

# PHYSICS

TARGET : JEE- Advanced 2023

# CAPS-17

## Nuclear Physics

### SCQ (Single Correct Type) :

- Nuclei of radioactive element A are produced at rate ' $t^2$ ' at any time  $t$ . The element A has decay constant  $\lambda$ . Let  $N$  be the number of nuclei of element A at any time  $t$ . At time  $t = t_0$ ,  $\frac{dN}{dt}$  is minimum. Then the number of nuclei of element A at time  $t = t_0$  is

(A)  $\frac{\lambda t_0^2 - 2t_0}{\lambda^2}$       (B)  $\frac{t_0 - \lambda t_0^2}{\lambda^2}$       (C)  $\frac{2t_0 - \lambda t_0^2}{\lambda}$       (D) None of these
- A radioactive sample decays by  $\beta$ -emission. In first two seconds ' $n$ '  $\beta$ -particles are emitted and in next 2 seconds, ' $0.25n$ '  $\beta$ -particles are emitted. The half life of radioactive nuclei is

(A) 2 sec      (B) 4 sec      (C) 1 sec      (D) none
- A factory produces a radionuclide at a constant rate  $R$ . Initially there were no nuclide present. Then activity of the sample produced by factory just after an average life of nuclide passes away is :

(A)  $R$       (B)  $Re^{-1}$       (C)  $R(1 - e^{-1})$       (D)  $\frac{R}{2} (1 - e^{-1})$
- A radionuclide with decay constant  $\lambda$  is produced in a reactor at a constant rate  $\alpha$  nuclei per second. During each decay energy  $E_0$  MeV is released. If the production of radio nuclides started at  $t = 0$ , total energy released upto time  $t$  is given by equation

(A)  $E_0\alpha (1 - e^{-\lambda t})$       (B)  $E_0 \frac{\alpha}{\lambda} (1 - e^{-\lambda t})$       (C)  $E_0(\alpha t - \frac{\alpha}{\lambda} e^{-\lambda t})$       (D)  $E_0 \left( \alpha t - \frac{\alpha}{\lambda} (1 - e^{-\lambda t}) \right)$
- The count rate observed from a radioactive source at ' $t$ ' second was  $N_0$  and at  $4t$  second it was  $\frac{N_0}{16}$ . The count rate observed, at  $\left(\frac{11}{2}\right)t$  second will be

(A)  $\frac{N_0}{128}$       (B)  $\frac{N_0}{64}$       (C)  $\frac{N_0}{32}$       (D) None of these
- The count rate from  $100 \text{ cm}^3$  of a radio-active liquid is  $C$ . Some of this liquid is now discarded. The count rate of the remaining liquid is found to be  $C/10$  after three half-lives. The volume of the remaining liquid in  $\text{cm}^3$  is

(A) 20      (B) 40      (C) 60      (D) 80

7. An isolated nucleus which was initially at rest, disintegrates into two nuclei due to internal nuclear forces and no  $\gamma$  rays are produced. If the ratio of their kinetic energy is found to be  $\frac{64}{27}$  then :
- (A) Ratio of their de-broglie wavelength is  $\frac{\sqrt{64}}{\sqrt{27}}$  respectively
- (B) Ratio of their speed is  $\frac{64}{37}$  respectively
- (C) Ratio of their nuclear radius is  $\frac{3}{4}$  respectively
- (D) Ratio of their nuclear radius is  $\frac{4}{3}$  respectively

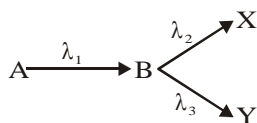
**MCQ (One or more than one correct) :**

8. Two radioactive nuclei A and B are present in equal numbers to begin with. Three day later, number of A nuclei are 3 times number of B nuclei. Choose the correct statement.
- (A)  $\lambda_B - \lambda_A = \frac{\ln 3}{3 \text{ days}}$
- (B)  $\lambda_A - \lambda_B = \frac{\ln 3}{3 \text{ days}}$
- (C) the ratio of activity rate of A and B after 3 days is  $\frac{3}{1}$
- (D) the ratio of activity rate of A and B after 3 days is less than  $\frac{3}{1}$ .
9.  ${}^{64}_{29}\text{Cu}$  can decay by  $\beta^-$  or  $\beta^+$  emission, or electron capture. It is known that  ${}^{64}_{29}\text{Cu}$  has a half life of 12.8 hrs with 40% probability of  $\beta^-$  decay, 20% probability of  $\beta^+$  decay and 40% probability of electron capture. The mass of  ${}^{64}_{29}\text{Cu}$  is 63.92977 amu while  ${}^{64}_{30}\text{Zn}$  is 63.92914 amu and  ${}^{64}_{28}\text{Ni}$  is 63.92796 amu. What is the half life for electron capture?
- (A) 5.12 Hrs.                      (B) 32 Hrs.                      (C) 2.56 Hrs.                      (D) 16 Hrs.

**Comprehension Type Question:**

**Comprehension#1**

The rate at which a particular decay process occurs in a radio active sample, is proportional to the number of radio active nuclei present. If N is the number of radio active nuclei present at some instant, the rate of change of N is  $\frac{dN}{dt} = -\lambda N$ .

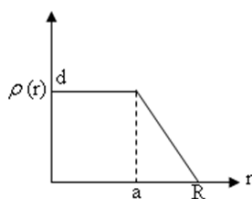


Consider radioactive decay of A to B which may further decay either to X or to Y,  $\lambda_1, \lambda_2$  and  $\lambda_3$  are decay constants for A to B decay, B to X decay and B of Y decay respectively. If at  $t = 0$  number of nuclei of A, B, X and Y are  $N_0, N_0, \text{zero and zero}$  respectively and  $N_1, N_2, N_3, N_4$  are number of nuclei A, B, X and Y at any instant.

10. Rate of accumulation of B of any instant will be –  
 (A)  $N_1\lambda_1 + N_2\lambda_2 + N_3\lambda_3$  (B)  $N_1\lambda_1 - N_3\lambda_2 - N_4\lambda_3$   
 (C)  $N_1\lambda_1 - N_2\lambda_2 - N_2\lambda_3$  (D)  $N_1\lambda_1 + N_2\lambda_2 - N_3\lambda_3$
11. The number of nuclei of B will first increase then after a maximum value, it will decrease, if–  
 (A)  $\lambda_1 > \lambda_2 + \lambda_3$  (B)  $\lambda_1 = \lambda_2 = \lambda_3$   
 (C)  $\lambda_1 = \lambda_2 + \lambda_3$  (D) For any values of  $\lambda_1, \lambda_2$  and  $\lambda_3$
12. At  $t = \infty$ , which of the following is incorrect ?  
 (A)  $N_2 = 0$  (B)  $N_3 = \frac{N_0\lambda_2}{\lambda_2 + \lambda_3}$  (C)  $N_4 = \frac{2N_0\lambda_3}{\lambda_2 + \lambda_3}$  (D)  $N_3 + N_4 + N_1 + N_2 = 2N_0$

### Comprehension # 2

The nuclear charge ( $Ze$ ) is non-uniformly distributed within a nucleus of radius  $R$ . The charge density  $\rho(r)$  (charge per unit volume) is dependent only on the radial distance  $r$  from the centre of the nucleus, as shown. The electric field is only along radial direction



13. The electric field at  $r = R$  is :  
 (A) independent of  $a$  (B) directly proportional to  $a$   
 (C) directly proportional to  $a^2$  (D) inversely proportional to  $a$
14. For  $a = 0$ , the value of  $d$  (maximum value of  $\rho$  as shown in the figure) is :  
 (A)  $\frac{3Ze}{4\pi R^3}$  (B)  $\frac{3Ze}{\pi R^3}$  (C)  $\frac{4Ze}{3\pi R^3}$  (D)  $\frac{Ze}{3\pi R^3}$

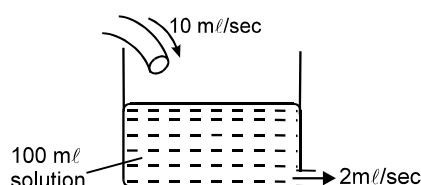
### Numerical based Questions :

15. A radioactive sample contains two radio nucleoids A and B having decay constant  $\lambda \text{ hr}^{-1}$  and  $2\lambda \text{ hr}^{-1}$ . Initially 25% of total decay comes from A. How long (in hr) will it take before 75% of total decay comes from A. [Take  $\lambda = \ln 3$ ]
16. The ratio of the components in a mixture, which consists of two elements, is to be determined. The atomic number of the elements are big, their atomic mass numbers are the same and the amount of the sample is only 58 mg. We know that both elements  $\beta$  decay when they are bombarded by neutrons. They behave similarly when absorbing neutrons. The half-life of element A is half an hour and the half-life of element B is an hour. Right after the neutron irradiation the  $\beta$  emission is measured. At this time 80 particles are measured in 2 seconds, and after an hour only 29 particles are measured during 2 seconds. Determine the mass of the element A (in mg) in the sample. Assume that spontaneous decay statistical law is obeyed by nuclei of both the elements.

17. In a sample initially there are equal number of atoms of two radioactive isotopes A and B. 3 days later the number of atoms of A is twice that of B. Half life of B is 1.5 days. What is half life of isotope A ? (in days)

**Subjective Type Questions :**

18. The nucleus of  ${}^{230}_{90}\text{Th}$  is unstable against  $\alpha$ -decay with a half-life of  $7.6 \times 10^3$  years. Write down the equation of the decay and estimate the kinetic energy of the emitted  $\alpha$ -particle from the following data :  $m({}^{230}_{90}\text{Th}) = 230.0381$  amu,  $m({}^{226}_{88}\text{Ra}) = 226.0254$  amu and  $m({}^4_2\text{He}) = 4.0026$  amu.
19. Radium being a member of the uranium series occurs in uranium ores. If the half lives of uranium and radium are respectively  $4.5 \times 10^9$  and 1620 years calculate the  $\frac{N_{\text{radium}}}{N_{\text{uranium}}}$  in Uranium ore at equilibrium.
20. Find the binding energy of a nucleus consisting of equal numbers of protons and neutrons and having the radius one and a half time smaller than that of  $\text{Al}^{27}$  nucleus. [atomic mass of  ${}^8_4\text{Be} = 8.0053$  u,  ${}^1_1\text{H} = 1.007826$ u,  ${}^1_0\text{n} = 1.008665$  u]
21. A 100 ml solution having activity 50 dps is kept in a beaker. It is now constantly diluted by adding water at a constant rate of 10 ml/sec and 2 ml/sec of solution is constantly being taken out. Find the activity of 10 ml solution which is taken out, assuming half life to be effectively very large.



22. A charged capacitor of capacity C is discharged through a resistance R. A radioactive sample decays with an average life J. If the ratio of electrostatic field energy stored in the capacitor to the activity of the radioactive sample remains constant with time then  $R = (xJ / C)$ . Where x is \_\_\_\_\_
23. A radio nuclide with half-life T days emits  $\beta^-$  particles of average kinetic energy E J. The radionuclide is used as a source in a machine which generates electric energy with efficiency 25%. The number of moles of the nuclide required to generate electrical energy at an initial rate P is  $n = \frac{yTP}{E N \ln(2)}$  where 'y' is. (Here N is Number of active nuclei in 1 mole)

- 24.** A radioactive isotope is being produced at a constant rate  $dN/dt = R$  in an experiment. The isotope has a half-life  $t_{1/2}$ . Show that after a time  $t \gg t_{1/2}$ , the number of active nuclei will become constant. Find the value of this constant. Suppose the production of the radioactive isotope starts at  $t = 0$ . Find the number of active nuclei at time  $t$ .
- 25.** Nuclei of radioactive element A are being produced at a constant rate  $\alpha$ . The element has a decay constant  $\lambda$ . At time  $t = 0$ , there are  $N_0$  nuclei of the element.
- (a) Calculate the number  $N$  of nuclei of A at time  $t$ .
- (b) If  $\alpha = 2N_0\lambda$ , calculate the number of nuclei of A after one half-life of A and also the limiting value of  $N$  as  $t \rightarrow \infty$ .