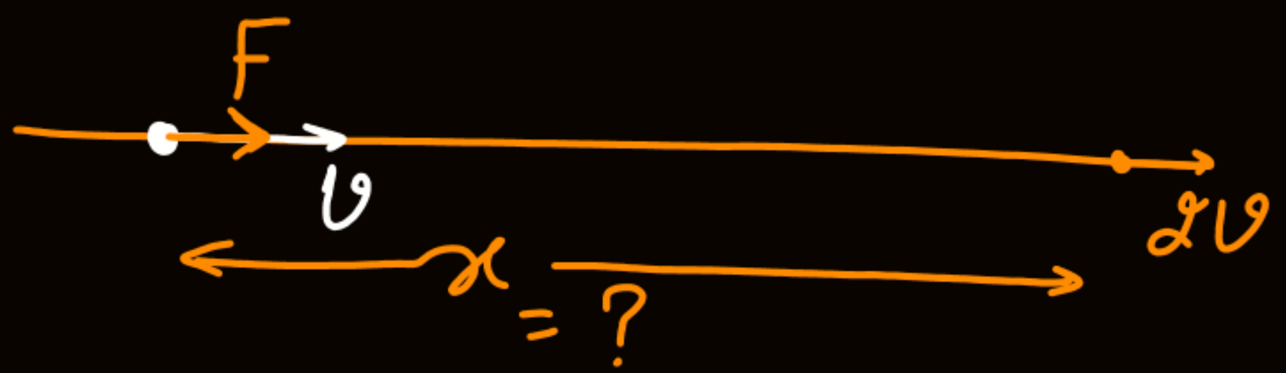
Sol:-



m

A particle is moving on a straight line and all the forces acting on it produce a constant power P calculate the distance travelled by the particle in the interval its speed increase from V to $2V$.

$$\boxed{x = \frac{7mv^3}{3P}}$$

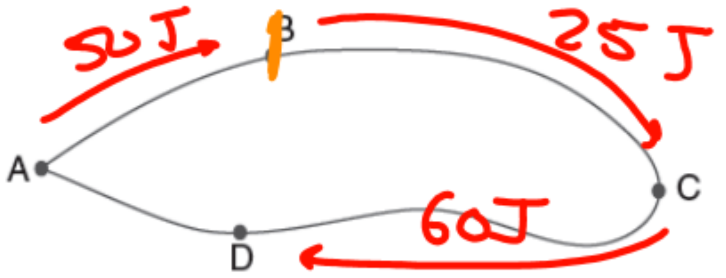
$$P = Fv\cos 0$$

$$= \left(m v \frac{dv}{dx}\right) v$$

$$P\int_0^x dx = m\int_v^{2v} v^2 dv$$

$$Px = m\left\{\frac{8v^3 - v^3}{3}\right\}$$

A particle moves along the loop $A–B–C–D–A$ while a conservative force acts on it. Work done by the force along the various sections of the path are – $W_{A \to B} = -50\ J$; $W_{B \to C} = 25\ J$; $W_{C \to D} = 60\ J$. Assume that potential energy of the particle is zero at A. Write the potential energy of particle when it is at B and D.



$$-50 + 25 + 60 + W_{DA} = 0$$

$$\underline{W_{DA} = -35\ J}$$

$$\Delta U = U_B - 0 = -W_{AB}$$

$$= 50J$$

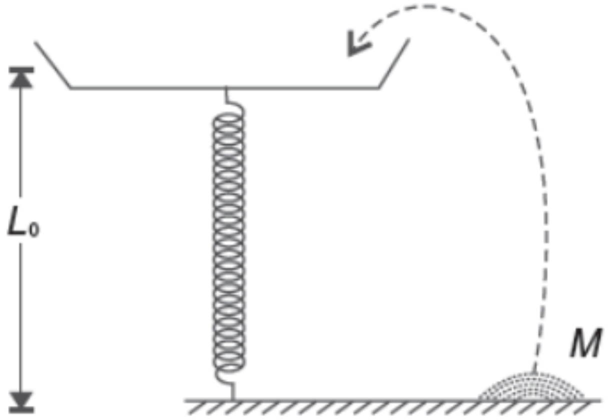
$$\boxed{U_B = 50J}$$

$$\Delta U = U_D - 0 = -W_{AD}$$

$$= -(35)$$

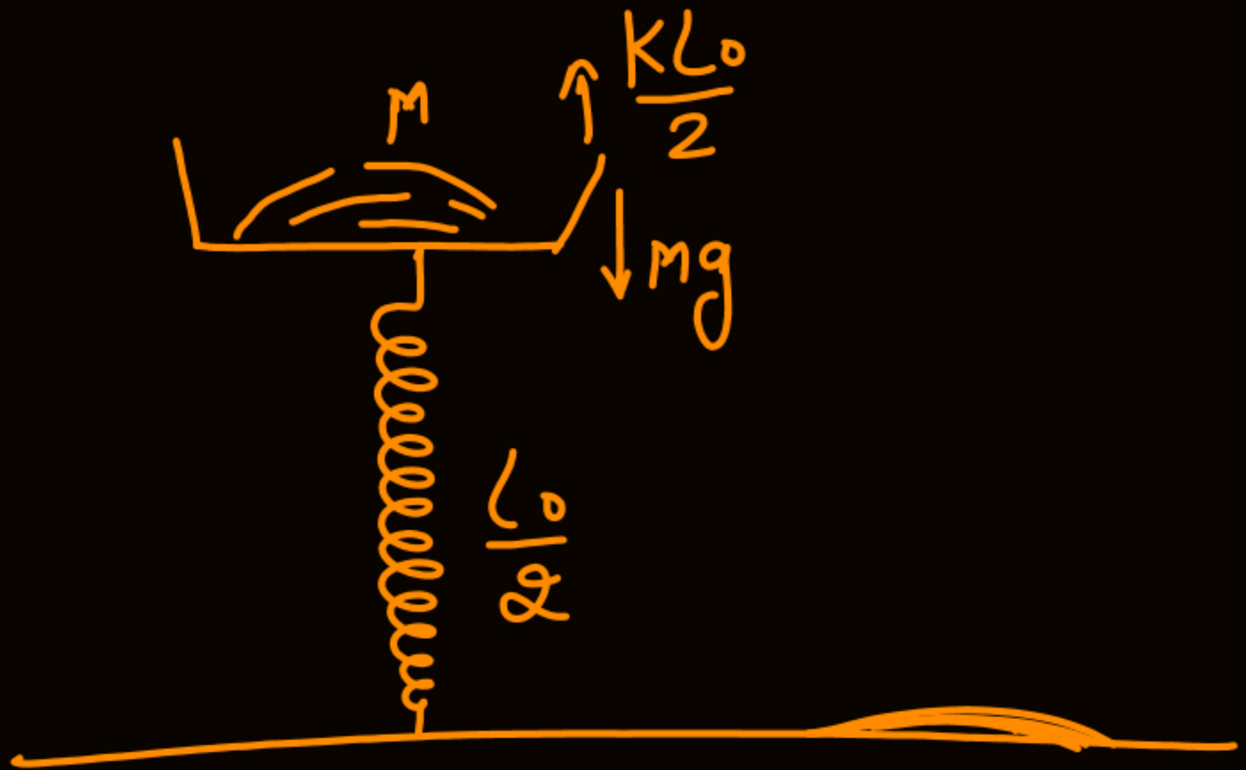
$$\boxed{U_D = -35}\ J$$

A pan of negligible mass is supported by an ideal spring which is vertical. Length of the spring is L_o. A mass M of sand is lying nearby on the floor. A boy lifts a small quantity of sand and gently puts it into the pan. This way he slowly transfers the entire sand into the pan. The spring compresses by $\frac{L_o}{2}$. Assume that height of the sand heap on the floor as well as in the pan is negligible. Calculate the work done by the boy against gravity in transferring the entire sand into the pan.



Sol:-

$$\frac{KL_0}{2} = Mg \quad \text{—} \ (1)$$



$$W_{ext} = \Delta TME$$

$$= \underbrace{\Delta K}_{\to 0} + \Delta U$$

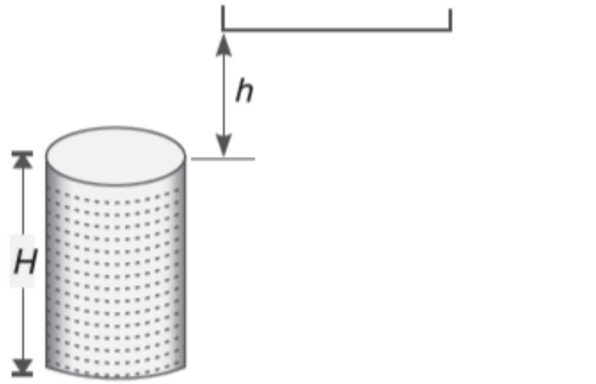
$$= \frac{MgL_0}{2} + \frac{1}{2}K\frac{L_0^2}{4}$$

$$= \frac{MgL_0}{2} + \frac{(KL_0)L_0}{8}$$

$$= \frac{MgL_0}{2} + \frac{MgL_0}{4}$$

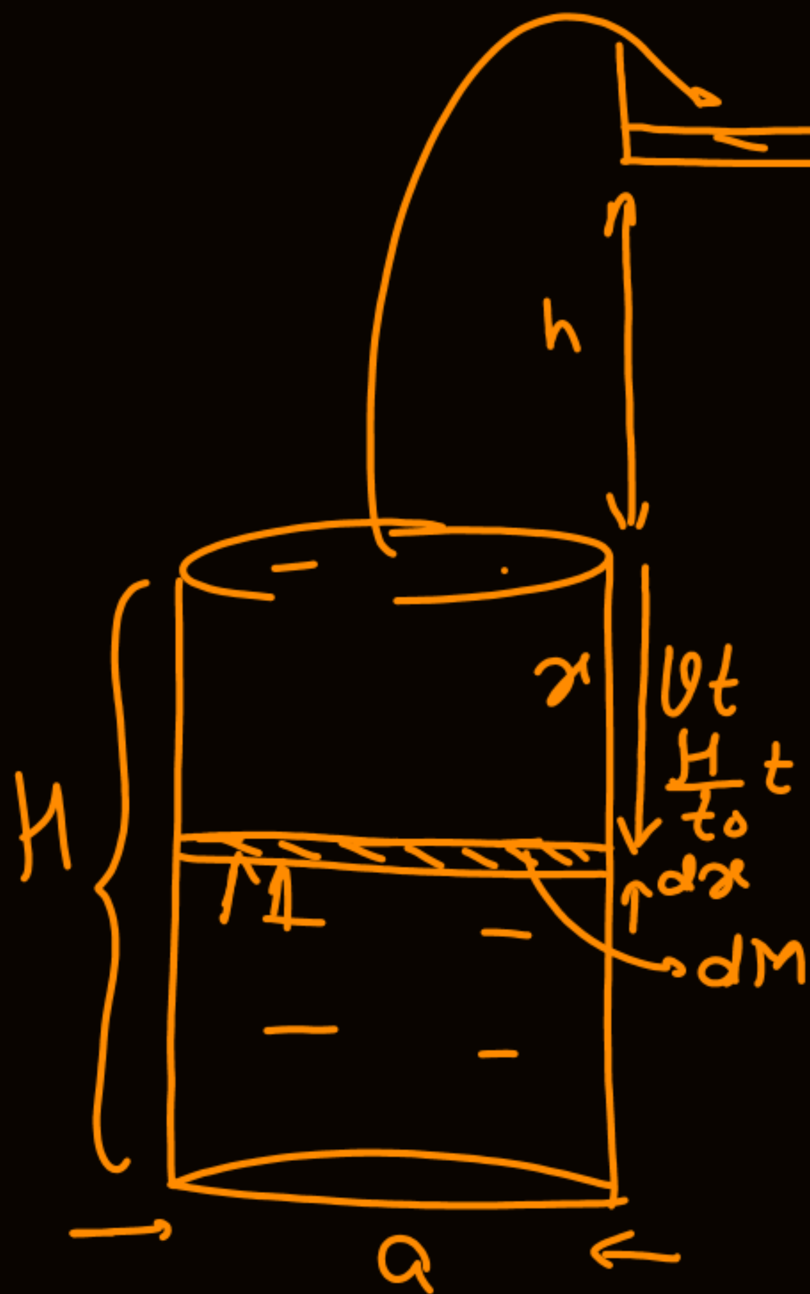
$$= \frac{3MgL_0}{4}$$

A completely filled cylindrical tank of height H contains water of mass M. At a height h above the top of the tank there is another wide container. The entire water from the tank is to be transferred into the container in time t_0 such that level of water in tank decreases at a uniform rate. How will the power of the external agent vary with time?



Sol:-

$$aH = AH' \to 0$$



$$v = \frac{H}{t_0}$$

$$dM = \frac{M}{H}dx$$

$$x = vt$$

$$dx = v\,dt$$

$$dw = \left(\frac{M}{H}dx\right)g(h+x)$$

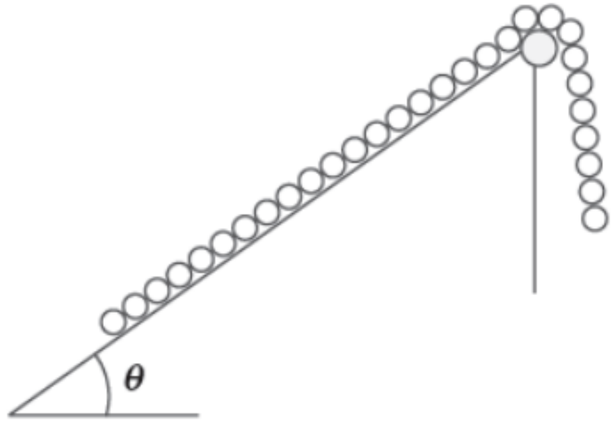
$$dw = \frac{M}{H}v\,dt\; g\{h+vt\}$$

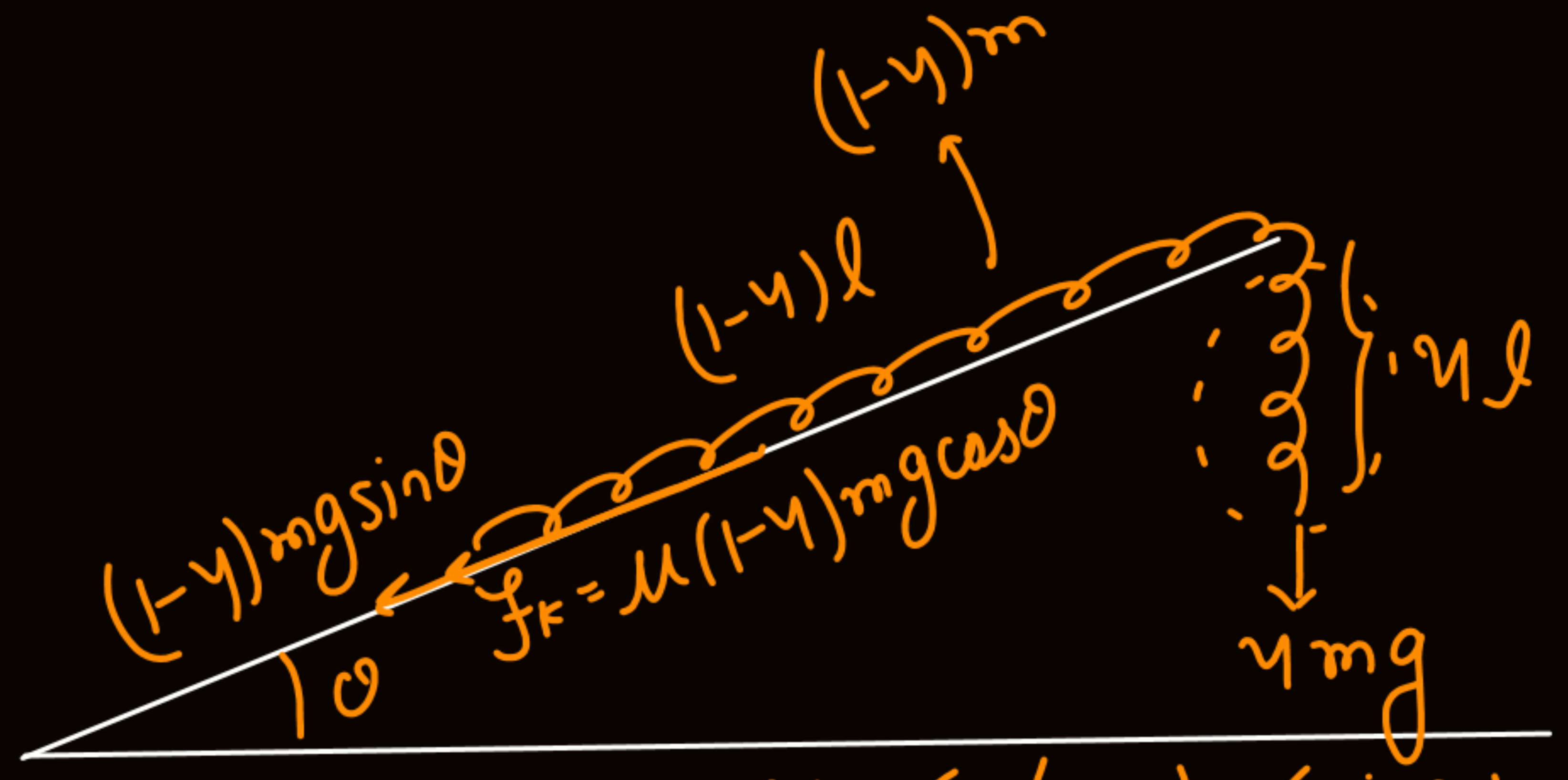
$$P = \frac{dw}{dt} = \frac{Mgv}{H}\{h+vt\}$$

$$= \frac{Mg}{H}\frac{H}{t_0}\left\{h+\frac{H}{t_0}t\right\}$$

$$\frac{Mg}{t_0}\left\{h+\frac{H}{t_0}t\right\}$$

A uniform chain of mass m_0 and length l rests on a rough incline with its part hanging vertically as shown in the fig. The chain starts sliding up the incline (and hanging part moving down) provided the hanging part equals η times the chain length ($\eta < 1$). What is the work performed by the friction force by the time chain slides completely off the incline. Neglect the dimension of pulley and assume it to be smooth.





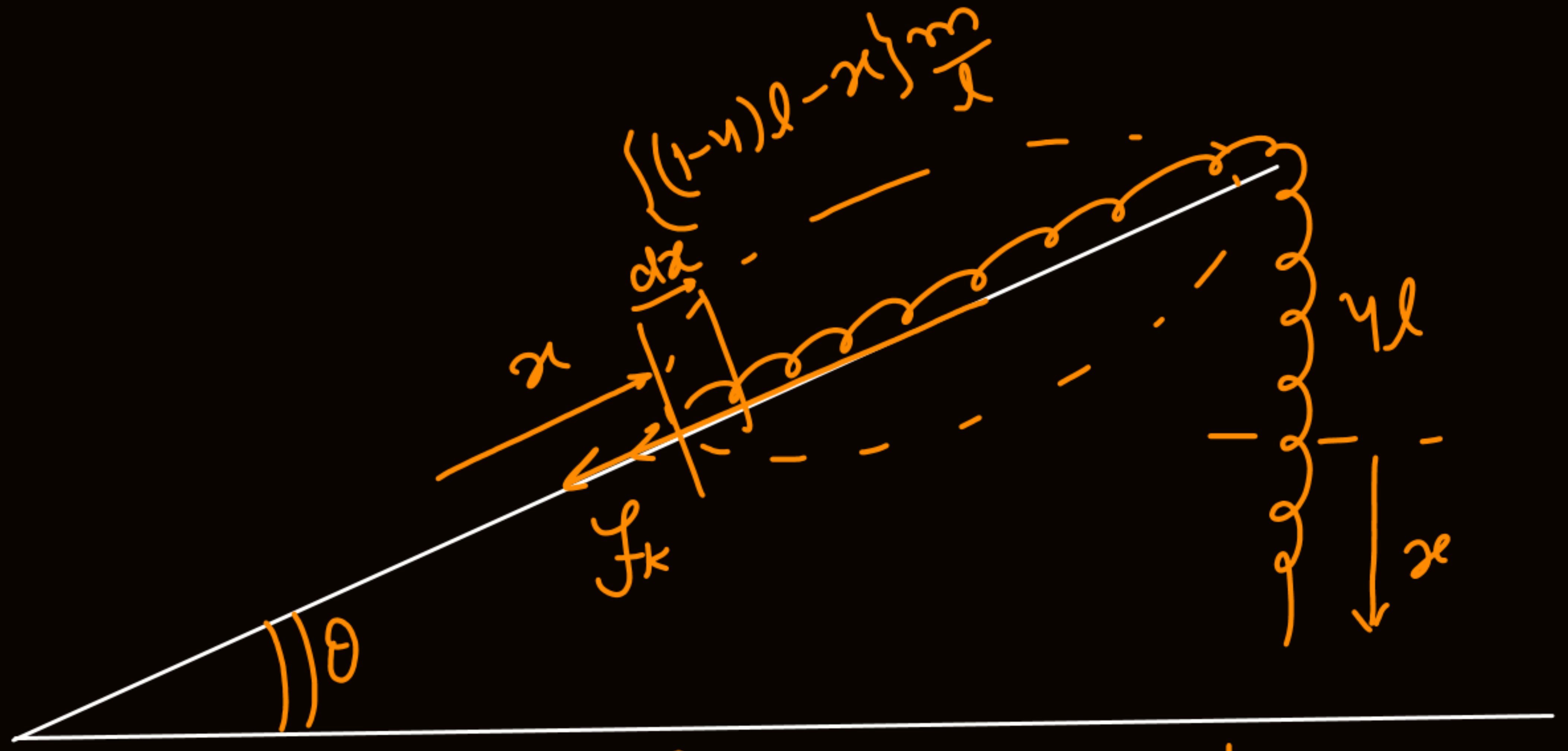
$l \to m$

$yl \to ym$

$$y\cancel{mg} = (1-y)\cancel{mg}\sin\theta + \mu(1-y)\cancel{mg}\cos\theta$$

$$y = (1-y)\sin\theta + \mu(1-y)\cos\theta$$

$$\boxed{\mu = \frac{y + (y-1)\sin\theta}{(1-y)\cos\theta}}$$

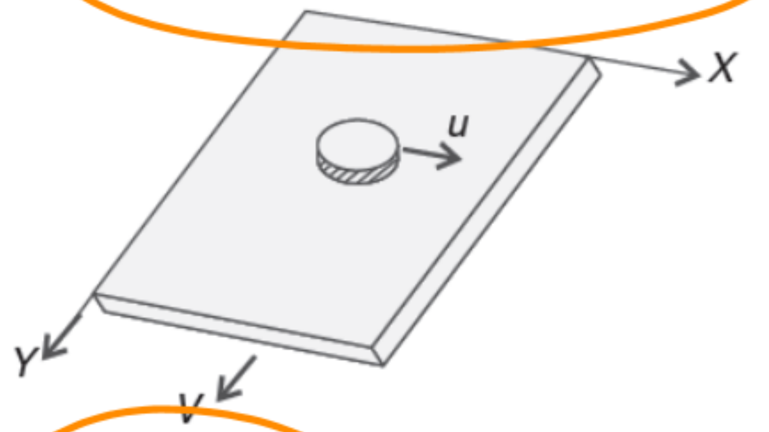


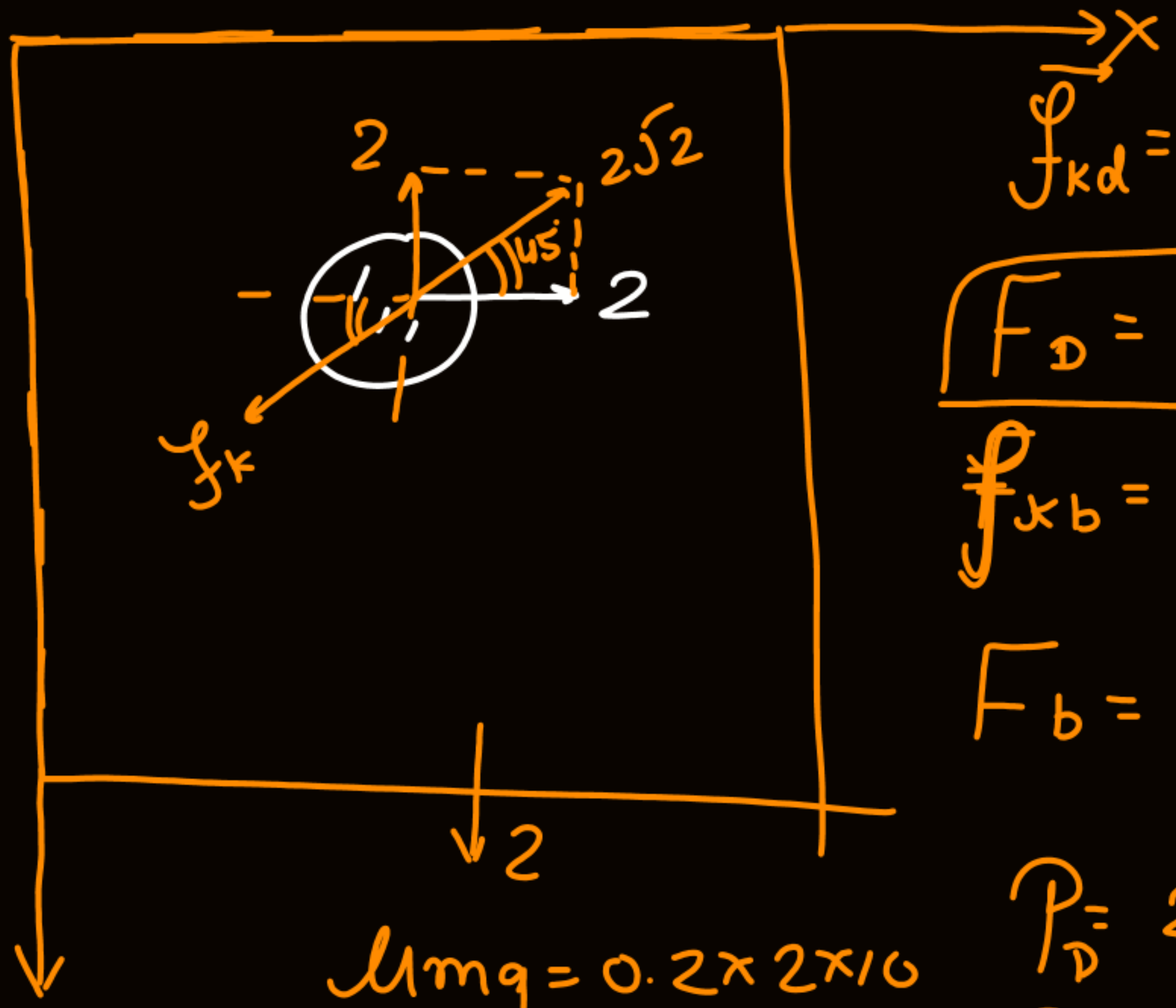
$$f_k = \mu \frac{m}{l}\{(1-\eta)l - x\} g\cos\theta$$

$$dW_f = f_k\,dx\cos\pi$$

$$= -f_k\,dx = -\frac{\mu m}{l} g \int_0^{(1-\eta)l} \{(1-\eta)l - x\}\,dx \Rightarrow \underline{\text{Ans.}}$$

A large flat board is lying on a smooth ground. A disc of mass $m = 2\ kg$ is kept on the board. The coefficient of friction between the disc and the board is $\mu = 0.2$. The disc and the board are moved with velocity $\vec{u} = 2\hat{i}\ ms^{-1}$ and $\vec{V} = 2\hat{j}\ ms^{-1}$ respectively [in reference frame of the ground]. Calculate the power of the external force applied on the disc and the force applied on the board. At what rate heat is being dissipated due to friction between the board and the disc? [$g = 10\ ms^{-2}$]





$f_{kd} = 4\left\{\frac{-\hat{i}+\hat{j}}{\sqrt{2}}\right\}$

$\boxed{F_D = 2\sqrt{2}\{\hat{i}-\hat{j}\}}$

$f_{kb} = 4\left\{\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right\}$

$F_b = 2\sqrt{2}\{-\hat{i}+\hat{j}\}$

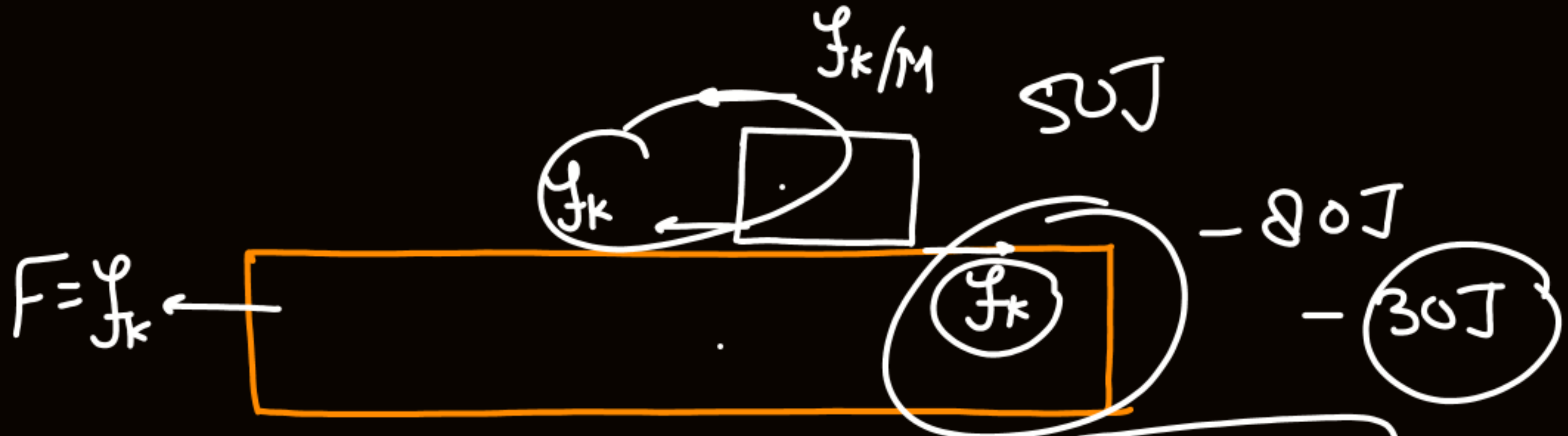
$P_D = 2\sqrt{2}\{\hat{i}-\hat{j}\}\cdot 2\hat{i} = 4\sqrt{2}$ Watt

$P_B = 2\sqrt{2}\{-\hat{i}+\hat{j}\}\cdot 2\hat{j} = 4\sqrt{2}$ Watt

$\mu mg = 0.2 \times 2 \times 10$

$\frac{8\sqrt{2}\ J}{sec} = 8\sqrt{2}W$

Sol:-



$$v = 0 + \frac{f_k}{M} t \Rightarrow \boxed{t = \frac{vM}{f_k}}$$

$$W = F x \cos 0$$

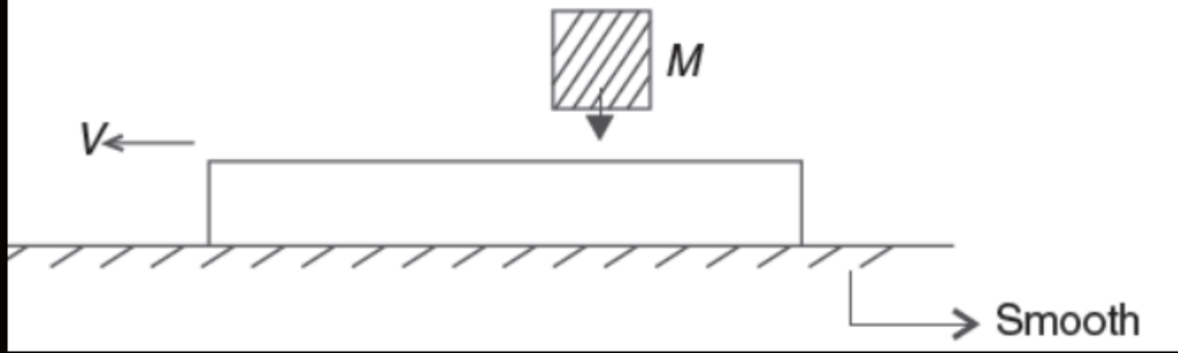
$$= f_k (v t) \times 1$$

$$= f_k \cdot v \frac{Mv}{f_k} = \underline{Mv^2}$$

A plank is moving along a smooth surface with a constant speed V. A block of mass M is gently placed on it. Initially the block slips and then acquires the constant speed (V) same as the plank. Throughout the period, a horizontal force is applied on the plank to keep its speed constant.

(a) Find the work performed by the external force.

(b) Find the heat developed due to friction between the block and the plank.



$$W_{ext} = \Delta K + \Delta U + Heat$$

$$Mv^2 = \frac{1}{2} Mv^2 + 0 + Heat$$

$$\boxed{Heat = \frac{1}{2} Mv^2}$$

A particle of mass $m = 1\ kg$ is free to move along x axis under influence of a conservative force. The potential energy function for the particle is

$$U = a\left[\left(\frac{x}{b}\right)^4 - 5\left(\frac{x}{b}\right)^2\right] \text{joule}$$

Where $b = 1.0\ m$ and $a = 1.0\ J$. If the total mechanical energy of the particle is zero, find the co-ordinates where we can expect to find the particle and also calculate the maximum speed of the particle.

Sol:-

$U = x^4 - 5x^2$

$x \to -\infty \quad U \to \infty$

$x \to \infty \quad U \to \infty$

$x^2 = t \quad U = t^2 - 5t = 0$

$t = 0 \quad t = 5$

$x^2 = 0 \quad x^2 = 5$

$\boxed{x = 0, +\sqrt{5}, -\sqrt{5}}$

$\frac{dU}{dx} = 4x^3 - 10x = 0$

$2x\{2x^2 - 5\} = 0$

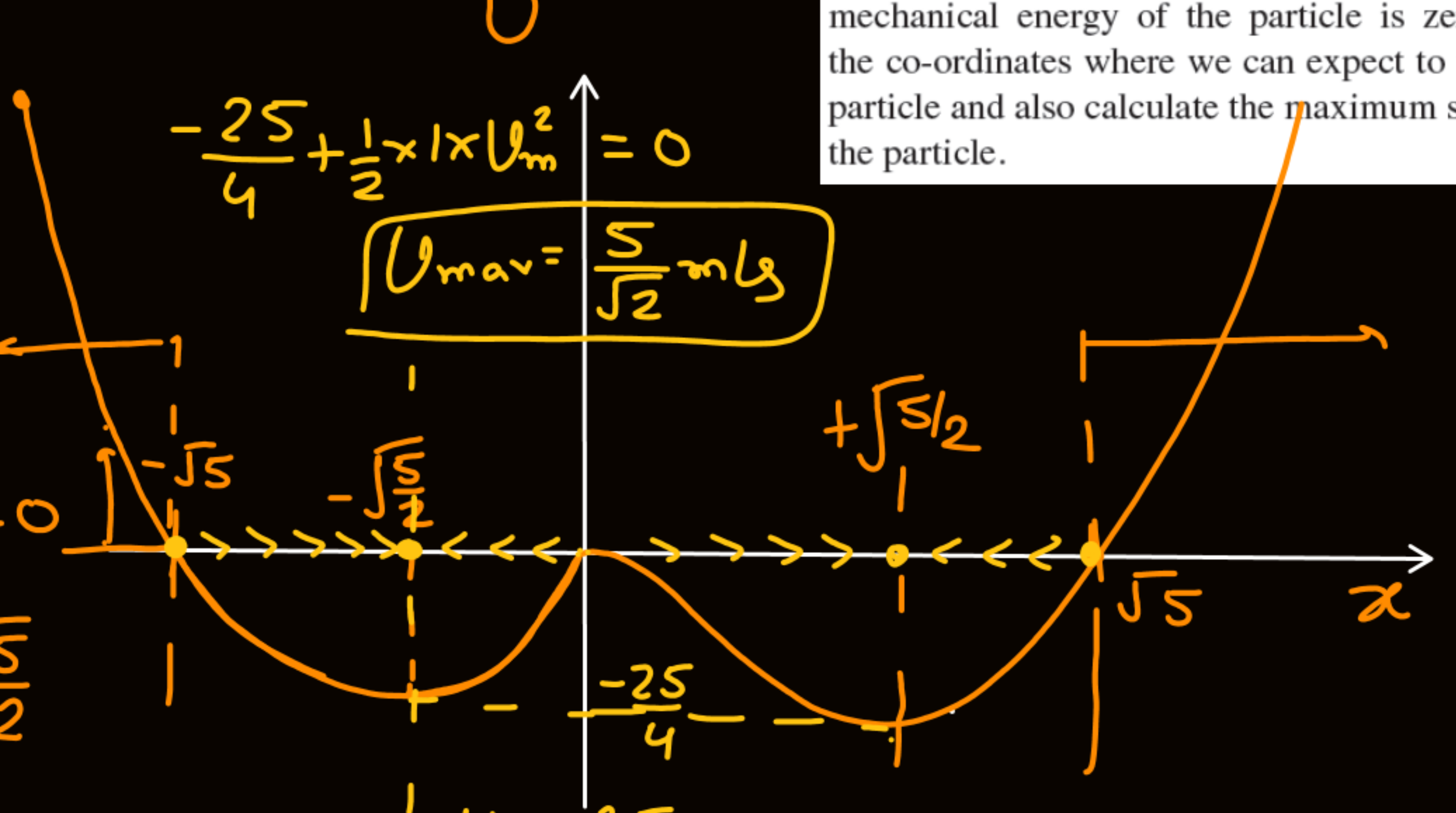
$x = 0, -\sqrt{\frac{5}{2}}, +\sqrt{\frac{5}{2}}$

$TE = U + K$

$0 = U + K$

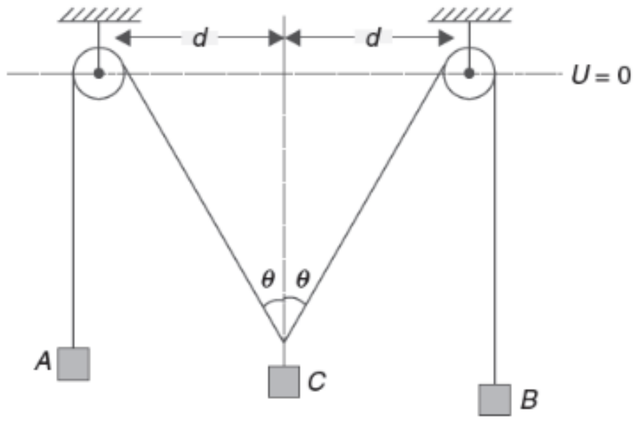
$-\frac{25}{4} + \frac{1}{2} \times 1 \times v_m^2 = 0$

$\boxed{v_{max} = \frac{5}{\sqrt{2}} \text{ m/s}}$



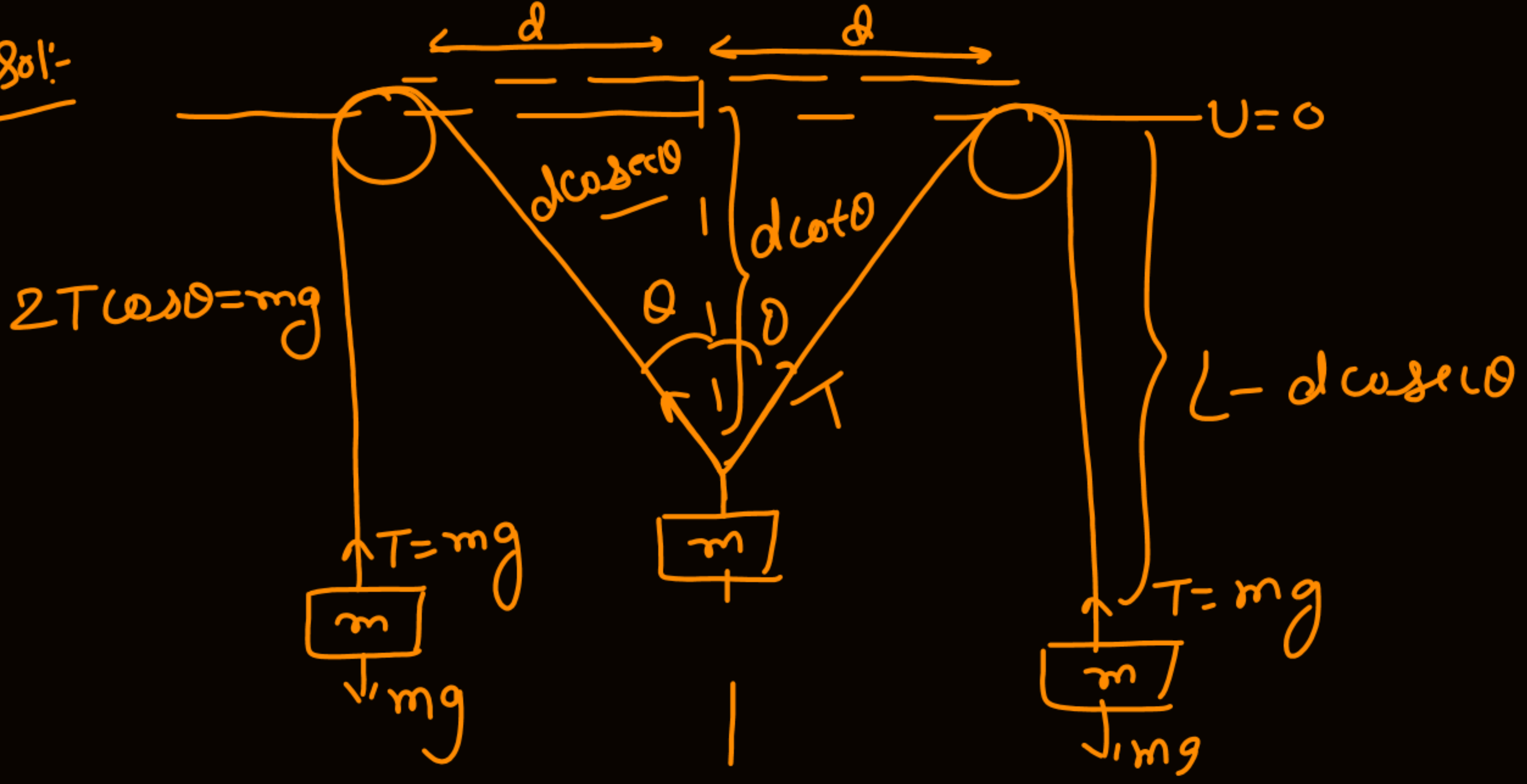
$U = \frac{25}{4} - 5 \times \frac{5}{2} = -\frac{25}{4}$

In the arrangement shown in the fig. all the three blocks have equal mass m. The length of the strings connecting A to C and B to C is L each. Assume the gravitational potential energy of any mass at the level of the pulleys to be zero. Neglect dimension of the pulley and treat the strings to be massless. Distance between the pulleys is $2d$.



(a) Write the potential energy of the system as a function of angle θ.
(b) Knowing that potential energy of the system will be maximum or minimum in equilibrium position, find value of θ for equilibrium.
(c) Tell if the equilibrium is stable or unstable.

Sol:-



$$U = -mgd\cot\theta - mg\{L - d\,\mathrm{cosec}\,\theta) \times 2$$

$$\frac{dU}{d\theta} = 0 = +mgd\,\mathrm{cosec}^2\theta - mg\left\{0 + d\cot\theta\,\mathrm{cosec}\,\theta\right\}$$

$$\mathrm{cosec}^2\theta = 2\cot\theta\,\mathrm{cosec}\,\theta$$

$$\frac{1}{\sin\theta} = 2\frac{\cos\theta}{\sin\theta}$$

$$\cos\theta = \frac{1}{2} \quad \boxed{\theta = 60^\circ}$$

$$\left(\frac{d^2U}{d\theta^2}\right)_{\theta=60^\circ} = (+) \rightarrow \text{stable eq}^m$$