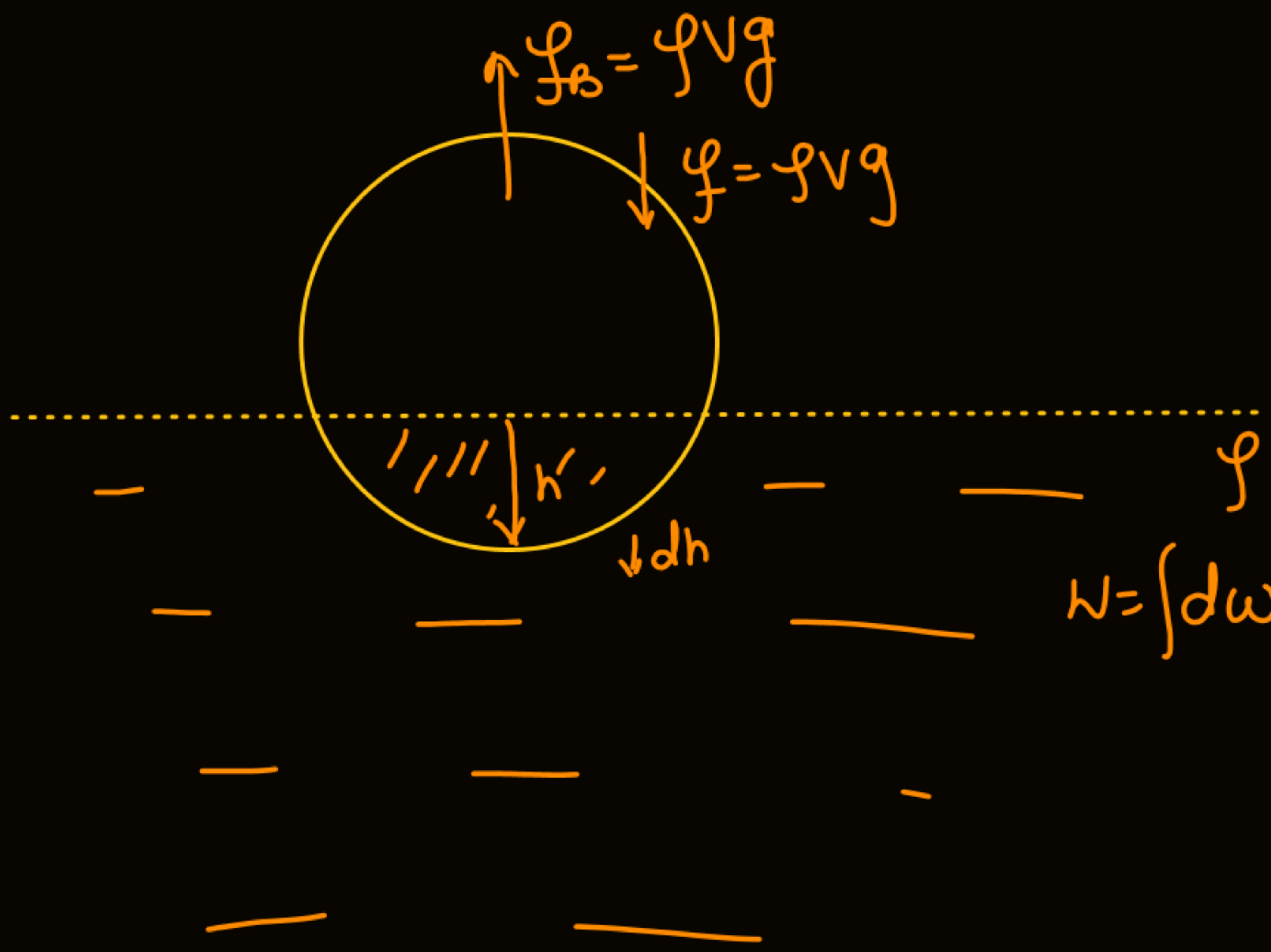
A sphere of radius R and having negligible mass is floating in a large lake. An external agent slowly pushes the sphere so as to submerge it completely. How much work was done by the agent? Density of water is ρ.

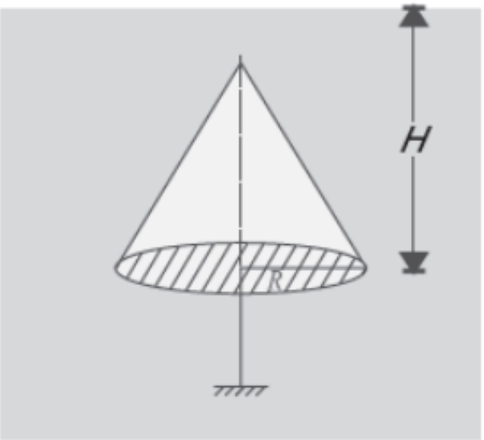


$$W = \int dw = \int F\, dh$$

$\left(\rho \frac{4}{3}\pi R^3\right)$

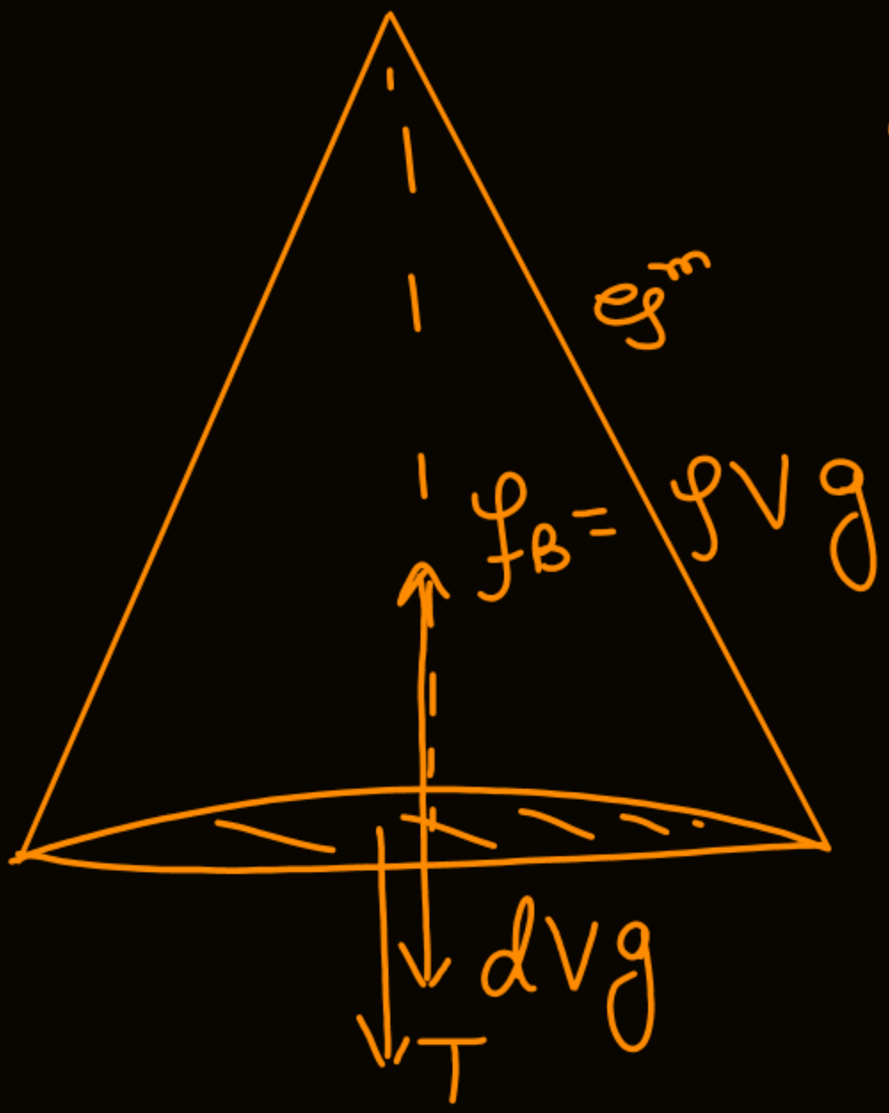
$$mgh = \left(\rho \frac{4}{3}\pi R^3\right) g R = \underline{W_{ext}}$$

A solid wooden cone has been supported by a string inside water as shown in the figure. The radius of the circular base of the cone is R and the volume of the cone is v. In equilibrium the base of the cone is at a depth H below the water surface. Density of wood is d ($< \rho$, density of water).



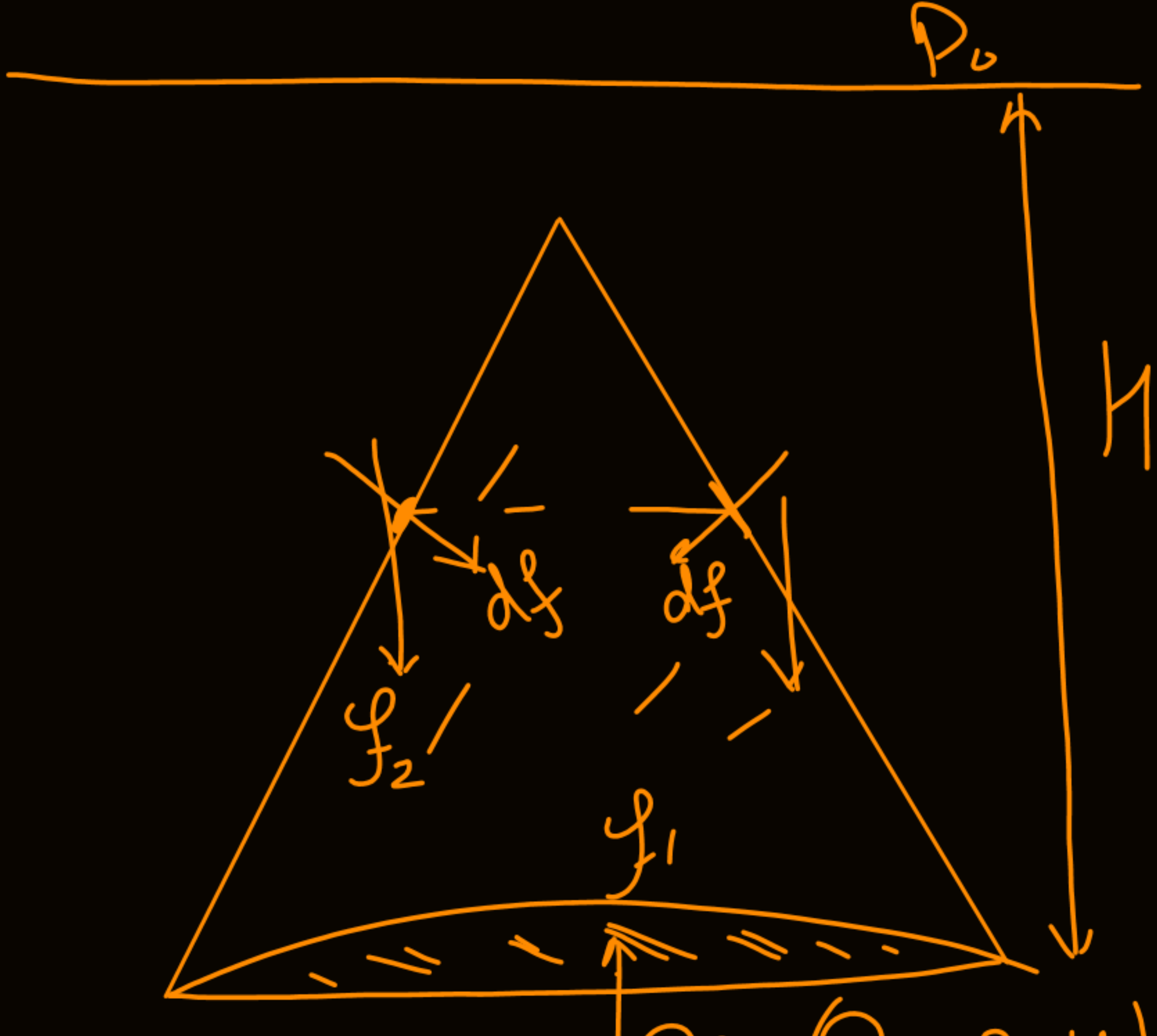
(a) Find tension in the string.

(b) Find the force applied by the water on the slant surface of the cone. Take atmospheric pressure to be P_0



$$\rho v g = T + d v g$$

$$\boxed{T = (\rho - d) v g}$$



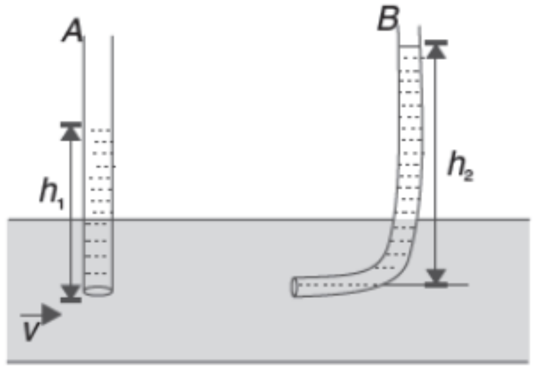
$$PA = (P_0 + \rho g H)\pi R^2$$

$$f_1 - f_2 = f_B$$

$$f_2 = f_1 - f_B$$

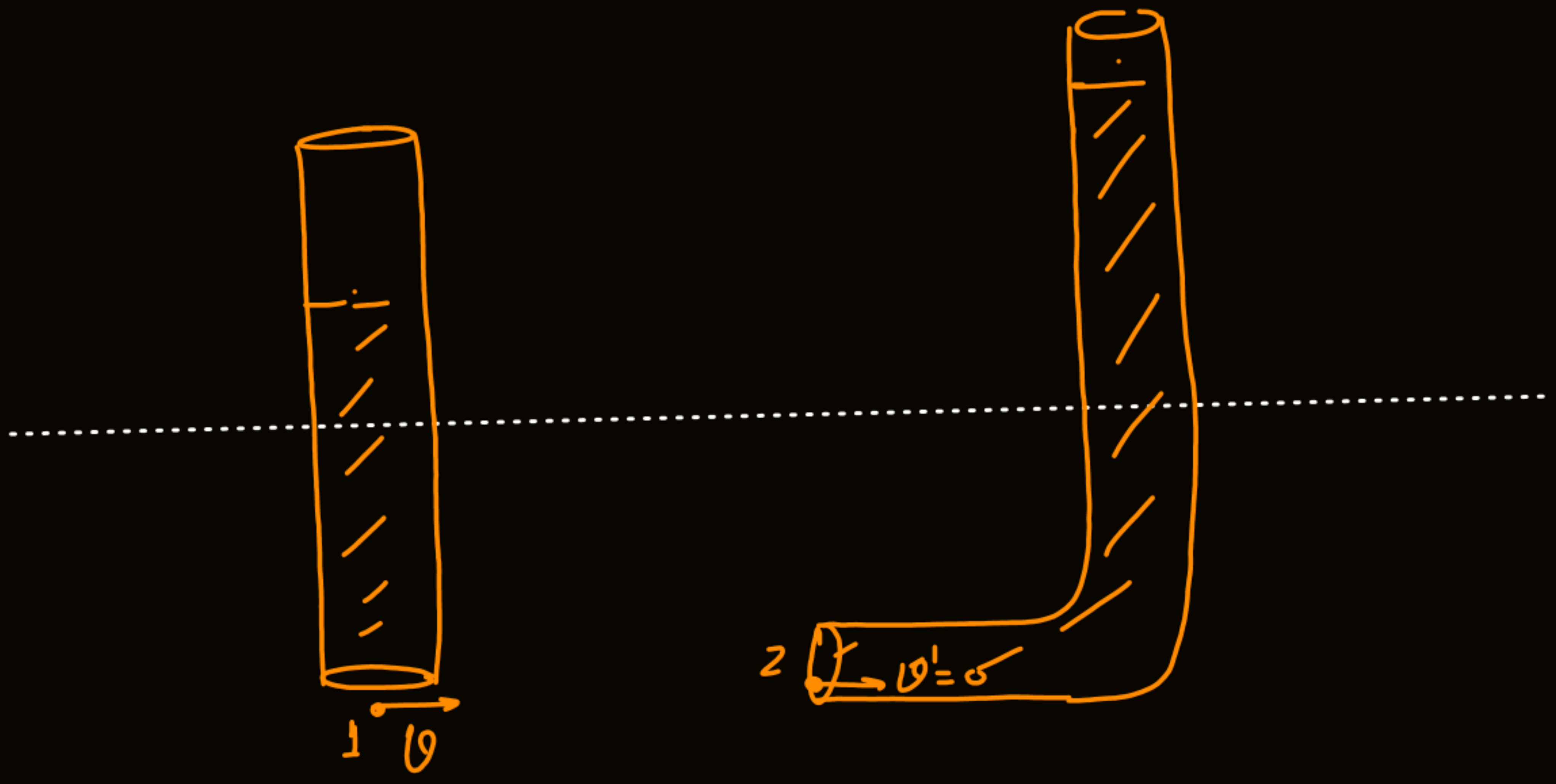
$$= (P_0 + \rho g H)\pi R^2 - \rho V g$$

A liquid is flowing in a horizontal pipe of uniform cross section at a speed v. Two tubes A an B are inserted into the pipe as shown. Assume the flow to remain streamline inside the pipe.



(a) The diagram depicts that height of liquid in tube B $(= h_2)$ is more than the height of liquid in tube A $(= h_1)$. Is it correct?

(b) Calculate the difference in height of the liquid in two tubes.



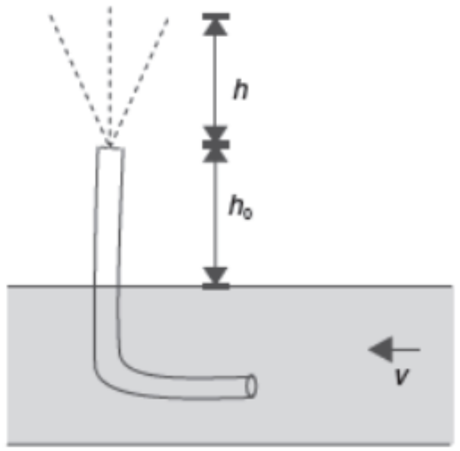
BE $\quad P_1 + \frac{1}{2}\rho v^2 = P_2 + 0$

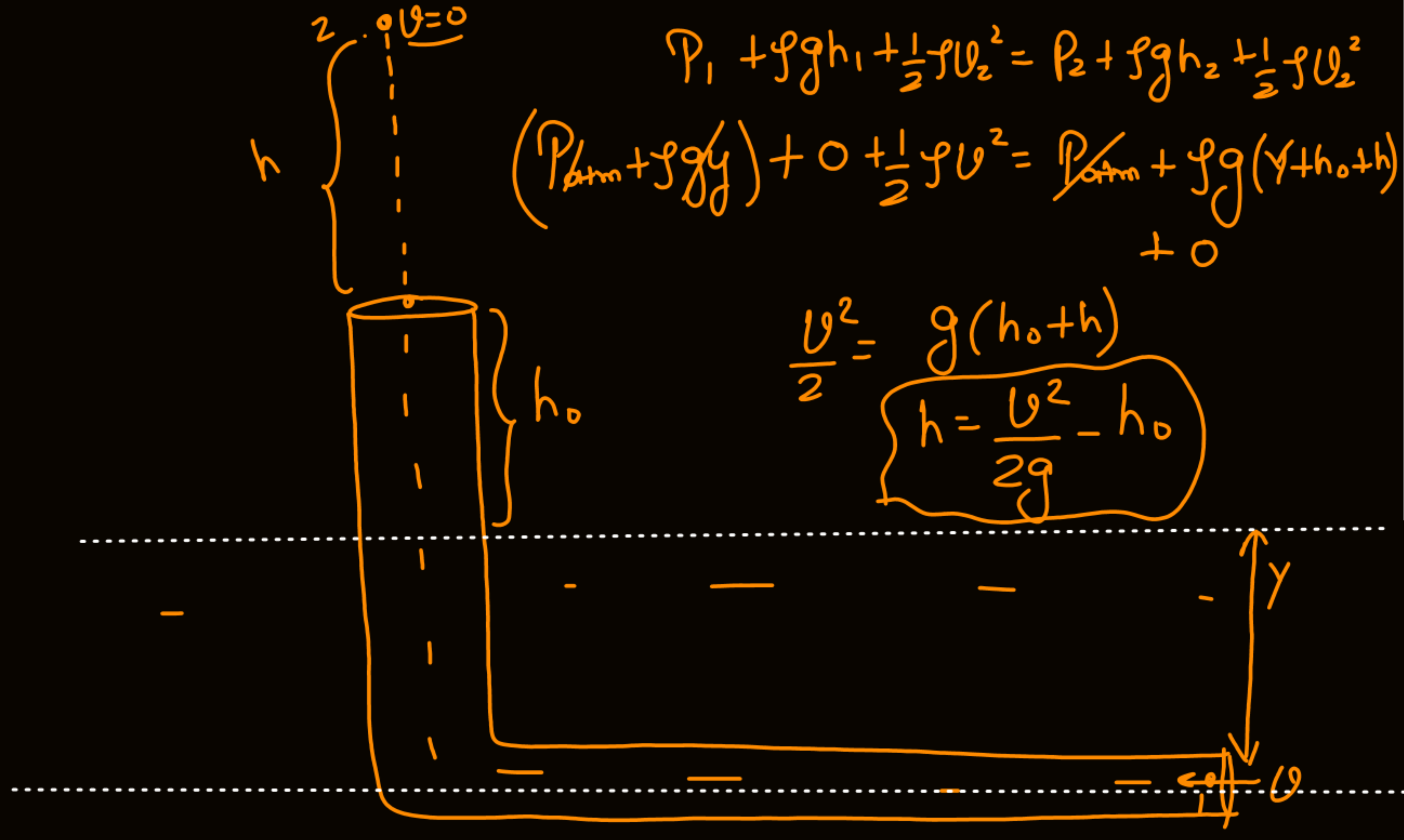
$$P_2 - P_1 = \frac{1}{2}\rho v^2 = \rho g(h_2 - h_1) \Rightarrow h_2 - h_1 = \frac{v^2}{2g}$$

$$P_2 = P_0 + \rho g h_2$$

$$P_1 = P_0 + \rho g h_1$$

Water is flowing in a stream at speed v. A L shaped tube is lowered into the stream as shown. The upper end of the tube is held at a height h_0 above the surface of the water. To what height 'h' above the upper end of the tube, will the water jet spurt? Assume that flow remains ideal.





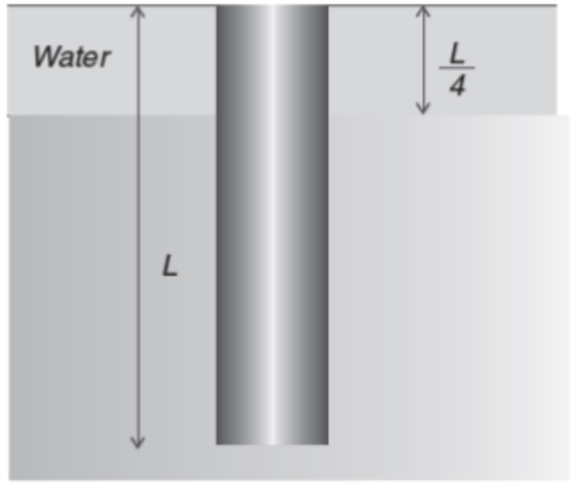
$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_2^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$$\left(\cancel{P_{atm}} + \rho g \cancel{y}\right) + 0 + \frac{1}{2}\rho v^2 = \cancel{P_{atm}} + \rho g(\cancel{y} + h_0 + h) + 0$$

$$\frac{v^2}{2} = g(h_0 + h)$$

$$\boxed{h = \frac{v^2}{2g} - h_0}$$

A cylinder of length L floats with its entire length immersed in two liquids as shown in the fig.



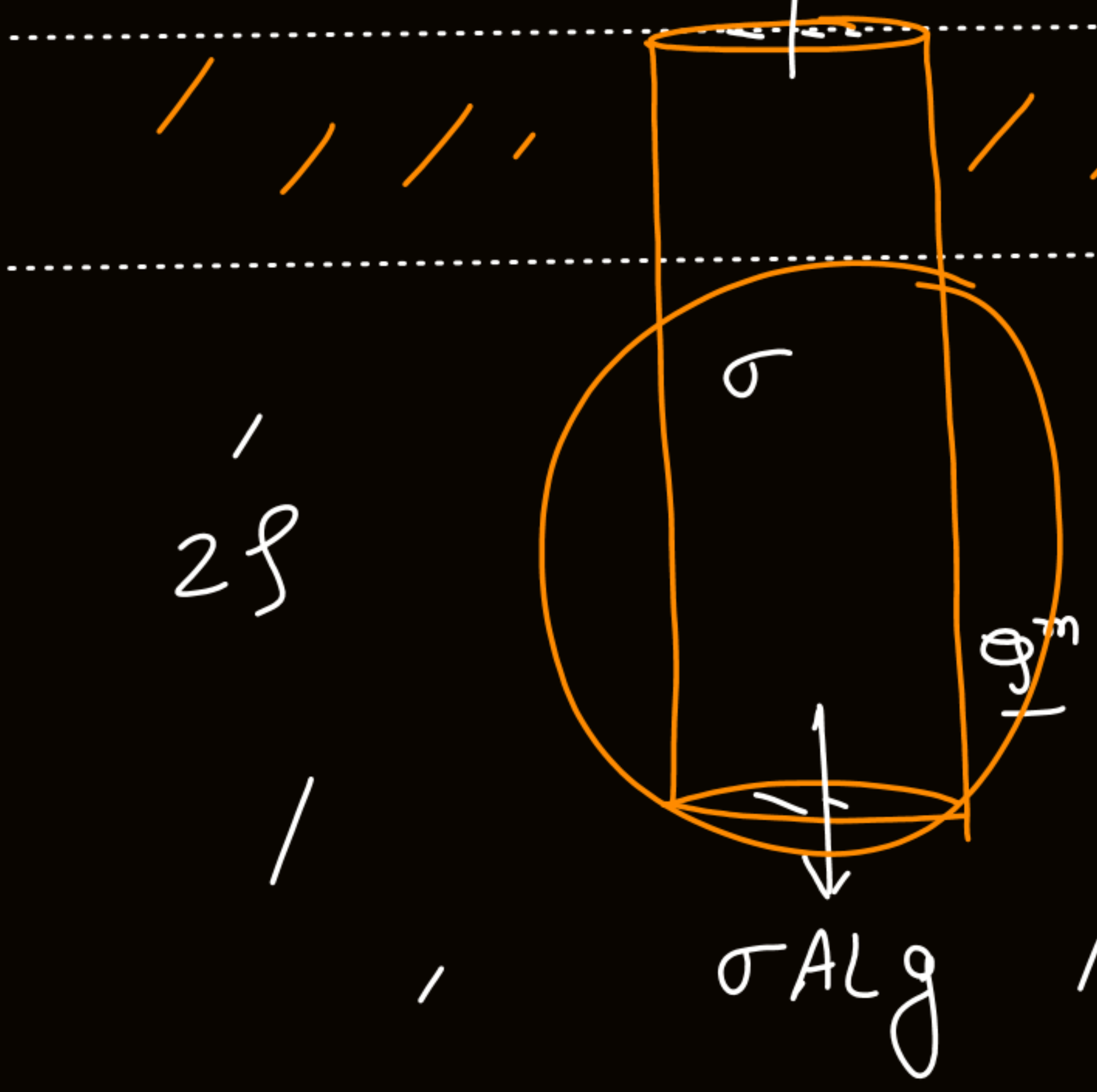
The upper liquid is water and the lower liquid has density twice that of water. The two liquid are immiscible. The cylinder is in equilibrium with its $\frac{3}{4}$ length in the denser liquid and $\frac{1}{4}$ of its length in water. The thickness of water layer is $\frac{L}{4}$ only. Find

① find s.g. of cylinder

② If cylinder is displaced slightly in vertical dirn find time period of its oscillation.

Sol:

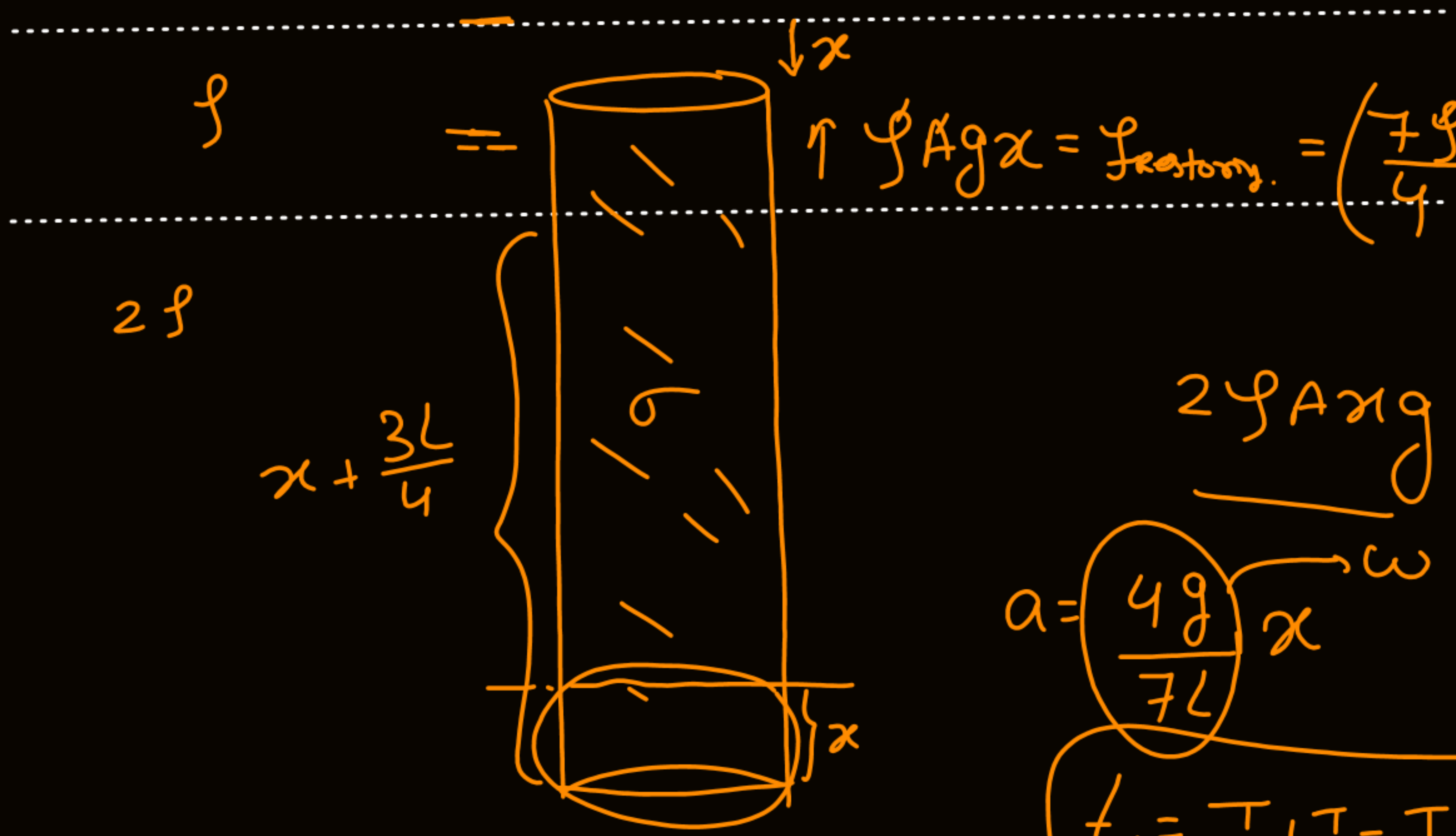
$$F_B = \rho A \frac{L}{4} g + 2\rho A \frac{3L}{4} g$$



$$\frac{7}{4}\rho \cancel{ALg} = \sigma \cancel{ALg}$$

$$\boxed{\frac{\sigma}{\rho} = 7/4} = s.g.$$

$$\boxed{\sigma = \frac{7}{4}\rho}$$



$\uparrow \rho A g x = f_{restoring} = \left(\frac{7\rho}{4} A L\right) a$

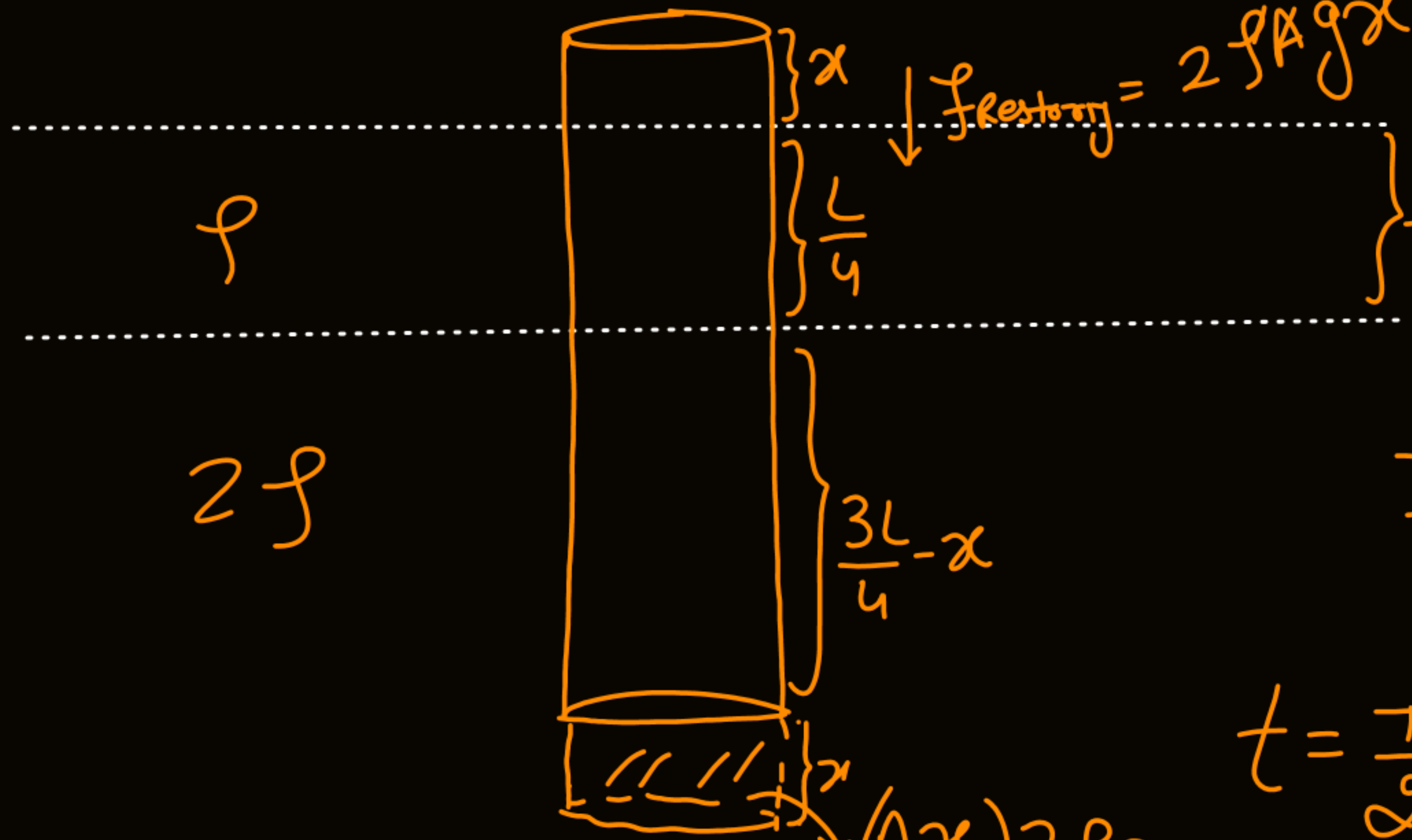
$\underline{2\rho A x g} - (\rho A x) g$

$a = \frac{4g}{7L} x \rightarrow \omega$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{7L}{4g}}$

$t_1 = \frac{T}{4} + \frac{T}{4} = \frac{T}{2} = \pi\sqrt{\frac{7L}{4g}}$

$$\downarrow F_{restoring} = 2\rho A g x = \frac{7}{4}\rho A L\, a$$



$$a = \frac{8g}{7L}\,x$$

$$\frac{8g}{7L} \to \omega'$$

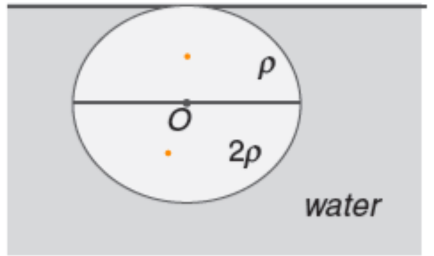
$$\frac{T'}{2} = \frac{\pi}{\omega'} = \pi\sqrt{\frac{7L}{8g}}$$

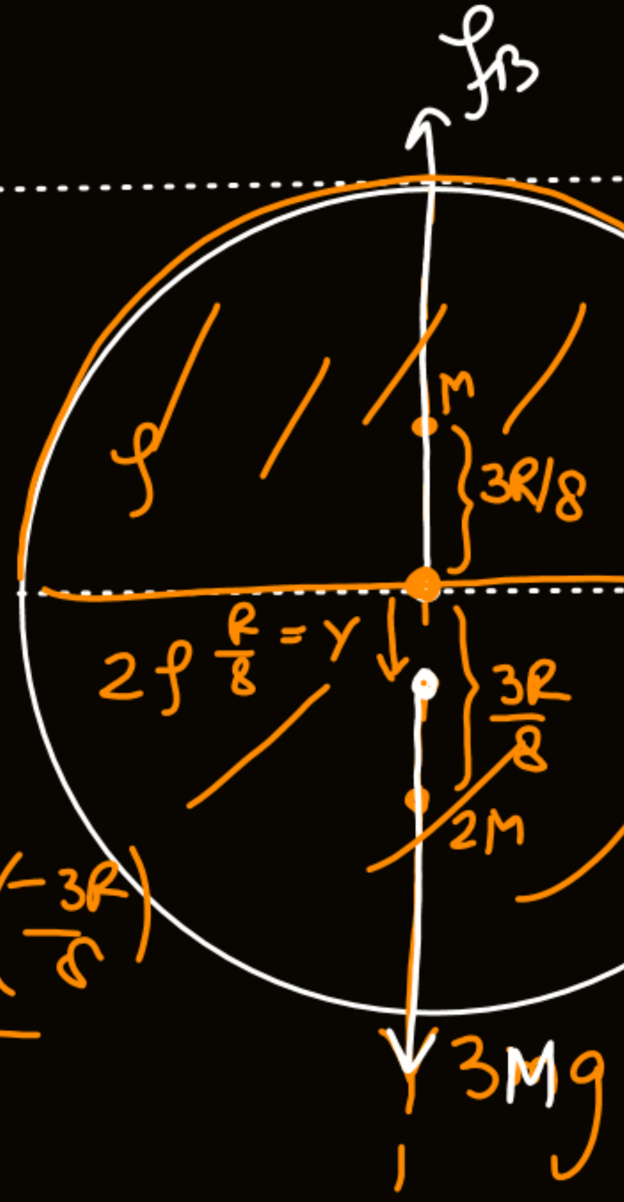
$$t = \frac{T}{2} + \frac{T'}{2} = \pi\sqrt{\frac{7L}{4g}} + \pi\sqrt{\frac{7L}{8g}}$$

A spherical ball of radius R is made by joining two hemispherical parts. The two parts have density ρ and 2ρ. When placed in a water tank, the ball floats while remaining completely submerged.

(a) If density of water is ρ_0, find ρ

(b) Find the time period of small angular oscillations of the ball about its equilibrium position. Neglect viscous forces.





$$\frac{2}{5}MR^2 + \frac{2}{5}2MR^2$$

$$\frac{6}{5}MR^2$$

$$Y_{com} = \frac{M\left(\frac{3R}{8}\right) + 2M\left(-\frac{3R}{8}\right)}{3M}$$

$$= -R/8$$

$$\rho \frac{\cancel{V}}{2}\cancel{g} + 2\rho\frac{\cancel{V}}{2}\cancel{g} = \rho_0 \cancel{V}\cancel{g}$$

$$\boxed{\frac{3\rho}{2} = \rho_0}$$

$$\rho = \frac{2\rho_0}{3}$$

$$\frac{3MgR}{8}\theta = \frac{369}{320} MR^2 \alpha$$

$$\frac{g\theta}{R} = \frac{123}{40}\alpha \Rightarrow \alpha = \overbrace{\frac{40g}{123R}}^{\omega^2}\theta$$

$$I = I_{com} + mx^2$$

$$\frac{6}{5}MR^2 = I_{com} + 3M\frac{R^2}{64}$$

$$I_{com} = MR^2\left(\frac{6}{5} - \frac{3}{64}\right)$$

$$I_{com} = MR^2\,\frac{384-15}{320}$$

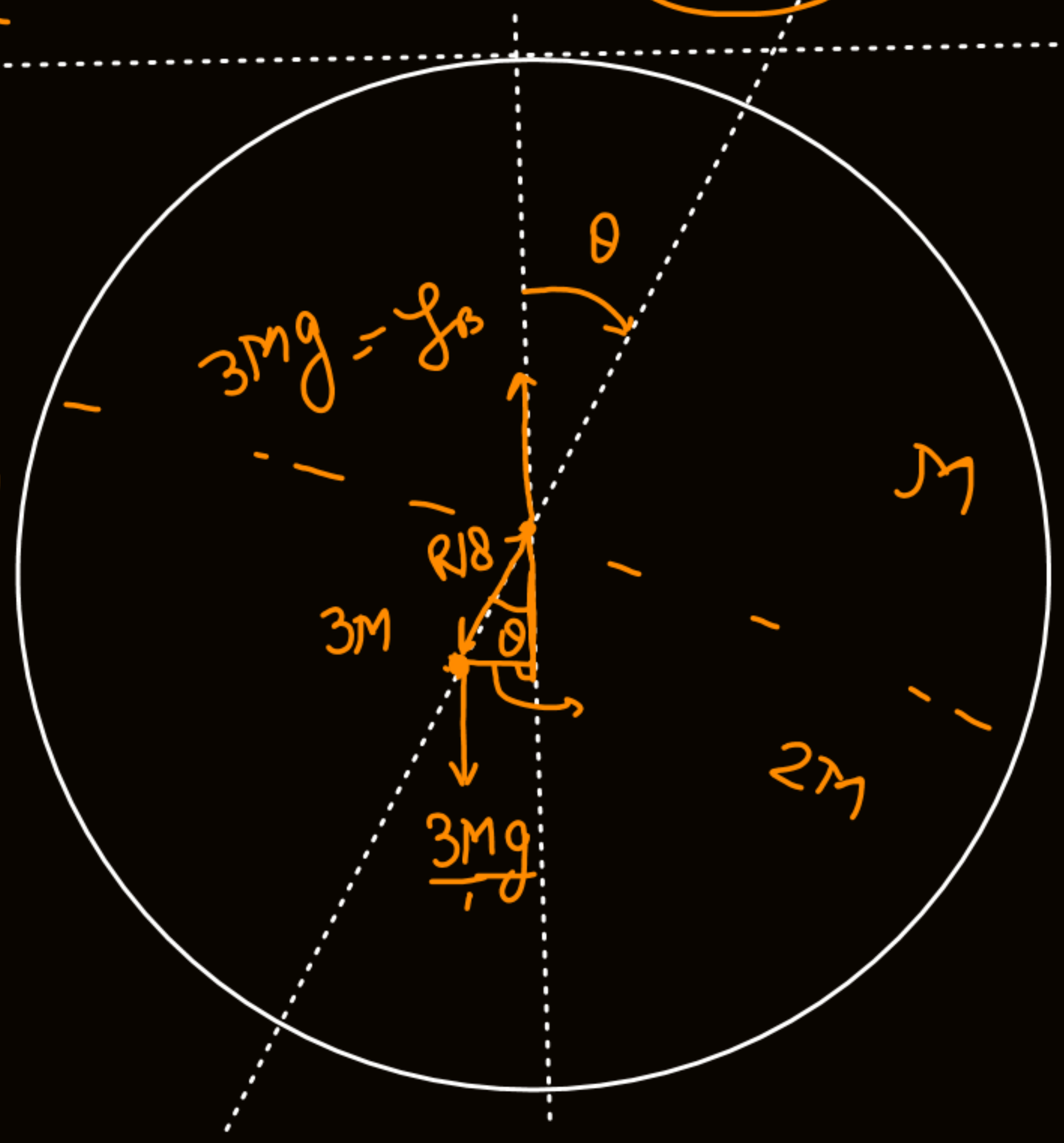
$$I_{com} = \frac{369}{320}MR^2$$

$$\tau = f_B r_\perp$$

$$= 3Mg\frac{R}{8}\sin\theta$$

$$= \frac{3MgR}{8}\theta = I\alpha$$

= Angular SHM



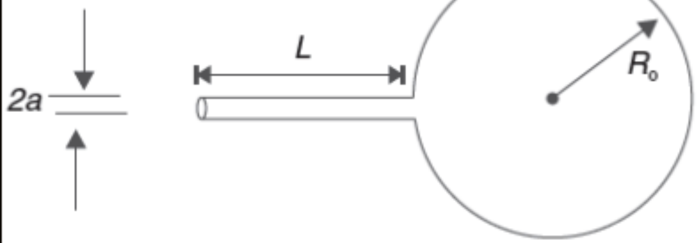
$$\omega = \sqrt{\frac{40g}{123R}}$$

$$\boxed{T = 2\pi\sqrt{\frac{123R}{40g}}}$$

$$\frac{dV}{dt} = \frac{\pi a^4 T}{2\eta L R}$$

$P_0 + \frac{4T}{R}$

$8\eta L$



Where ΔP is pressure difference at the two ends of the tube and η is coefficient of viscosity. Assume that the bubble remains spherical.

$$4\pi R^2 \frac{dR}{dt} = \frac{-\pi a^4 T}{2\eta L R}$$

$$V = \frac{4\pi R^3}{3}$$

$$\frac{dV}{dt} = \frac{4}{3}\pi 3R^2 \frac{dR}{dt} = 4\pi$$

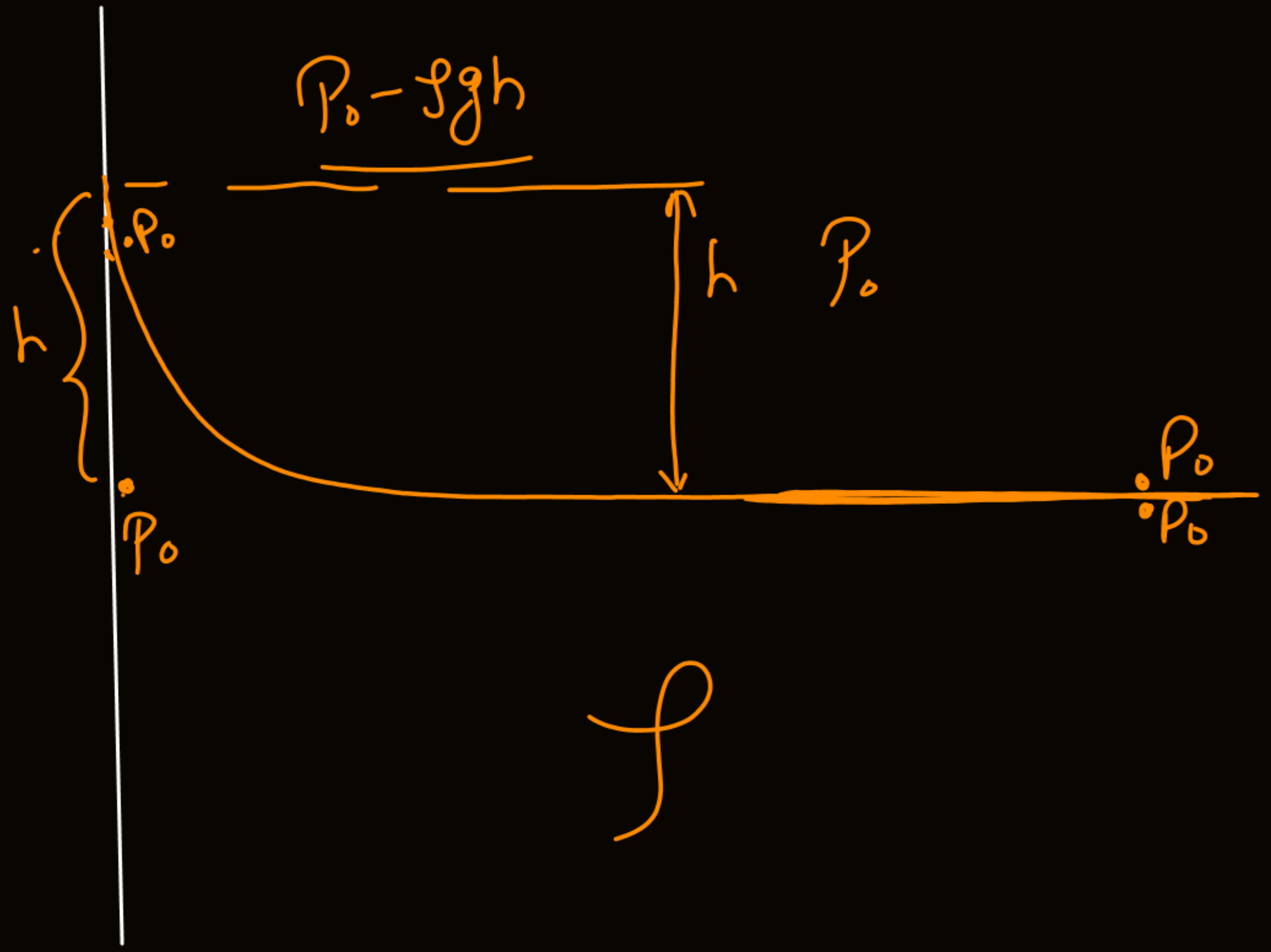
$$-\frac{8\pi\eta L}{\pi a^4 T}\int_{R_0}^{R} R^3\,dR = \int_0^t dt$$

$$-\frac{2\eta L}{a^4 T}\left(R^4 - R_0^4\right) = t$$

$$R^4 - R_0^4 = -\frac{a^4}{2\eta}$$

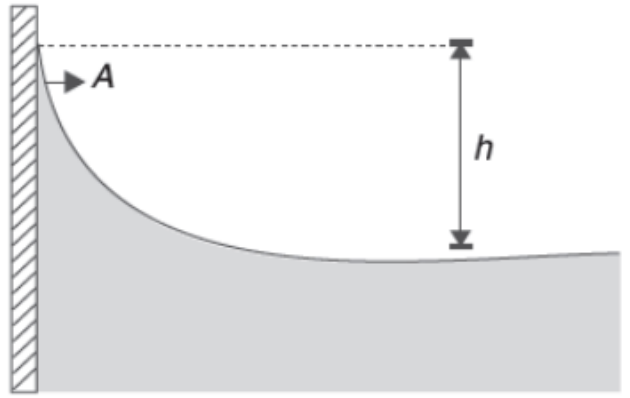
$$R^4 = \left(R_0^4 - \frac{a^4}{2\eta}\right.$$

$$\left(\quad t\right)^{1/4}$$



A liquid having surface tension T and density ρ is in contact with a vertical solid wall. The liquid surface gets curved as shown in the figure. At the bottom the liquid surface is flat.

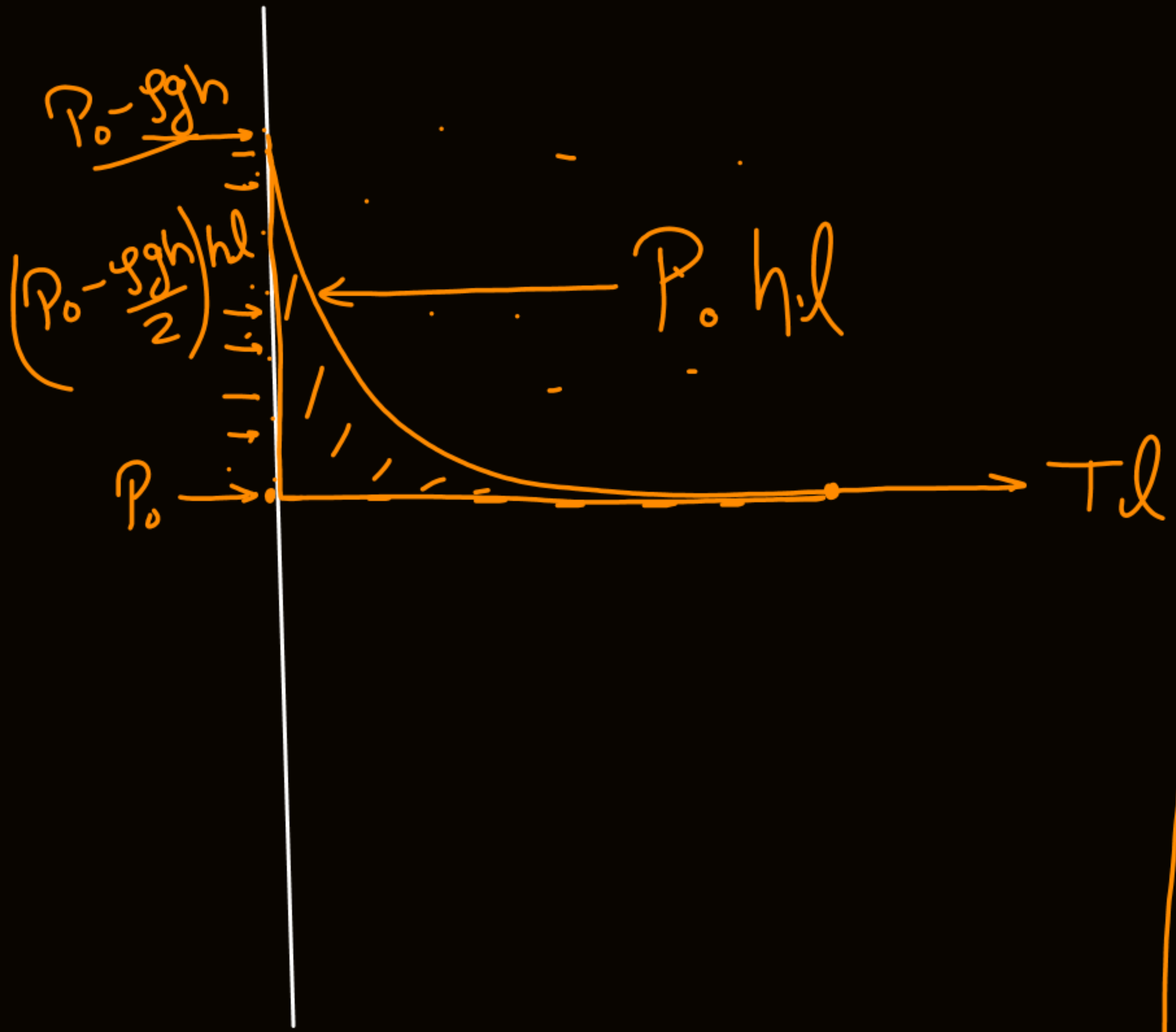
The atmospheric pressure is P_o.



(i) Find the pressure in the liquid at the top of the meniscus (i.e. at A)

(ii) Calculate the difference in height (h) between the bottom and top of the meniscus.

$$\frac{2T}{R} = \frac{2T}{\infty} = 0$$



$$Tl + \left(\cancel{P_0} - \frac{\rho g h}{2}\right) hl = \cancel{P_0 hl}$$

$$Tl - \frac{\rho g h}{2} hl = 0$$

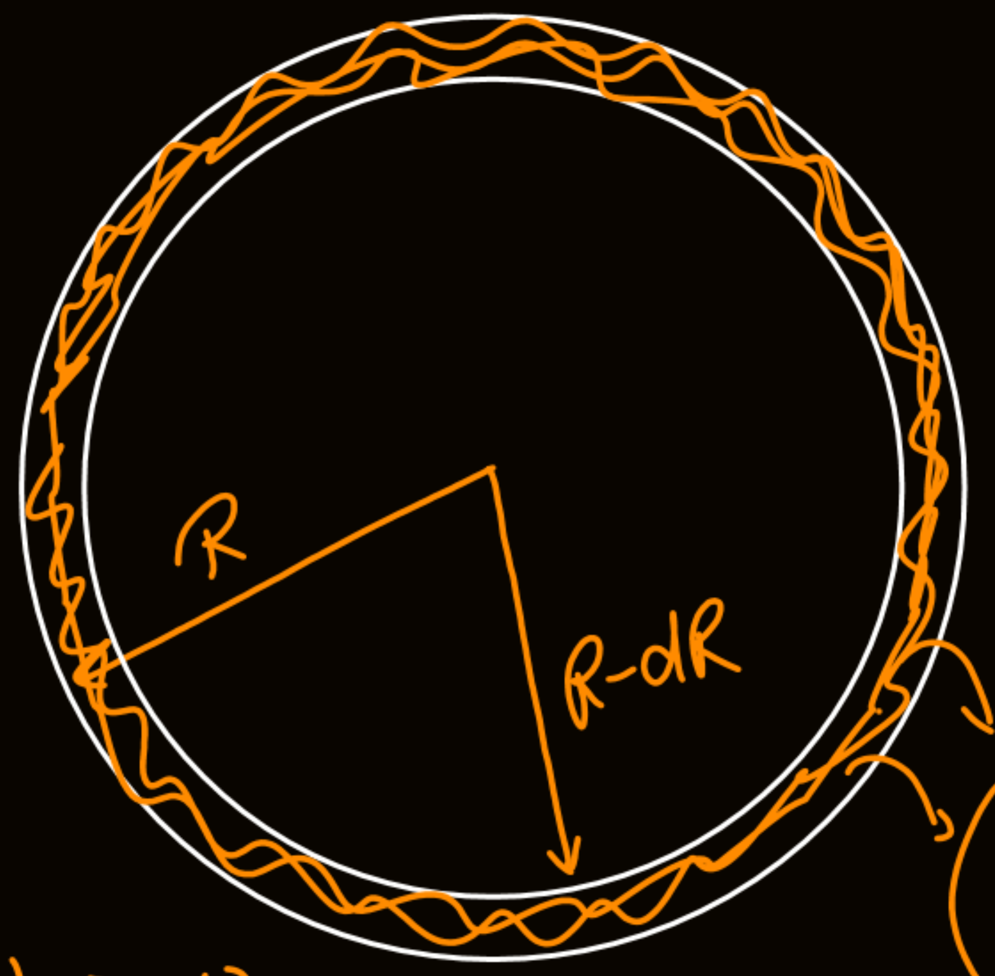
$$T = \frac{\rho g h^2}{2}$$

$$\boxed{h = \sqrt{\frac{2T}{\rho g}}}$$

$4\pi R^2$ $\quad A = 4\pi R^2$

$dA = 4\pi 2R\,dR = 8\pi R\,dR$

Is it possible that water evaporates from a spherical drop of water just by means of surface energy supplying the necessary latent heat of vaporisation? The drop does not use its internal thermal energy and does not receive any heat from outside. It is known that water drops of size less than $10^{-6}\ m$ do not exist. Latent heat of vaporisation of water is $L = 2.3 \times 10^6\ Jkg^{-1}$ and surface tension is $T = 0.07\ Nm^{-1}$.



dm

$(4\pi R^2)T - 4\pi(R-dR)^2 T$

$(8\pi R\,dR)T = U_1$

$(\rho 4\pi R^2 dR)L = U_2$

$U_1 \geq U_2$

$2(8\pi R\,dR)T \geq \rho 4\pi R^2 dR \cdot L$

$\boxed{R \leq \frac{2T}{\rho L}}$ ✓