Q. 4: The focal length of a spherical mirror is given by $f = \frac{R}{2}$, where R is radius of curvature of the mirror. For a given spherical mirror made of steel the focal length is $f = 24.0$ cm. Find its new focal length if temperature increases by 50°C. Given $\alpha_{steel} = 1.2 \times 10^{-5}\,°C^{-1}$

$$f = \frac{R}{2} = 24\,cm$$

$$f' = \frac{R'}{2} = \frac{R\{1+\alpha\Delta T\}}{2}$$

$$= \underline{24\,cm\{1+1.2\times10^{-5}\times50\}}$$

Q. 6: A pendulum based clock keeps correct time in an aeroplane flying uniformly at a height h above the surface of the earth. The cabin temperature inside the plane is 10°C. The same pendulum keeps correct time on the surface of the earth when temperature is 30°C. Find the coefficient of linear expansions of the material of the pendulum. You can assume that $h << R$ (radius of the earth)

h

$$\alpha = \frac{2h}{R\Delta T} = \frac{2h}{R \times 20} = \frac{h}{10R}$$

$$T = 2\pi\sqrt{\frac{l}{g\left(1-\frac{2h}{R}\right)}} = 2\pi\sqrt{\frac{l(1+\alpha\Delta T)}{g}}$$

$$\left(1-\frac{2h}{R}\right)^{-1} = (1+\alpha\Delta T)$$

$$\cancel{1}+\frac{2h}{R} = \cancel{1}+\alpha\Delta T$$

V_{c0}

V_0

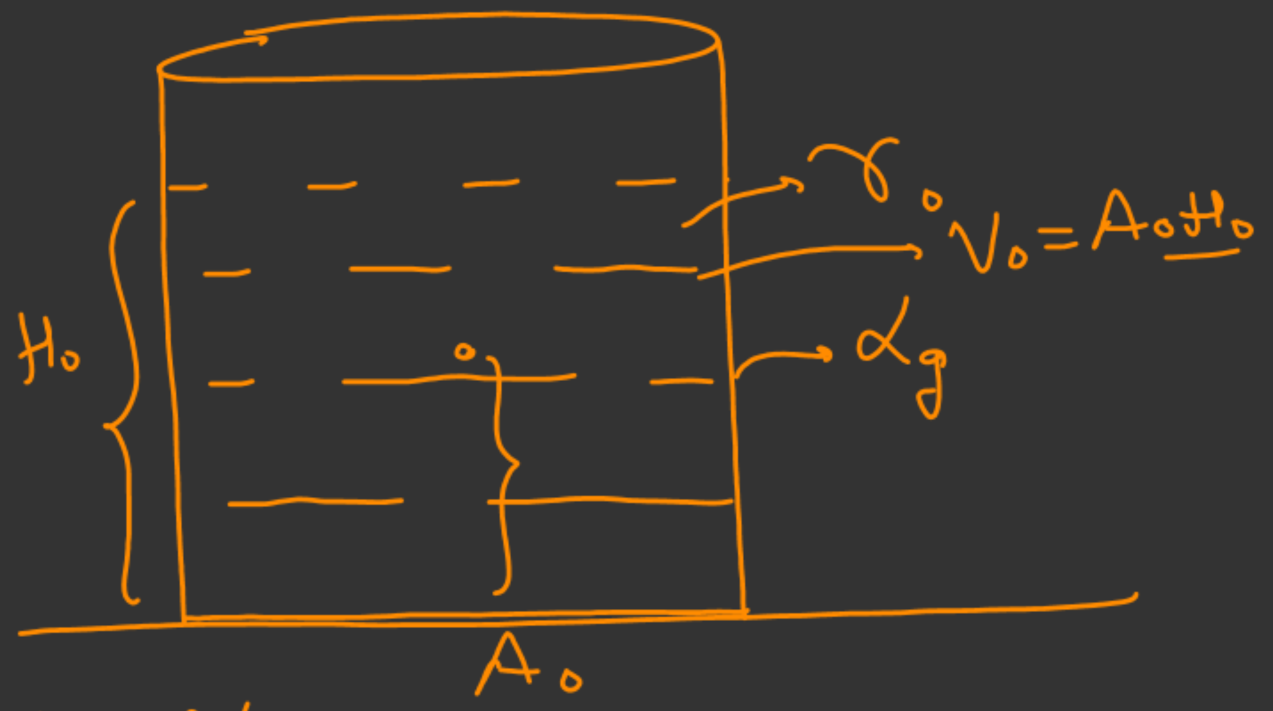
$$\frac{V_0}{V_{c0}} = \frac{V_0(1+\gamma_0\Delta T)}{V_{c0}\{1+3\alpha_g\Delta T\}}$$

$$\boxed{\gamma_0 = 3\alpha_g}$$

Q. 7: A liquid having coefficient of volume expansion γ_0 is filled in a cylindrical glass vessel. Glass has a coefficient of linear expansion of α_g. The liquid along with the container is heated to raise their temperature by ΔT. Mass of the container is negligible.

(a) Find relationship between α_g and γ_0 if it was found that the centre of mass of the system did not move due to heating.

(b) Find relationship between α_g and γ_0 if the fraction of volume of the container occupied by the liquid does not change due to heating.



$$H_w = \frac{V_0}{A_0}$$

$$H = \frac{V}{A} = \frac{V_0(1+\gamma_0\Delta T)}{A_0\{1+2\alpha_g\Delta T\}}$$

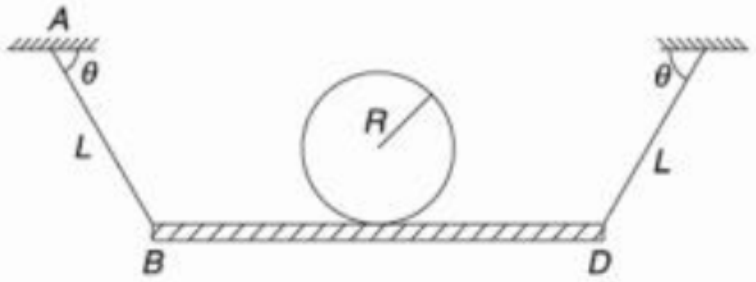
$$H = \frac{H_0\{1+\gamma_0\Delta T\}}{(1+2\alpha_g\Delta T)}$$

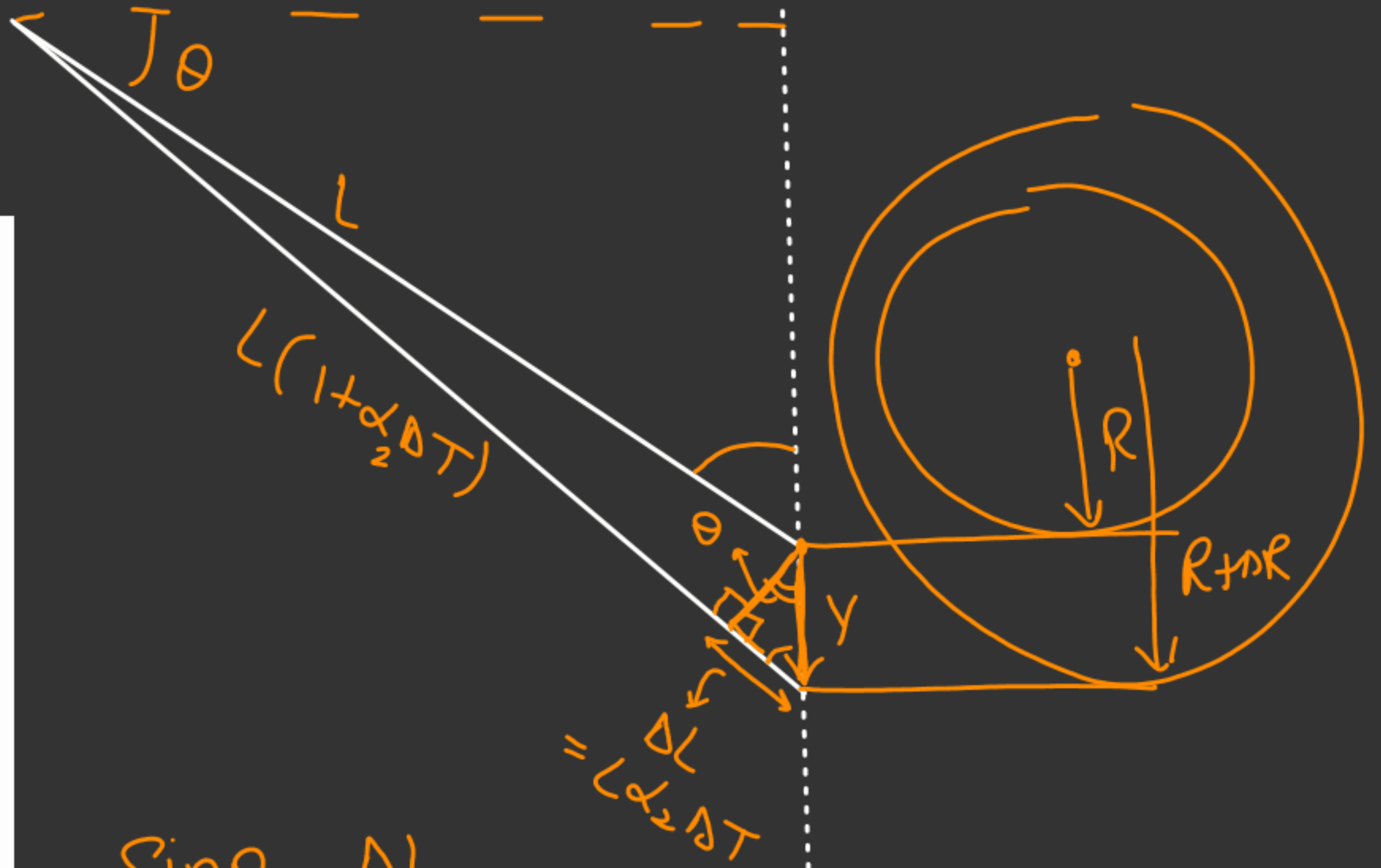
$$\boxed{\gamma_0 = 2\alpha_g}$$

$$2L\alpha_2 = R\alpha_1$$

$$\boxed{\frac{\alpha_1}{\alpha_2} = \frac{2L}{R} = 8}$$

Q. 23: A metal cylinder of radius R is placed on a wooden plank BD. The plank is kept horizontal suspended with the help of two identical string AB and CD each of length L. The temperature coefficient of linear expansion of the cylinder and the strings are α_1 and α_2 respectively. Angle θ shown in the figure is 30°. It was found that with change in temperature the centre of the cylinder did not move. Find the ratio $\frac{\alpha_1}{\alpha_2}$, if it is know that $L = 4R$. Assume that change in value of θ is negligible for small temperature changes.





$$\sin\theta = \frac{\Delta L}{Y}$$

$$Y = \frac{\Delta L}{\sin\theta} = \frac{L\alpha_2\cancel{\Delta T}}{1/2} = \Delta R = R\alpha_1\cancel{\Delta T}$$

$P = 420\,\text{Watt}$
$= 420\,\frac{J}{sec}$

$t = 0$ sec, IW $0°C$ — 150 sec, IW $0°C$ — 273 sec, W $10°C$ — 378 sec, W $20°C$

M – total mass
m – ice
$M - m = \text{Water} = 1050 - 210 = 840\,gm$

$MS\Delta T = Pt$

$M \times \frac{4.2\,J}{gm\,°C} \times 10\,°C = 420\,\frac{J}{sec} \times 105\,sec$

$$\boxed{M = 1050\,gm}$$

0 – 273 sec

$m \times 336\,\frac{J}{gm} + 1050 \times 4.2 \times 10 = 420\,\frac{J}{sec} \times 273\,sec$

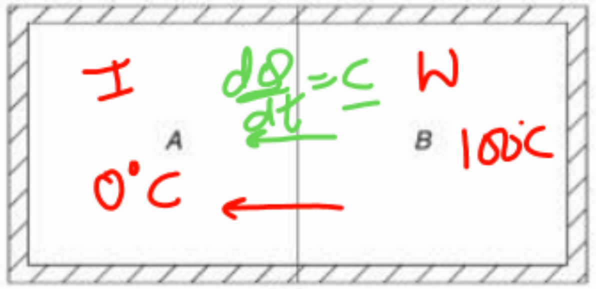
$m = \frac{420 \times 273 - 1050 \times 4.2 \times 10}{336} = 210\,gm$

Q. 15: A well insulated container has a mixture of ice and water, at 0 °C. The mixture is supplied heat at a constant rate of 420 watt by switching on an electric heater at time $t = 0$. The temperature of the mixture was recorded at time $t = 150s$, $273s$ and $378s$ and the readings were 0 °C, 10 °C and 20 °C respectively. Calculate the mass of water and ice in the mixture.

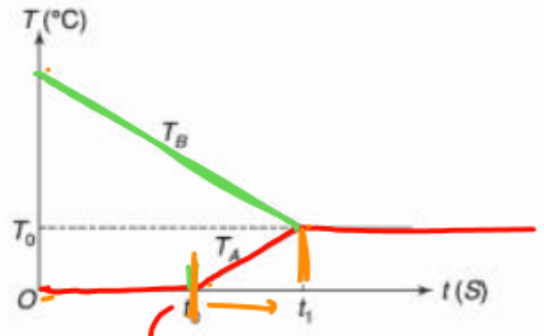
Specific heat of water = 4.2 J g^{-1} °C^{-1}, Specific latent heat of fusion of ice = 336 J g^{-1}.

Assume that the mixture is stirred slowly to maintain a uniform temperature of its content.

T_A and T_B versus time (t) [Fig. (b)]. Assume that temperature inside each compartment remains uniform.



(a)



(b)

(a) Is it correct to assert that the conducting wall conducts heat at a uniform rate, irrespective of the temperature difference between A and B?

(b) Find the value of time t_1 and temperature T_0 shown in the graph, if it is known that $t_0 = 200$ s.
Specific heat of ice = 0.5 cal g^{-1} °C^{-1}
Specific heat of water = 1.0 cal g^{-1} °C^{-1}
Latent heat of fusion of ice = 80 cal g^{-1}

$$100 - T_0 = 80 + T_0$$

$$2T_0 = 20$$

$$\boxed{T_0 = 10°C}$$

$$100\ gm \times 80\ \frac{cal}{gm} = P \times 200\ sec$$

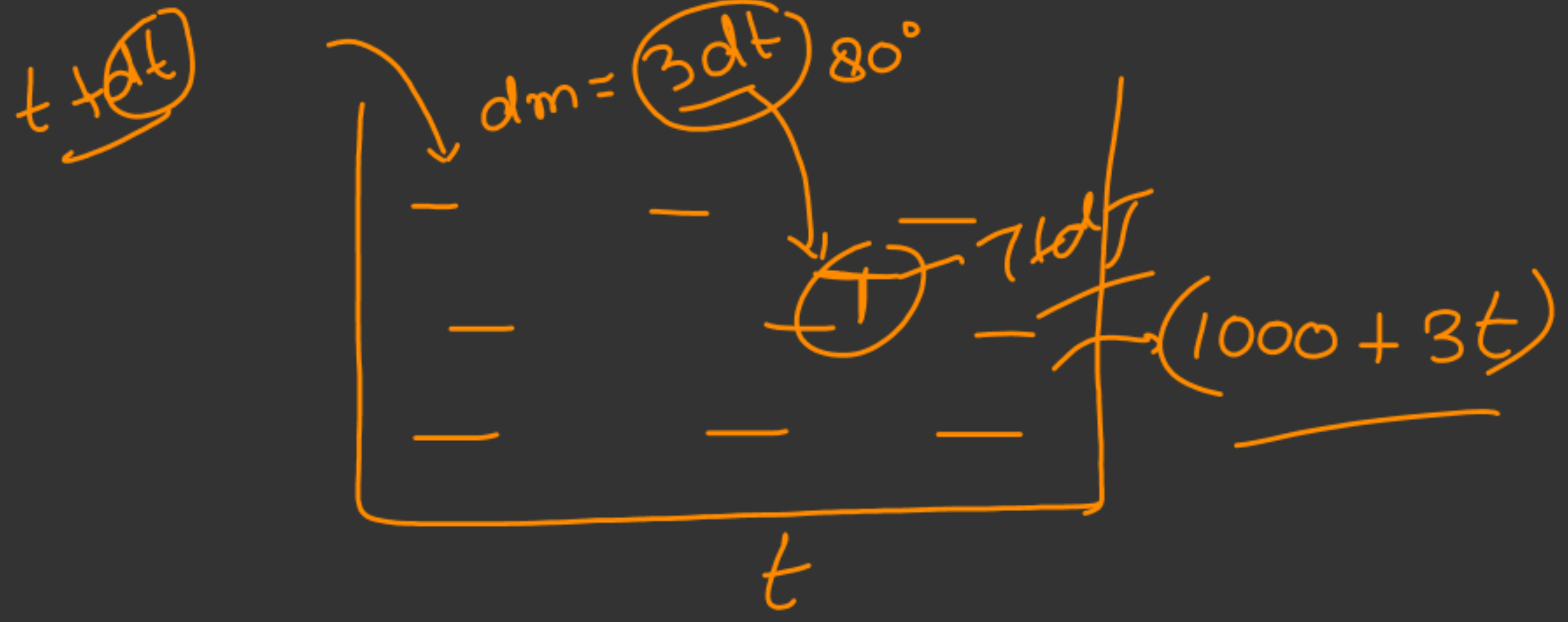
$$\boxed{P = 40\ \frac{cal}{sec}}$$

$$100\ gm \times 1\ \frac{cal}{gm\ °C} \times 90°C = 9000\ cal = 40\ \frac{cal}{sec}\ t_2$$

$$\boxed{t_2 = \frac{900}{4} = 225\ sec}$$

$$100 \times 1 \times 10$$

$$= 1000 = 40t \Rightarrow t = \frac{1000}{40} = 25\ sec$$



$$(3dt)\cancel{s}(80-T) = (100+3t)\cancel{s}(\cancel{T+dT})$$

$$dT$$

$$3\int_0^t \frac{dt}{100+3t} = \int_{20}^{T} \frac{dT}{80-T}$$

$$\frac{\cancel{3}}{3}\ln(100+3t)\Big|_0^t = -1\ln(80-T)\Big|_{20}^{T}$$

$$\ln\frac{100+3t}{100} = \ln\frac{60}{80-T}$$

$$(100+3t)(80-T) = 6000$$

$$80-T = \frac{6000}{100+3t}$$

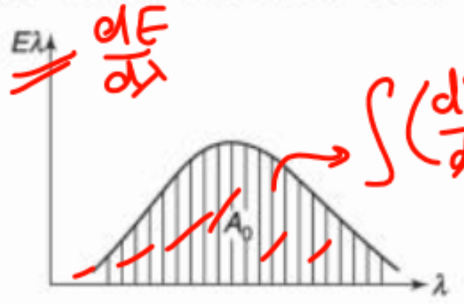
$$\boxed{T = 80 - \frac{6000}{100+3t}}$$

Q. 25: Water from a reservoir maintained at a constant temperature of 80°C is added at a slow and steady rate of $\mu = 3\ \text{gs}^{-1}$ to a calorimeter initially containing 1000 g of water at 20°C. The water in the calorimeter is stirred slowly to make the temperature uniform. Assume heat loss to the surrounding and work done in stirring is negligible and heat capacity of the calorimeter is negligible. Write the temperature of water in the calorimeter as a function of time.

$\lambda_m \to$ Red

1000 K

Q. 15: An iron ball is heated to 727°C and it appears bright red. The plot of energy density distribution versus wavelength is as shown. The graph encloses an area A_0 under it. Now the ball is heated further and it appears bright yellow. Find the area (A) of the energy density graph now.



$\frac{dE}{d\lambda}$

$\int \left(\frac{dE}{d\lambda}\right) d\lambda = E$

$= \sigma T^4$

$\lambda_m T = b$

If the given that wavelengths for red and yellow light are 8000 Å and 6000 Å respectively.

$\lambda_m \to$ Yellow

$\lambda \propto \frac{1}{T}$

$T\uparrow \quad \lambda\downarrow$

σT^4

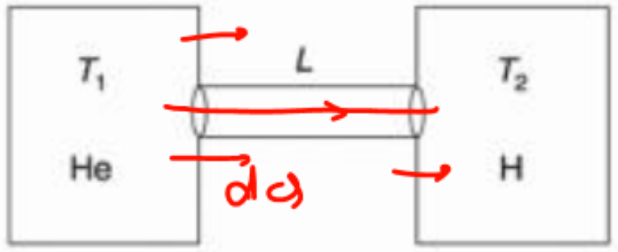
$$\frac{A}{A_0} = \left(\frac{T'}{T}\right)^4 = \left(\frac{\lambda_R}{\lambda_y}\right)^4$$

$$A = A_0\left(\frac{8\cancel{000}}{6\cancel{000}}\right)^4$$

$$A = A_0\left(\frac{4}{3}\right)^4$$

$$A = A_0 \frac{256}{81}$$

capacity are connected by conducting rod of length L and cross sectional area A. Thermal conductivity of the rod is k and its curved cylindrical surface is well insulated from the surrounding. Heat capacity of the rod is also negligible. One container is filled with n moles of helium at temperature T_1 and the other one is filled with equal number of moles of hydrogen at temperature $T_2(<T_1)$. Calculate the time after which the temperature difference between two gases will becomes half the initial difference.



t $\qquad \theta_1 \qquad \theta_2$

$$\theta = \theta_1 - \theta_2$$

$$d\theta = d\theta_1 - d\theta_2$$

$$i_{th} = \frac{kA(\theta_1-\theta_2)}{l} = \frac{kA\theta}{l} = \frac{dQ}{dt}$$

$$dQ = -\frac{3}{2}nR\,d\theta_1 = \frac{5}{2}nR\,d\theta_2$$

$$d\theta_1 = -\frac{2}{3nR}dQ$$

$$d\theta_2 = \frac{2}{5nR}dQ$$

$$d\theta = d\theta_1 - d\theta_2 = \left(\frac{-2}{3nR} - \frac{2}{5nR}\right)dQ$$

$$= \frac{-10-6}{15nR}dQ$$

$$d\theta = \frac{-16}{15nR}dQ = \frac{-16}{15nR}\frac{kA}{l}\theta\,dt$$

$$\int_{T_1-T_2}^{\frac{T_1-T_2}{2}} \frac{d\theta}{\theta} = \frac{-16kA}{15nRl}\int_0^t dt$$

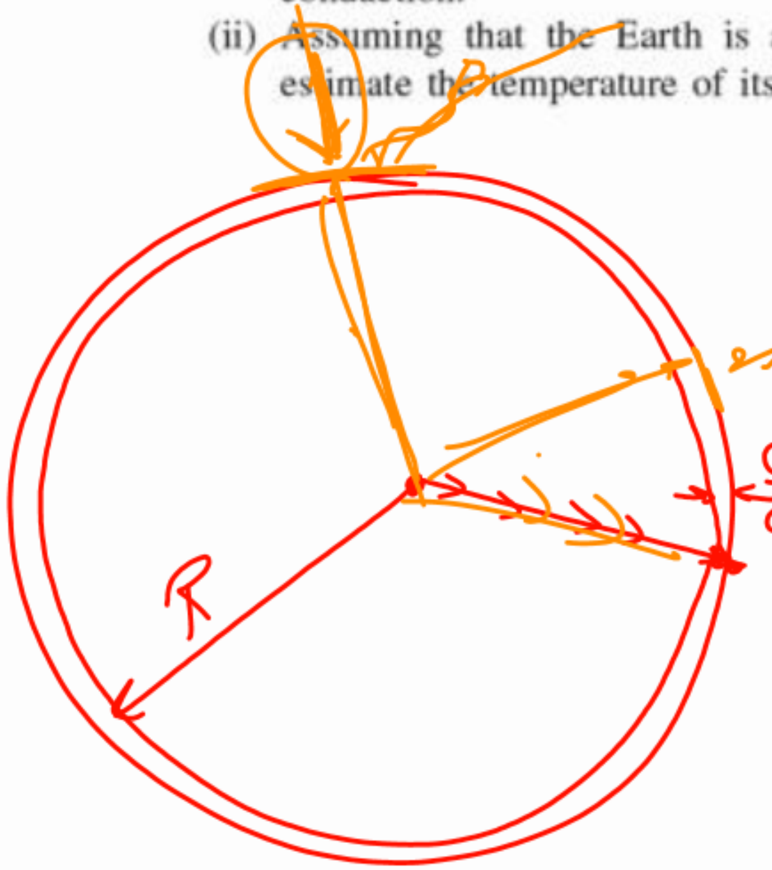
$$\left(\ln\theta\right)_{T_1-T_2}^{\frac{T_1-T_2}{2}} = -\ln 2 = \frac{-16KA}{15nRl}t$$

$$\boxed{t = \frac{15\ln 2\, nRL}{16KA}}$$

Q. 42: Heat received by the Earth due to solar radiations is 1.35 KWm^{-2}. It is also known that the temperature of the Earth's crust increases 1°C for every 30 m of depth. The average thermal conductivity of the Earth's crust is $K = 0.75\ J(msK)^{-1}$ and radius of the Earth is $R = 6400$ km.

(i) Calculate rate of heat loss by the Earth's core due to conduction.

(ii) Assuming that the Earth is a perfect block body estimate the temperature of its surface.

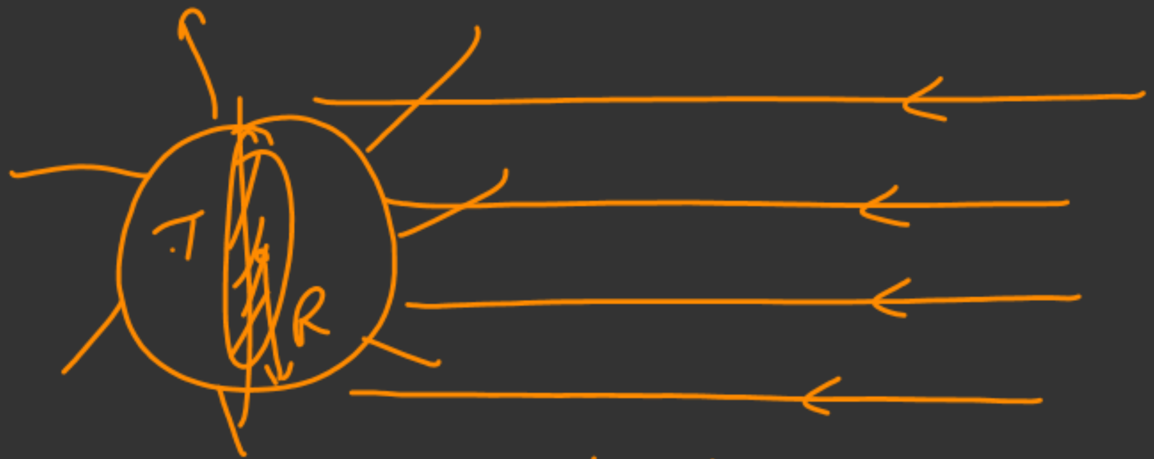


$$i_{th} = KA\frac{dT}{dR} = K4\pi R^2\frac{dT}{dR}$$

$$\frac{dT}{dR} = \frac{1°C}{30m}$$

$$i_{th} = 0.75 \times 4\pi \times (6400{,}000)^2 \times \frac{1}{30}$$

$$= 1.29 \times 10^{13}\ \text{Watt.}$$



$$P_{in} = 1.35 \times 10^3 \times \pi R^2$$

$$P_{out} = 1.\ 4\pi R^2 \sigma T^4$$

$$1.35 \times 10^3 = 4\sigma T^4$$

$$\boxed{T = 275\ \text{kelvin}}$$