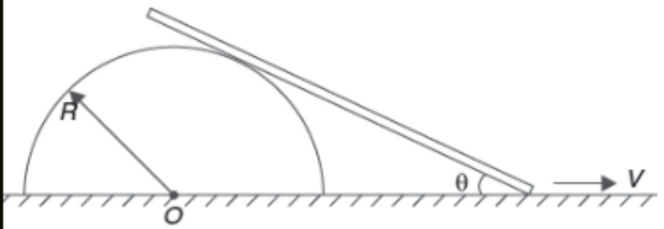
There is a fixed half cylinder of radius $R$ on a horizontal table. A uniform rod of length $2R$ leans against it as shown. At the instant shown, $\theta = 30°$ and the right end of the rod is sliding with velocity $v$.

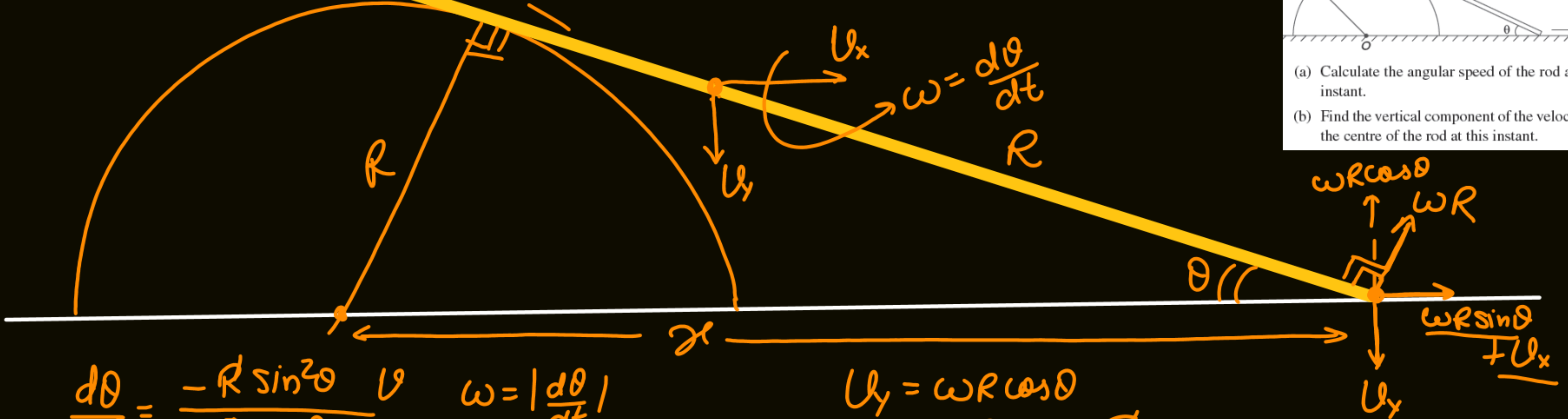



(a) Calculate the angular speed of the rod at this instant.

(b) Find the vertical component of the velocity of the centre of the rod at this instant.

$$\sin\theta = R/x$$

$$\cos\theta \frac{d\theta}{dt} = \frac{-R}{x^2}\frac{dx}{dt}$$




$$\frac{d\theta}{dt} = \frac{-R\sin^2\theta}{R^2\cos\theta}v$$

$$\frac{d\theta}{dt} = \frac{-v}{R}\frac{\sin^2\theta}{\cos\theta} \Rightarrow$$

$$\omega = \left|\frac{d\theta}{dt}\right|$$

$$= +\frac{v}{R}\times\frac{1\times 2}{4\times\sqrt{3}}$$

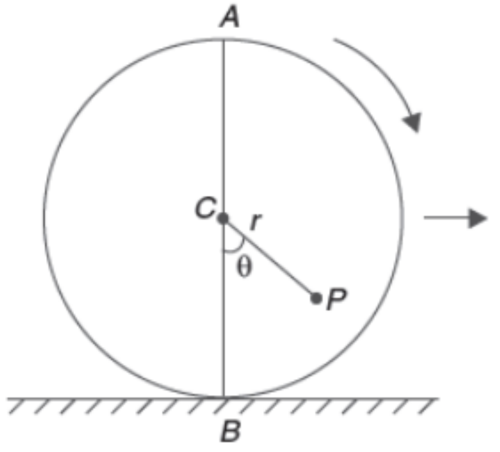
$$= \frac{v}{2\sqrt{3}R}$$

$$v_y = \omega R\cos\theta$$

$$= \frac{v}{2\sqrt{3}R}R\times\frac{\sqrt{3}}{2}$$

$$= \frac{v}{4}$$

A uniform disc of radius $R = 2\sqrt{3}\ m$ is moving on a horizontal surface without slipping. At some instant its angular velocity is $\omega = 1$ rad/s and angular acceleration is $\alpha = \sqrt{3}\ rad/s^2$.




(a) Find acceleration of the top point $A$.

(b) Find acceleration of contact point $B$.

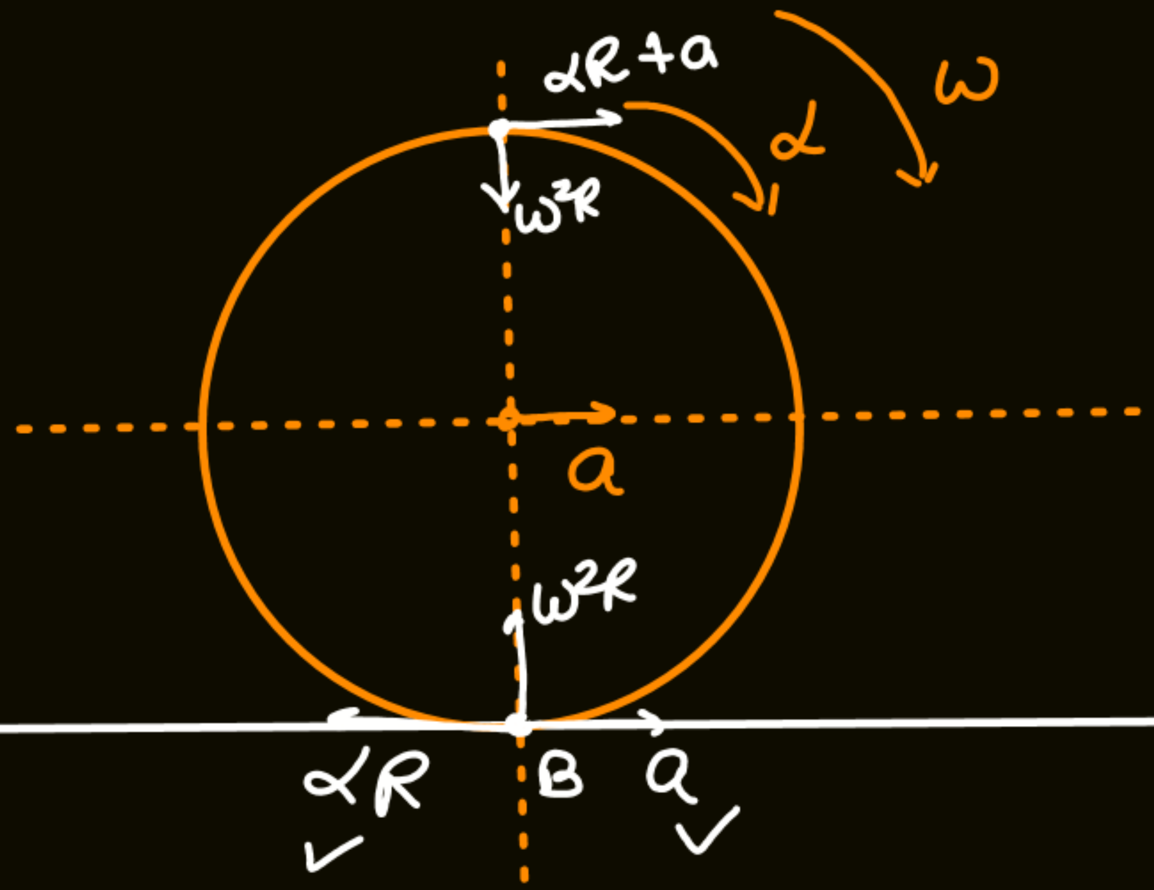
(c) Find co - oridnates $(r, \theta)$ for a point $P$ which has zero acceleration.


$2\alpha R = 2\times 2\sqrt{3}\times\sqrt{3} = 12$

$\omega^2 R = 2\sqrt{3}$ m/s

$a_A = \sqrt{144+12}$

$= \sqrt{156}$ m/s$^2$




$a = \alpha R$

$= \sqrt{3}\times 2\sqrt{3}$

$= 6$

$a_B = \omega^2 R$

$= 2\sqrt{3}$ m/s$^2$

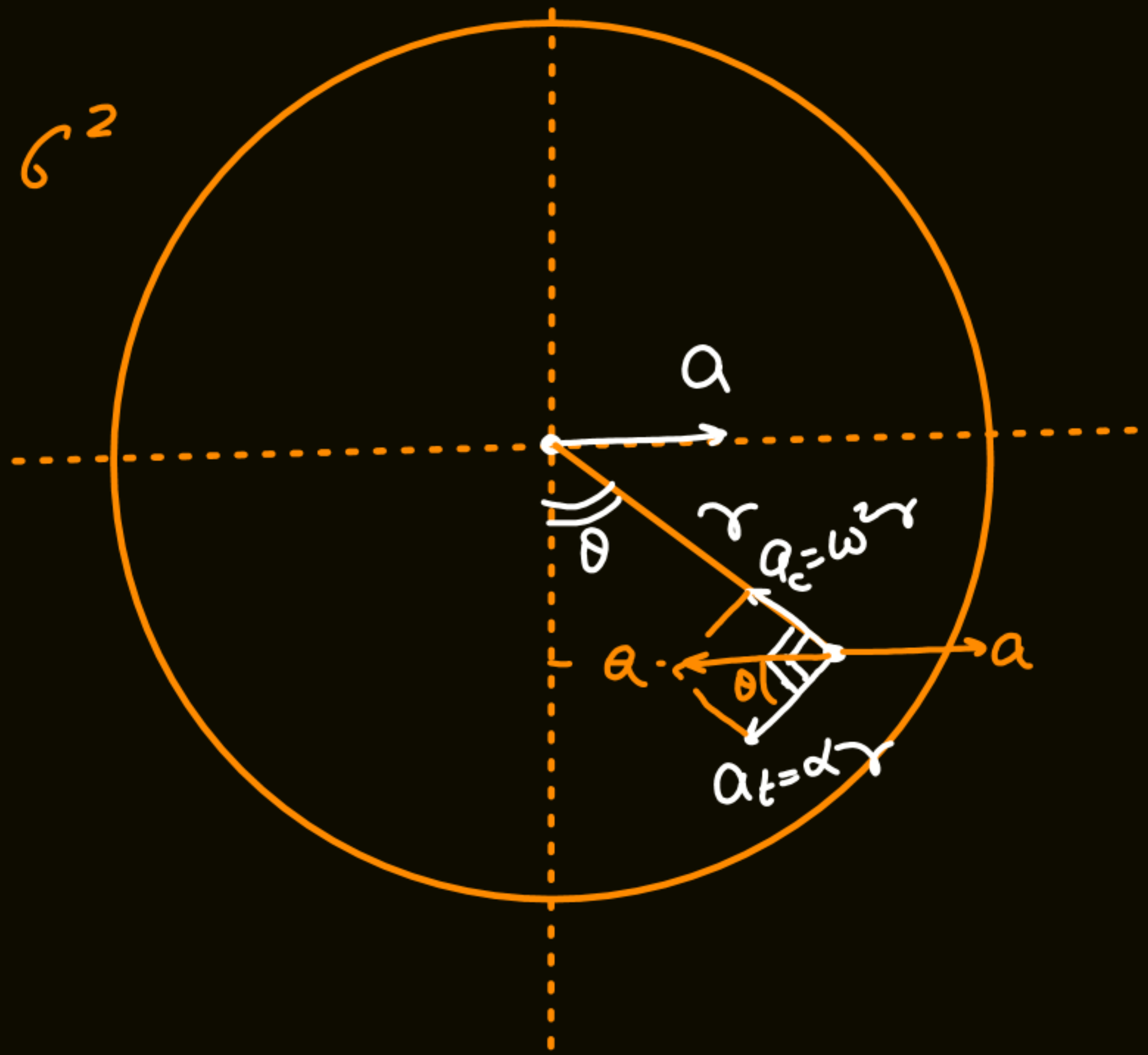
$a_c^2 + a_t^2 = a^2$

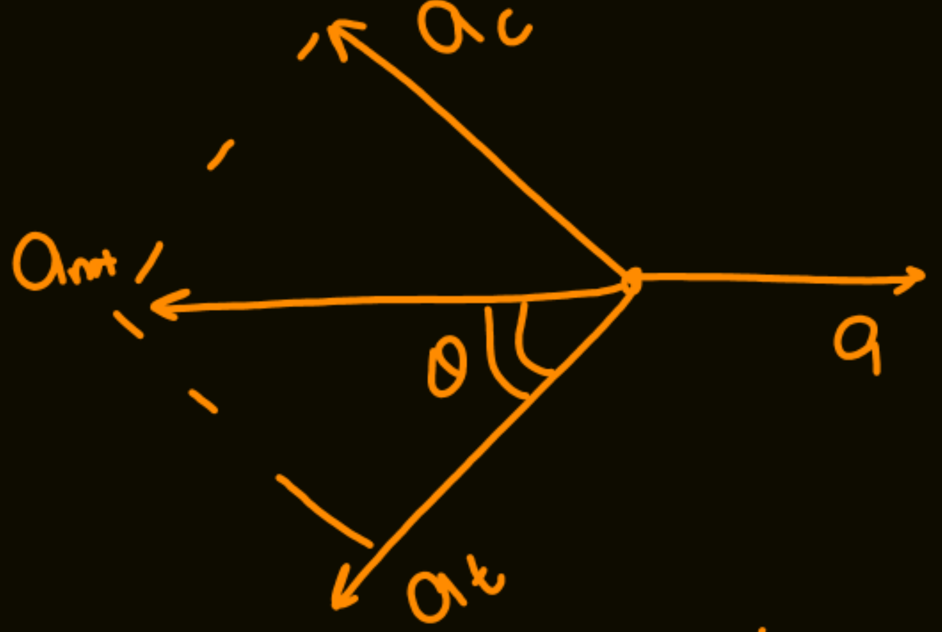
$(1^2 r)^2 + (\sqrt{3} r)^2 = 6^2$

$4r^2 = 36$

$r^2 = 9$

$\boxed{r = 3m}$

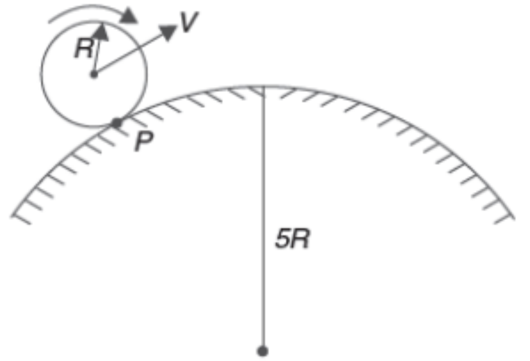



$\tan\theta = \frac{a_c}{a_t} = \frac{1^2 r}{\sqrt{3} r} = \frac{1}{\sqrt{3}}$

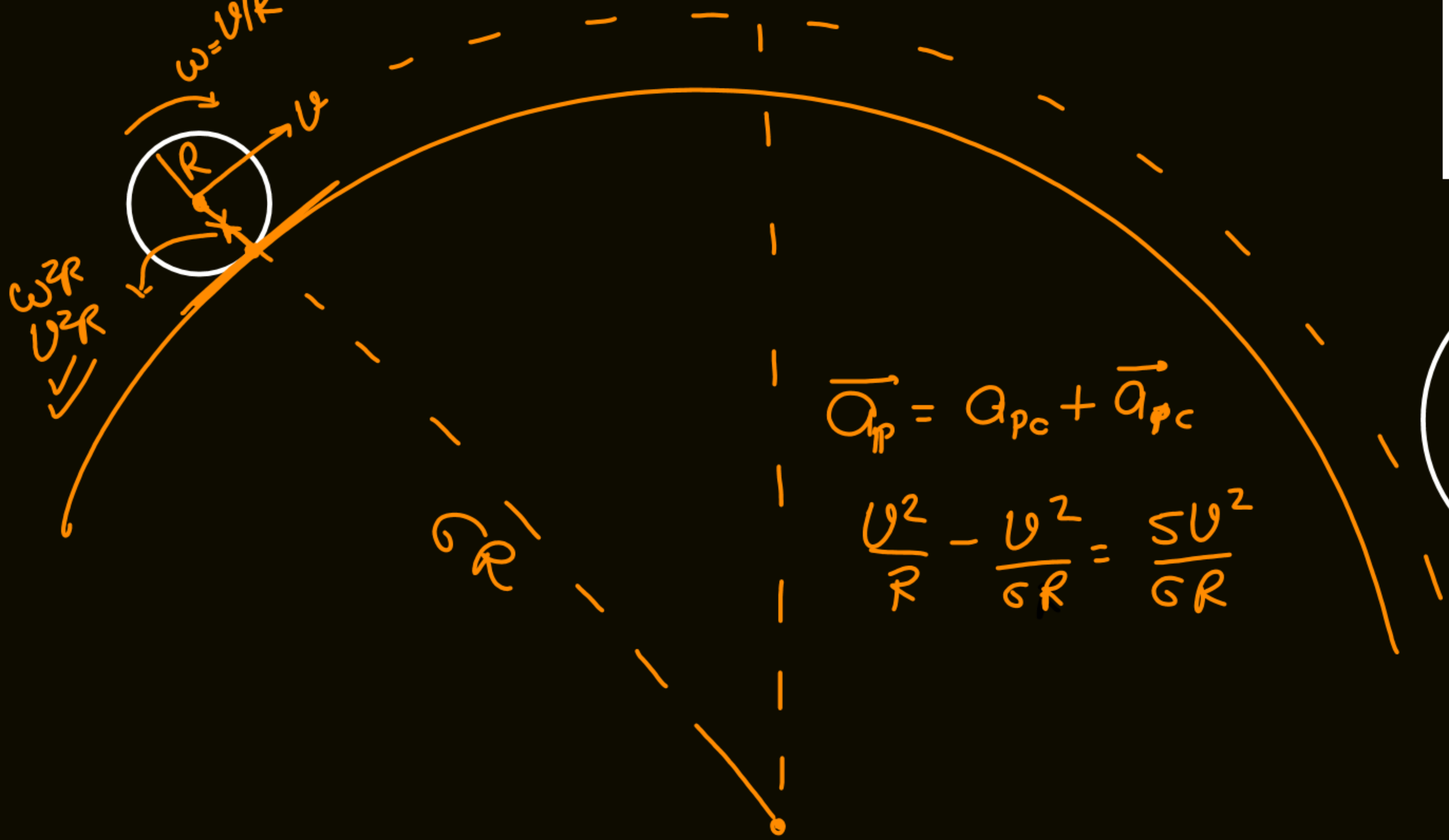
$\underline{\theta = 30^\circ}$

A convex surface has a uniform radius of curvature equal to $5R$. A wheel of radius $R$ is rolling without sliding on it with a constant speed $v$. Find the acceleration of the point ($P$) of the wheel which is in contact with the convex surface.



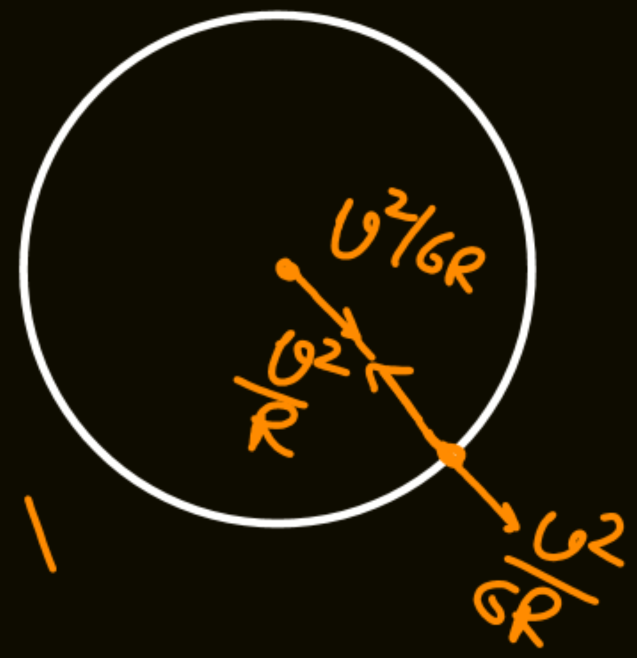

$$\frac{v^2}{6R} = a_{com}$$

$\omega = v/R$

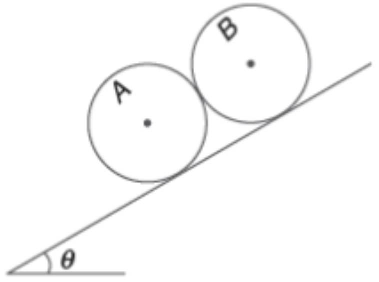


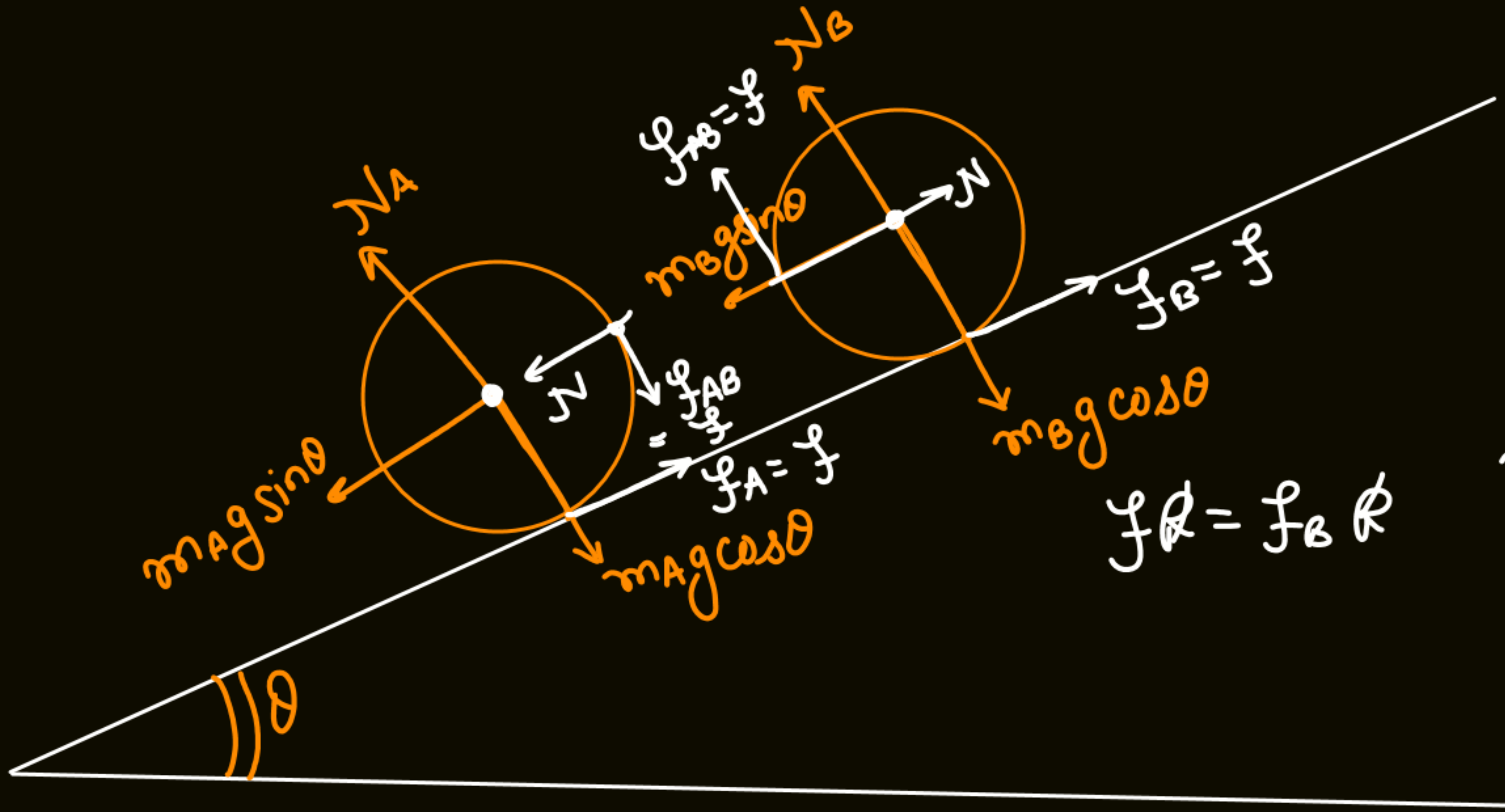

$$\vec{a}_P = a_{PC} + \vec{a}_{C}$$

$$\frac{v^2}{R} - \frac{v^2}{6R} = \frac{5v^2}{6R}$$



Two cylinders $A$ and $B$ have been placed in contact on an incline. They remain in equilibrium. The dimensions of the two cylinders are same. Which cylinder has larger mass?




$$f\cancel{R} = f_B \cancel{R}$$

$$m_A g \sin\theta + N = f$$

$$m_B \sin\theta g = N + f$$

$$m_A g \sin\theta = f - N$$

$$\boxed{m_B > m_A}$$

$$f\cancel{R} = f_{AB} \cancel{R}$$

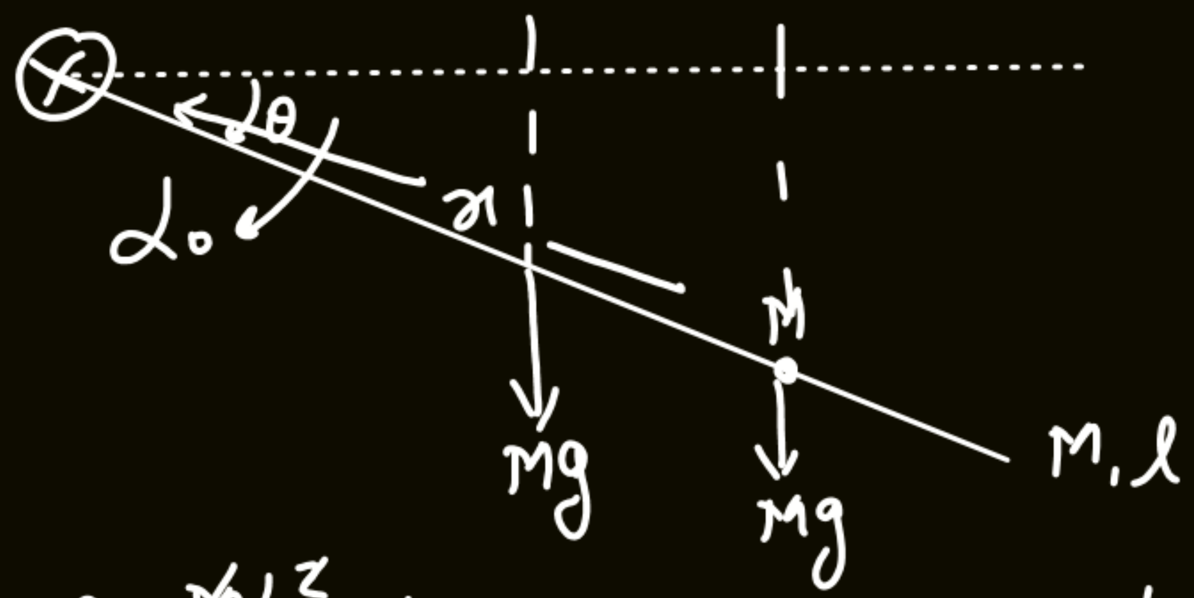
$$m_A g \sin\theta + N = m_B g \sin\theta - N$$

$$N = \frac{(m_B - m_A) g \sin\theta}{2} > 0$$


A rod of mass $M$ and length $L$ is hinged about its end $A$ so that it can rotate in vertical plane. When the rod is released from horizontal position it takes $t_0$ time for it to become vertical.

(a) A particle of mass $M$ is stuck at the end $B$ of the rod and the rod is once again released from its horizontal position. Will it take more time or less time (than $t_0$) for the rod to become vertical from its horizontal position.

(b) At what distance $x$ from end $A$ shall the particle of mass $M$ be stuck so that it takes minimum time for the rod to become vertical from its horizontal position.




$$Mg\frac{L}{2}\cos\theta = \frac{ML^2}{3}\alpha_0$$

$$\boxed{\alpha_0 = \frac{3g}{2L}\cos\theta}$$

$\underline{x = L}$ $\quad \alpha = \dfrac{3g\left(\frac{3L}{2}\right)\cos\theta}{4L^2}$

$$= \frac{9g}{8L}\cos\theta$$

$\alpha_0 > \alpha$

more time

$$\tau = Mg\frac{L}{2}\cos\theta + Mgx\cos\theta = \left(\frac{ML^2}{3} + Mx^2\right)\alpha$$

$$\alpha = \frac{3g\left\{\frac{L}{2} + x\right\}\cos\theta}{\left(L^2 + 3x^2\right)}$$

$t\downarrow \quad \underline{\alpha\uparrow}$

$$\alpha = \frac{3g\cos\theta\left\{\frac{L}{2}+x\right\}}{\left(L^2+3x^2\right)}$$

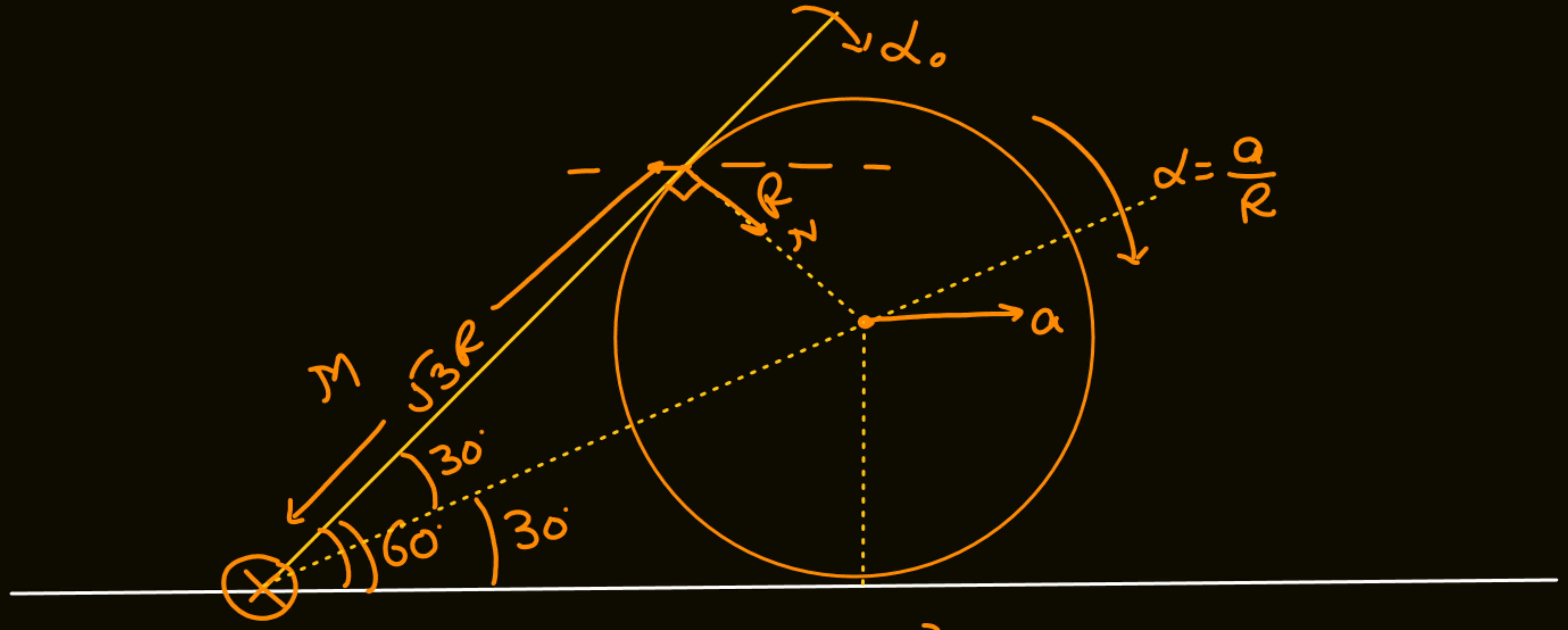
$$\frac{d\alpha}{dx} = 0 = 3g\cos\theta\,\frac{\left(L^2+3x^2\right)(1)-\left(\frac{L}{2}+x\right)(6x)}{\left(L^2+3x^2\right)^2} = 0$$

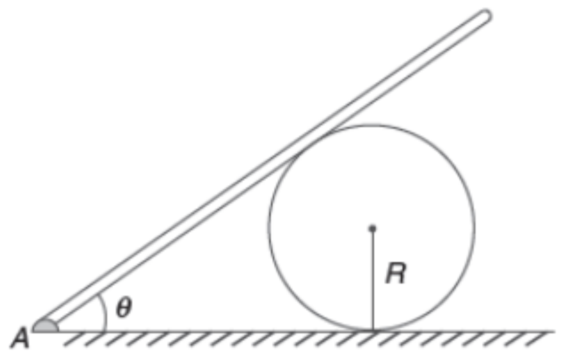
$$L^2+3x^2 = 3xL+6x^2$$

$$3x^2+3xL-L^2=0$$

$$x = \frac{-3L+\sqrt{9L^2+4\times 3L^2}}{2\times 3}$$

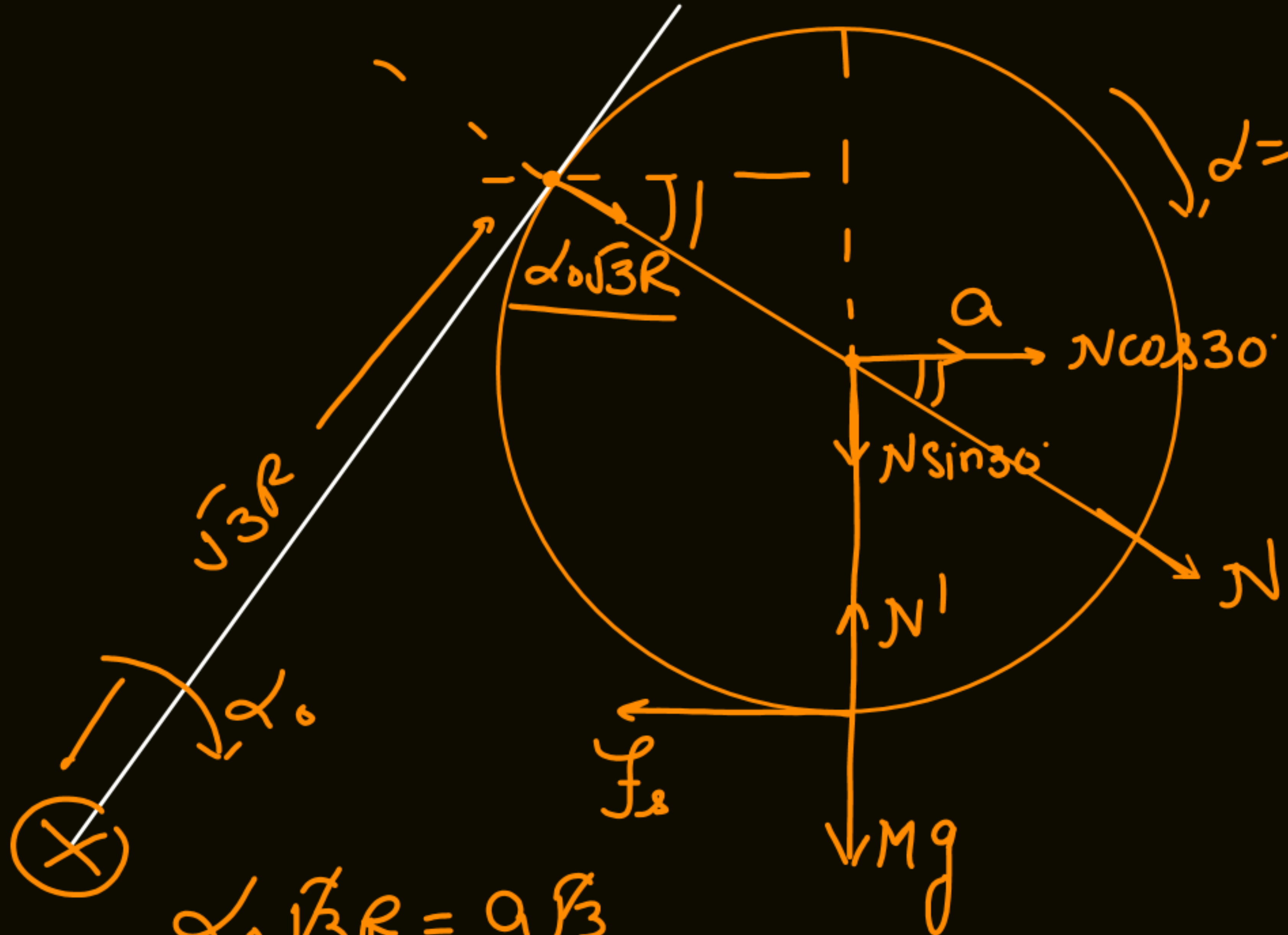
$$x = \frac{-3L+\sqrt{21}L}{6} = \left(\frac{\sqrt{21}}{6}-\frac{1}{2}\right)L$$




A ring of mass $M$ and radius $R$ is held at rest on a rough horizontal surface. A rod of mass $M$ and length $L = 2\sqrt{3}R$ is pivoted at its end A on the horizontal surface and is supported by the ring. There is no friction between the ring and the rod. The ring is released from this position. Find the acceleration of the ring immediately after the release if $\theta = 60°$. Assume that friction between the ring on the horizontal surface is large enough to prevent slipping of the ring.

$$M\frac{(2\sqrt{3}R)^2}{3} = \frac{12MR^2}{3} = 4MR^2$$



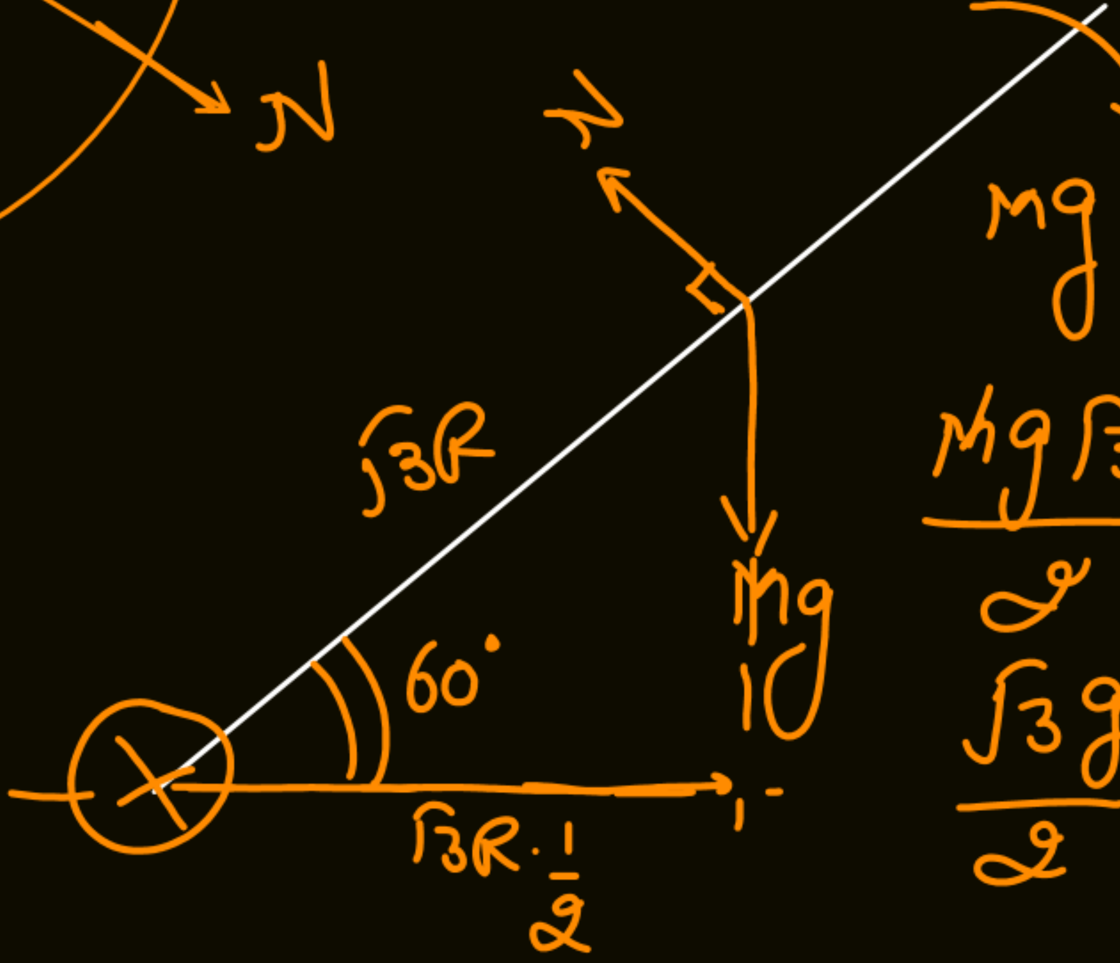

$$\alpha_0\sqrt{3}R = a\frac{\sqrt{3}}{2}$$

$$\boxed{\alpha_0 = \frac{a}{2R}}$$

$$\frac{N\sqrt{3}}{2} - f_s = Ma \quad (1)$$

$$f_s R = \frac{MR^2}{2}\frac{a}{R} \quad (2)$$

$$\boxed{\frac{N\sqrt{3}}{2} = 2Ma}$$




$$Mg\frac{\sqrt{3}R}{2} - N\sqrt{3}R = 4MR^2\alpha_0$$

$$\frac{Mg\sqrt{3}R}{2} - 4MaR = 4MR^2\frac{a}{2R}$$

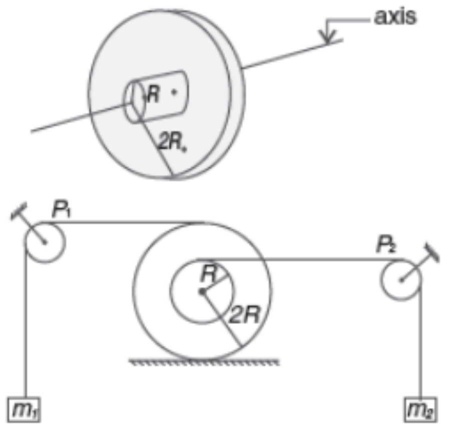
$$\frac{\sqrt{3}g}{2} = 6a$$

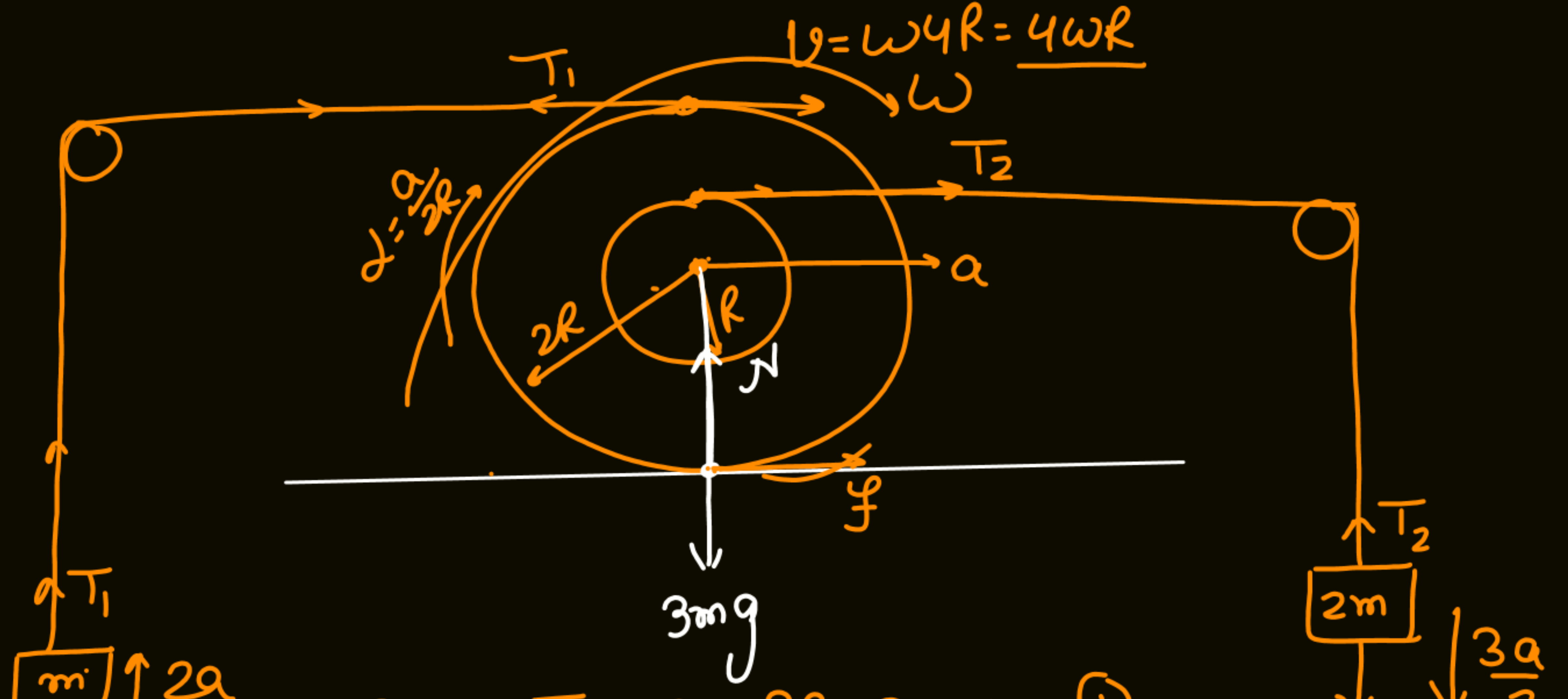
$$a = \frac{\sqrt{3}g}{12} = \frac{g}{4\sqrt{3}}$$

A spool has the shape shown in figure. Radii of inner and outer cylinders are $R$ and $2R$ respectively. Mass of the spool is $3m$ and its moment of inertia about the shown axis is $2mR^2$. Light threads are tightly wrapped on both the cylindrical parts. The spool is placed on a rough surface with two masses $m_1 = m$ and $m_2 = 2m$ connected to the strings as shown. The string segment between spool and the pulleys $P_1$ and $P_2$ are horizontal. The centre of mass of the spool is at its geometrical centre. System is released from rest.

(a) What is minimum value of coefficient of friction between the spool and the table so that it does not slip?

(b) Find the speed of $m_1$ when the spool completes one rotation about its centre.




$$2mg - T_2 = 2m\frac{3a}{2} = 3ma \quad \text{—(1)}$$

$$T_1 - mg = m2a \quad \text{—(2)}$$

$$T_2\cancel{R} - T_1 2\cancel{R} - f2\cancel{R} = \cancel{2}mR^{\cancel{2}}\frac{a}{\cancel{2}\cancel{R}} = ma\cancel{R} \quad \text{—(3)}$$

$$T_2 + f - T_1 = 3ma \quad \text{—(4)}$$

$$f = -\frac{mg}{3}$$

$$f = \frac{\cancel{m}g}{3} \le \mu\, 3\cancel{m}g$$

$$\boxed{\mu \geq \frac{1}{9}}$$

$$\boxed{\mu_{min} = \frac{1}{9}}$$

$$\omega^2 = 0^2 + 2\alpha(2\pi)$$

$$\omega^2 = 4\pi\alpha$$

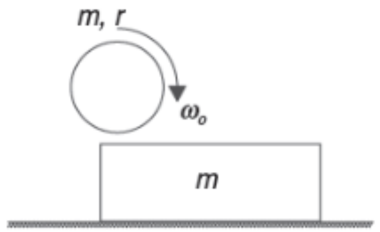
$$= 4\pi\frac{a}{2R} = \frac{2\pi a}{R}$$

$$\omega = \sqrt{\frac{2\pi a}{R}}$$

$$v = 4\omega R$$

$$= 4\sqrt{2\pi a R}$$

In the figure shown a plank of mass $m$ is lying at rest on a smooth horizontal surface. A cylinder of same mass $m$ and radius $r$ is rotated to an angular speed $w_0$ and then gently placed on the plank. It is found that by the time the slipping between the plank and the cylinder cease, 50% of total kinetic energy of the cylinder and plank system is lost. Assume that plank is long enough and $\mu$ is the coefficient of friction between the cylinder and the plank.



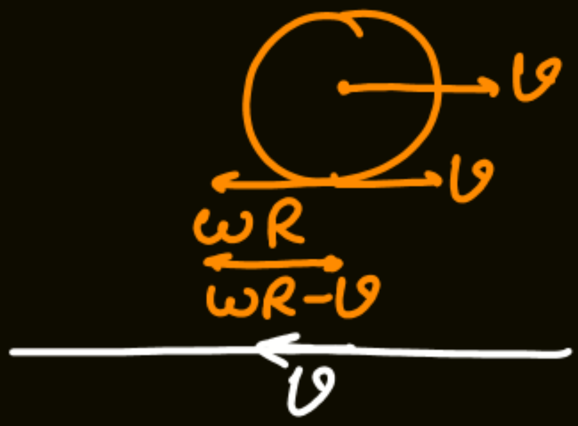

(a) Find the final velocity of the plank.

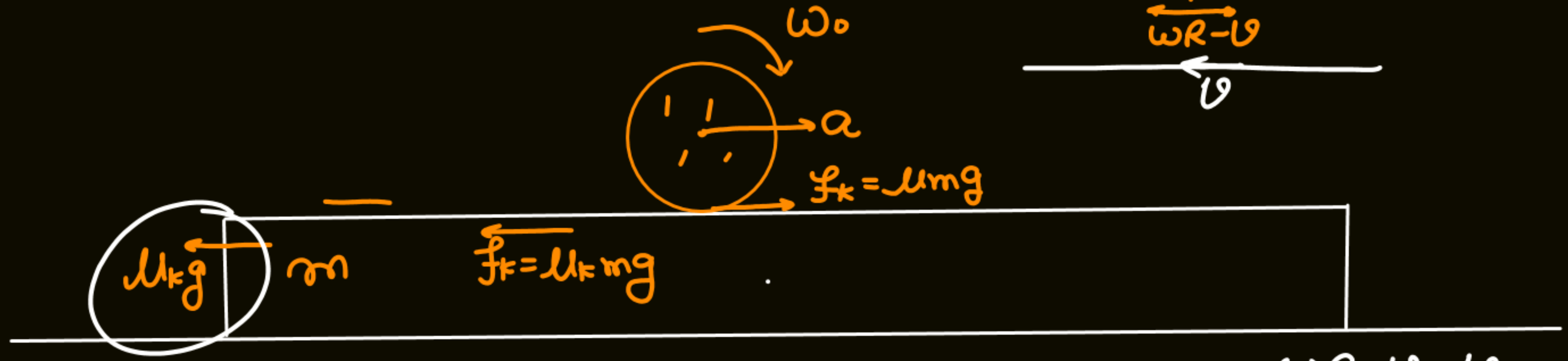
(b) Calculate the magnitude of the change in angular momentum of the cylinder about its centre of mass.

(c) Distance moved by the plank by the time slipping ceases between cylinder and plank.

$$\cancel{m} v_1 = \cancel{m} v_2$$

$$\boxed{v_1 = v_2}$$

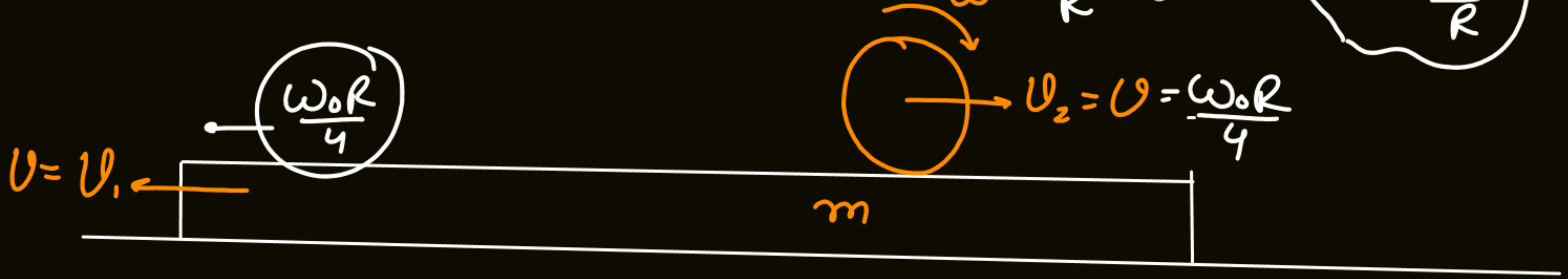



$$\omega R - v = v$$

$$\boxed{\omega = \frac{2v}{R}}$$

$$\omega = \frac{2v}{R} = \frac{\omega_0}{2}$$



$$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{mR^2}{2}\cdot\frac{4v^2}{R^2} = \frac{1}{2}\left\{\frac{1}{2}\cdot\frac{mR^2}{2}\omega_0^2\right\}$$

$$2\cancel{m}v^2 = \frac{1}{8}\cancel{m}\omega_0^2R^2$$

$$\boxed{v = \frac{\omega_0 R}{4}}$$

$$v^2 = u^2 + 2aS$$

$$\left(\frac{\omega_0 R}{4}\right)^2 = 0^2 + 2\mu g S$$

$$\boxed{S = \frac{\omega_0^2 R^2}{32\mu g}}$$

$$\Delta L = L_f - L_i$$

$$= \frac{mR^2}{2}\left(\frac{\omega_0}{2}\right) - \frac{mR^2}{2}\omega_0$$

$$= -\frac{mR^2\omega_0}{4}$$