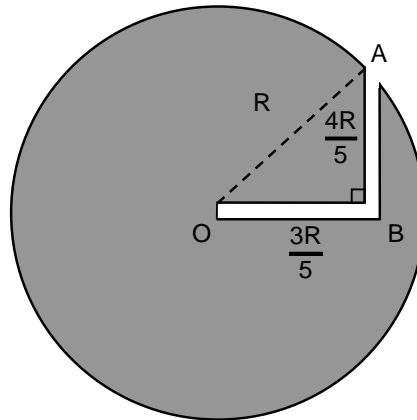


- (A) $r_1 v_1$ (B) $r_2 v_2$ (C) $\frac{1}{2} \frac{v_1 r_1^2}{r_2}$

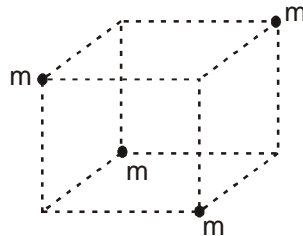
(D) dependent on the position of planet from sun

8. Suppose the earth suddenly shrinks in size, still remaining spherical and mass unchanged (All gravitational forces pass through the centre of the earth).
- (A) The days will become shorter.
 (B) The kinetic energy of rotation about its own axis will increase
 (C) The duration of the year will increase.
 (D) The magnitude of angular momentum about its axis will increase.

9. A narrow smooth tunnel of L-shape is made into earth up to center as shown in the figure. A small ball which just fits in tunnel is released from 'A'. The collision of ball is inelastic. (g is acceleration due to gravity on surface of planet)

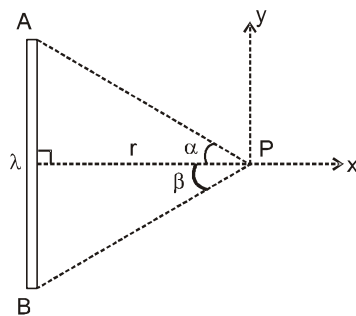


- (A) time to fall from A to B is $\frac{\pi}{2} \sqrt{\frac{R}{g}}$ (B) time to fall from B to O is $\frac{\pi}{4} \sqrt{\frac{R}{g}}$
- (C) speed just before hitting B is $\frac{4R}{5} \sqrt{\frac{g}{R}}$ (D) speed just before hitting O is $\frac{R}{5} \sqrt{\frac{g}{R}}$
10. Four point masses each of mass m are kept at four corners of a cube of side length ' a '. Then



- (A) the gravitational potential energy of system is $= -\frac{3\sqrt{2}Gm^2}{a}$.
- (B) the gravitational potential energy of system is $= -\frac{4\sqrt{2}Gm^2}{a}$.
- (C) the magnitude of force on one of the particle due to the remaining masses is equal to $\frac{\sqrt{6}Gm^2}{a^2}$.
- (D) the magnitude of force on one of the particle due to others is equal to $\sqrt{\frac{3}{2}} \frac{Gm^2}{a^2}$.

11. A uniform rod AB lying in xy plane has linear mass density λ . A point P lies at perpendicular distance r from the rod and E_x and E_y are the components of the gravitational field at point P along x, y direction respectively then



- (A) If $\alpha = \beta$ then $E_y = 0$
 (B) If $\alpha = \beta = 45^\circ$, then $E_x = \frac{2G\lambda}{r}$
 (C) Gravitational potential at point P is not defined if $\alpha = \beta = \frac{\pi}{2}$
 (D) If $\alpha = \beta$ then $E_x = 0$ and $E_y = 0$

Comprehension Type Question:

The minimum and maximum distances of a satellite from the center of earth are $2R$ and $4R$ respectively, here R is the radius of earth and M is the mass of the earth.

12. The minimum and maximum speeds are :

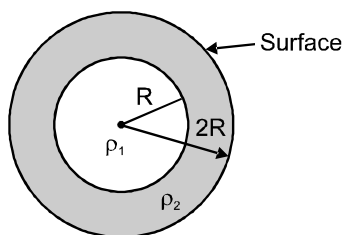
(A) $\sqrt{\frac{GM}{9R}}, \sqrt{\frac{2GM}{R}}$ (B) $\sqrt{\frac{GM}{5R}}, \sqrt{\frac{3GM}{2R}}$ (C) $\sqrt{\frac{GM}{6R}}, \sqrt{\frac{2GM}{3R}}$ (D) $\sqrt{\frac{GM}{3R}}, \sqrt{\frac{5GM}{2R}}$

13. The radius of curvature at the point of minimum distance is :

(A) $\frac{8R}{3}$ (B) $\frac{5R}{3}$ (C) $\frac{4R}{3}$ (D) $\frac{7R}{3}$

Numerical based Questions :

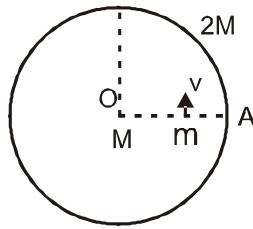
14. An earth satellite is revolving in a circular orbit of radius a with a velocity v_0 . A gun in the satellite is directly aimed toward earth. A bullet is fired from the gun with muzzle velocity $\frac{v_0}{2}$. Find the ratio of distance of farthest and closest approach of bullet from centre of earth. (Assume that mass of the satellite is very-very large with respect to the mass of the bullet)
15. A planet is made of two materials of density ρ_1 and ρ_2 as shown in figure.



The acceleration due to gravity at surface of planet is same as a depth 'R'. The ratio of

$\frac{\rho_1}{\rho_2}$ is $\frac{x}{3}$. Find the value of x .

16. A particle of mass M is fixed at the centre (point O) of a uniform shell of mass $2M$ and radius R . Another particle of mass m ($\ll M$) is projected from mid point of radius OA with velocity V perpendicular to the radius. The maximum value of v so that the particle does not strike the surface of the shell is $\sqrt{\frac{NGM}{3R}}$, calculate N .



Subjective Type Questions :

17. A planet of mass ' m ' revolves around the sun (of mass M) in an elliptical orbit. The semi major axis of the orbit is ' a '. Find out the total mechanical energy of the planet.
18. A star in its prime age is said to be under equilibrium due to gravitational pull and outward radiation pressure (p). Consider the shell of thickness dr as in the figure of question (13). If the pressure on this shell is dp then the correct equation is (G is universal gravitational constant)