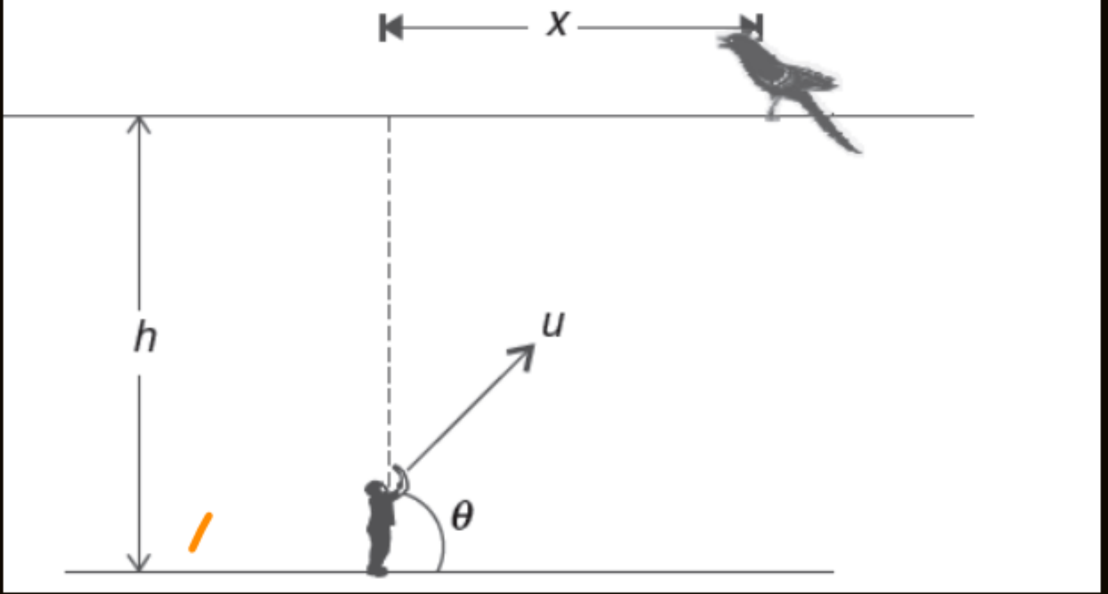
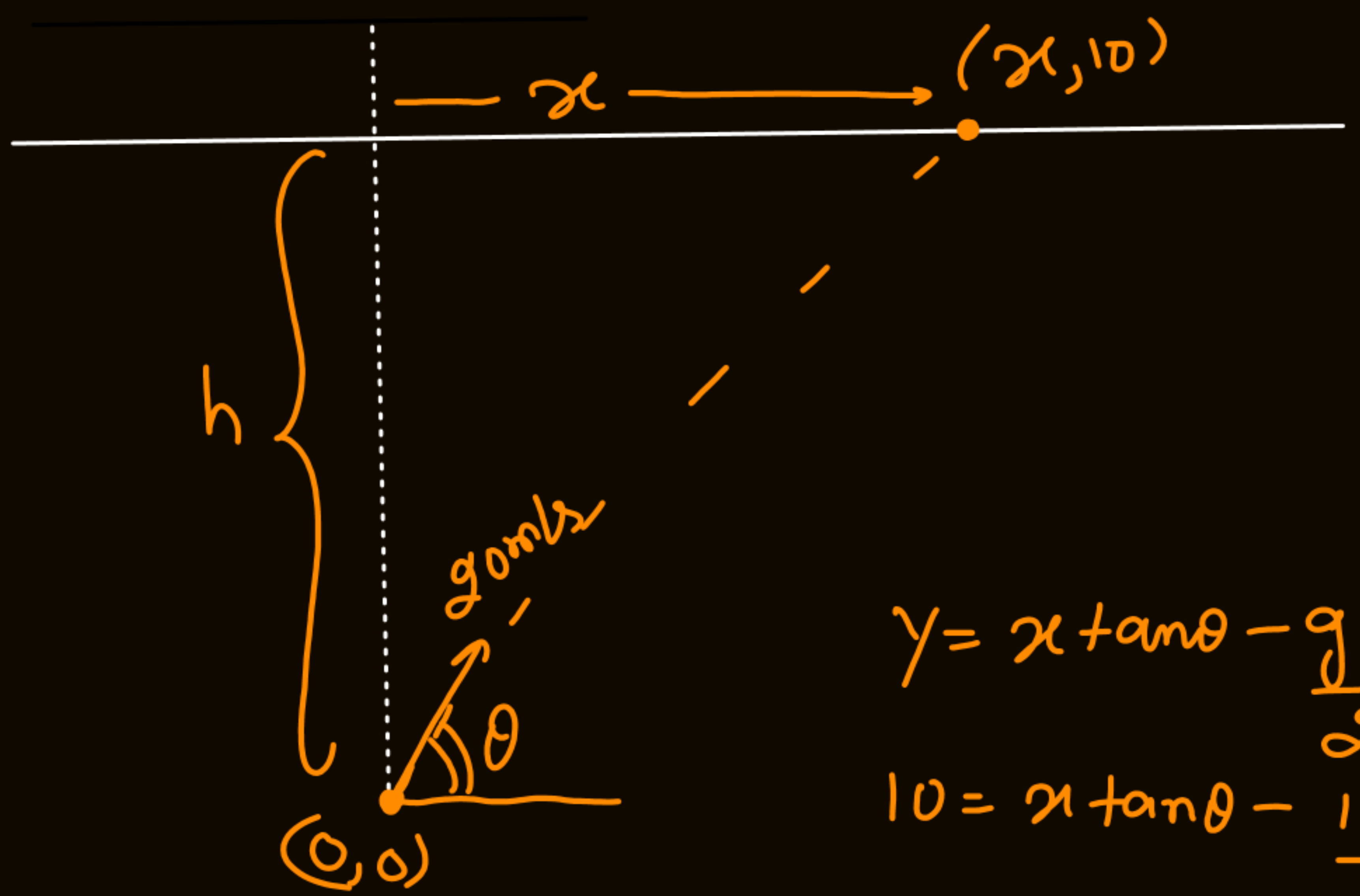
A horizontal electric wire is stretched at a height $h = 10\ m$ above the ground. A boy standing on the ground can throw a stone at a speed $u = 20\ ms^{-1}$. Find the maximum horizontal distance x at which a bird sitting on the wire can be hit by the stone.



Sol.



$$y = x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta)$$

$$10 = x\tan\theta - \frac{10\times x^2}{2\times 20\times 20}\{1+\tan^2\theta\}$$

$$800 = 80x\tan\theta - x^2 - x^2\tan^2\theta$$

$$x^2\tan^2\theta - 80x\tan\theta + (800+x^2) = 0$$

$$b^2 - 4ac \geq 0$$

$$6400x^2 - 4x^2(800+x^2) \geq 0$$

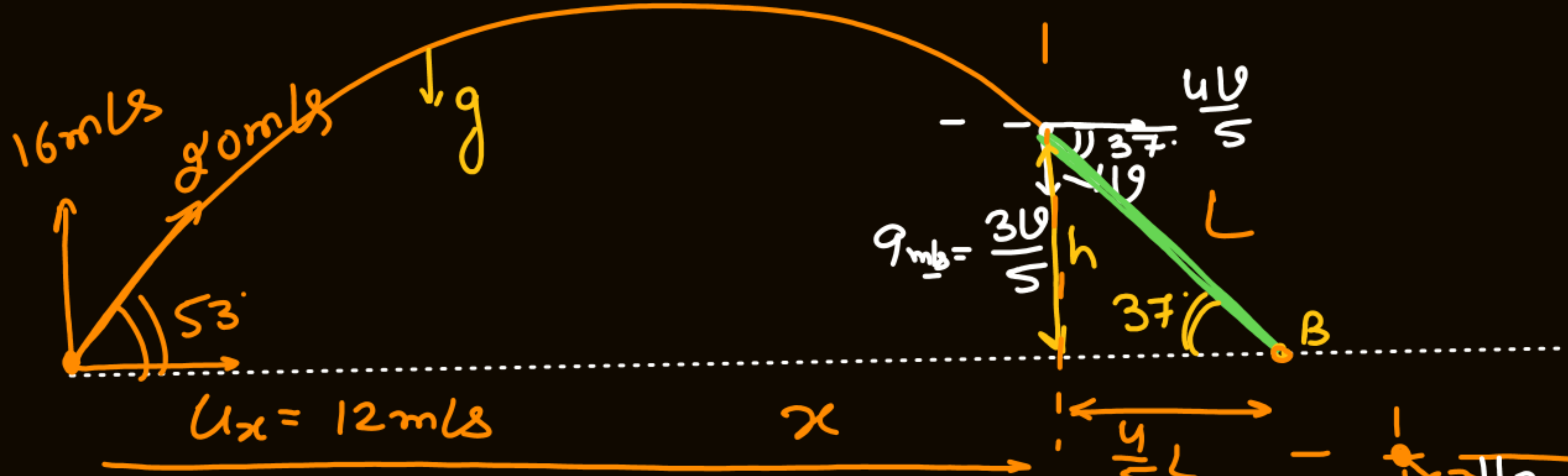
$$1600 \geq 800+x^2$$

$$800 \geq x^2$$

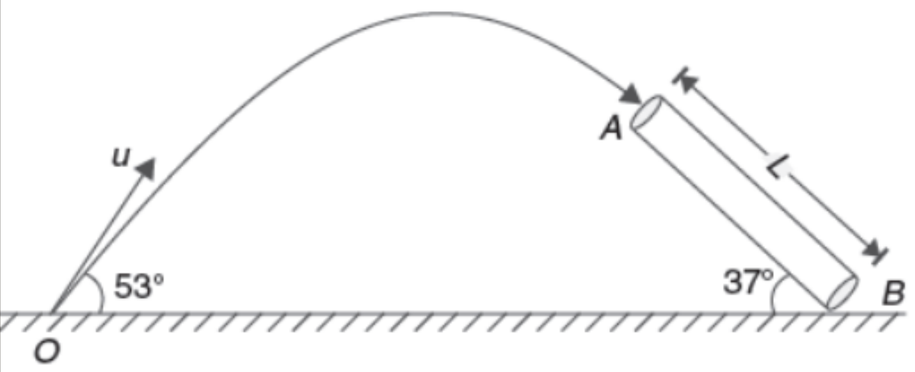
$$x \leq 20\sqrt{2}\,m$$

$$x_{max} = 20\sqrt{2}\,m$$

Sol:-

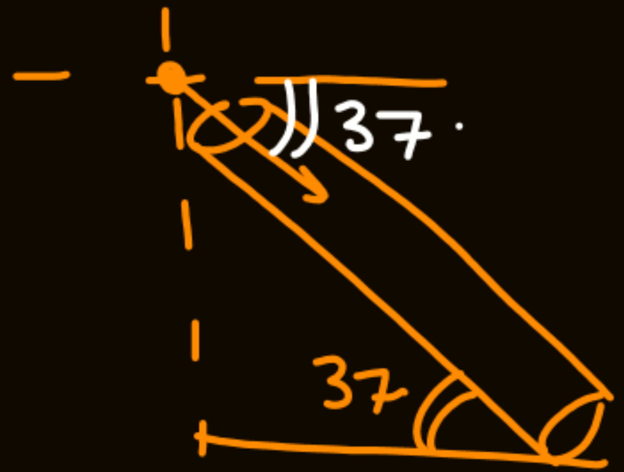


AB is a pipe fixed to the ground at an inclination of 37°. A ball is projected from point O at a speed of $u = 20m/s$ at an angle of 53° to the horizontal and it smoothly enters into the pipe with its velocity parallel to the axis of the pipe. [Take $g = 10\ ms^{-2}$]



(a) Find the length L of the pipe

(b) Find the distance of end B of the pipe from point O.



$$12 = \frac{4v}{5}$$

$$\frac{3}{5} \times 15$$

$$v = 15\ m/s$$

$$v_y^2 = u_y^2 + 2a_y y$$

$$81 = 256 + 2(-10)h$$

$$h = \frac{175}{20} = \frac{17.5}{2} = 8.75$$

$$\sin 37° = \frac{3}{5} = \frac{8.75}{L}$$

$$L = \frac{8.75 \times 5}{3}$$

$$OB = \underline{x} + \frac{4}{5} L$$

$$= 12\,m/s \times 2.5 + \frac{4}{5} L$$

$$= 30 + \frac{4}{5} L$$

$$V_y = U_y + a_y t$$

$$-9 = 16 - 10t$$

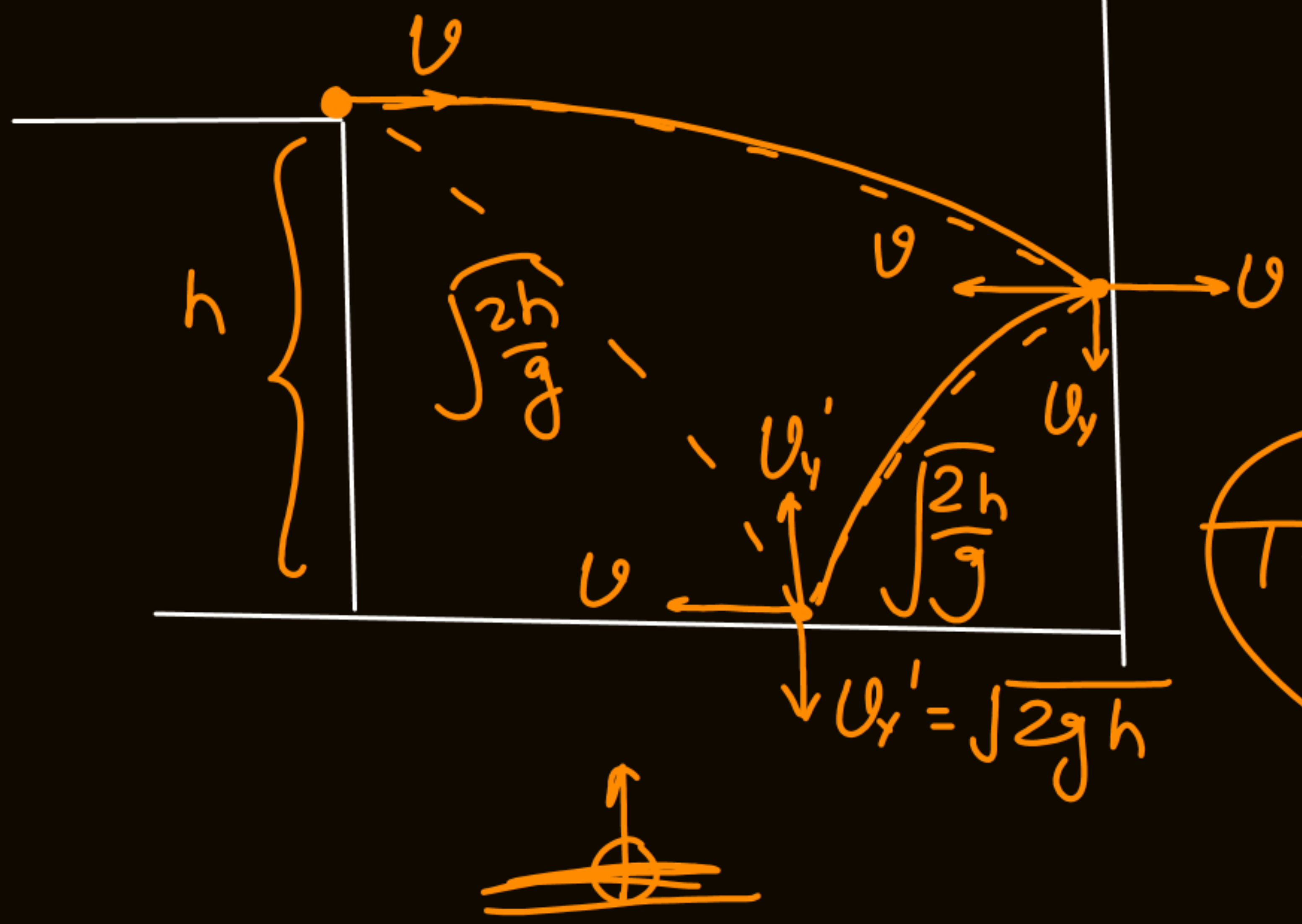
$$\underline{t = 2.5\,sec}$$

@

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$h = 0 + \frac{1}{2} g t^2$$

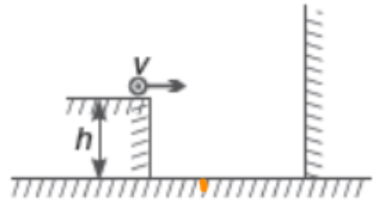
$$t = \sqrt{\frac{2h}{g}}$$



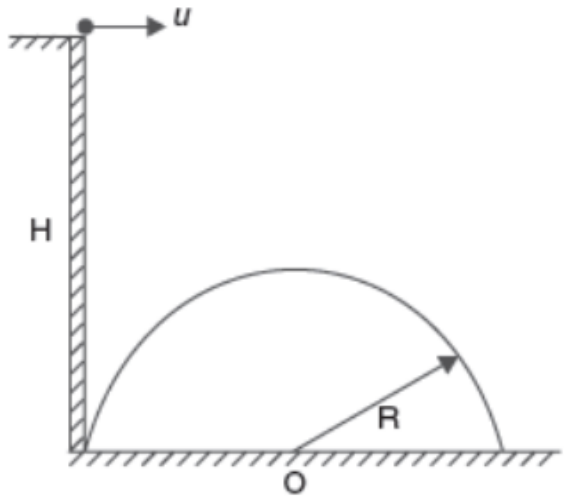
$$v_y' = \sqrt{2gh}$$

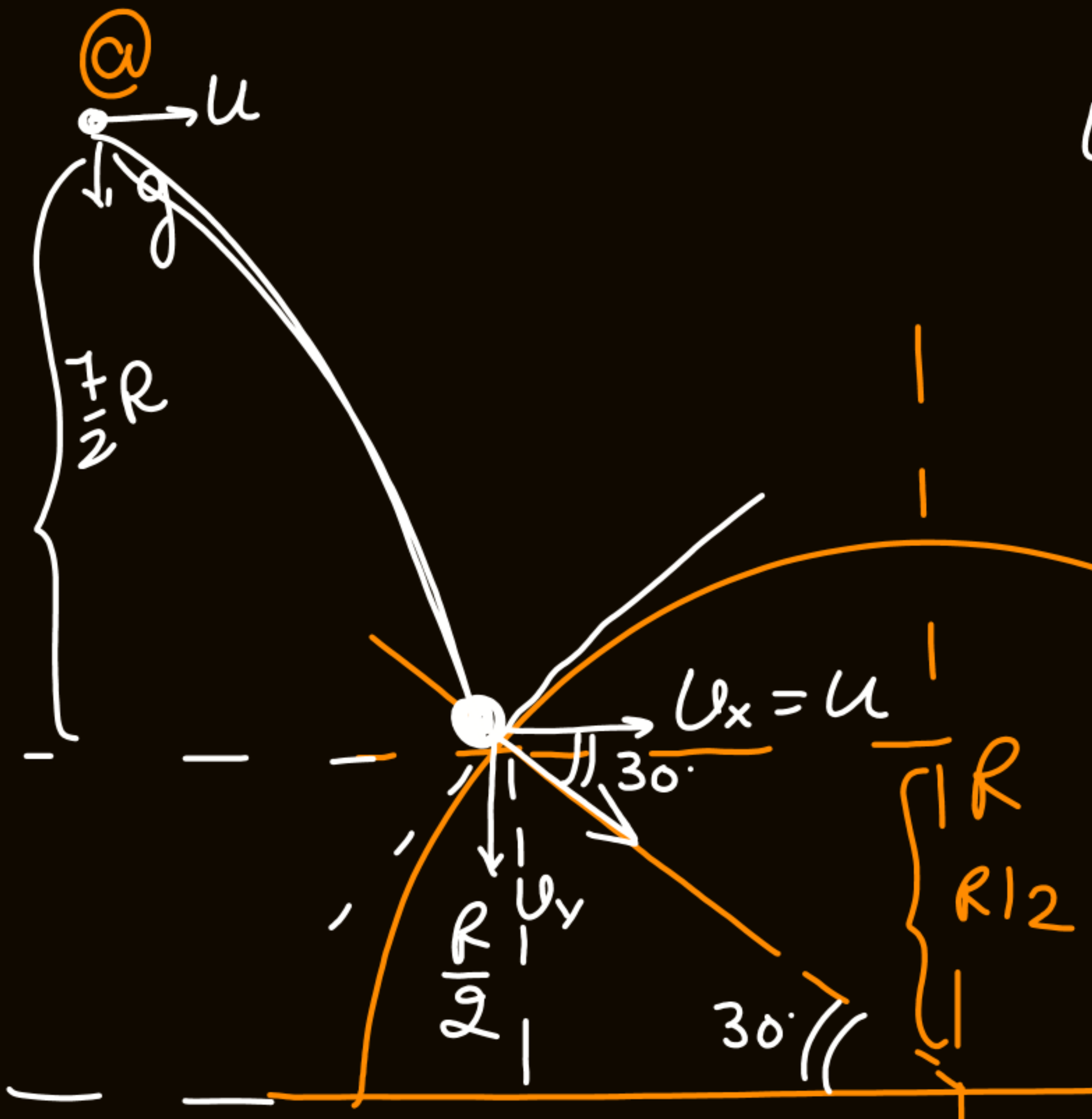
$$T = 2\sqrt{\frac{2h}{g}}$$

(a) A particle is thrown from a height h horizontally towards a vertical wall with a speed v as shown in the figure. If the particle returns to the point of projection after suffering two elastic collisions, one with the wall and another with the ground, find the total time of flight. [Elastic collision means the velocity component perpendicular to the surface gets reversed during collision.]



(b) Touching a hemispherical dome of radius R there is a vertical tower of height $H = 4R$. A boy projects a ball horizontally at speed u from the top of the tower. The ball strikes the dome at a height $\frac{R}{2}$ from ground and rebounds. After rebounding the ball retraces back its path into the hands of the boy. Find u.

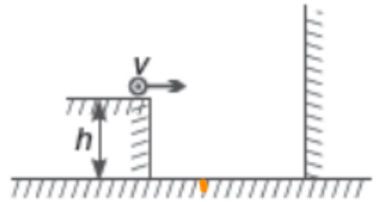




$$u\cos 30^\circ = u$$
$$u\sin 30^\circ = U_y = \sqrt{7gR}$$
$$\tan 30^\circ = \frac{\sqrt{7gR}}{u}$$
$$u = \sqrt{21gR}$$

$$U_y^2 = 0^2 + 2g\frac{7}{2}R \Rightarrow U_y = \sqrt{7gR}$$

(a) A particle is thrown from a height h horizontally towards a vertical wall with a speed v as shown in the figure. If the particle returns to the point of projection after suffering two elastic collisions, one with the wall and another with the ground, find the total time of flight. [Elastic collision means the velocity component perpendicular to the surface gets reversed during collision.]



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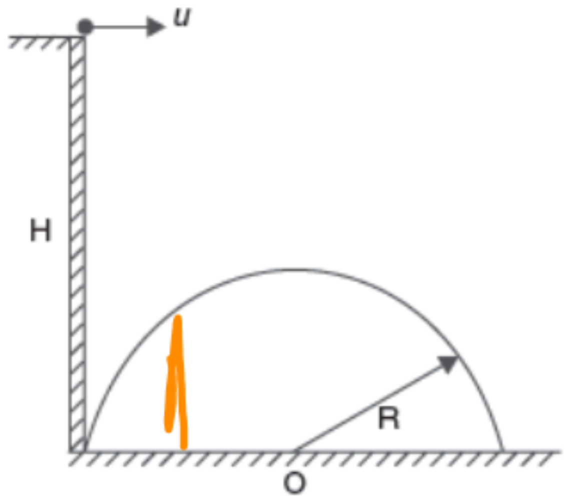
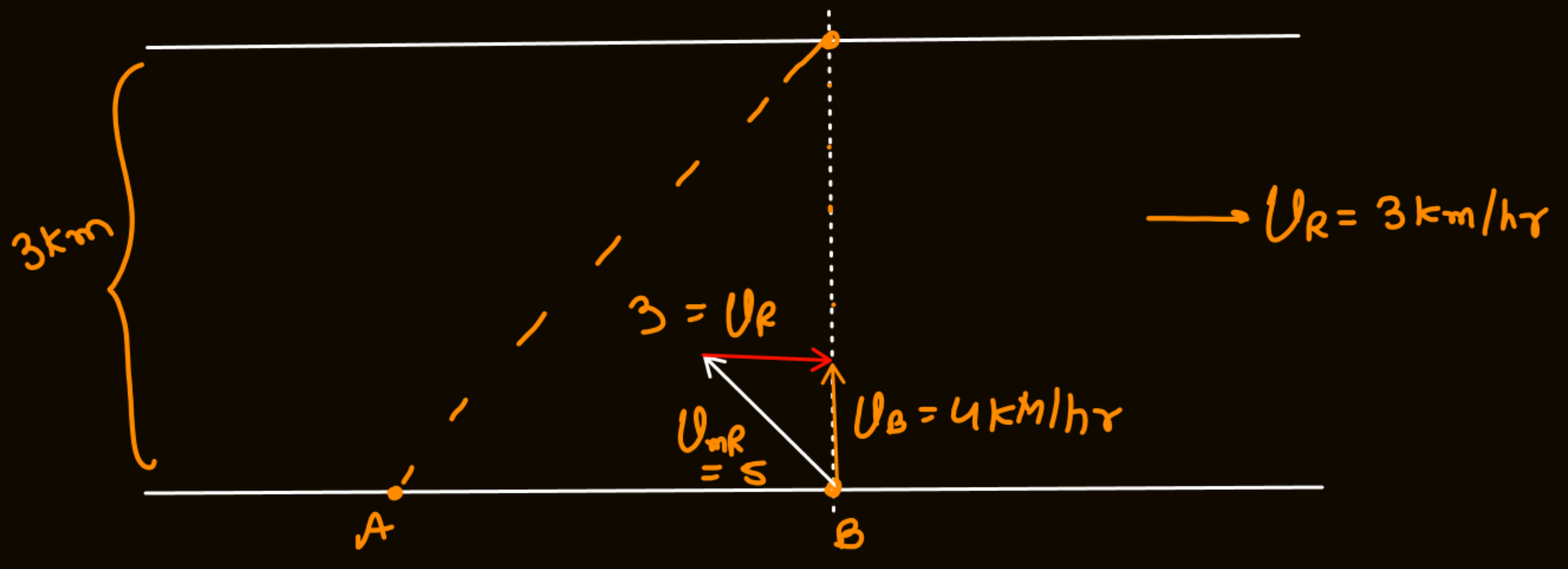
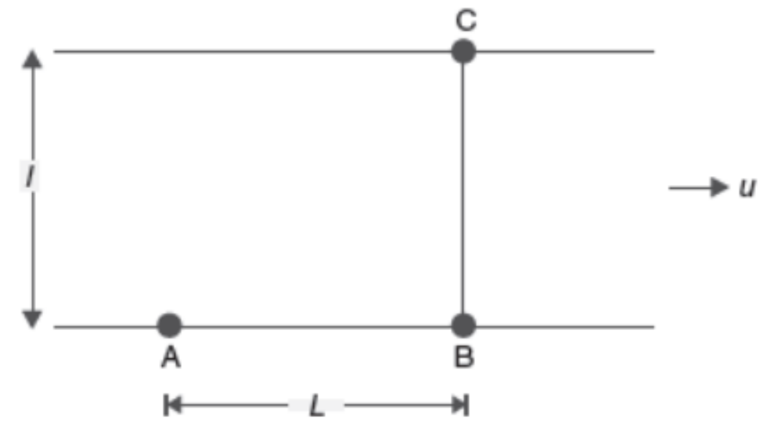


Sol:



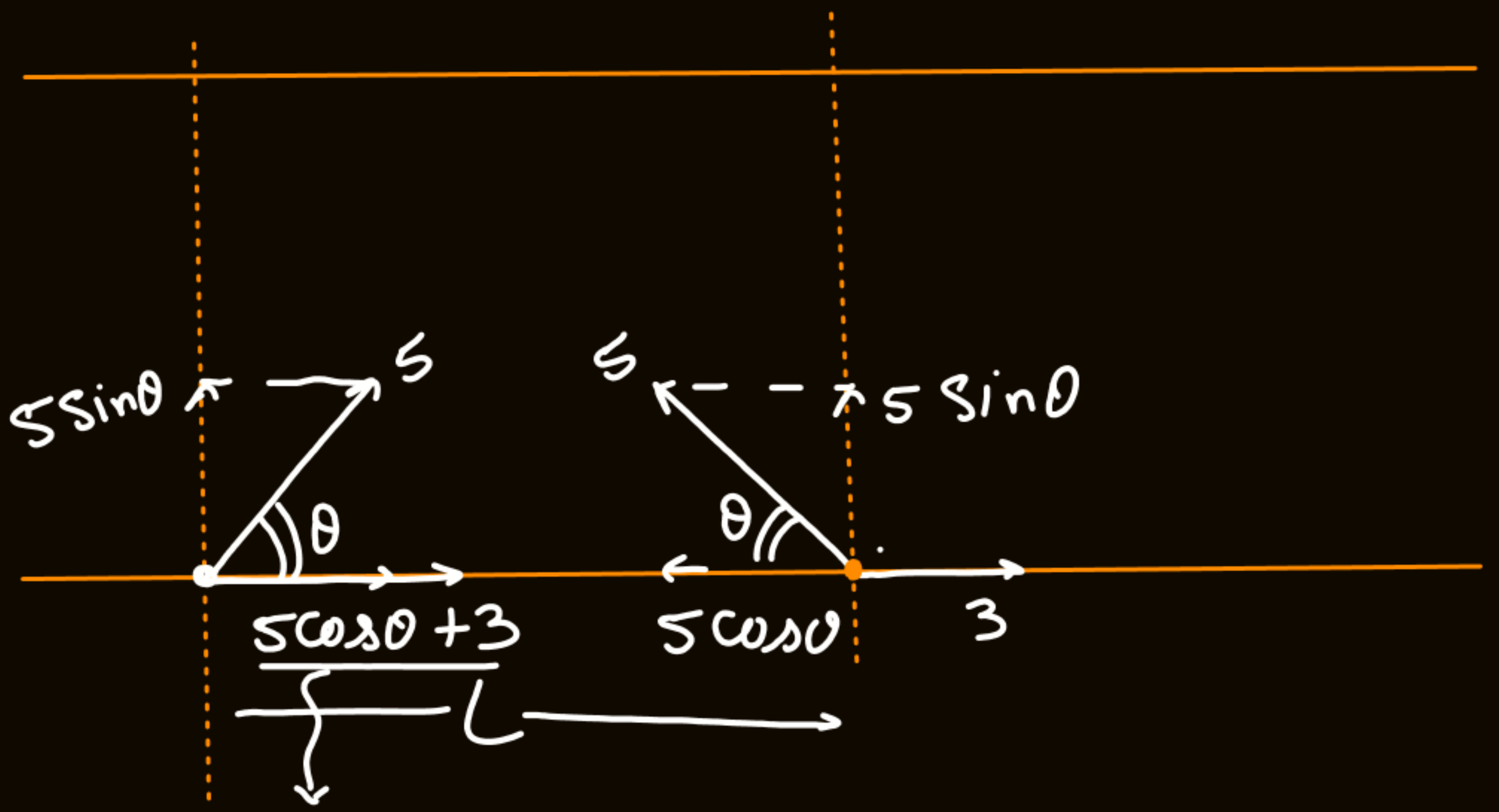
$$t = \frac{3\,km}{4\,km/hr} = \frac{3}{4}\,hr = 45\ minutes.$$



Two friends A and B are standing on a river bank L distance apart. They have decided to meet at a point C on the other bank exactly opposite to B. Both of them start rowing simultaneously on boats which can travel with velocity $V = 5\ km/hr$ in still water. It was found that both reached at C at the same time. Assume that path of both the boats are straight lines. Width of the river is $l = 3.0\ km$ and water is flowing at a uniform speed of $u = 3.0\ km/hr$.

(a) In how much time the two friends crossed the river.

(b) Find L.



$5\cos\theta = 3$

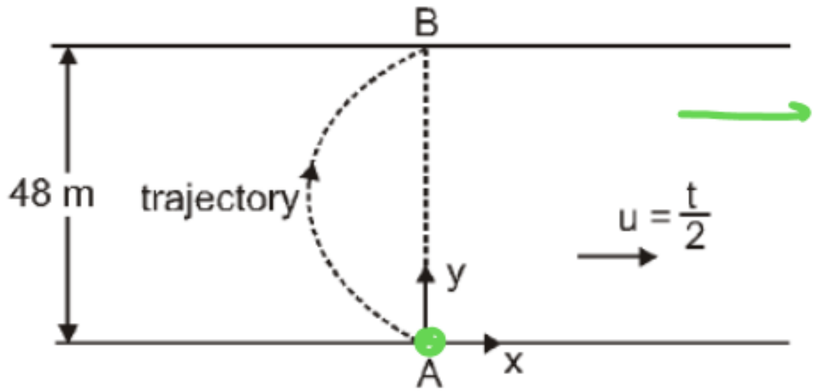
$v_x = 6\,km/hr$

$x = v_x t$

$= 6\,km/hr \times \frac{3}{4}\,hr = 4.5\,km = L$

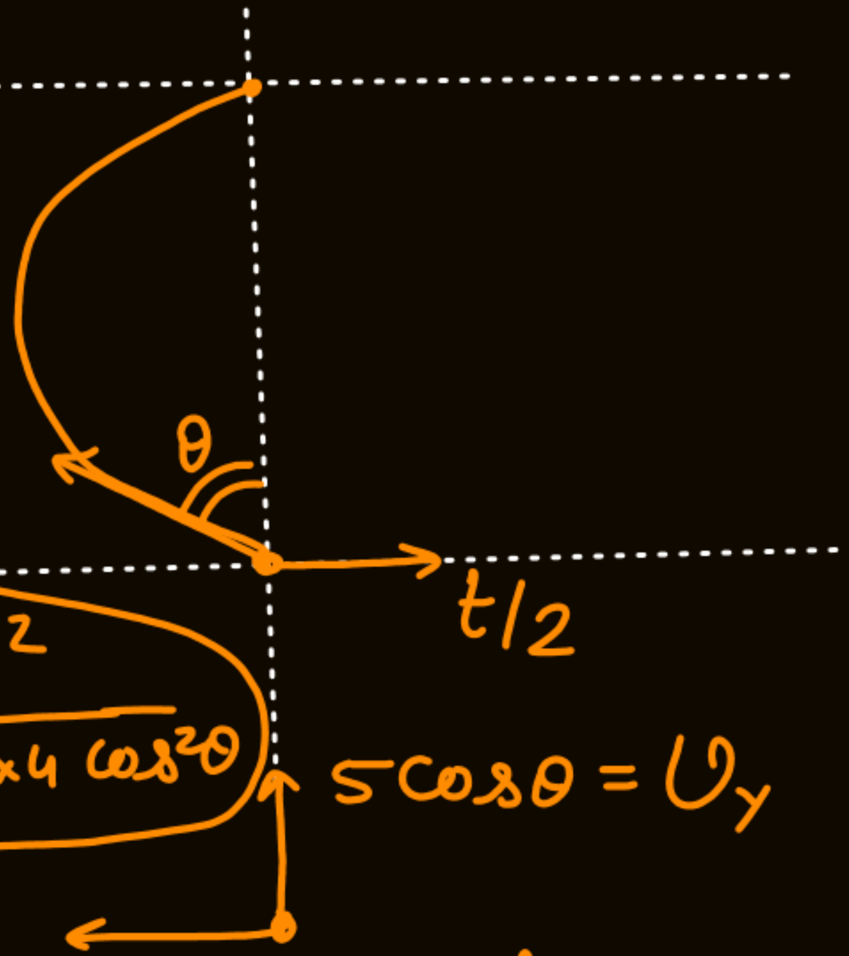
Ex

A man starts swimming at time t = 0 from point A on the ground and he wants to reach the point B directly opposite the point A. His velocity in still water is 5 $\frac{m}{sec}$ and width of river is 48 m. River flow velocity 'u' varies with time t (in seconds) as $u = \frac{t}{2}\frac{metre}{sec}$. He always tries to swim in particular fixed direction with river flow. Find the (Given $\sin^{-1}\left(\frac{24}{25}\right) = 74^\circ$)



$\longrightarrow a = \frac{du}{dt} = 0.5\ m/s^2$

(a) direction (with line AB) in which he should make stroke and the time taken by man to cross the river.
(b) trajectory of path.



$y = (5\cos\theta)t$

$x = -5\sin\theta\, t + \frac{t^2}{4}$

$x = -\frac{5\sin\theta \cdot y}{5\cos\theta} + \frac{y^2}{25\times4\cos^2\theta}$

$5\cos\theta = v_y$

$v_x = \left(5\sin\theta - \frac{t}{2}\right) = \frac{dx}{dt}$

$x = (5\sin\theta)t - \frac{t^2}{4} = 0$

$t = 20\sin\theta$

$(5\cos\theta)t = 48$

$5\cos\theta \cdot 20\sin\theta = 48$

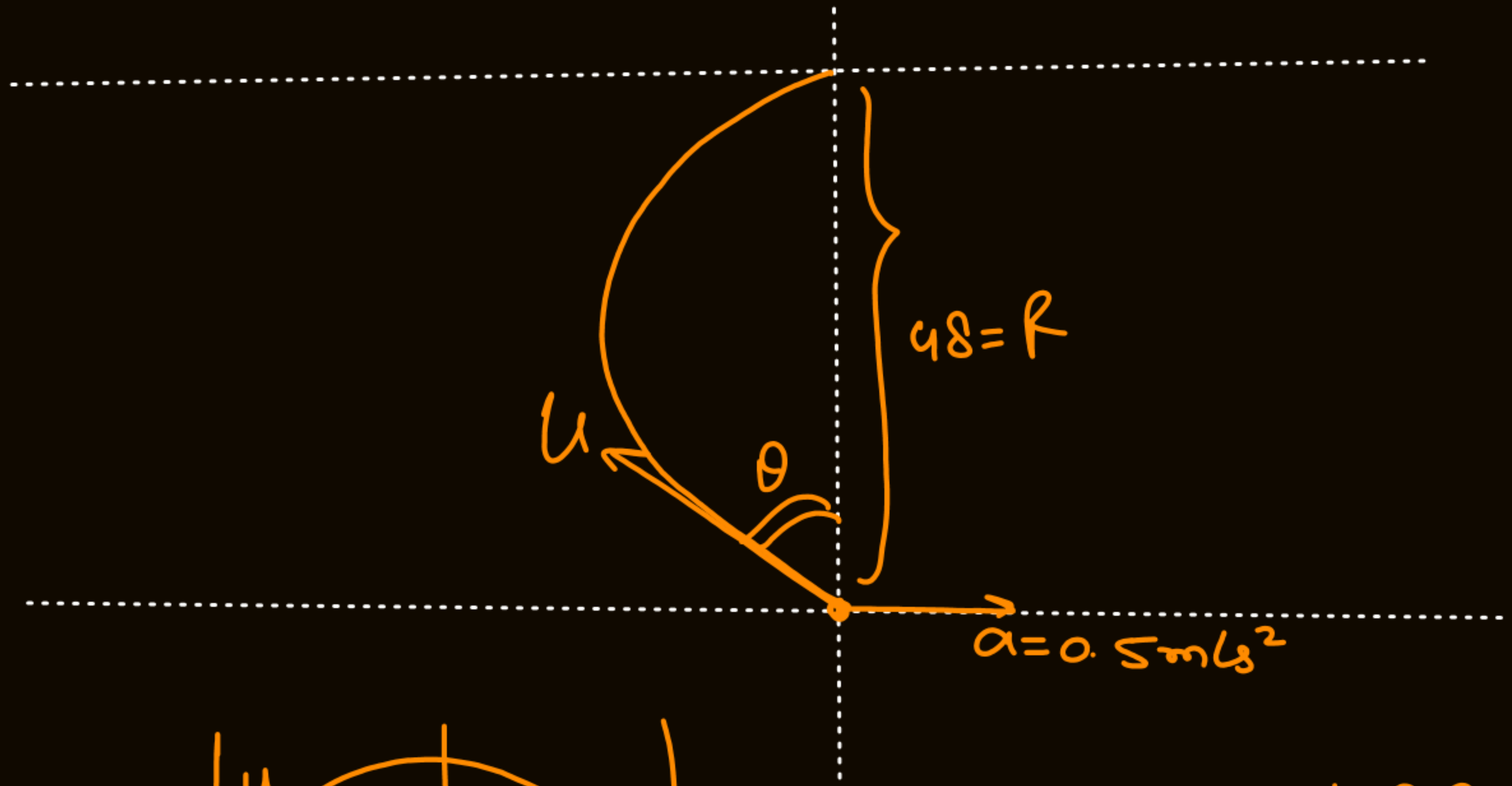
$50(\sin2\theta) = 48$

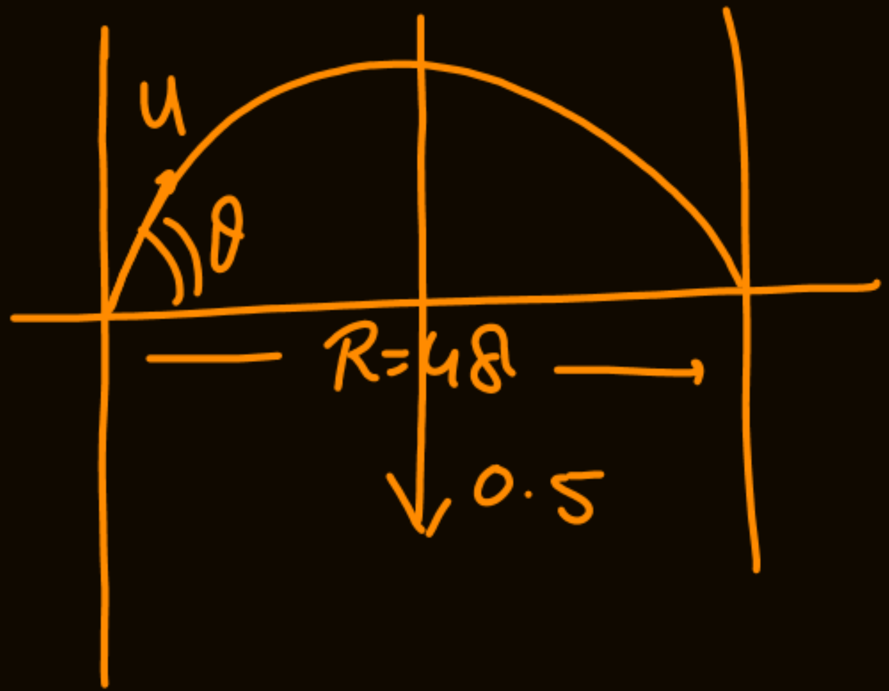
$\sin2\theta = \frac{24}{25}$

$2\theta = 74^\circ$

$\theta = 37^\circ$

$t = 20\times\frac{3}{5} = 12\ sec$

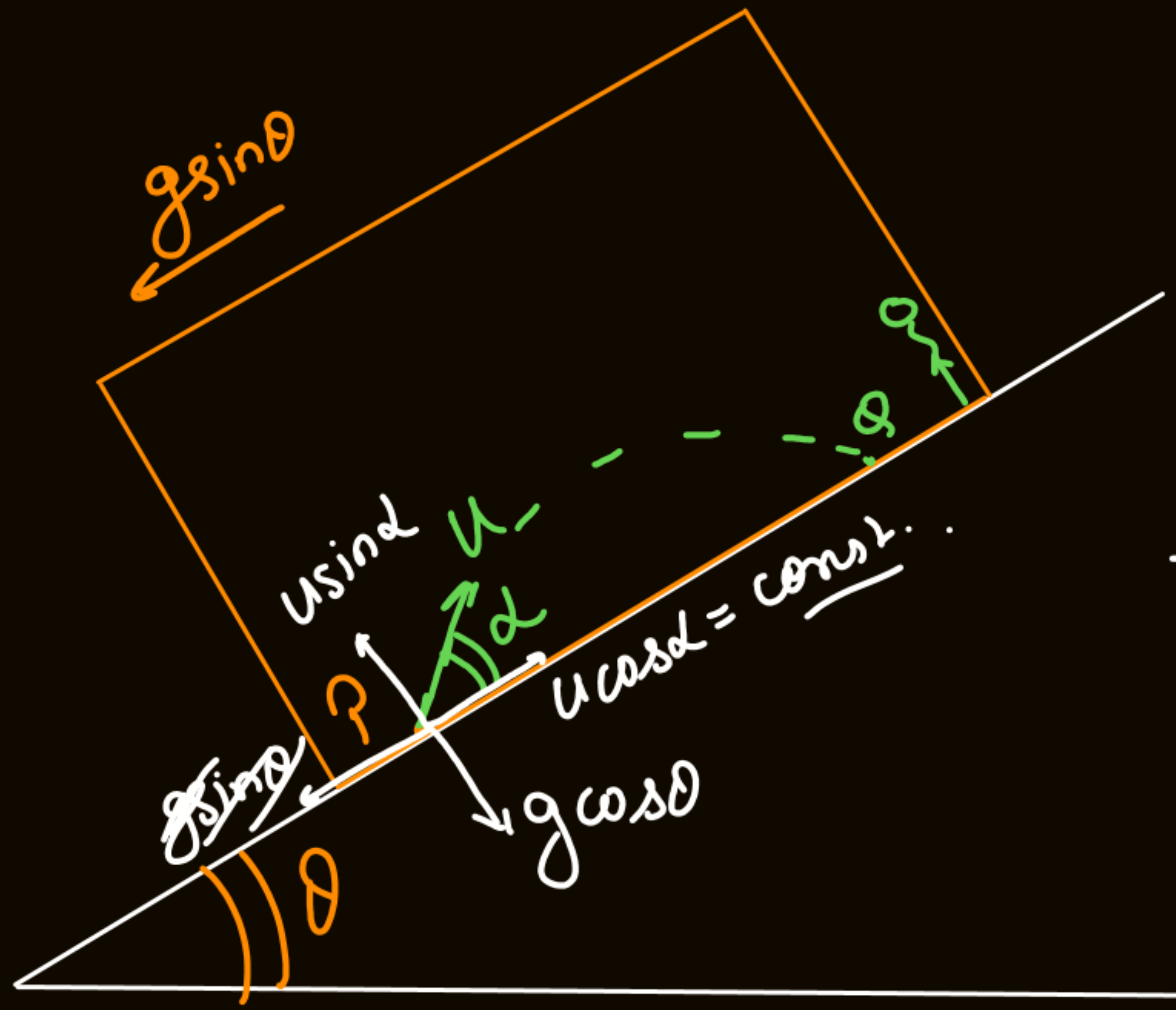




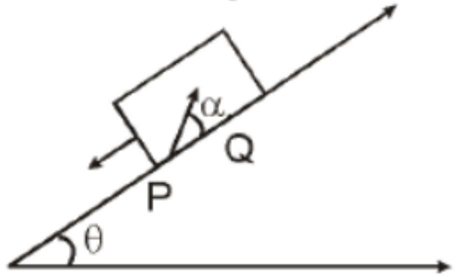
$$R = 48 = \frac{25 \sin 2\theta}{0.5}$$

$$\boxed{24 = 25 \sin 2\theta}$$

Sol:-



A large heavy box is sliding without friction down a smooth plane of inclination θ From a point P on the bottom of the box, a particle is thrown inside box. The initial speed of the particle with respect to the box is u and the direction of projection makes an angle α with the bottom as shown in the figure :



(a) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands (Assume that the particle does not hit any other surface of the box. Neglect air resistance.)

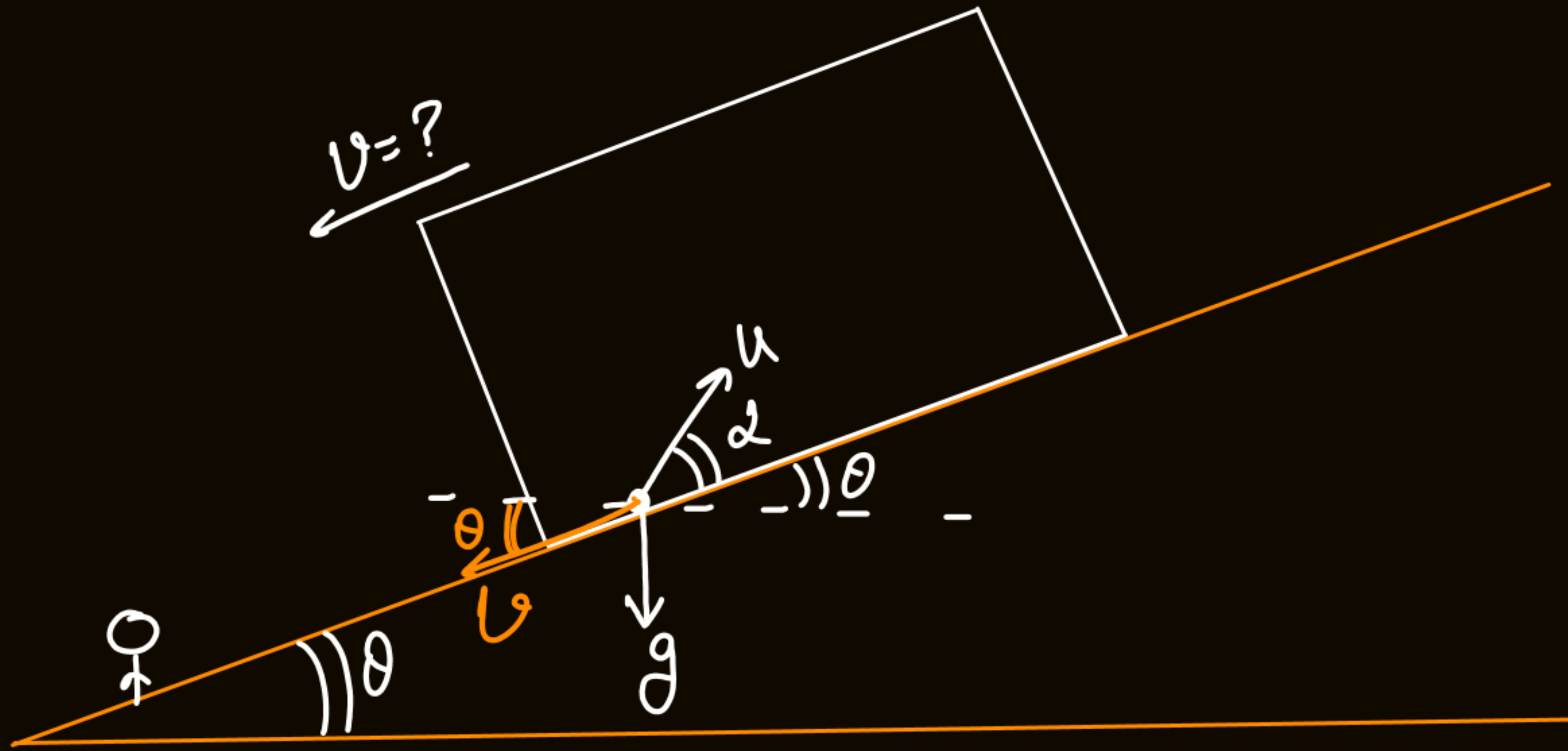
(b) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when the particle was projected.

$$T = \frac{2u_\perp}{g_\perp} = \frac{2u\sin\alpha}{g\cos\theta}$$

$$PQ = x = u_x t$$

$$= u\cos\alpha \cdot \frac{2u\sin\alpha}{g\cos\theta}$$

$$= \frac{u^2\sin 2\alpha}{g\cos\theta}.$$



$$vt + \frac{1}{2} g\sin\theta\, t^2 = \frac{u^2\sin 2\alpha}{g\cos\theta}$$

$$t = \frac{2u\sin\alpha}{g\cos\theta}$$

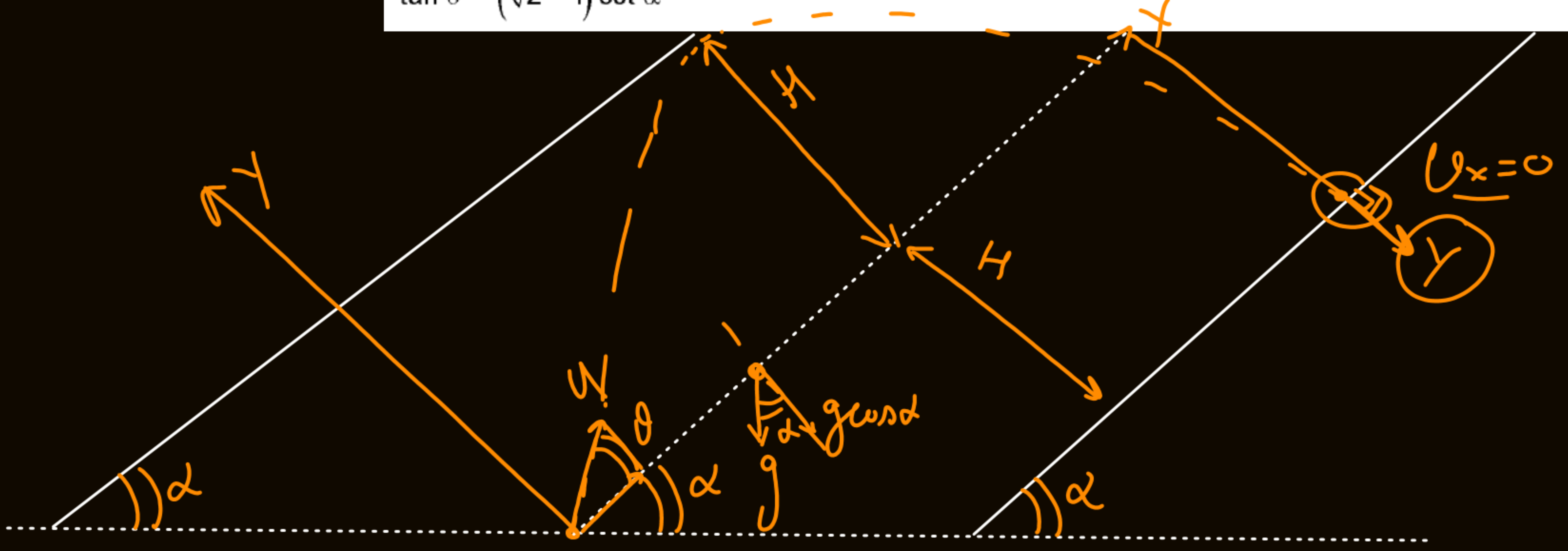
$$\vec{U}_B = \vec{U}_{B/Box} + \vec{U}_{Box}$$

$$u\cos(\alpha+\theta) = v\cos\theta$$

$$\boxed{v = \frac{u\cos(\alpha+\theta)}{\cos\theta}}$$

Two parallel straight lines are inclined to the horizontal at an angle α. A particle is projected from a point mid way between them so as to graze one of the lines and strikes the other at right angles. Show that if θ is the angle between the direction of projection and either of lines, then

$\tan\theta = \left(\sqrt{2}-1\right)\cot\alpha$



$u_x = u\cos\theta$

$a_x = -g\sin\alpha$

$$H = \frac{u_\perp^2}{2g_\perp} = \frac{u^2 \sin^2\theta}{2g\cos\alpha}$$

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$-H = u\sin\theta\, t - \frac{1}{2} g\cos\alpha \cdot t^2$$

$$v_x = u_x + a_x t$$

$$0 = u\cos\theta - g\sin\alpha \cdot t$$

$$t = \frac{u\cos\theta}{g\sin\alpha}$$

$$-\frac{u^2\sin^2\theta}{2g\cos\alpha} = \frac{u\sin\theta\cos\theta}{g\sin\alpha} - \frac{g\cos\alpha}{2}\,\frac{u^2\cos^2\theta}{g^2\sin^2\alpha}$$

$$-\frac{\sin^2\theta}{2\cos\alpha} = \frac{\sin\theta\cos\theta}{\sin\alpha} - \frac{\cos\alpha\cos^2\theta}{2\sin^2\alpha}$$

multiply with $\frac{2\cos\alpha}{\cos^2\theta}$