CAPS-10

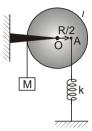
SIMPLE HARMONIC MOTION

PHYSICS

TARGET: JEE- Advanced 2023

SCQ (Single Correct Type):

In the system shown in figure find the angular frequency of small oscillations of block of mass M. Here pulley has moment of inertia I about horizontal axis passing through O. In equilibrium, line OA is horizontal



(A)
$$\frac{R}{2} \sqrt{\frac{k}{1 + MR^2}}$$
 (B) $\frac{1}{2} \sqrt{\frac{k}{M}}$

(B)
$$\frac{1}{2}\sqrt{\frac{k}{M}}$$

(C)
$$R\sqrt{\frac{k}{I}}$$

(D)
$$\frac{R}{2}\sqrt{\frac{k}{2I+MR^2}}$$

2. A metre stick swinging in vertical plane about an fixed horizontal axis passing through its one end undergoes small oscillation of frequency fo. If the bottom half of the stick were cut off, then its new frequency of small oscillation would become:



(B)
$$\sqrt{2} f_0$$

(D)
$$2\sqrt{2} f_0$$

3. A torsional pendulum is formed by attaching a wire to the center of a meter stick with a mass of 6 kg. If the resulting period is 2 second, what is the torsion constant (in SI unit) for the wire? [Take : $\pi^2 = 10$]

- (A) 1
- (B)3
- (C)5
- (D) 15

A particle of mass 10 gm is placed in a potential field given by $V = (50 x^2 + 100) J/kg$. The 4. frequency of oscillation in cycle/sec is:

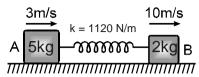
- (A) $\frac{10}{\pi}$
- (B) $\frac{5}{2}$
- (C) $\frac{100}{\pi}$
- (D) $\frac{50}{}$

5. A particle performs SHM of time period T, along a straight line. Find the minimum time interval to go from position A to position B. At A both potential energy and kinetic energy are same and at B the speed is half of the maximum speed.

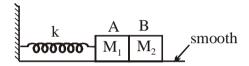
- (A) $\frac{T}{8}$
- (B) $\frac{T}{12}$
- (C) $\frac{T}{24}$
- (D) $\frac{T}{4}$

MCQ (One or more than one correct):

6. Two blocks A (5kg) and B(2kg) attached to the ends of a spring constant 1120N/m are placed on a smooth horizontal plane with the spring undeformed. Simultaneously velocities of 3m/s and 10m/s along the line of the spring in the same direction are imparted to A and B then



- (A) when the extension of the spring is maximum the velocities of A and B are zero.
- (B) the maximum extension of the spring is 25cm.
- (C) the first maximum compression occurs $3\pi/56$ seconds after start.
- (D) maximum extension and maximum compression occur alternately.
- 7. Initially spring is compressed by x_0 and blocks are in contact when system is released, then block starts moving and after some time contact between blocks breaks, then

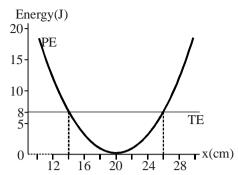


- (A) Blocks will separated at natural length of spring.
- (B) After seperation block A perform SHM of amplitude $x_0 \sqrt{\frac{m_1}{m_1 + m_2}}$.
- (C) After seperation maximum velocity of block A is $x_0 \sqrt{\frac{k}{m_1 + m_2}}$.
- (D) After separation block A will perform SHM of amplitude x₀.
- 8. A 20 gm particle is subjected to two simple harmonic motions $x_1 = 2 \sin 10 t$,

 $x_2 = 4 \sin (10 t + \frac{\pi}{3})$. where $x_1 \& x_2$ are in metre & t is in sec.

- (A) The displacement of the particle at t = 0 will be $2\sqrt{3}$ m.
- (B) Maximum speed of the particle will be $20\sqrt{7}$ m/s.
- (C) Magnitude of maximum acceleration of the particle will be $200 \sqrt{7}$ m/s².
- (D) Energy of the resultant motion will be 28 J.

9. Figure shows the potential-energy diagram and the total energy line of a particle oscillating on a spring.

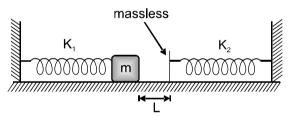


- (A) the amplitude of oscillation of particle is 6 cm.
- (B) the spring constant is 4.44 kN/m.
- (C) the mass of particle is 1kg.
- (D) the maximum kinetic energy of particle is at x = 20 cm.

Comprehension Type Question:

COMPREHENSION #1

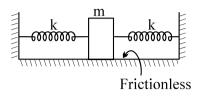
There are two ideal springs of force constants K_1 and K_2 respectively. When both springs are relaxed the separation between free ends is L. Now the particle of mass m attached to free end of left spring is displaced by distance 2L towards left and then released. Assuming the surface to be frictionless $\left(\frac{K_1}{K_2} = \frac{4}{3}\right)$.



- **10.** The time interval after which mass 'm' hits the right spring will be :
 - (A) $\frac{7\pi}{6}\sqrt{\frac{m}{K_1}}$
- (B) $\frac{4\pi}{3}\sqrt{\frac{m}{K_4}}$
- (C) $\frac{3\pi}{4} \sqrt{\frac{m}{K_1}}$
- (D) $\frac{7\pi}{4}\sqrt{\frac{m}{K_1}}$
- **11.** The maximum compression produced in right spring will be :
 - (A) $\frac{6L}{7}$
- (B) $\frac{7L}{6}$
- (C) $\frac{L}{3}$
- (D) $\frac{2L}{3}$
- **12.** Suppose mass m hits the right spring and sticks to it. The extension in left spring when mass 'm' is in equilibrium position during its motion is :
 - (A) $\frac{4L}{7}$
- (B) $\frac{3L}{7}$
- (C) L
- (D) $\frac{L}{2}$

COMPREHENSION #2

The block of mass m is attached to 2 springs as shown. In the equilibrium position, the springs are at their natural length, the mass oscillates along the line of springs with amplitude d. At t = 0, mass is at + d/2 from equilibrium and moving to right. The right spring is removed at that instant, without changing velocity of block.



Find the new time period of oscillation if original time period was T₀. 13.

- (A) $2T_0$
- (B) $\frac{T_0}{2}$
- (C) $\frac{T_0}{\sqrt{2}}$
- (D) $\sqrt{2} T_0$

14. The new amplitude of mass is

- (A) d
- (B) $\frac{\sqrt{3}d}{2}$
- (C) $\frac{\sqrt{7}d}{2}$
- (D) none

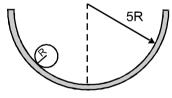
15. The velocity of the mass when it passes through equilibrium is

(A)
$$\frac{\sqrt{19}\pi d}{T_0}$$

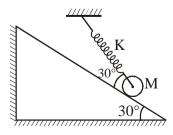
- (B) $\frac{\sqrt{2\pi d}}{T_a}$ (C) $\frac{\sqrt{14\pi d}}{T_a}$
- (D) None of these

Numerical based Questions:

16. A solid sphere (radius = R) rolls without slipping in a cylindrical vessel (radius = 5R). Find the angular frequency of small of oscillations of the sphere in s⁻¹. Take R = $\frac{1}{14}$ m and g = 10 m/s². (axis of cylinder is fixed and horizontal).



- 17. A car accelerates uniformly from rest. Initially, its door is slightly ajar (open). Calculate how far (in m) the car travels before the door slams shut. Assume the door has a frictionless hinge, a uniform mass distribution and a length of 1.2 m from front to back.
- 18. A sphere of mass M and radius R is on a smooth fixed inclined plane in equilibrium as shown in the figure. If now the sphere is displaced through a small distance along the plane, what will be the angular frequency (in radian/sec) of the resulting SHM? (Given k = 4M/3).



19. A small body of mass m is connected to two horizontal springs of elastic constant k, natural length $\frac{3d}{4}$. In the equilibrium position both springs are stretched to length d, as shown in figure. What will be the ratio of period of the motion $\left(\frac{T_a}{T_b}\right)$ if the body is displaced **horizontally**

$$\begin{array}{c|c} d & d \\ \hline k & m \\ k \end{array}$$

 T_a is time period when the particle oscillates along the line of springs and T_b is time period when the particle oscillates perpendicular to the plane of the figure? **Neglect effects of gravity**

Matrix Match Type:

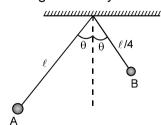
by a small distance where

20. A particle of mass m = 1 kg executes SHM about mean position O with angular frequency $\omega = 1.0$ rad/s and total energy 2J. x is positive if measured towards right from O. At t = 0, particle is at O and moves towards right. Match the condition in column-I with the position of the particle in column-II and indicate your answer by darkening appropriate bubbles in the 4×4 matrix given in the OMR.

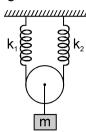
	O S P
Column-I	Column-II
(A) speed of particle is $\sqrt{2}$ m/s at	(p) $x = + 1m$
(B) Kinetic energy of the particle is 1J at	(q) $x = -1m$
(C) At $t = \pi/6$ s particle is at	(r) $x = +\sqrt{2} \text{ m}$
(D) Kinetic energy is 1.5 J at	(s) $x = -\sqrt{2} \text{ m}$

Subjective Type Questions:

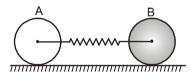
21. Two simple pendulums A and B having lengths ℓ and $\ell/4$ respectively are released from the position as shown in figure. Calculate the time after which the release of the two strings become parallel for the first time. Angle θ is very small.



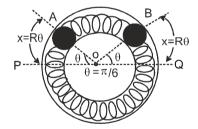
22. Mass 'm' is suspended from the massless pulley which itself is supported by two springs connected to each other as shown. If the mass is slightly displaced from its equilibrium position, find the frequency of resulting SHM of mass 'm'.



23. Two spheres A and B of the same mass m and the same radius are placed on a rough horizontal surface. A is a uniform hollow sphere and B is uniform solid sphere. A and B can roll without sliding on the floor. They are also tied centrally to a light spring of spring constant k. A and B are released when the extension in the spring is x₀. Find the amplitude of SHM of the spheres and the frequency of oscillation of the spheres



24. Two identical balls A and B, each of mass 0.1 kg, are attached to two identical mass less springs. The spring-mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in the figure. The pipe is fixed in a horizontal plane. The centres of the balls can move in a circle of radius 0.06 m. Each spring has a natural length of 0.06π metre and spring constant 0.1 N/m. Initially, both the balls are displaced by an angle $\theta = \pi/6$ radian with respect to the diameter PQ of the circle (as shown in fig.) and released from rest.



- (i) Calculate the frequency of oscillation of ball B.
- (ii) Find the speed of ball A when A and B are at the two ends of the diameter PQ.
- (iii) What is the total energy of the system?
- 25. Two wheels which are rotated by some external source with constant angular velocity in opposite directions as shown in figure. A uniform plank of mass M is placed on it symmetrically. The friction co-efficient between each wheel and the plank is μ. Find the frequency of oscillations, when plank is slightly displaced along its length and released.

