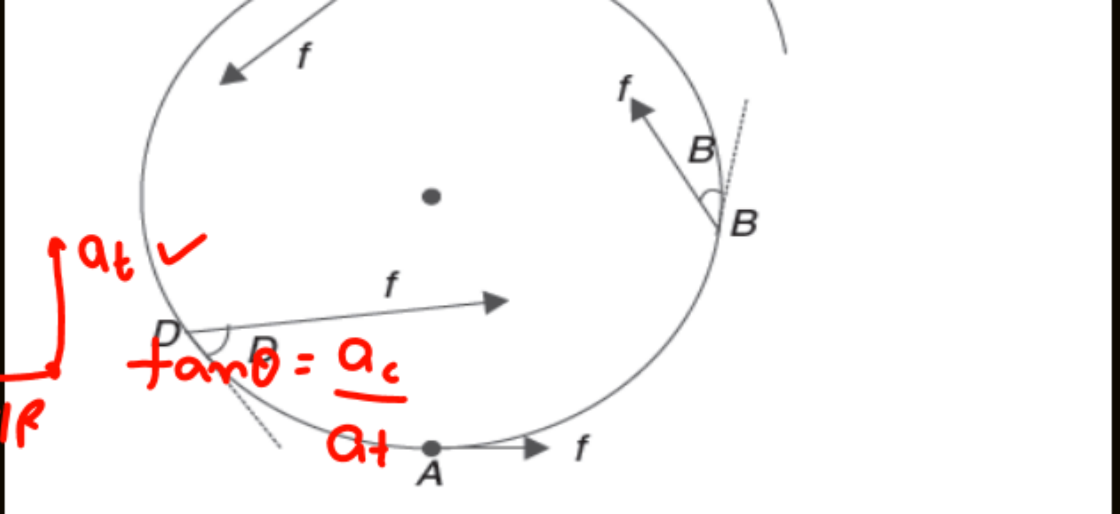
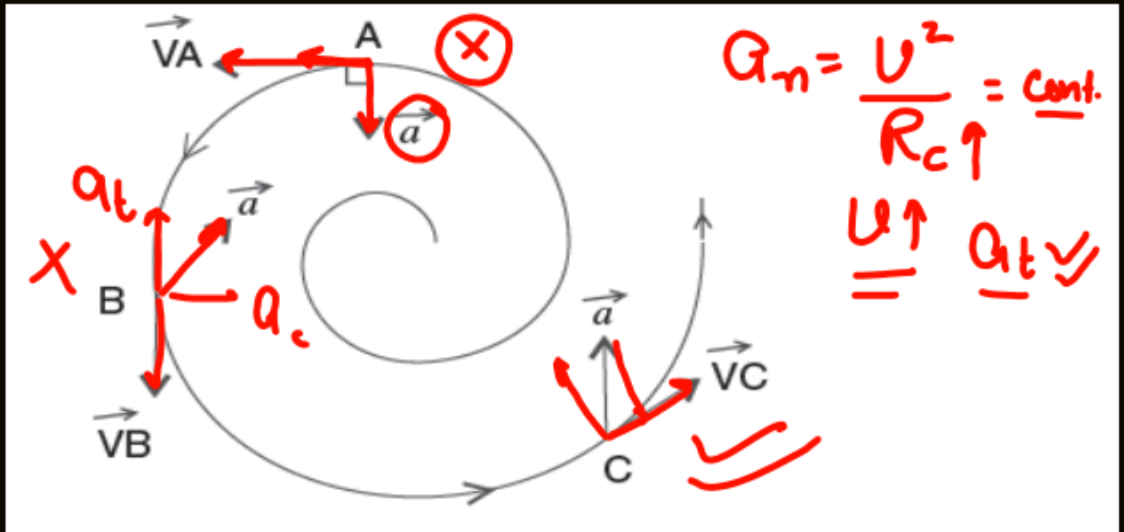
(a) A car starts moving (at point A) on a horizontal circular track and moves in anticlockwise sense. The speed of the car is made to increase uniformly. The car slips just after point D. The figure shows the friction force (f) acting on the car at points A, B, C and D. The length of the arrow indicates the magnitude of the friction and it is given that $\angle D > \angle B > \angle C$. At which point ($A$, B, C or D) the friction forces represented is certainly wrong ?



(b) A particle is moving along an expanding spiral (shown in fig) such that the normal force on the particle [i.e., component of force perpendicular to the path of the particle] remains constant in magnitude. The possible direction of acceleration $(\vec{a})$ of the particle has been shown at three points A, B and C on its path. At which of these points the direction of acceleration has been represented correctly.



(c) A particle is moving in XY plane with a velocity. $\vec{v} = 4\hat{i} + 2t\hat{j}\ \text{ms}^{-1}$. Calculate its rate of change of speed and normal acceleration at $t = 2\ s$.

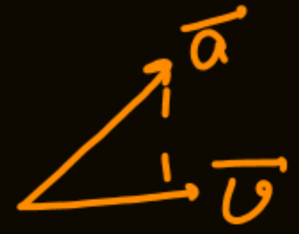
$$\vec{v} = 4\hat{i} + 2t\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 2\hat{j}$$

$$a_t = \frac{d|\vec{v}|}{dt} = \frac{d}{dt}\sqrt{16+4t^2}$$

$$= \frac{1}{2}\frac{8t}{\sqrt{16+4t^2}} = \frac{4t}{\sqrt{16+4t^2}}$$

$$= \frac{8}{4\sqrt{2}} = \sqrt{2}\ m/s^2$$

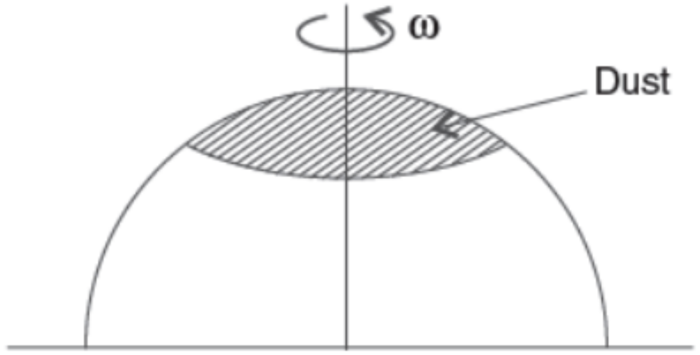


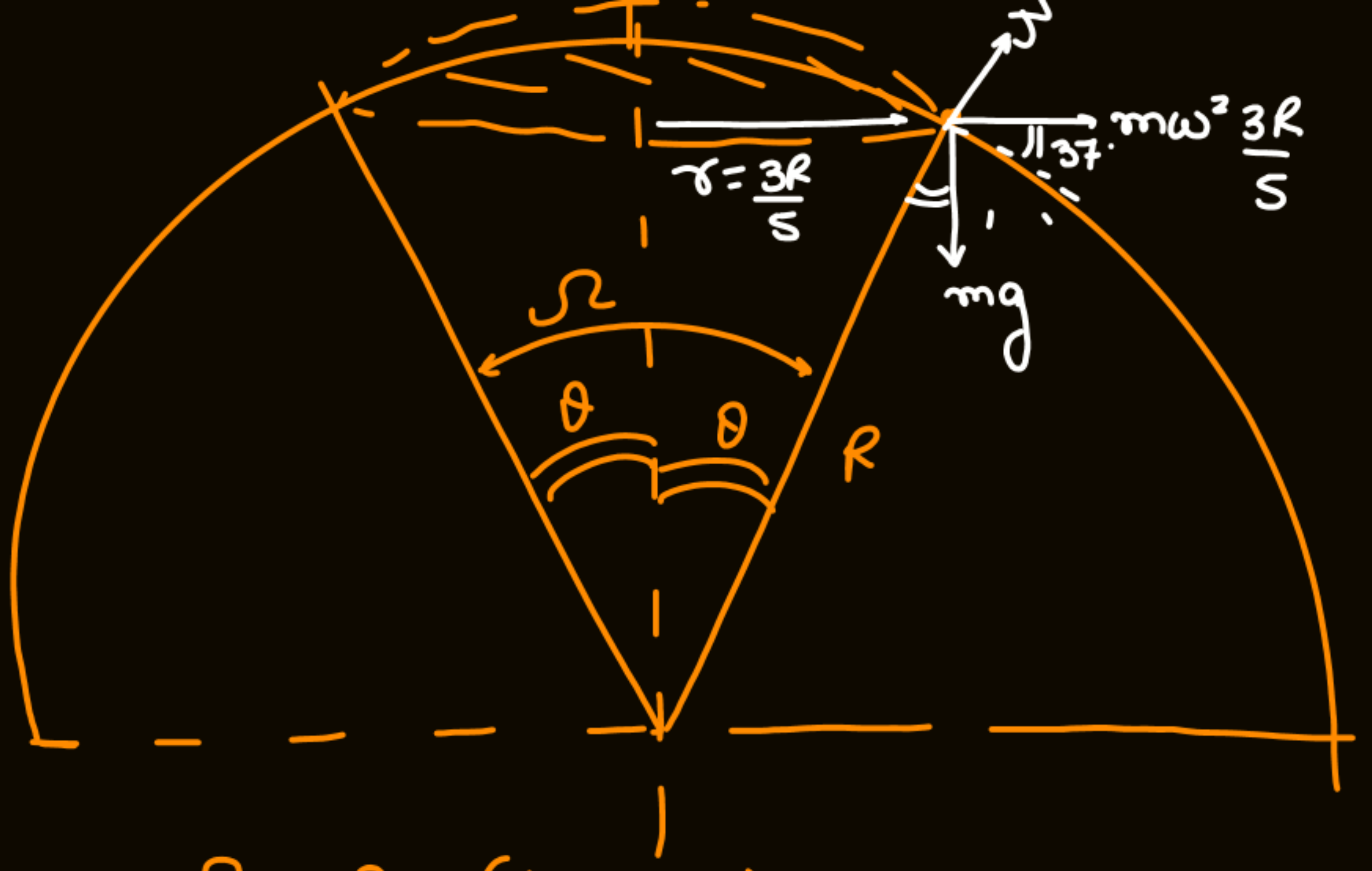
$$\vec{a}_t = \frac{\vec{a}\cdot\vec{v}}{|\vec{v}|} = \frac{4t}{\sqrt{16+4t^2}} = \frac{8}{4\sqrt{2}} = \sqrt{2}\ m/s^2$$

$$a^2 = a_t^2 + a_n^2$$

$$2^2 = (\sqrt{2})^2 + a_n^2 \Rightarrow a_n = \sqrt{2}\ m/s^2$$

surface. The sphere is rotated about a vertical axis passing through its centre at angular speed $\omega = 10\ rad\ s^{-1}$. Now the dust is visible only on top 20% area of the curved hemispherical surface. Radius of the hemisphere is $R = 0.1\ m$. Find the coefficient of friction between the dust particle and the hemisphere [$g = 10\ ms^{-2}$].





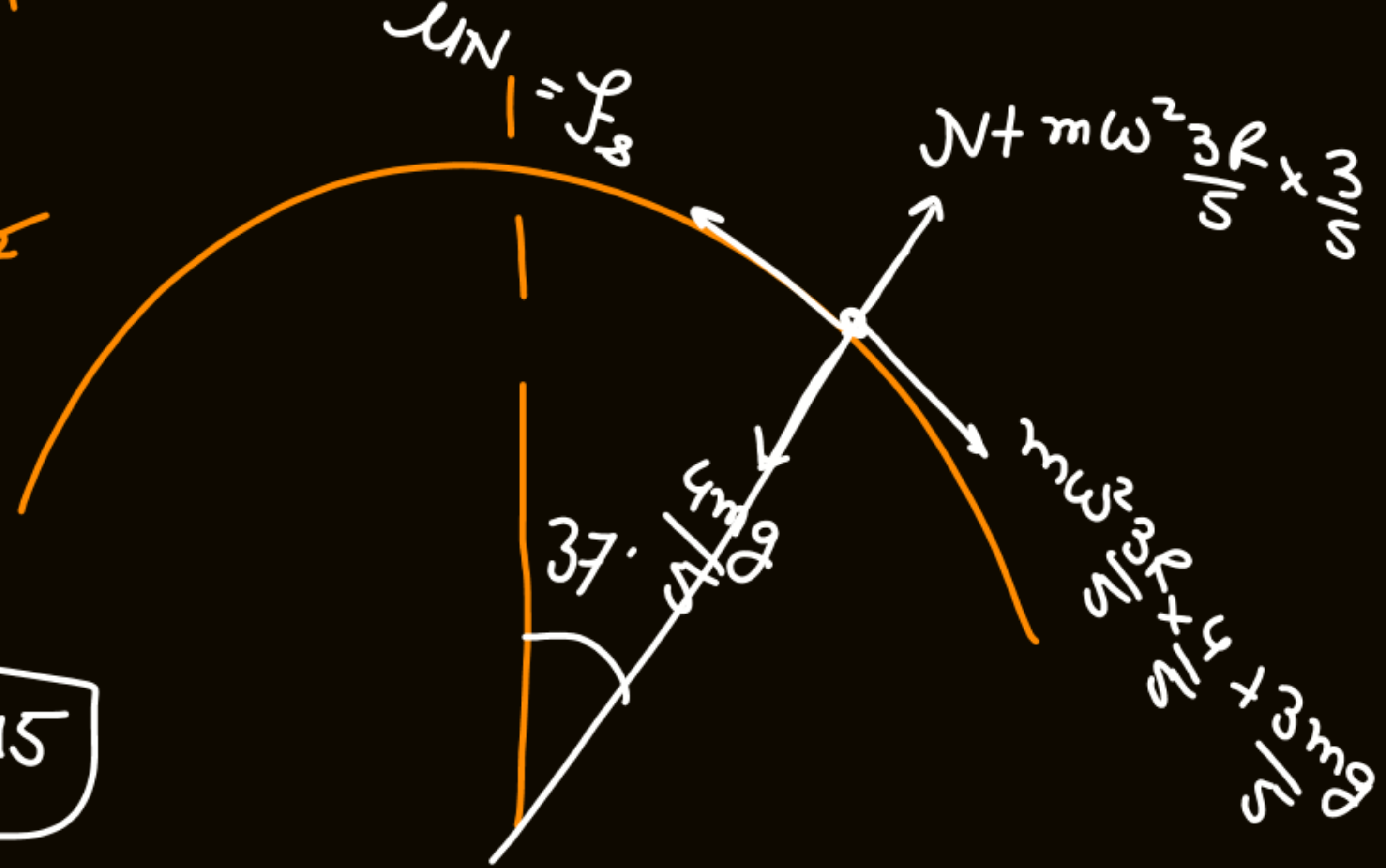
$\Omega = 2\pi(1-\cos\theta)$

$A = \Omega R^2 = \cancel{2\pi R^2}\{1-\cos\theta\} = 0.2 \times \cancel{2\pi R^2}$

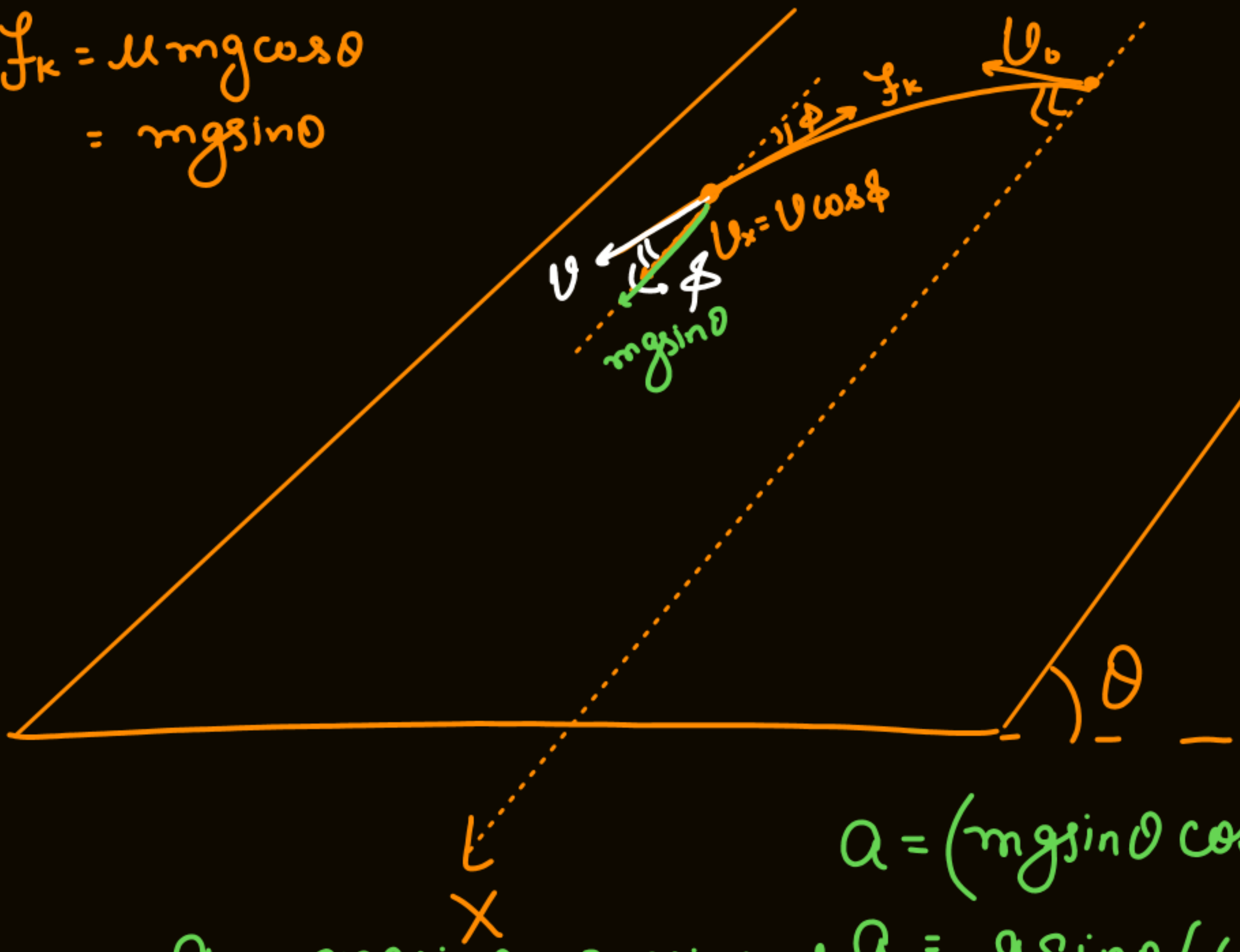
$\cos\theta = \frac{4}{5}$ $\quad \theta = 37°$

$N + m\omega^2 R \frac{9}{25} = 4mg/5$

$\mu N = \frac{3mg}{5} + \frac{12 m\omega^2 R}{25} \Rightarrow \boxed{\mu = 2.45}$



$f_k = \mu mg\cos\theta$
$= mg\sin\theta$



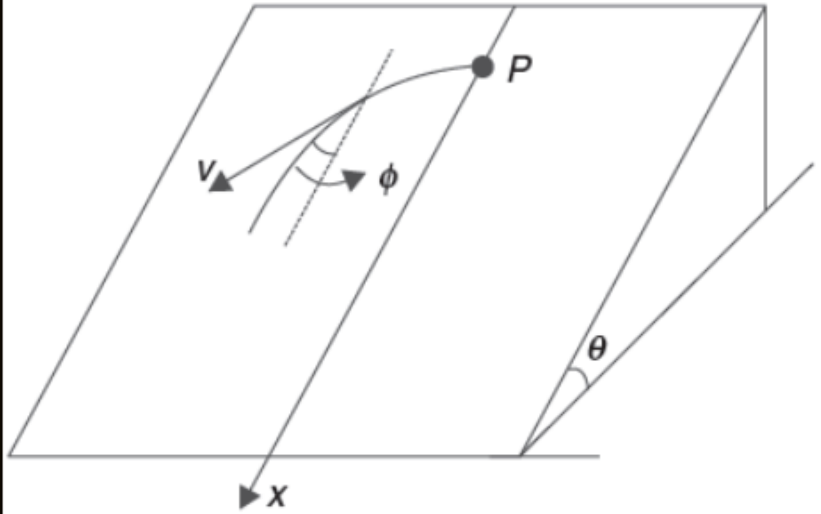
$$a_x = \frac{mg\sin\theta - mg\sin\theta\cos\phi}{m}$$

$$\boxed{a_x = g\sin\theta\,(1-\cos\phi)}$$

$$a = (mg\sin\theta\cos\phi - mg\sin\theta)/m$$

$$a = g\sin\theta\{\cos\phi - 1\} = \frac{dv}{dt}$$

A small disc P is placed on an inclined plane forming an angle θ with the horizontal and imparted an initial velocity v_0. Find how the velocity of disc depends on the angle ϕ which its velocity vector makes with the x axis (see figure). The coefficient of friction is $\mu = \tan\theta$ and initially $\phi_0 = \frac{\pi}{2}$.



$$a + a_x = 0$$

$$\int a\,dt + \int a_x\,dt = \int 0\,dt$$

$$\boxed{v + v_x = C}$$

$$v + v\cos\phi = C$$

$$v\{1 + \cos\phi\} = C$$

$$v = \frac{C}{(1+\cos\phi)}$$

$$\boxed{v = \frac{v_0}{1+\cos\phi}}$$

$$\phi = \frac{\pi}{2} \quad v = v_0$$

$$v_0 + v_0\cos\frac{\pi}{2} = C$$

$$C = v_0$$

$$\int_{v_0}^{v} a\,dt + \int_{0}^{v_x} a_x\,dt = \int_0^t 0\,dt$$

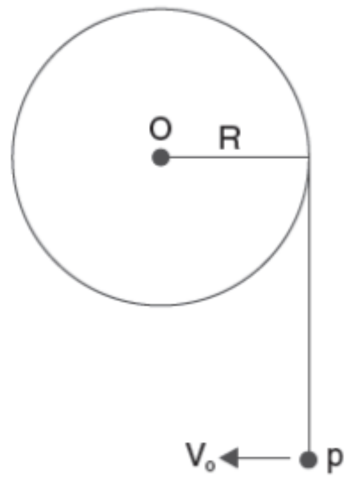
$$(v - v_0) + v_x - 0 = 0$$

$$v + v_x = v_0$$

$$v + v\cos\phi = v_0$$

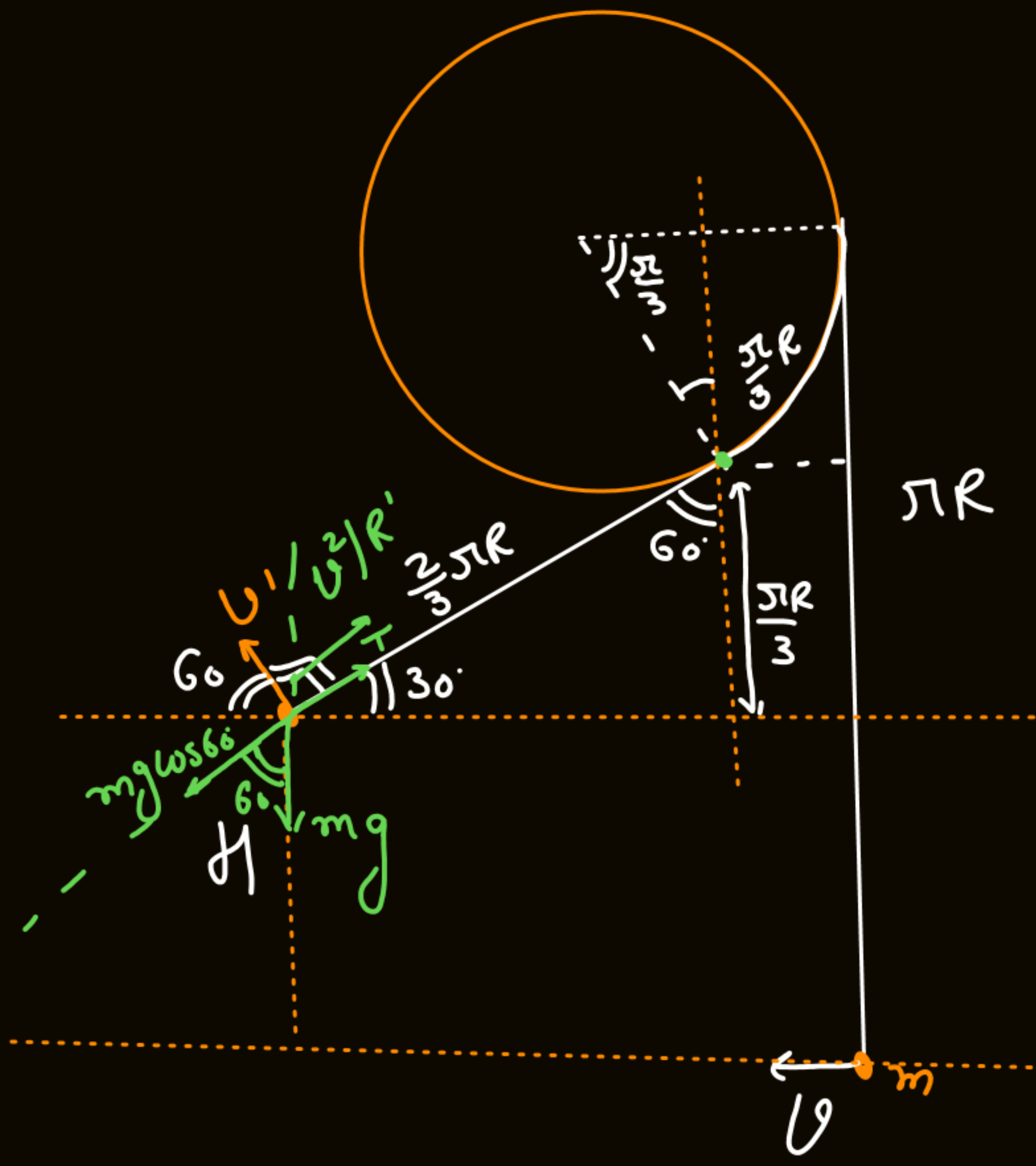
$$\boxed{v = \frac{v_0}{1+\cos\phi}}$$

A light thread is tightly wrapped around a fixed disc of radius R. A particle of mass m is tied to the end P of the thread and the vertically hanging part of the string has length πR. The particle is imparted a horizontal velocity $V = \sqrt{\frac{4\pi g R}{3}}$. The string wraps around the disc as the particle moves up. At the instant the velocity of the particle makes an angle of $\theta = 60°$ with horizontal, calculate.



(a) speed of the particle

(b) tension in the string



$$\pi R = \frac{\sqrt{3}R}{2} + \frac{\pi R}{3} + H$$

$$H = \frac{2}{3}\pi R - \frac{\sqrt{3}R}{2}$$

TMEC

$$\frac{1}{2}m\frac{4\pi g R}{3} + 0$$

$$= \frac{1}{2}mv'^2 + mg\left\{\frac{2}{3}\pi R - \frac{\sqrt{3}R}{2}\right\}$$

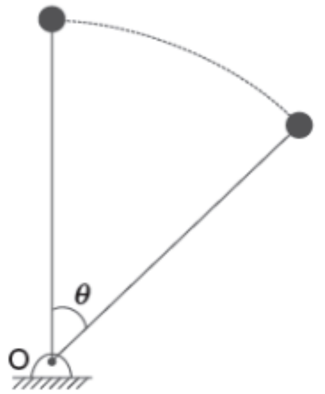
$$\boxed{v' = \sqrt{\sqrt{3}gR}}$$

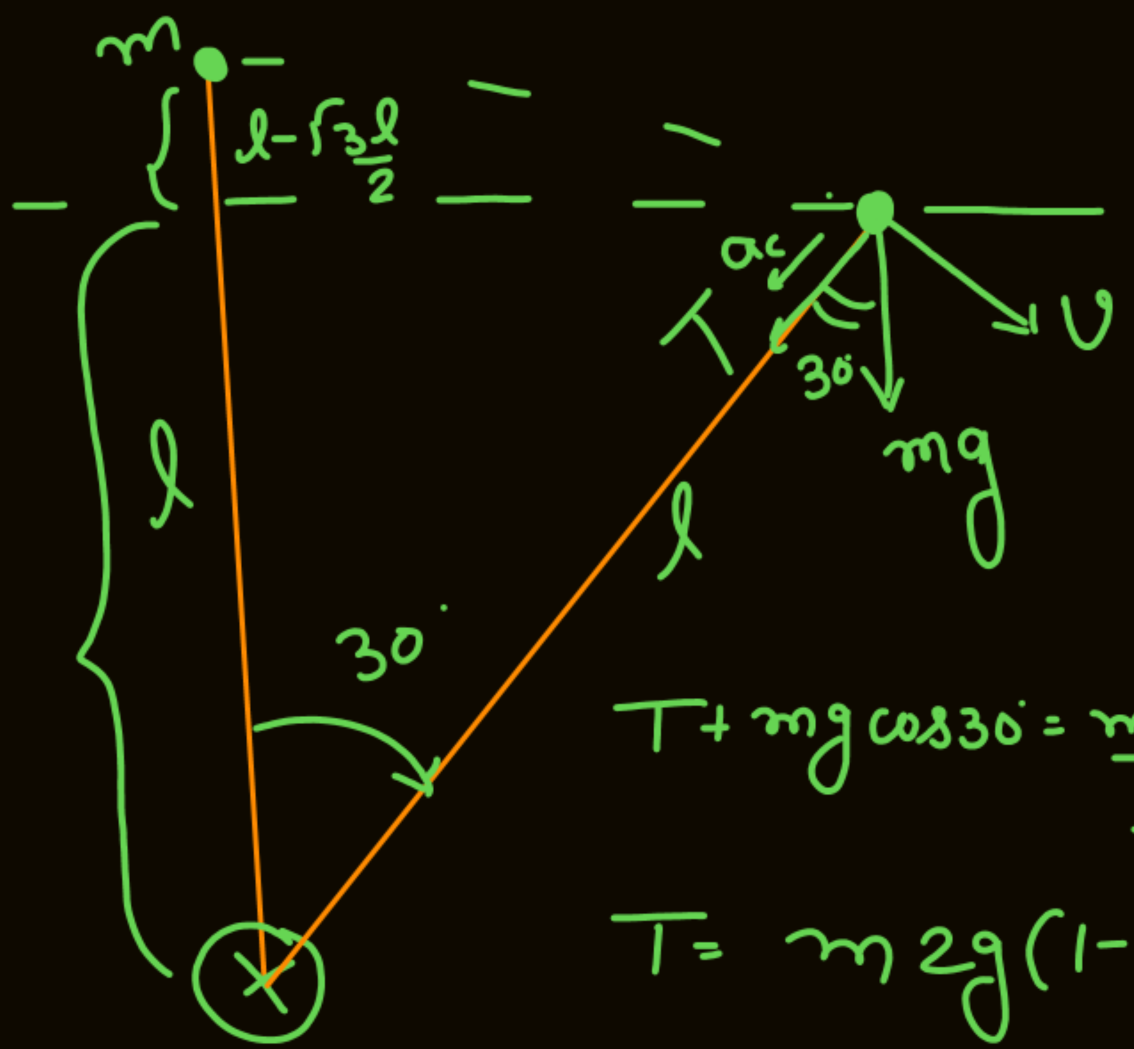
$$T - \frac{mg}{2} = \frac{m\sqrt{3}gR}{\frac{2}{3}\pi R}$$

A light rigid rod has a bob of mass m attached to one of its end. The other end of the rod is pivoted so that the entire assembly can rotate freely in a vertical plane. Initially, the rod is held vertical as shown in the figure. From this position it is allowed to fall.

(a) When the rod has rotated through $\theta = 30°$, what kind of force does it experience–compression or tension?

(b) At what value of θ the compression (or tension) in the rod changes to tension (or compression)?





$$v = \sqrt{2gl\left(1 - \frac{\sqrt{3}}{2}\right)}$$

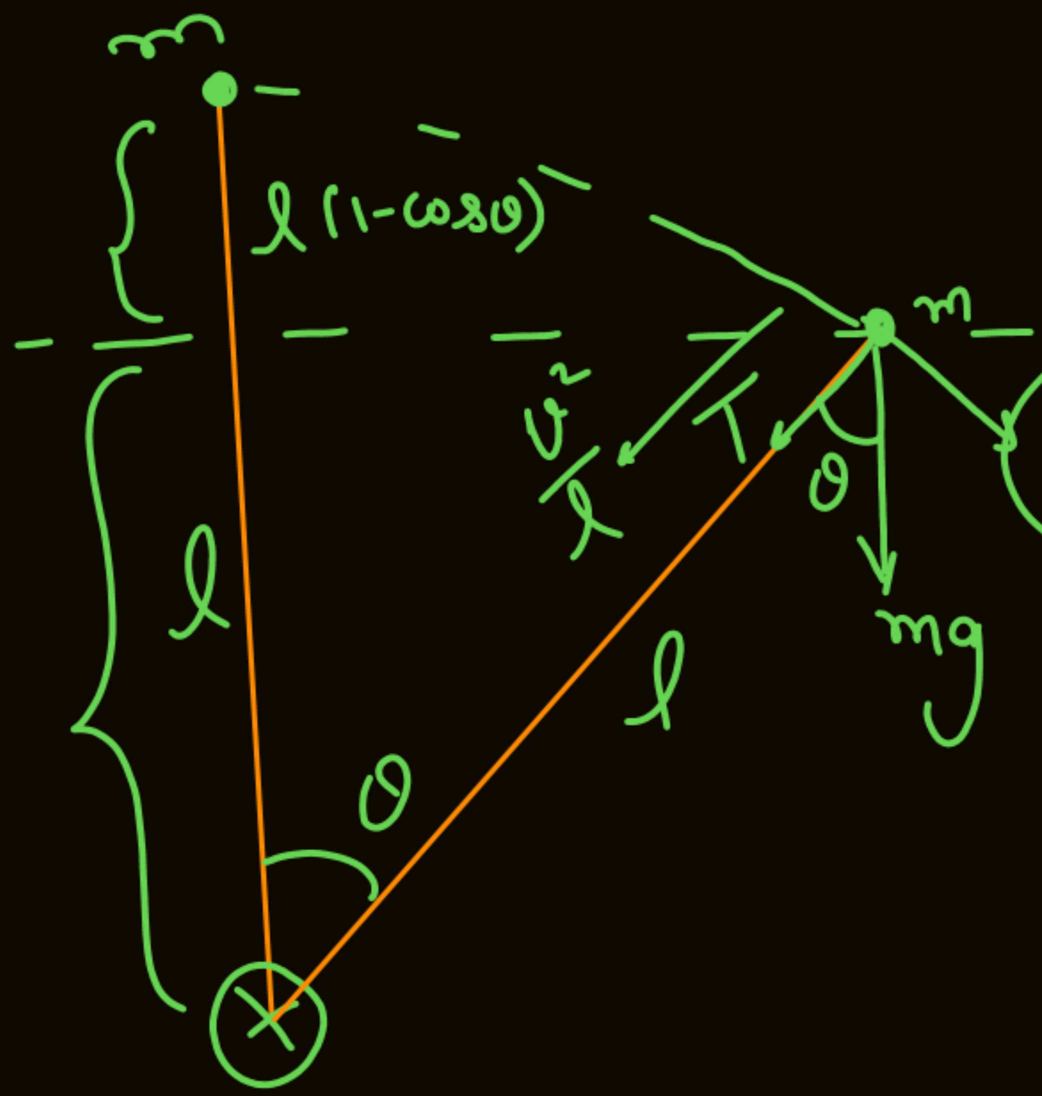
$$T + mg\cos 30° = \frac{mv^2}{l}$$

$$T = m2g\left(1 - \frac{\sqrt{3}}{2}\right) - mg\frac{\sqrt{3}}{2}$$

$$= 2mg - \sqrt{3}mg - \frac{\sqrt{3}mg}{2}$$

$$= 2mg - \frac{3}{2}\sqrt{3}mg = mg\left\{2 - \frac{3\sqrt{3}}{2}\right\} = \ominus$$

1.732



$$v = \sqrt{2gl(1-\cos\theta)}$$

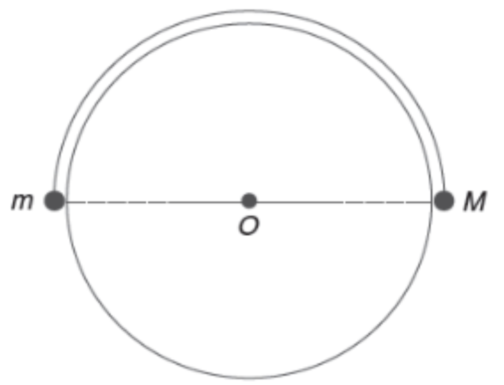
$$T + mg\cos\theta = m\,2g(1-\cos\theta)$$

$$T = 2mg - 3mg\cos\theta = 0$$

$$\cos\theta = \frac{2}{3}$$

Two particles of masses M and m ($M > m$) are connected by a light string of length πR.

The string is hung over a fixed circular frame of radius R.

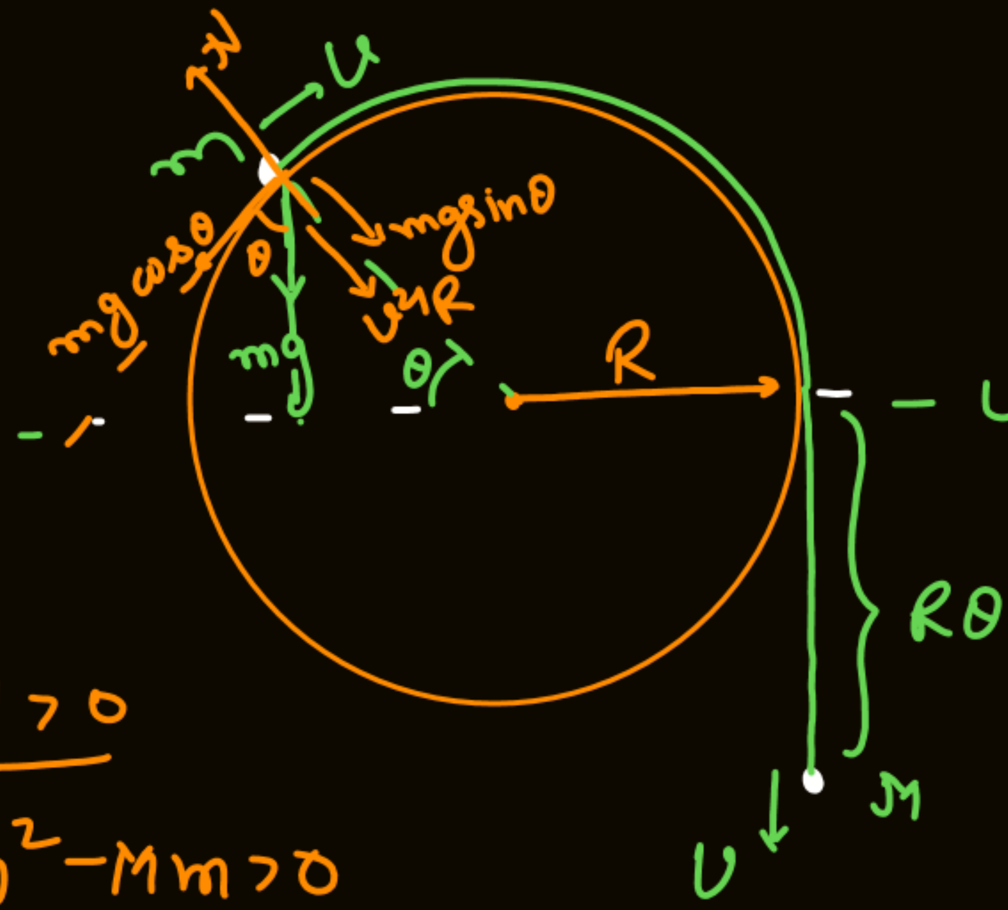


Initially the particles lie at the ends of the horizontal diameter of the circle (see figure). Neglect friction.

(a) If the system is released, and if m remains in contact with the circle, find the speed of the masses when M has descended through a distance $R\theta$ ($\theta < \pi$).

(b) Find the reaction force between the frame and m at this instant.

(c) Prove that m_1 will certainly remain in contact with the frame, just after the release, if $3m > M$.



$$0+0+0+0 = \frac{1}{2}(m+M)v^2 + mgR\sin\theta - MgR\theta$$

$$\boxed{v = \sqrt{2gR\frac{(M\theta - m\sin\theta)}{(M+m)}}}$$

$N > 0$

$3m^2 - Mm > 0$

$3m - M > 0$

$\boxed{3m > M}$

$$mg\sin\theta - N = m\,2g\frac{\{M\theta - m\sin\theta\}}{M+m}$$

$$\boxed{N = mg\sin\theta - 2mg\left\{\frac{M\theta - m\sin\theta}{M+m}\right\}}$$

$\theta \to 0$

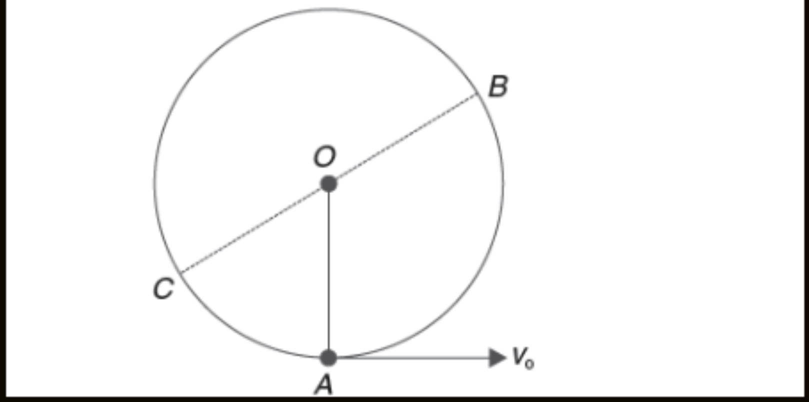
$\sin\theta \to \theta$

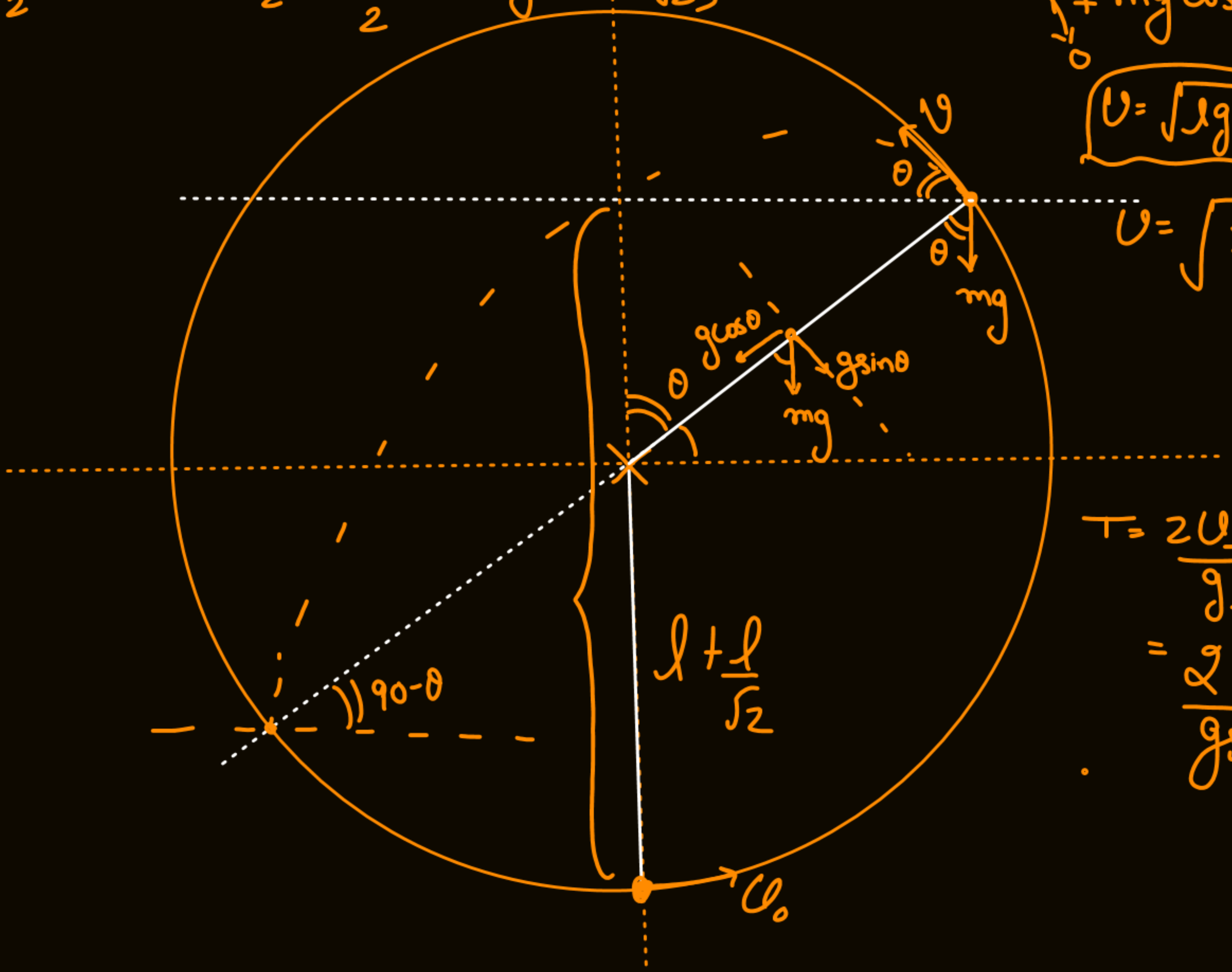
$$N = mg\theta - 2mg\frac{\{M\theta - m\theta\}}{M+m}$$

$$N = \frac{(Mm + m^2 - 2mM + 2m^2)g\theta}{M+m}$$

$$\frac{1}{2}mv_0^2 + 0 = \frac{1}{2}m\frac{lg}{2} + mgl\left\{1+\frac{1}{\sqrt{2}}\right\}$$

A heavy particle is attached to o[ne end of a] string of length l whose other [end is fixed at] O. The particle is projected horizontally with a velocity v_0 from its lowest position A. When the angular displacement of the string is more than 90°, the particle leaves the circular path at B. The string again becomes taut at C such that B,O,C are collinear. Find v_0 in terms of l and g.





$$\cancel{T}_{0} + mg\cos\theta = \frac{mv^2}{l}$$

$$\boxed{v = \sqrt{lg\cos\theta}}$$

$$v = \sqrt{\frac{lg}{\sqrt{2}}}$$

$$T = \frac{2v_1}{g_1} = \frac{2v}{g\sin\theta}$$

$$2l = 0t + \frac{1}{2}g\cos\theta\,\frac{4v^2}{g^2\sin^2\theta}$$

$$v^2 = lg\frac{\sin^2\theta}{\cos\theta} = lg\cos\theta$$

$$\underline{\theta = 45°}$$