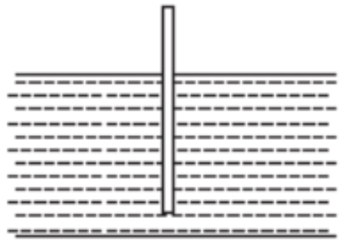
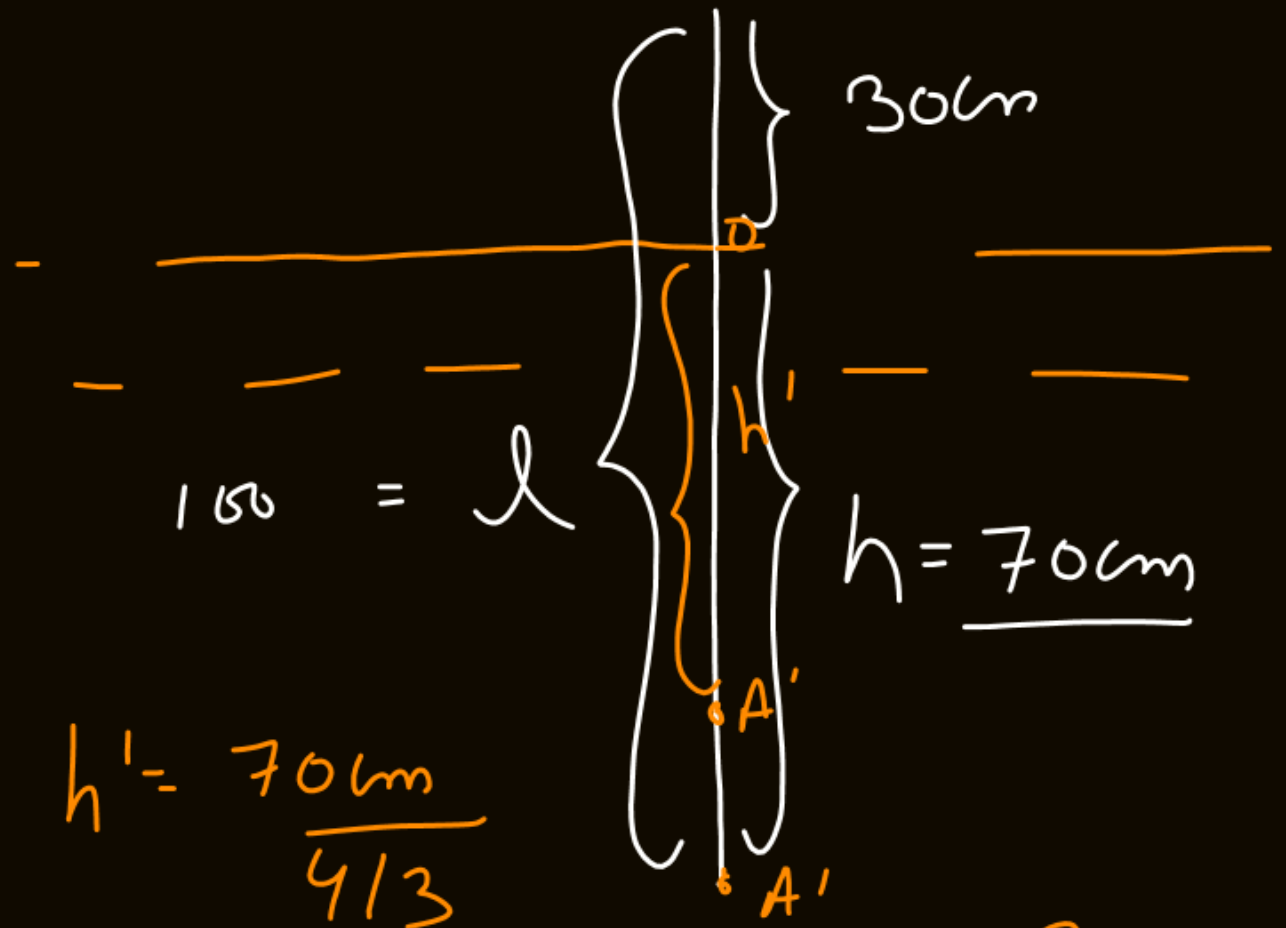
Q. 24: A wooden stick of length 100 cm is floating in water while remaining vertical. The relative density of the wood is 0.7. Calculate the apparent length of the stick when viewed from top (close to the vertical line along the stick) Refractive index of water $= \frac{4}{3}$.



Sol:-



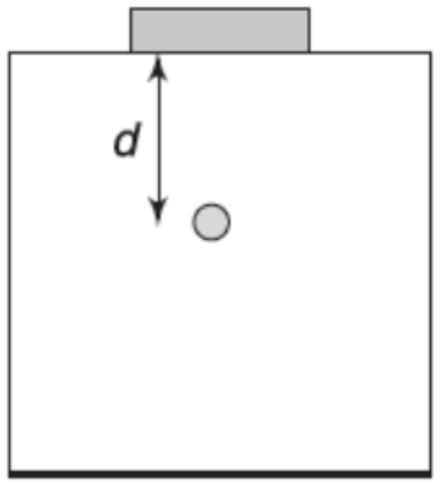
$$\frac{hA}{lA} = \frac{\rho_s}{\rho_w} = 0.7$$

$$h = 0.7l$$

$$h' = \frac{70\,cm}{4/3} = \frac{210}{4} = 52.5\,cm$$

$$30\,cm + 52.5\,cm = 82.5\,cm$$

Q. 30: A large transparent cube (refractive index = 1.5) has a small air bubble inside it. When a coin (diameter 2 cm) is placed symmetrically above the bubble on the top surface of the cube, the bubble cannot be seen by looking down into the cube at any angle. However, when a smaller coin (diameter 1.5 cm) is placed directly over it, the bubble can be seen by looking down into the cube. What is the range of the possible depths d of the air bubble beneath the top surface?

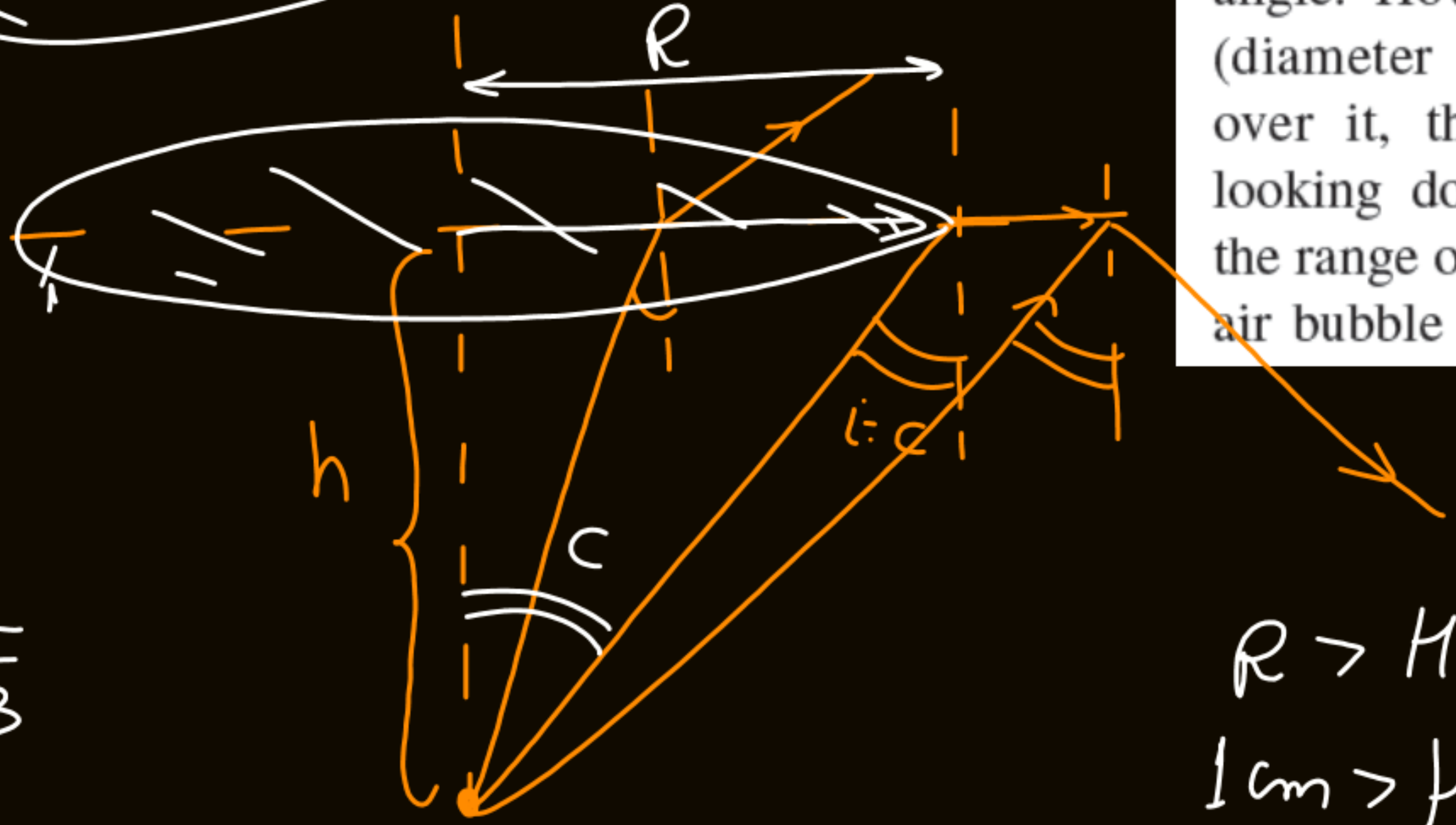


Circle of illuminance

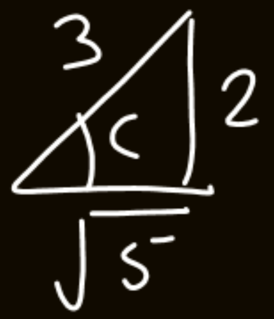
$H > \frac{3\sqrt{5}}{8}$

air

$\mu = \frac{3}{2}$



$\sin C = \frac{1}{\mu} = \frac{2}{3}$



$R < H \tan C$

$\frac{1.5}{2} < H \frac{2}{\sqrt{5}} \Rightarrow H > \frac{\sqrt{5} \times 3}{8}$

$R > H \tan C$

$1\,\text{cm} > H \frac{2}{\sqrt{5}}$

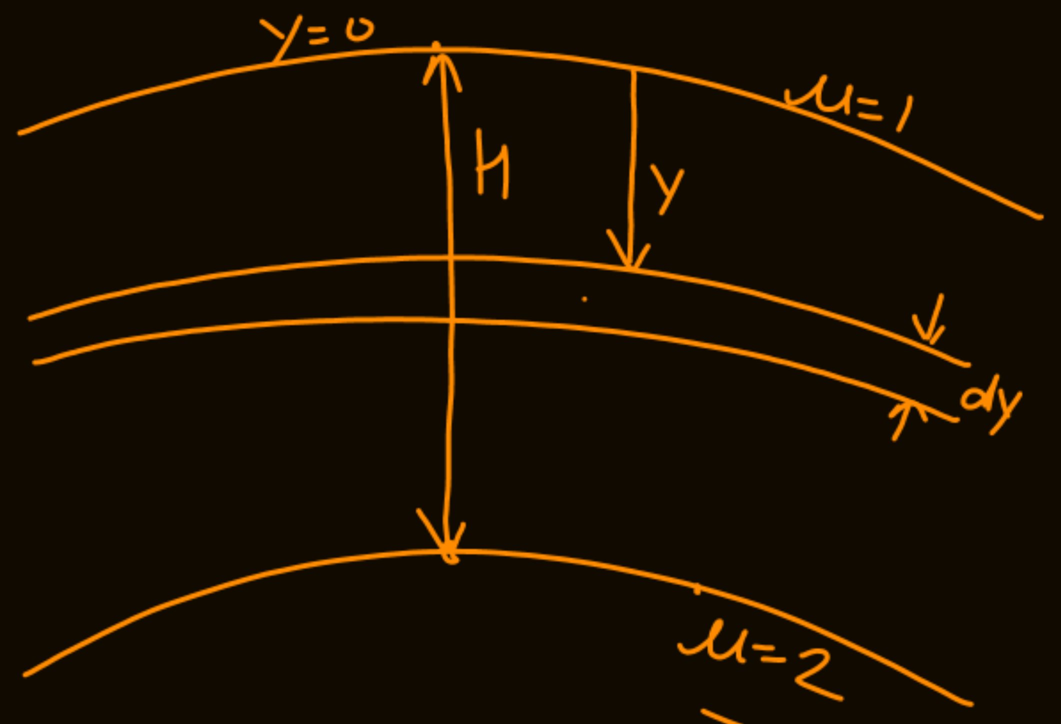
$H < \frac{\sqrt{5}}{2}\,\text{cm}$

Ray not coming out

$\frac{3\sqrt{5}}{8} < H < \frac{\sqrt{5}}{2}\,\text{cm}$

Q. 35: The atmosphere of earth extends upto height H and its refractive index varies with depth y from the top as $\mu = 1 + \frac{y}{H}$. Calculate the apparent thickness of the atmosphere as seen by an observer in space.

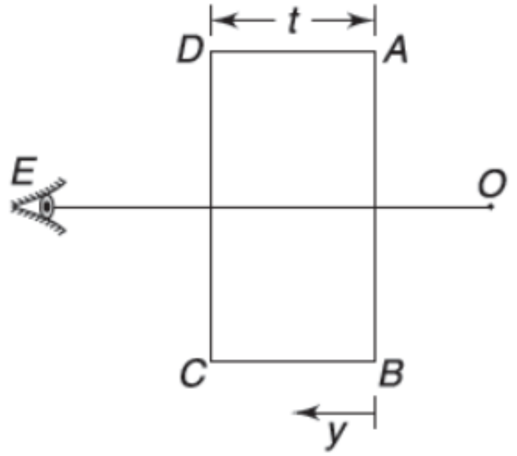
$$t_{app} = \frac{t_1}{\mu_1} + \frac{t_2}{\mu_2}$$

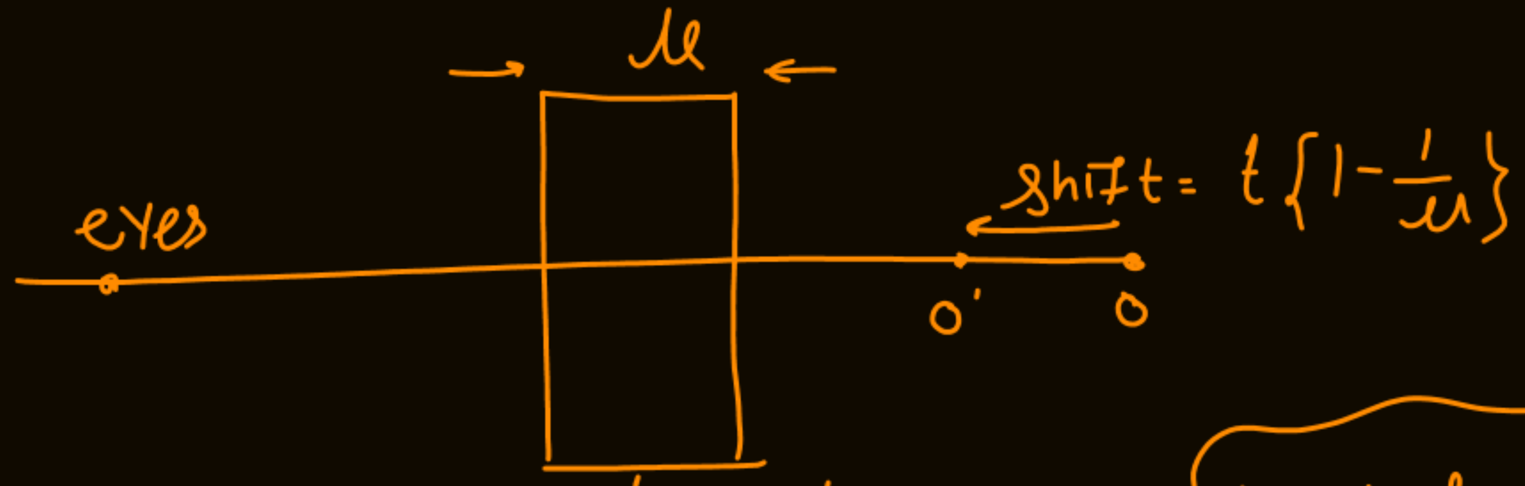


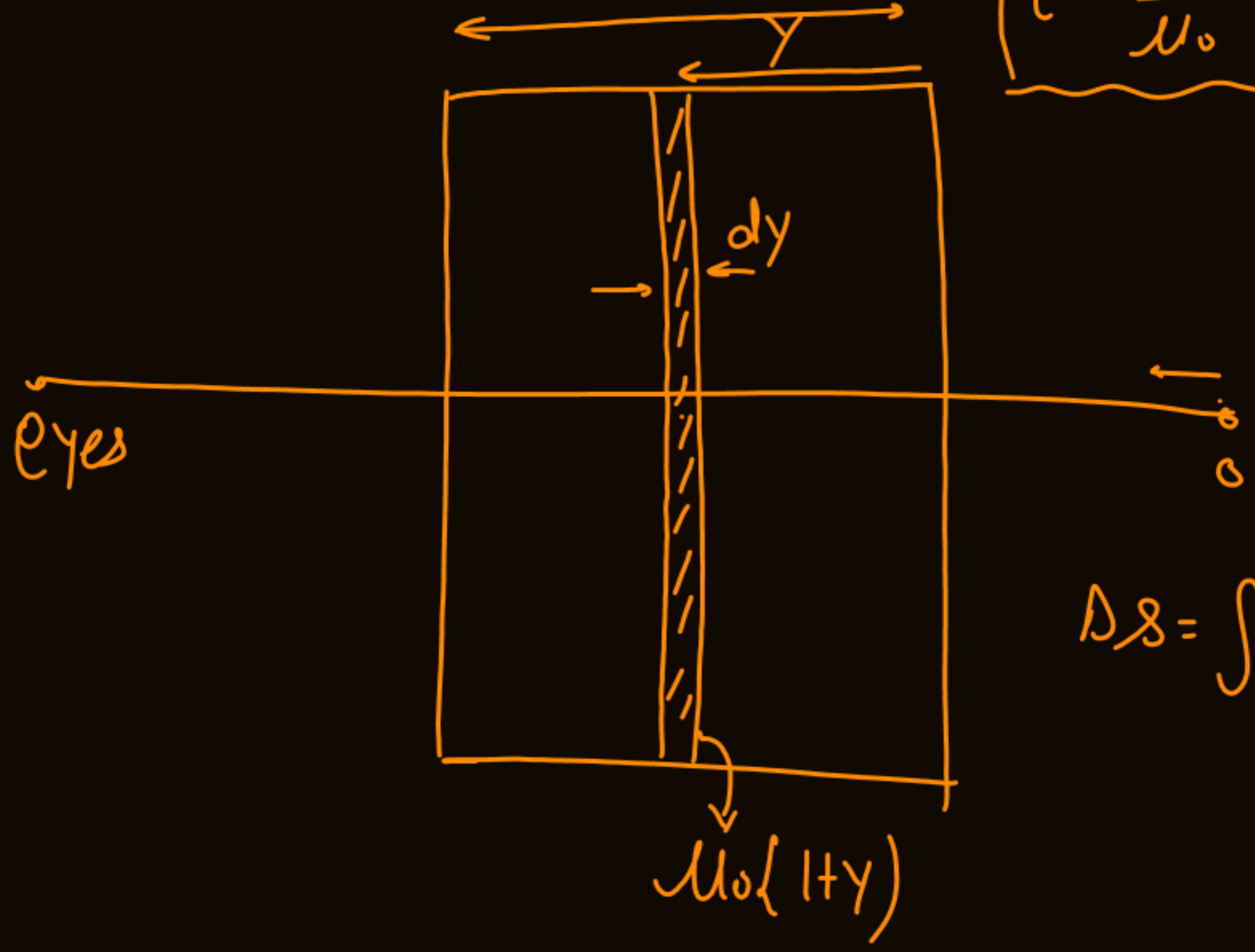
$$dy' = \int \frac{dy}{\mu/1} = \int_0^H \frac{dy\, H}{(H+y)}$$

$$= H \ln(H+y)\Big|_0^H = H\{\ln 2H - \ln H\} = \ln 2\, H = 0.693H$$

Q. 36: A glass slab is placed between an object (O) and an observer (E) with its refracting surfaces AB and CD perpendicular to the line OE. The refractive index of the glass slab changes with distance (y) from the face AB as $\mu = \mu_0(1 + y)$. Thickness of the slab is t. Find how much closer (compared to original distance) the object appears to the observer. Consider near normal incidence only.







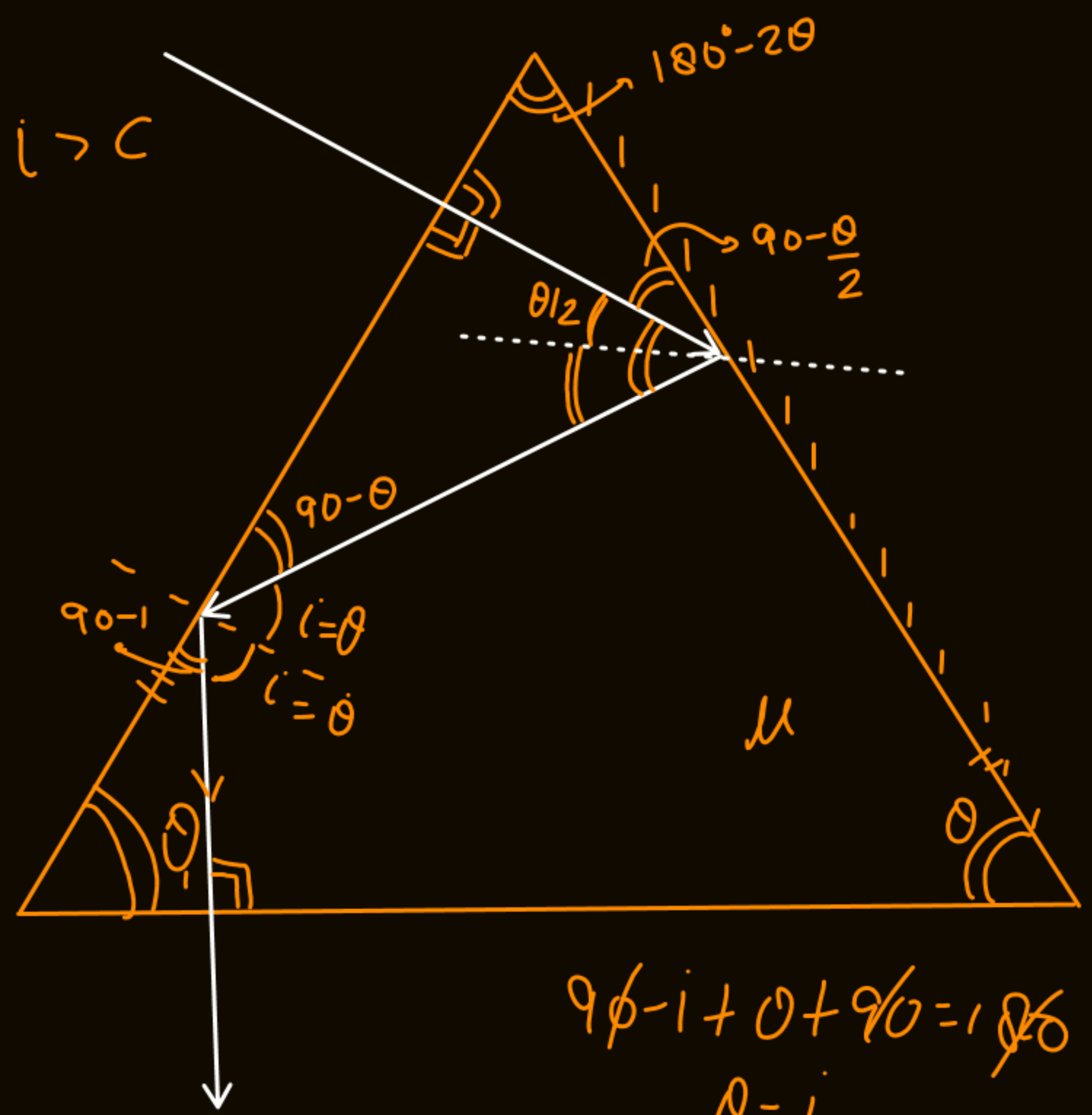
$$t - \frac{1}{\mu_0}\ln(1+t)$$

$$ds = dy\left\{1 - \frac{1}{\mu}\right\} = dy\left\{1 - \frac{1}{\mu_0(1+y)}\right\}$$

$$\Delta s = \int ds = \int_0^t dy - \frac{1}{\mu_0}\int_0^t \frac{dy}{(1+y)}$$

$$= t - \frac{1}{\mu_0}\ln(1+y)\Big|_0^t = t - \frac{1}{\mu_0}\left(\ln(1+t)\right)$$

Q. 43: An isosceles glass prism has one of its faces silvered. A light ray is incident normally on the other face which is identical in size to the silvered face. The light ray is reflected twice on the same sized faces and emerges through the base of the prism perpendicularly. Find the minimum value of refractive index of the material of the prism.



$90 - i + \theta + 90 = 180$

$\theta = i$

$90 - \theta + 90 + (\ ) = 180$

$(\ ) = \theta$

$90 + 180 - 2\theta + 90 - \frac{\theta}{2} = 180$

$180 = \frac{5\theta}{2}$

$\theta = 72° = i$

$i > C$

$72° > C$

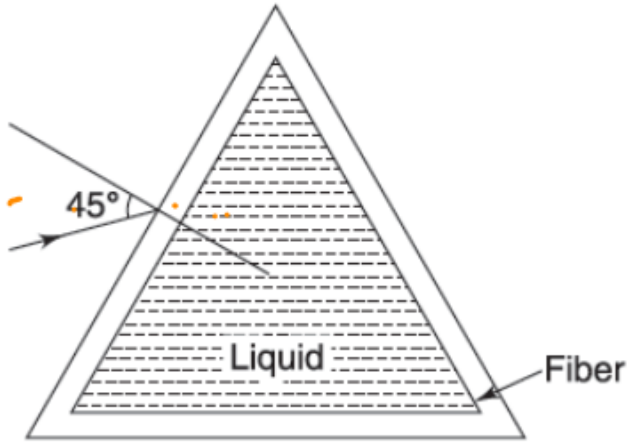
$\sin 72° > \sin C$

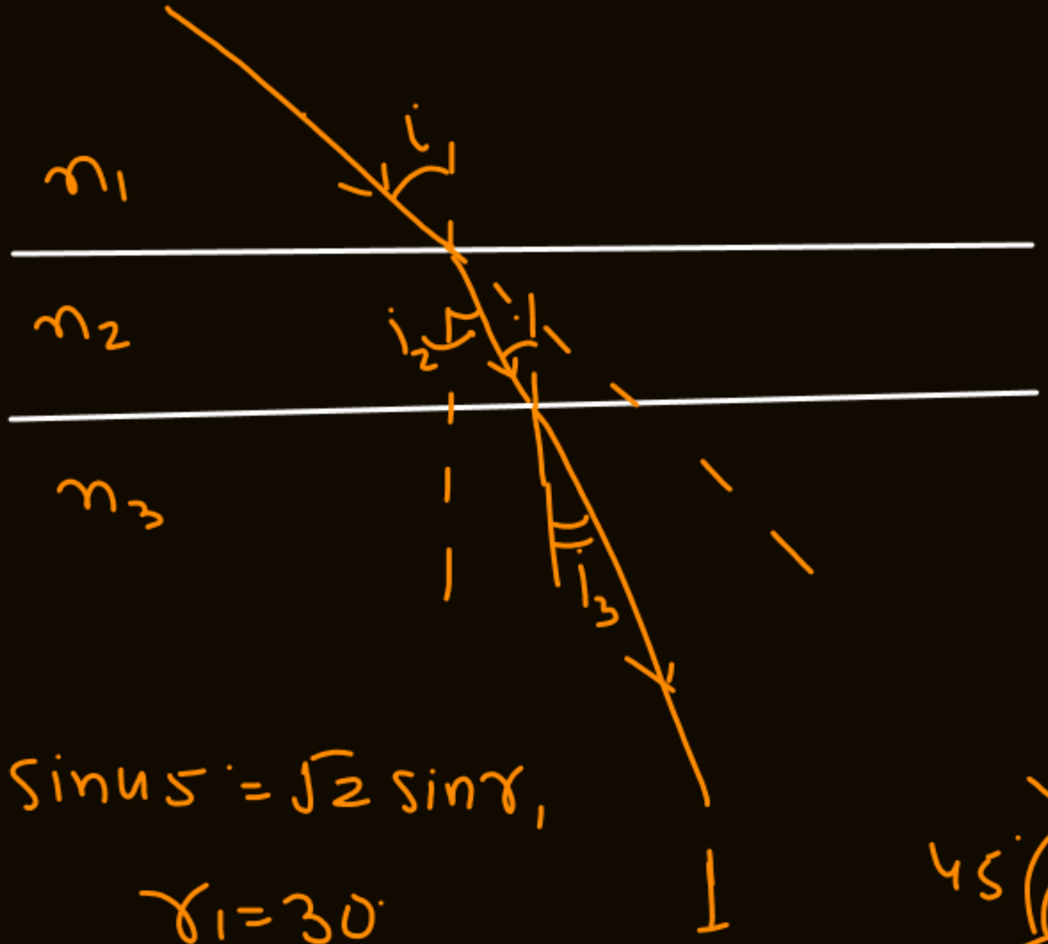
$\sin 72° > \frac{1}{\mu}$

$\boxed{\mu > \frac{1}{\sin 72°}}$

Ans.

Q. 52: An equilateral prism has its faces made of a transparent fiber sheet (having refractive index = 1.25) having thickness of 1 mm. The fibre prism is filled with a liquid of refractive index $\sqrt{2}$. Find the deviation of a light ray incident on one face of such a prism at an angle of 45°.

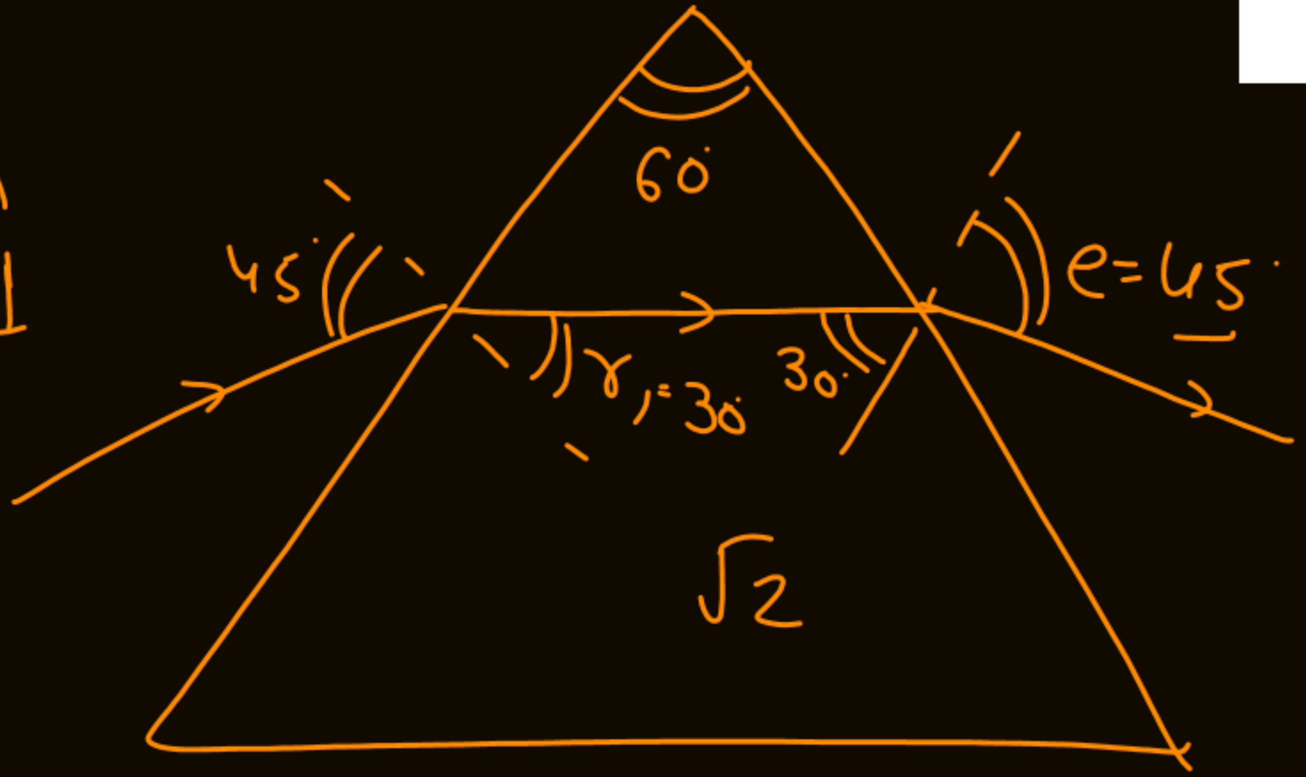




$$n_1 \sin i_1 = n_2 \sin i_2 = n_3 \sin i_3$$

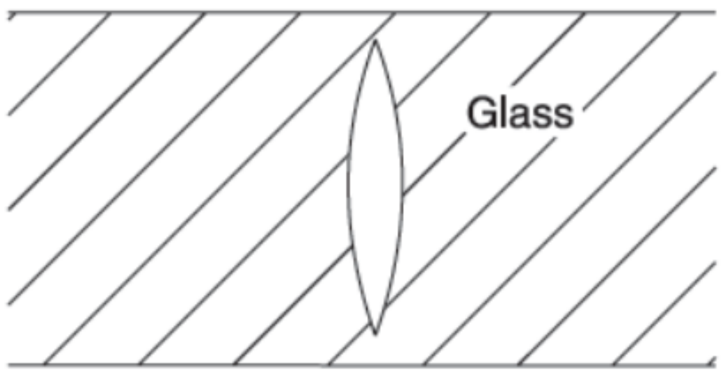
$$1 \times \sin 45^\circ = \sqrt{2} \sin r_1$$

$$r_1 = 30^\circ$$



$$\delta = i + e - A = 45^\circ + 45^\circ - 60^\circ = 30^\circ = \delta_{min}$$

Q. 70: There is an air lens in an extended glass medium. The radius of curvature of both the curved surfaces is R, and refractive index of the glass is $\frac{3}{2}$. Power of this air lens is P. Find the refractive index of the material to be filled inside the lens so that its power becomes $-P$.



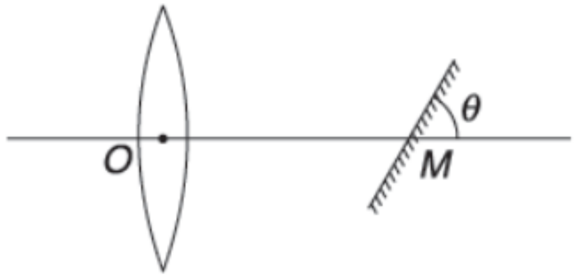
$$P = \frac{1}{f} = \left(\frac{\mu_l}{\mu_s} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

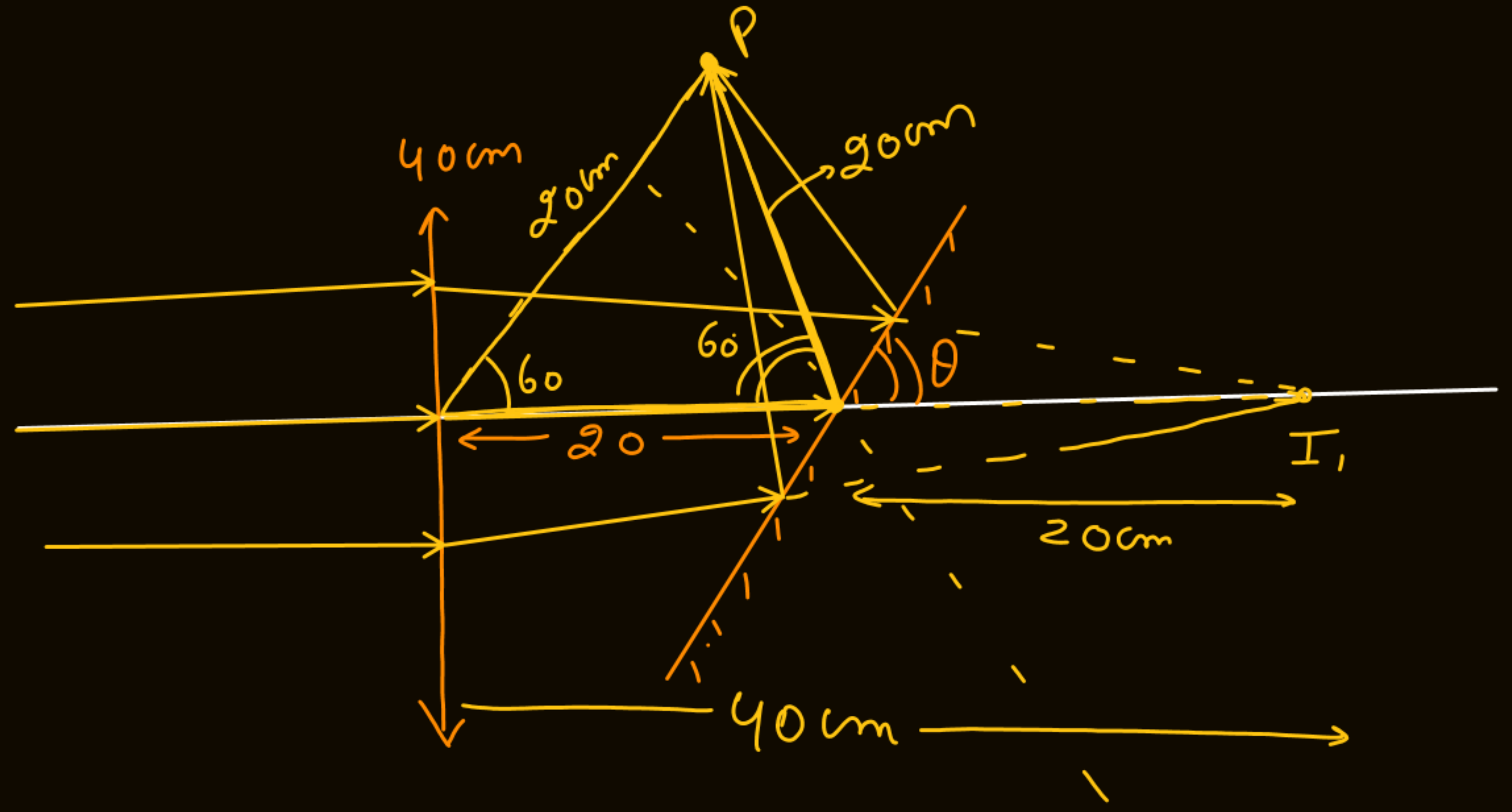
$$P = \left(\frac{1}{3/2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad -(1)$$

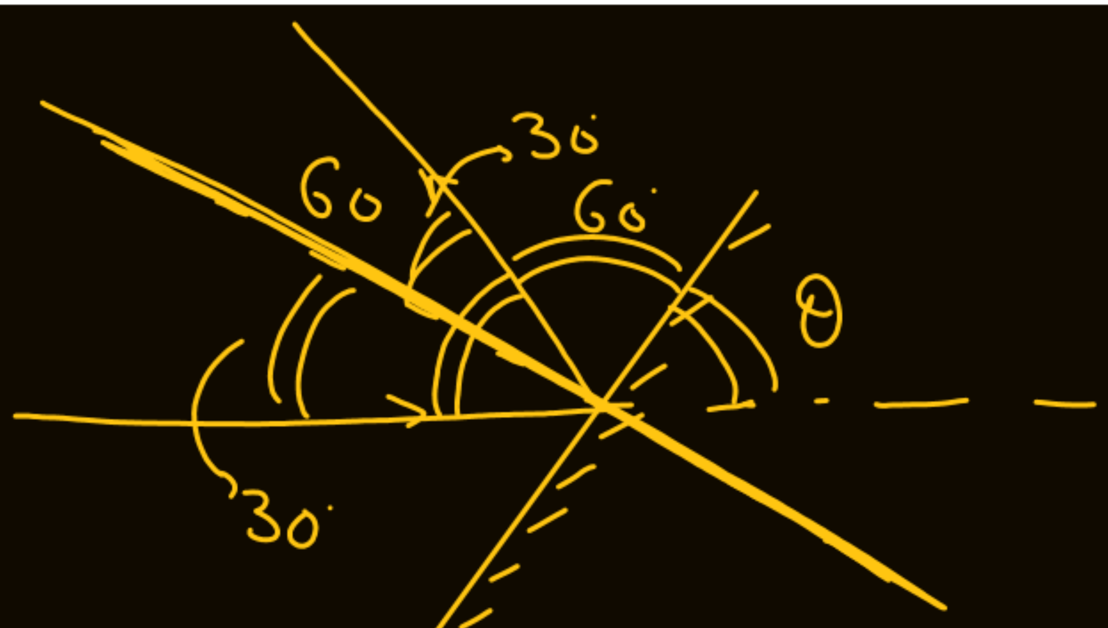
$$-P = \left(\frac{\mu_l}{3/2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad -(2)$$

$$\frac{(2)}{(1)} \Rightarrow \mu_l = 2$$

Q. 68: A horizontal parallel beam of light passes though a vertical convex lens of focal length 40 cm. Behind the lens there is a plane mirror making an angle θ with the principal axis of the lens. The mirror intersects the principal axis at M. Distance between the optical centre of the lens and point M is $OM = 20$ cm. The light beam reflected by the mirror converges at a point P. Distance OP is 20 cm. Find θ.



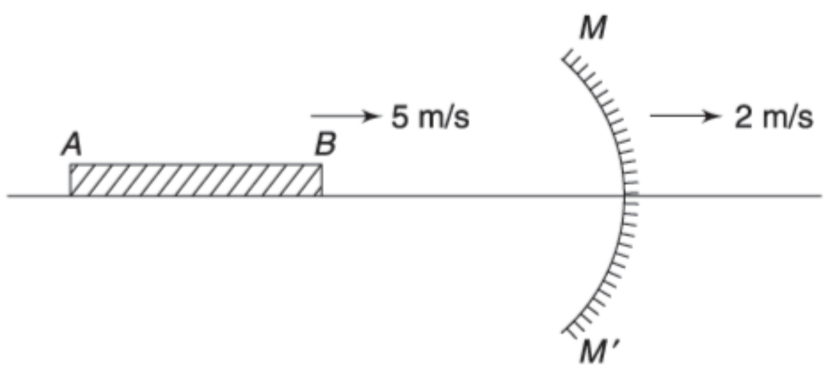




$60° + 60° + \theta = 180°$

$\theta = 60°$

Q. 88: A pencil (AB) of length 20 cm is moving along the principal axis of a concave mirror MM', with a velocity 5 m/s approaching the mirror. The mirror itself is moving away from the pencil at a speed of 2 m/s. Find the rate of change of length of the image of the pencil at the instant end A is at a distance of 60 cm from the mirror.

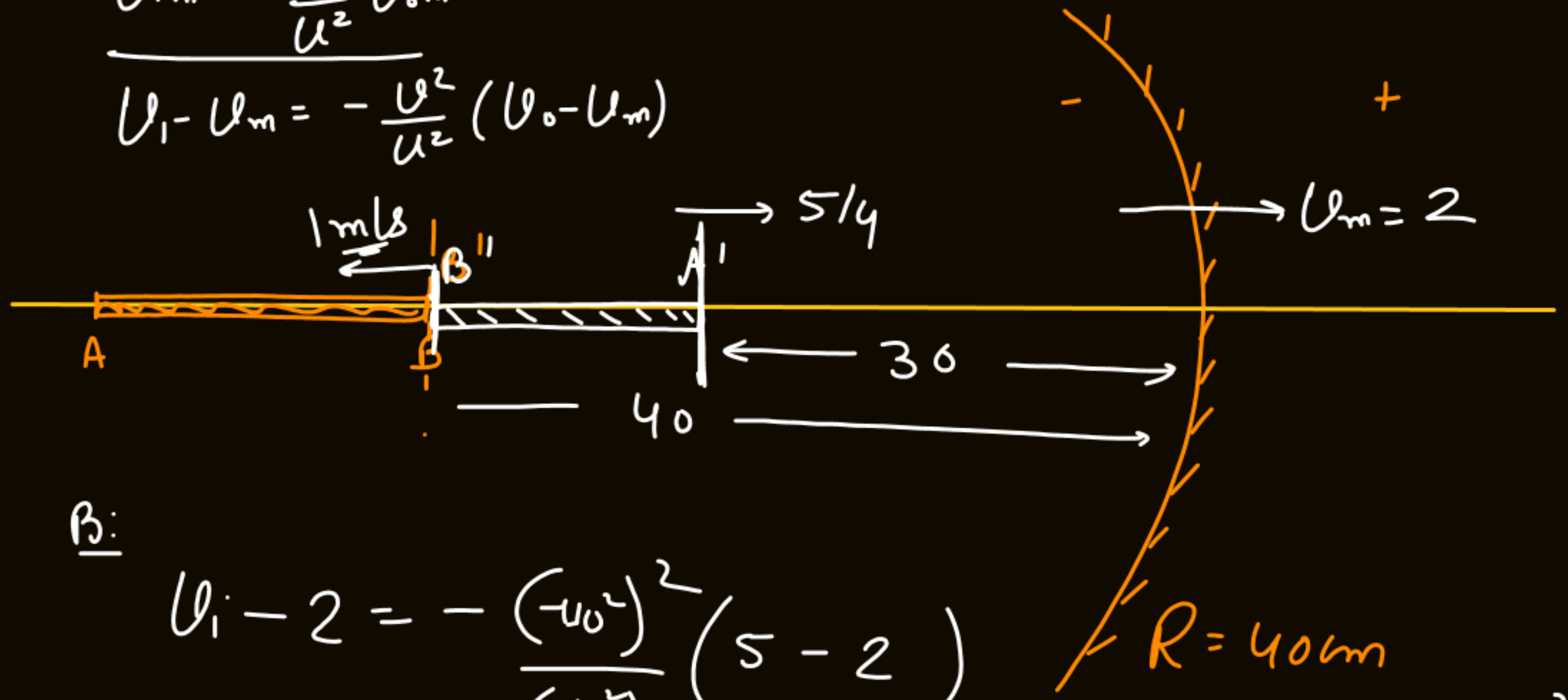


given $R_c = 40\,cm$

$B \to B'$

$$v_{im} = -\frac{v^2}{u^2} v_{om}$$

$$v_i - v_m = -\frac{v^2}{u^2}(v_o - v_m)$$



B:

$$v_i - 2 = -\frac{(-40)^2}{(-40)^2}(5-2)$$

$$v_i - 2 = -(+3)$$

$$v_i = -3 + 2 = -1$$

A:

$$v_i - 2 = -\frac{(30)^2}{(60)^2}(5-2)$$

$$v_i = 2 - \frac{1}{4} \times 3 = \frac{5}{4}$$

$$\frac{5}{4} + 1 = \frac{9}{4}\,m/s \quad \text{Ans}$$

$$\frac{1}{v} + \frac{1}{-60} = \frac{1}{-20}$$

$$\frac{1}{v} = \frac{1}{60} - \frac{1}{20} = \frac{1-3}{60}$$

$$v = -\frac{60}{2} = -30$$

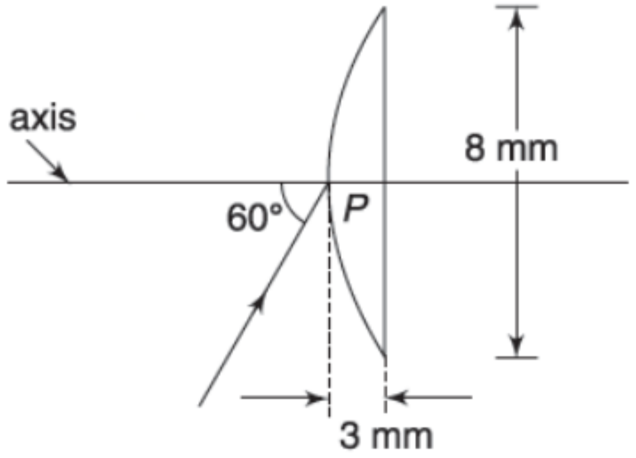
Q. 133: A plane convex lens has aperture diameter of 8 mm and thickness of the lens at the centre is 3 mm. The refractive index of the material of the lens is $\sqrt{3}$. A light ray is incident at mid point P of the curved surface at an angle of incidence of 60°.

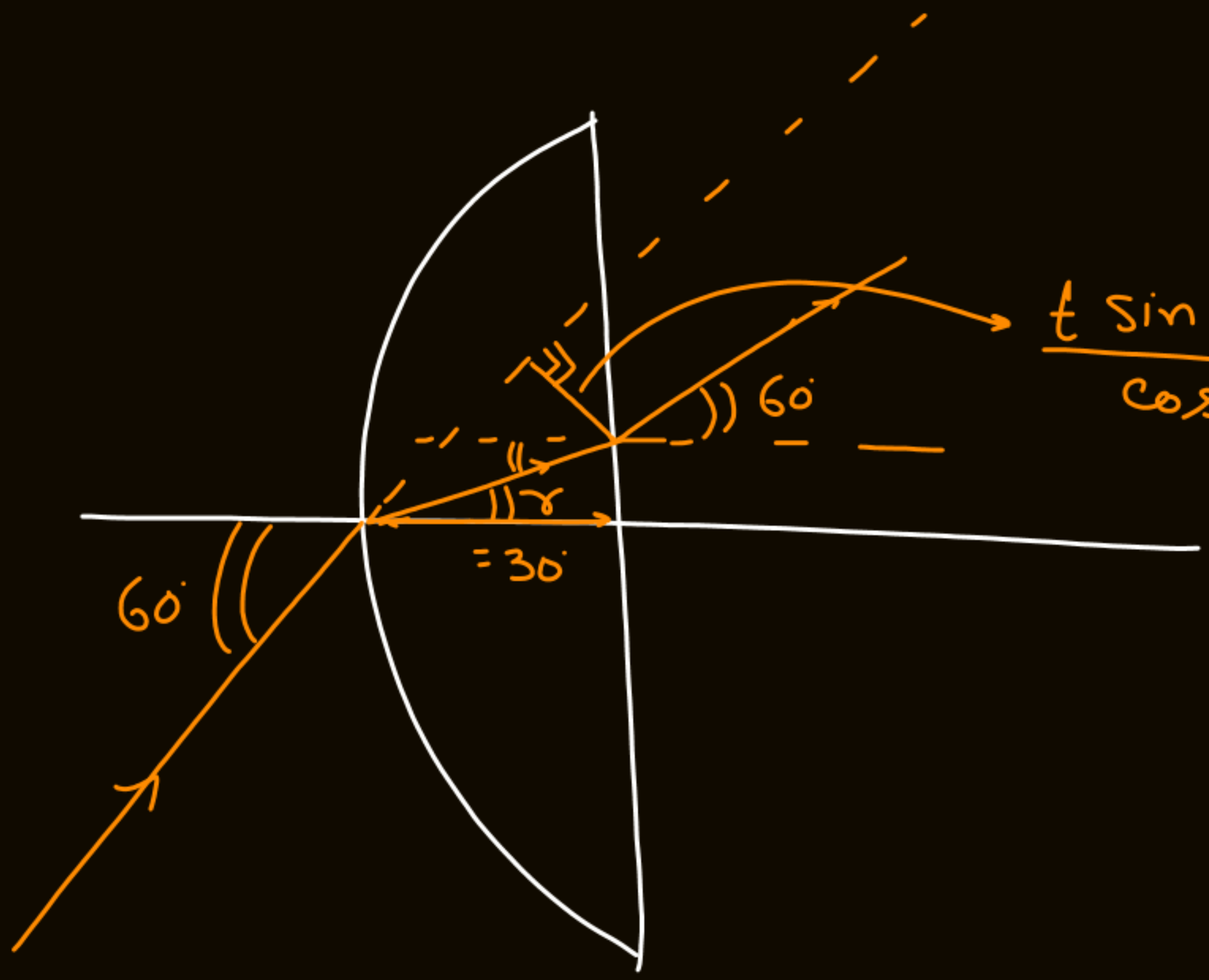
(a) Calculate the angular deviation suffered by the ray as it passes through the lens.

(b) Find the lateral shift in the path of the ray as it passes through the lens.

(c) Find the radius of curvature of the curved surface.

(d) If a narrow beam of light is incident at P parallel to the axis shown, where will it get focused. Take $2\sqrt{3}(\sqrt{3}-1) \simeq 2.5$





$\delta = 0$

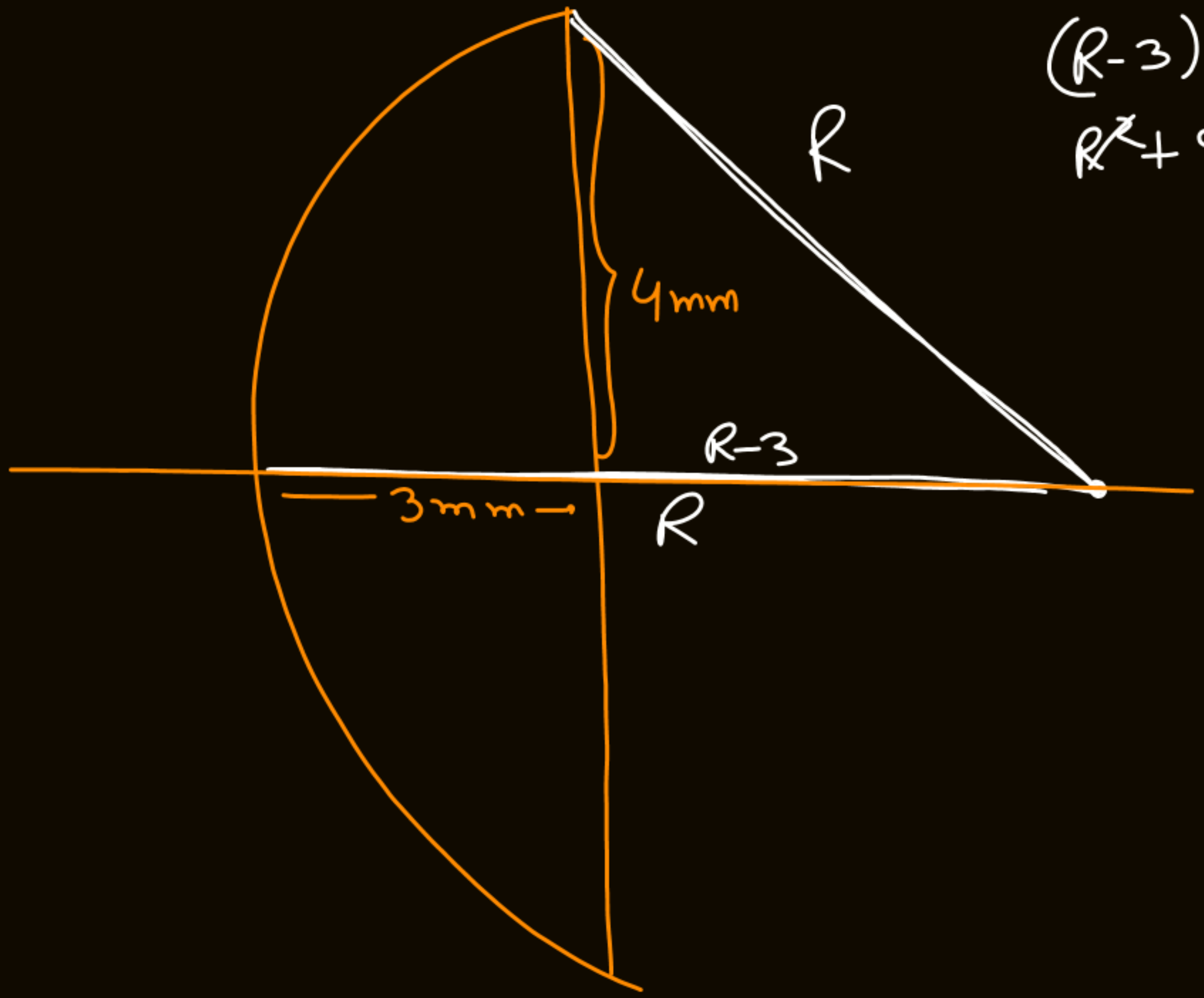
$\dfrac{t \sin(i-r)}{\cos r}$

$3\text{mm} \dfrac{\sin 30°}{\cos 30}$

$\sqrt{3}\text{ mm}$

$1 \times \sin 60° = \sqrt{3} \sin r$

$r = 30°$



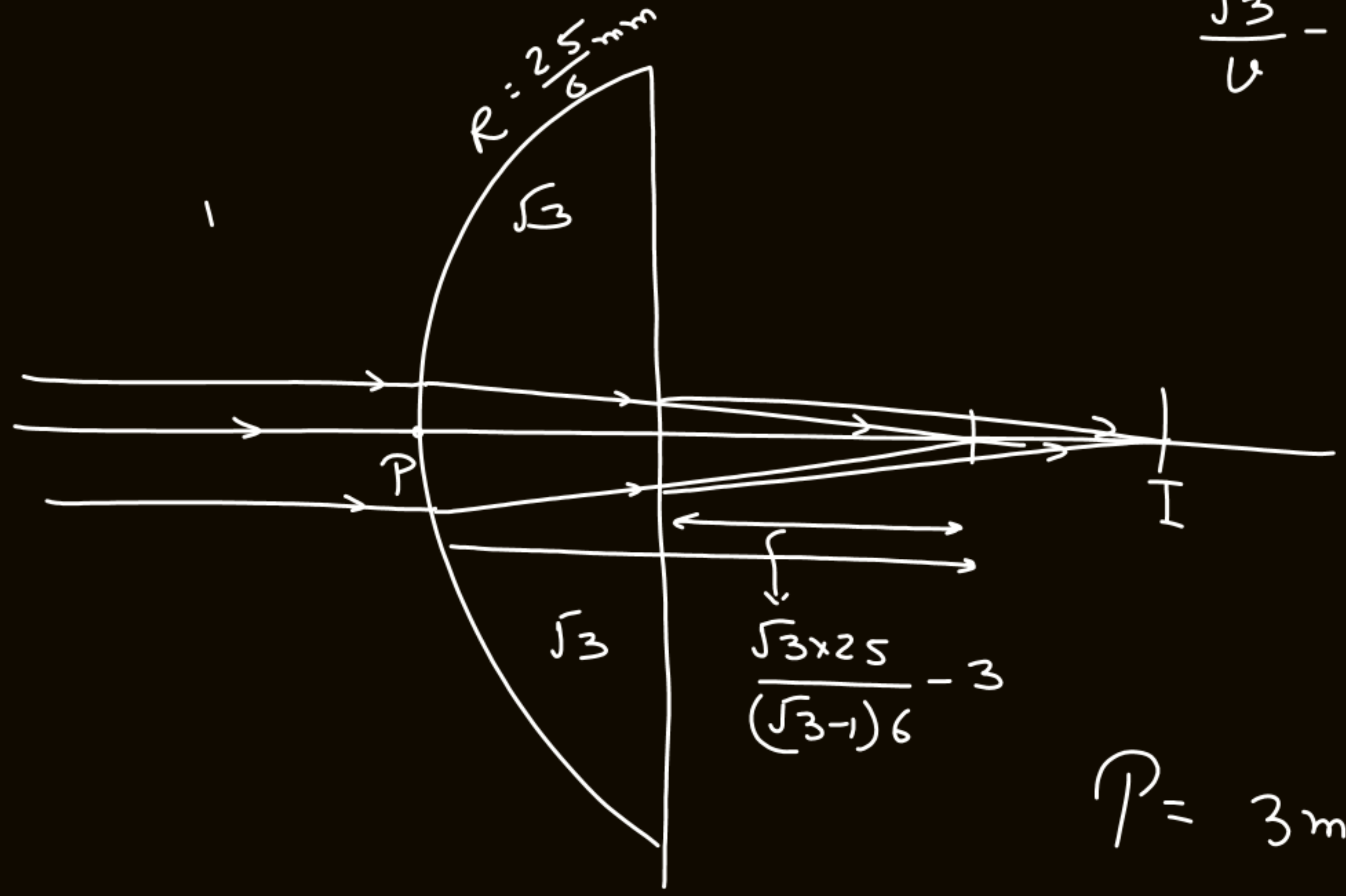
$$(R-3)^2+4^2=R^2$$

$$R^2+9-6R+16=R^2$$

$$6R=25$$

$$R=\frac{25}{6}\text{ mm}$$

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$



$$\frac{\sqrt{3}}{v} - \frac{1}{\infty} = \frac{\sqrt{3}-1}{R}$$

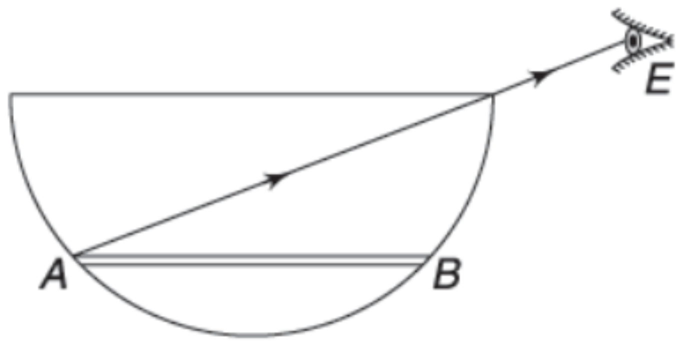
$$v = \frac{\sqrt{3}R}{\sqrt{3}-1} = \frac{\sqrt{3}\times 25}{(\sqrt{3}-1)\,6}\,mm$$

$$d' = \frac{d_0}{n_{Rel}}$$

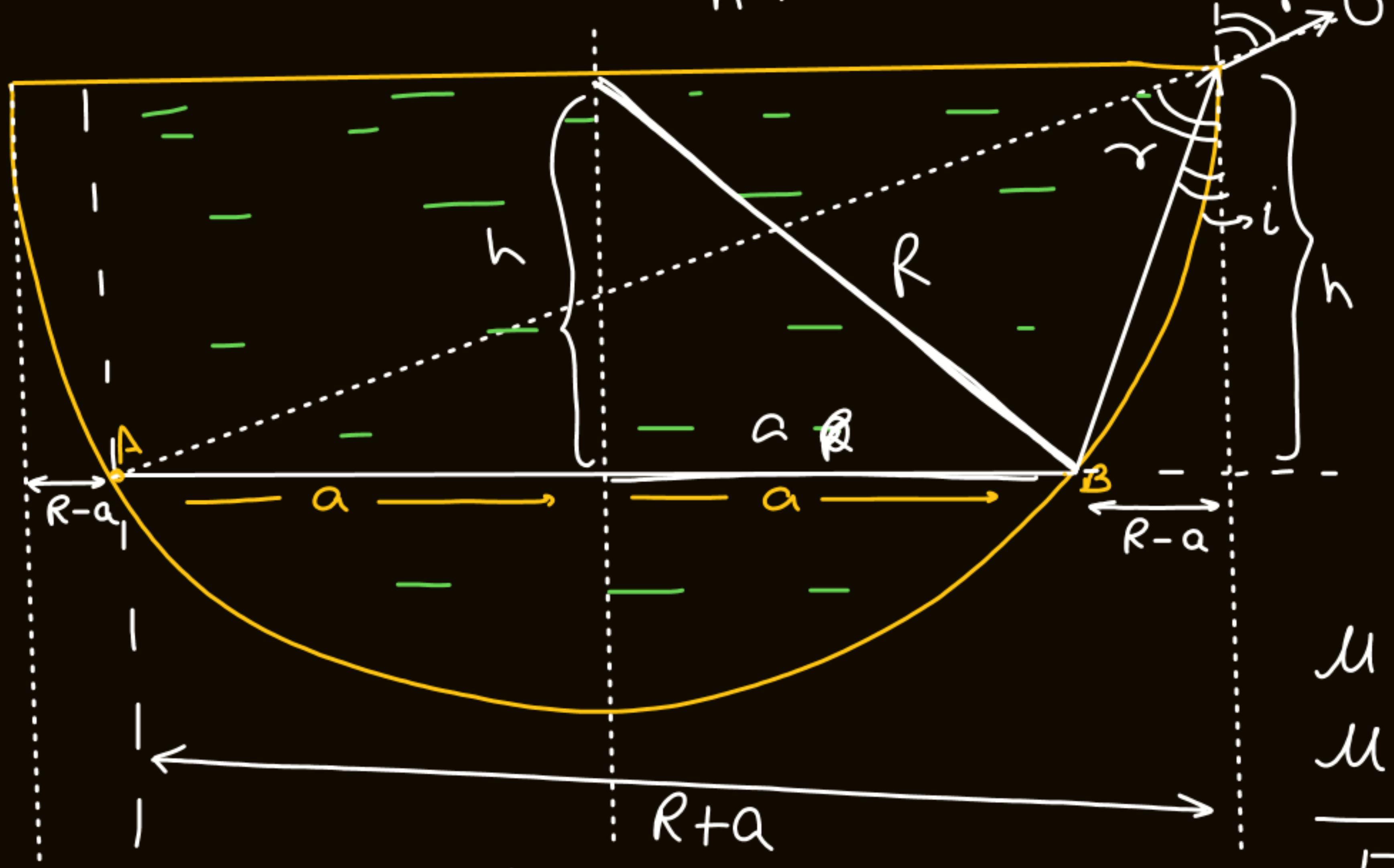
$$= \frac{\frac{25\sqrt{3}}{(\sqrt{3}-1)\times 6} - 3}{\sqrt{3}}$$

$$P = 3mm + d'$$

Q. 142: A stick is placed inside a hemispherical bowl as shown in Figure. The stick is horizontal and has a length of $2a$. Eye of an observer is located at E such that it can just see the end A of the stick. A liquid is filled upto edge of the bowl and the end B of the stick becomes visible to the observer. Radius of the bowl is R. Find the refractive index (μ) of the liquid.



$$h^2+a^2=R^2$$



$$\mu \sin i = 1 \sin r$$

$$\frac{\mu (R-a)}{\sqrt{(R-a)^2+h^2}} = 1 \times \frac{R+a}{\sqrt{(R+a)^2+h^2}}$$

$$\mu = \frac{R+a}{R-a} \frac{\sqrt{(R-a)^2+h^2}}{\sqrt{(R+a)^2+h^2}}$$

$$= \frac{R+a}{R-a}\sqrt{\frac{R^2+a^2-2Ra+R^2-a^2}{R^2+a^2+2Ra+R^2-a^2}} = \frac{R+a}{R-a}\sqrt{\frac{R-a}{R+a}} = \sqrt{\frac{R+a}{R-a}} \text{ Ans}$$