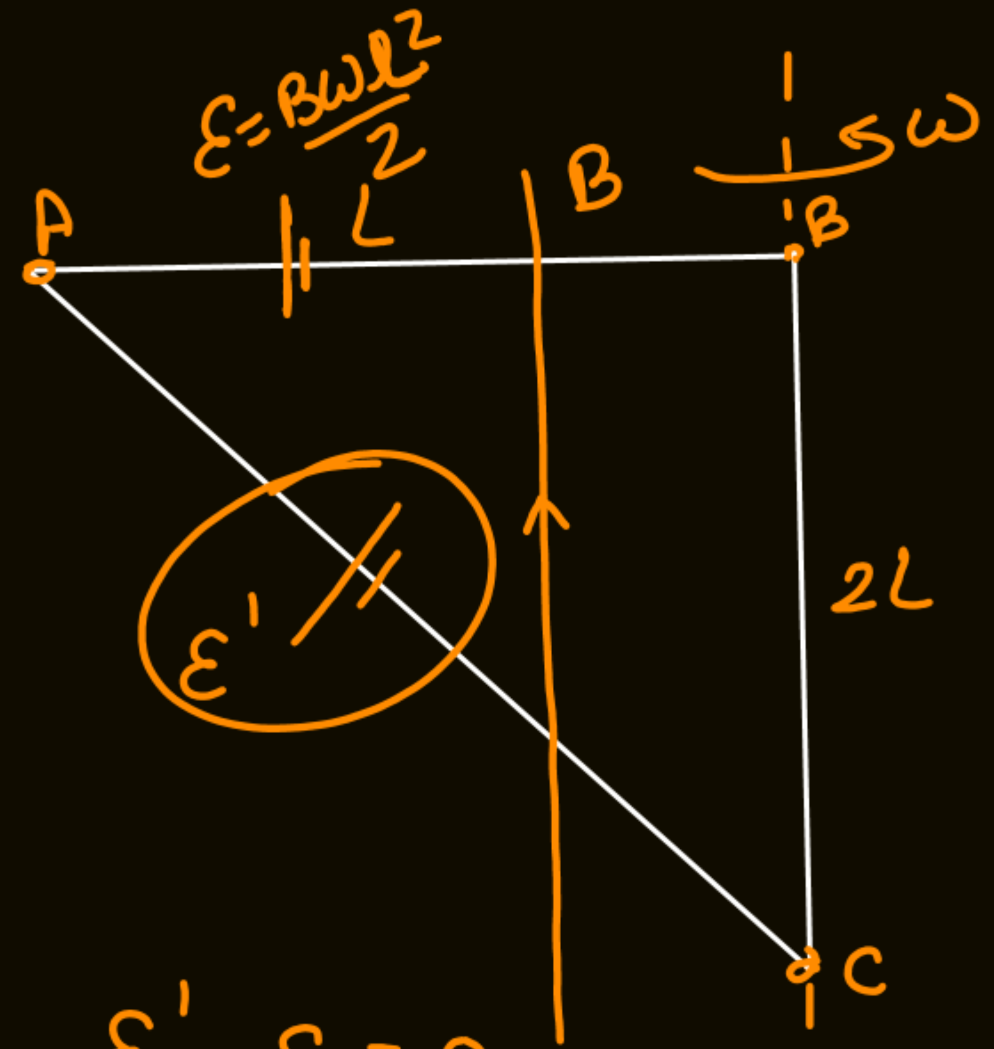
Q. 7: ABC is a triangular frame made of a conducting wire and is right angled at B. Its side BC is vertical and AB is horizontal. The frame is placed in a uniform magnetic field B in vertically upward direction. The frame is rotated about its side BC with constant angular speed ω. Resistance of the entire frame is R. Neglect gravity. Length $AB = L$; $BC = 2L$.

(a) Find the torque needed to keep the frame rotating with constant angular speed.

(b) Find the potential difference between points A and C.



$\theta = 90^\circ$

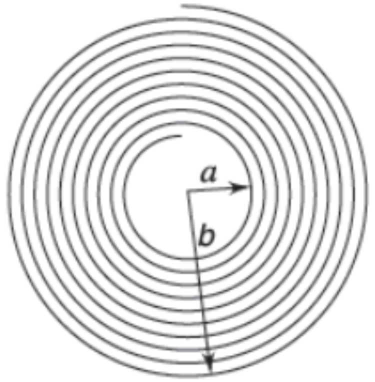
$\phi = 0 = \text{const.}$

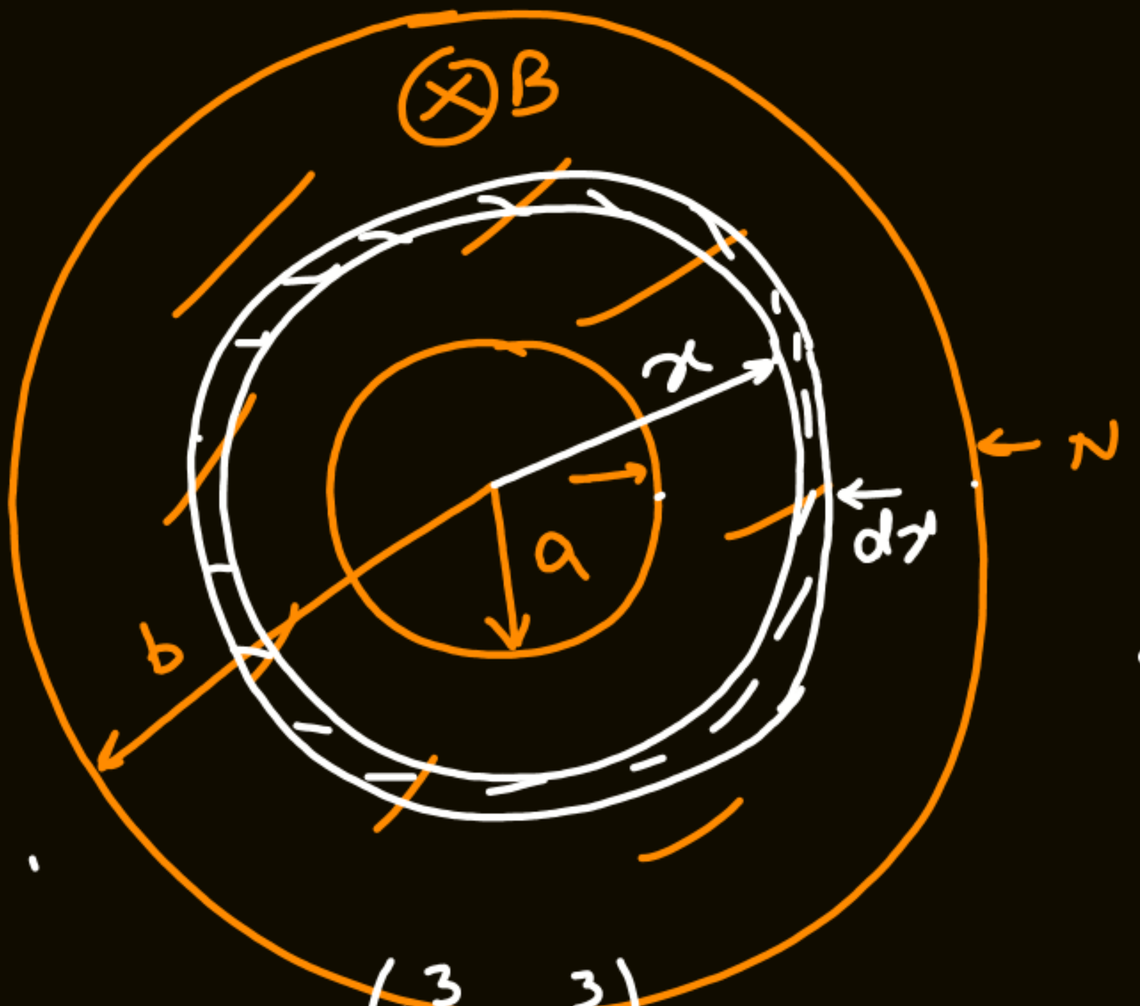
$\mathcal{E} = \frac{d\phi}{dt} = 0$

$\mathcal{E}_{net} = \mathcal{E}' - \mathcal{E} = 0$

$\mathcal{E}' = \mathcal{E} = \frac{B\omega L^2}{2}$

Q. 14: A flat coil, in the shape of a spiral, has a large number of turns N. The turns are wound tightly and the inner and outer radii of the coil are a and b respectively. A uniform external magnetic field (B) is applied perpendicular to the plane of the coil. Find the emf induced in the coil when the field is made to change at a rate $\frac{dB}{dt}$.





$b-a \rightarrow N$

$dx \rightarrow \frac{N}{b-a}dx$

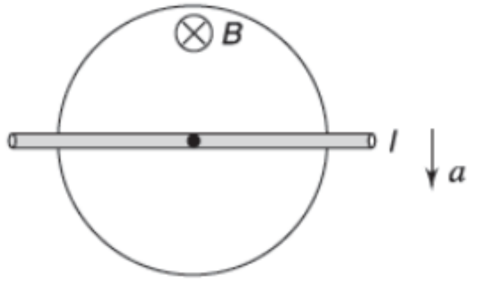
$$\phi = \int_a^b (\pi x^2 B)\frac{N}{b-a}dx$$

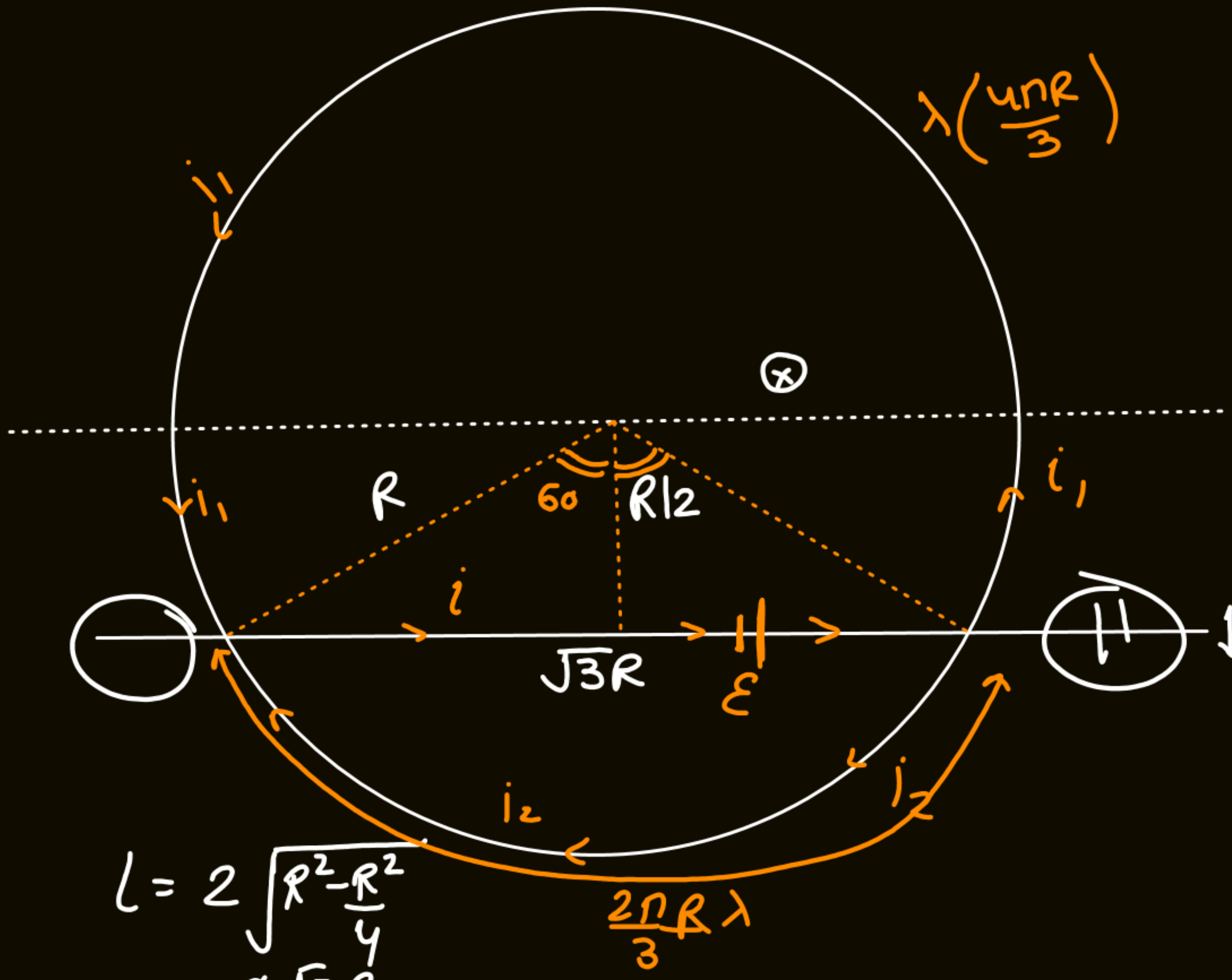
$$= \frac{\pi B N}{b-a}\left(\frac{x^3}{3}\right)_a^b$$

$$\phi = \frac{\pi B N (b^3-a^3)}{3(b-a)}$$

$$\boxed{\varepsilon = \left|\frac{d\phi}{dt}\right| = \frac{\pi N (b^3-a^3)}{3(b-a)}\frac{dB}{dt}}$$

Q. 17: A wire ring of radius R is fixed in a horizontal plane. The wire of the ring has a resistance of $\lambda \Omega m^{-1}$. There is a uniform vertical magnetic field B in entire space. A perfectly conducting rod (l) is kept along the diameter of the ring. The rod is made to move with a constant acceleration a in a direction perpendicular to its own length. Find the current through the rod at the instant it has travelled through a distance $x = \frac{R}{2}$.





$l = 2\sqrt{R^2 - \frac{R^2}{4}}$

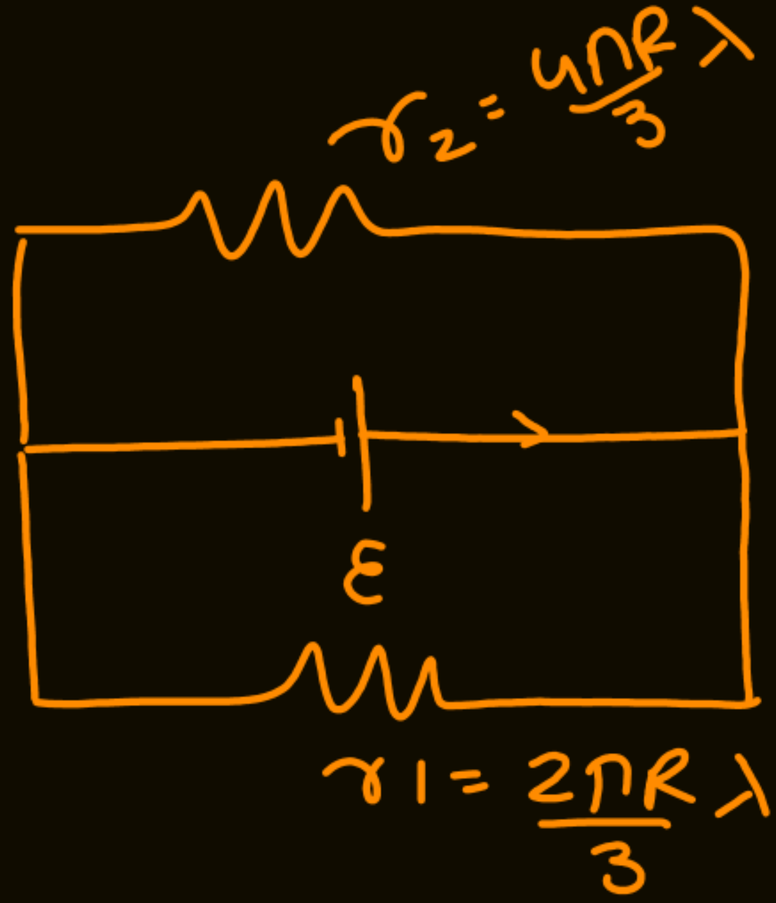
$= \cancel{2}\frac{\sqrt{3}R}{\cancel{2}}$

$v = \sqrt{2 \times a\, R/2} = \sqrt{aR}$

$\mathcal{E} = vBl$

$= \sqrt{aR}\cdot B\sqrt{3}R$

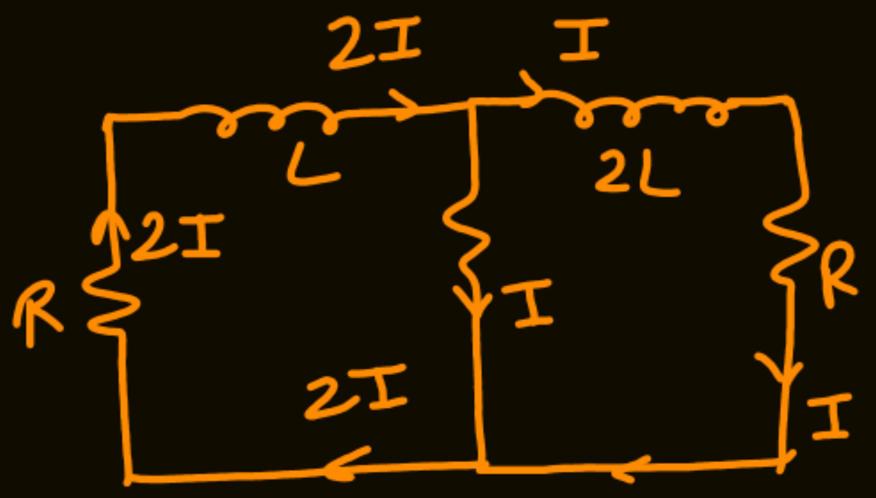
$$\boxed{\mathcal{E} = \sqrt{3a}\, B R^{3/2}}$$



$$r_{eq} = \frac{\frac{2\pi R\lambda}{3} \times \frac{\cancel{4}^{2}\cancel{\pi R\lambda}}{\cancel{3}}}{3\,\frac{\cancel{6}\cancel{\pi R\lambda}}{\cancel{3}}} = \frac{4\pi R\lambda}{9}$$

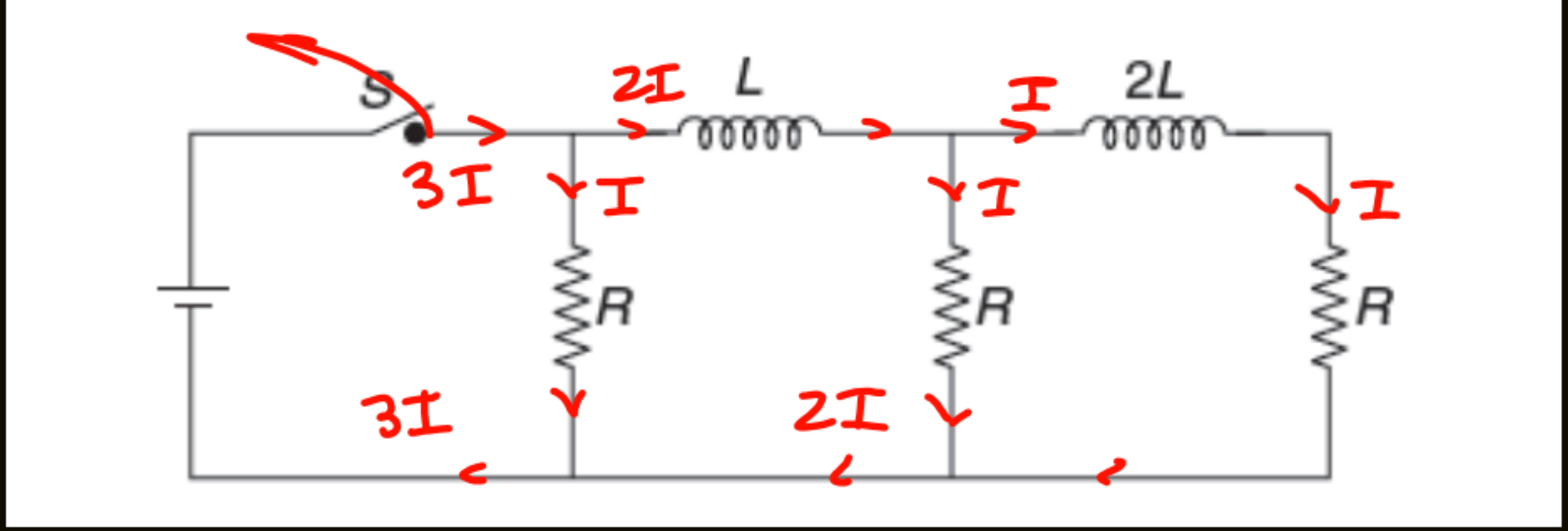
$$i = \frac{\varepsilon}{r_{eq}} = \frac{\sqrt{3a}\,B R^{3/2}\,9}{4\pi R\lambda}$$

$$= \frac{\sqrt{3aR}\;B\,9}{4\pi\lambda}$$

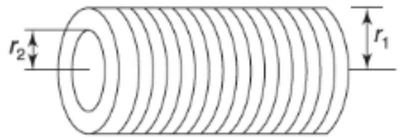


$$\frac{1}{2}L(2I)^2 + \frac{1}{2}\times 2LI^2$$
$$= \underline{3LI^2}$$

Q. 26: In the circuit shown in figure, the current through each resistor is I. Find the currents through the resistors immediately after the switch 'S' is opened. How much heat will get dissipated in the circuit after the switch is opened?



Q. 33: Two long co-axial solenoids have radii, and number of turns per unit length equal to r_1, r_2 and n_1, n_2 respectively where suffix 1 refers to the outer solenoid and 2 refers to the inner solenoid. Length of both is l. The current in the outer solenoid is made to grow as $I_1 = kt$ where t is time. Resistance of the wire used in inner solenoid is R. Write the current induced in the inner solenoid assuming that it is shorted.



$$B = \mu_0 n_1 i$$

$$\phi = (\mu_0 n_1 i)\pi r_2^2 n_2 l$$

$$\phi = \underbrace{(\mu_0 n_1 n_2 \pi r_2^2 l)}_{M} kt$$

$$\varepsilon = \frac{d\phi}{dt} = \mu_0 n_1 n_2 \pi r_2^2 l k$$

$$i = i_0\left\{1 - e^{-\frac{Rt}{L}}\right\}$$

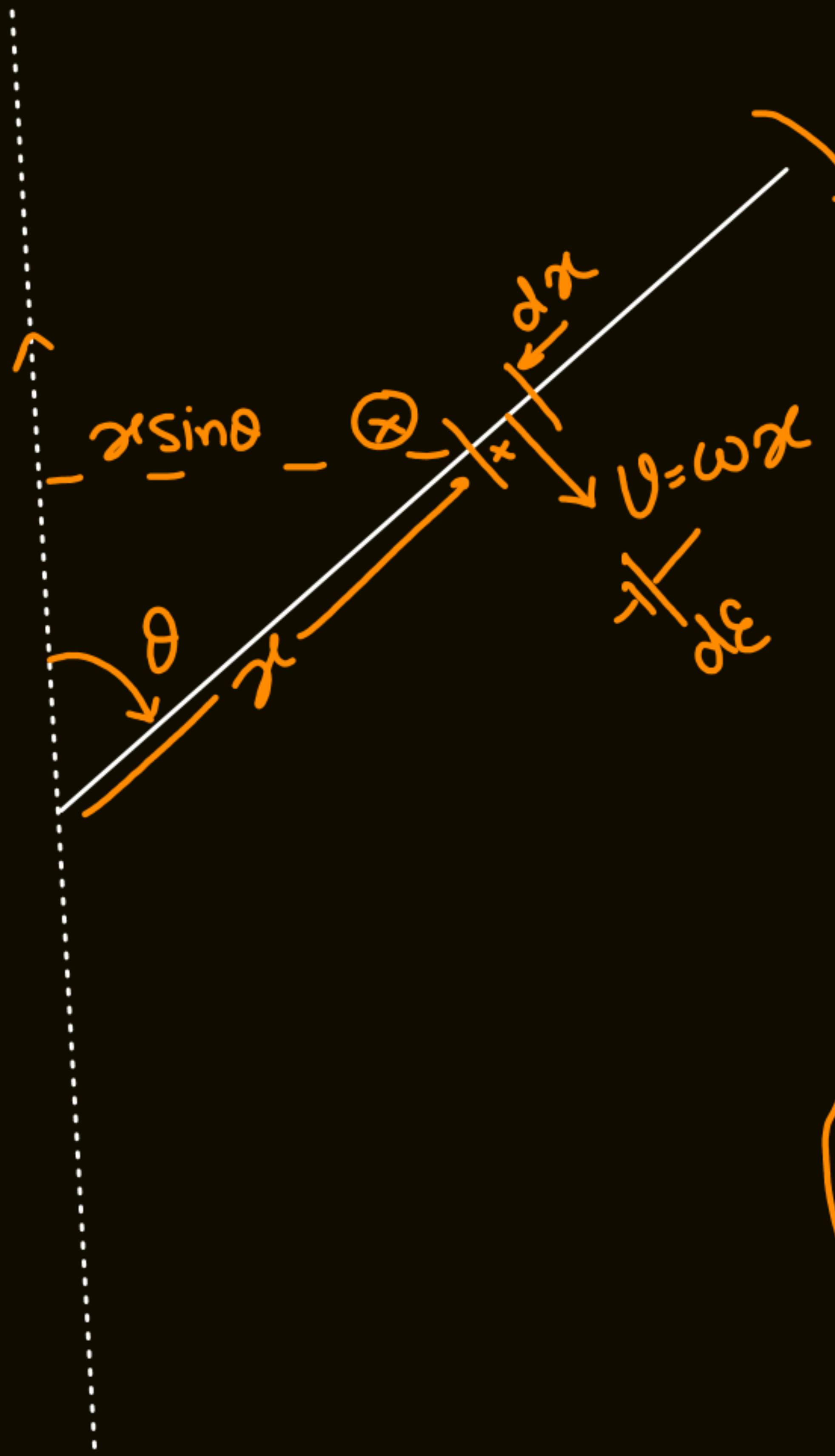
$$i = \frac{\varepsilon}{R}\left\{1 - e^{-\frac{Rt}{L}}\right\}$$

$$\boxed{L = \mu_0 n_2^2 \pi r_2^2 l}$$

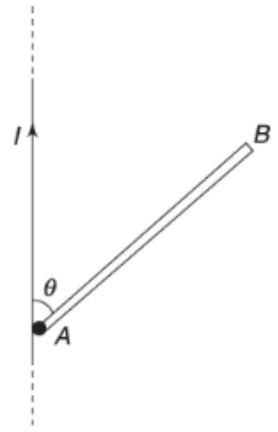
$$mg\frac{L}{2}(1-\cos\theta) = \frac{1}{2}\frac{mL^2}{3}\omega^2$$

$$\omega = \sqrt{\frac{3g}{L}(1-\cos\theta)}$$

I



equilibrium. Calculate the emf between the ends o
when it has rotated through an angle θ (see Figure



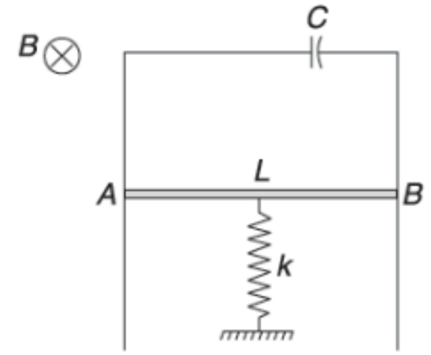
$$B = \frac{\mu_0 I}{2\pi x\sin\theta}$$

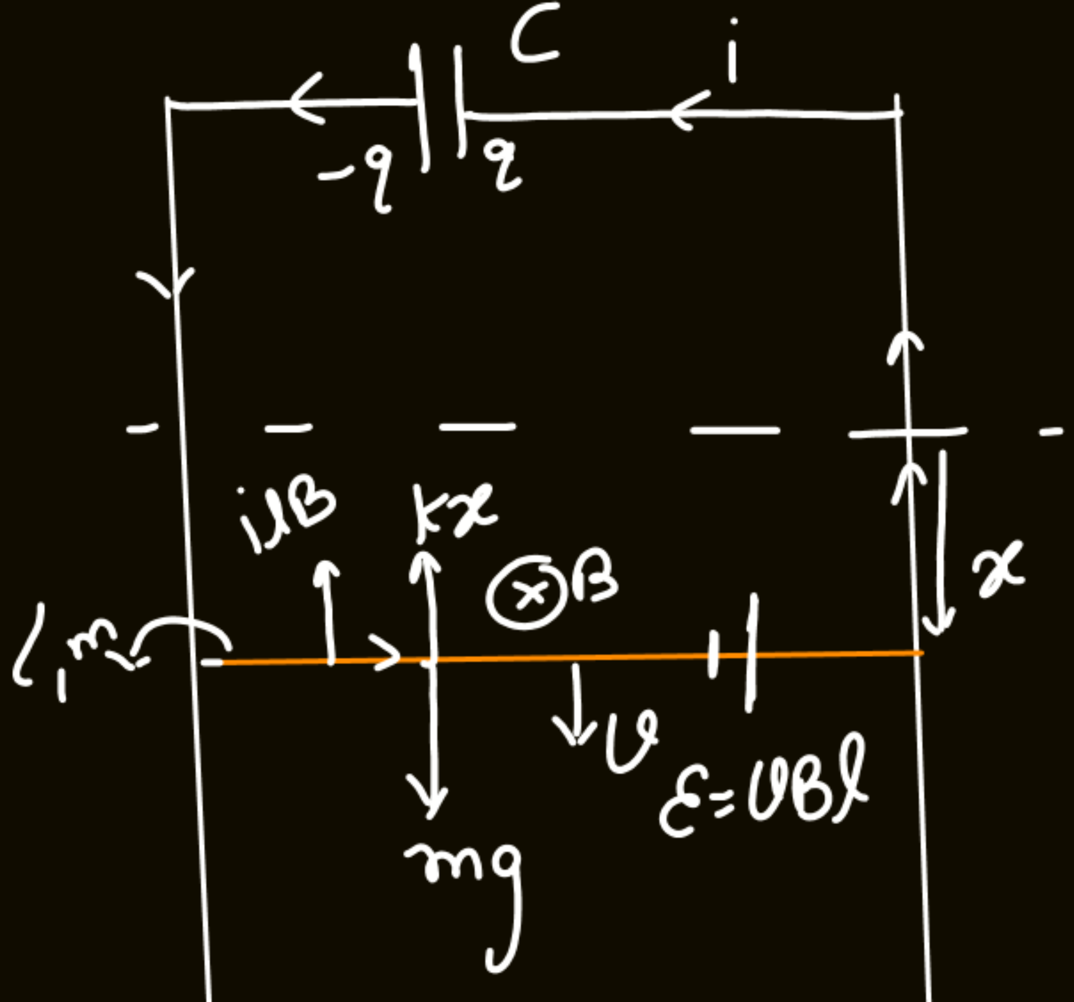
$$\mathcal{E} = \int d\varepsilon = \int \omega x \frac{\mu_0 I}{2\pi \cancel{x}\sin\theta}\,dx$$

$$= \frac{\omega\mu_0 I}{2\pi\sin\theta}\int_0^L dx$$

$$\boxed{\mathcal{E} = \frac{\omega\mu_0 I L}{2\pi\sin\theta}}$$

Q. 45: Two metal bars are fixed vertically and are connected on top by a capacitor of capacitance C. A sliding conductor AB can slide freely on the two bars. Length of conductor AB is L and its mass is m. It is connected to a vertical spring of force constant k. The conductor AB is released at time $t = 0$, from a position where the spring is relaxed. Taking initial position of the conductor as origin and downward direction as positive x axis, write the x co-ordinate of the conductor as a function of time. The entire space has a uniform horizontal magnetic field B. Neglect resistance and inductance of the circuit and assume that the bar AB always remains horizontal.





$$q = CvBl$$

$$i = \frac{dq}{dt} = CBl\frac{dv}{dt}$$

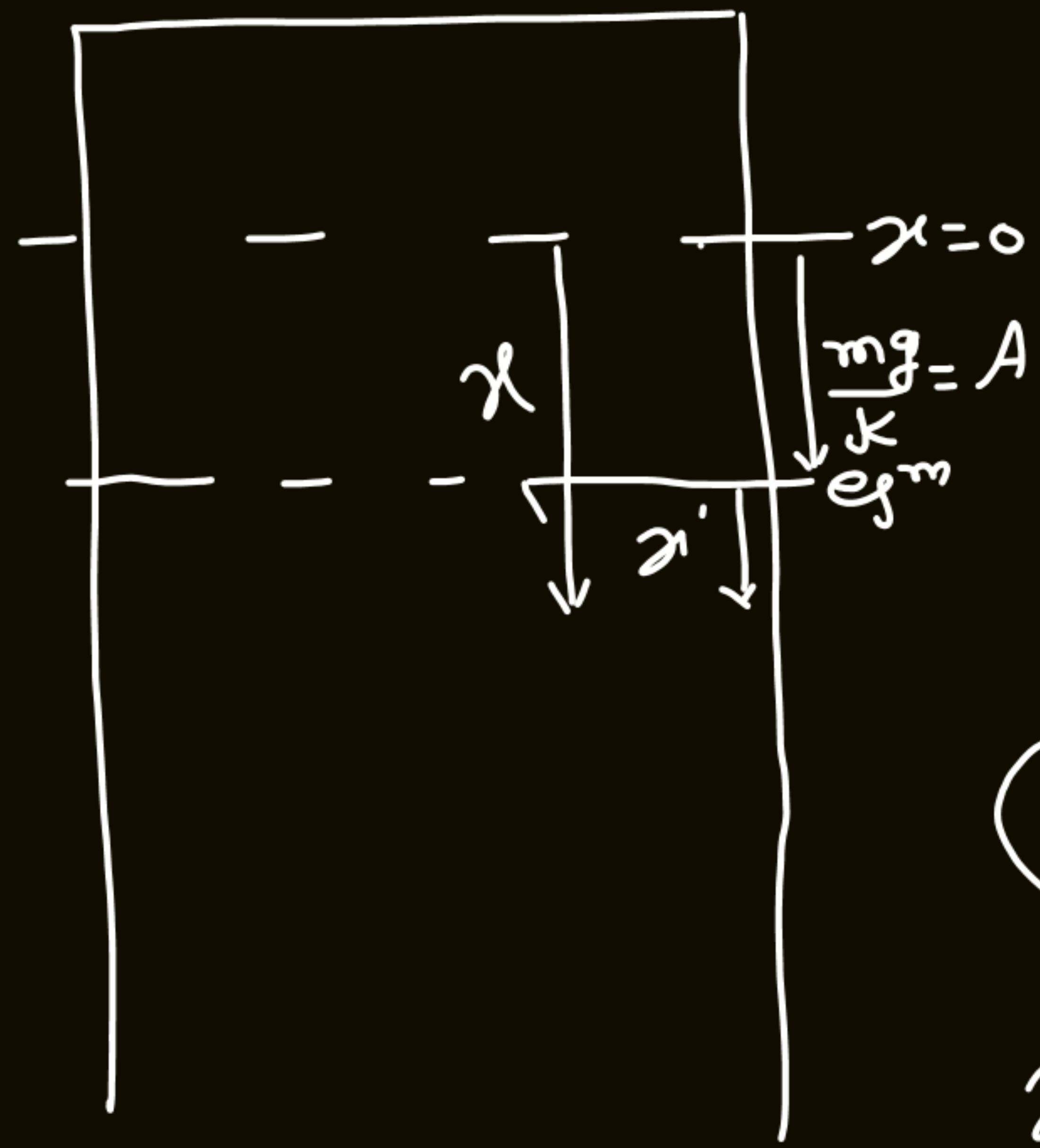
$$mg - kx - ilB = m\frac{dv}{dt}$$

$$mg - kx - CB^2l^2\frac{dv}{dt} = m\frac{dv}{dt}$$

$$\frac{dv}{dt}\left(m + CB^2l^2\right) = mg - kx \Rightarrow \text{SHM Proved}$$

$$\frac{dv}{dt} = \frac{mg}{m+CB^2l^2} - \frac{k}{m+CB^2l^2}x \rightarrow \omega^2$$

$$\frac{dv}{dt} = 0 \Rightarrow x = \frac{mg}{k}$$



$$\omega = \sqrt{\frac{k}{m + CB^2l^2}}$$

$$x' = -A\cos\omega t$$

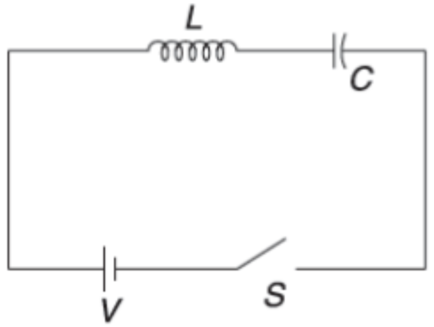
$$x' = -\frac{mg}{k}\cos\omega t$$

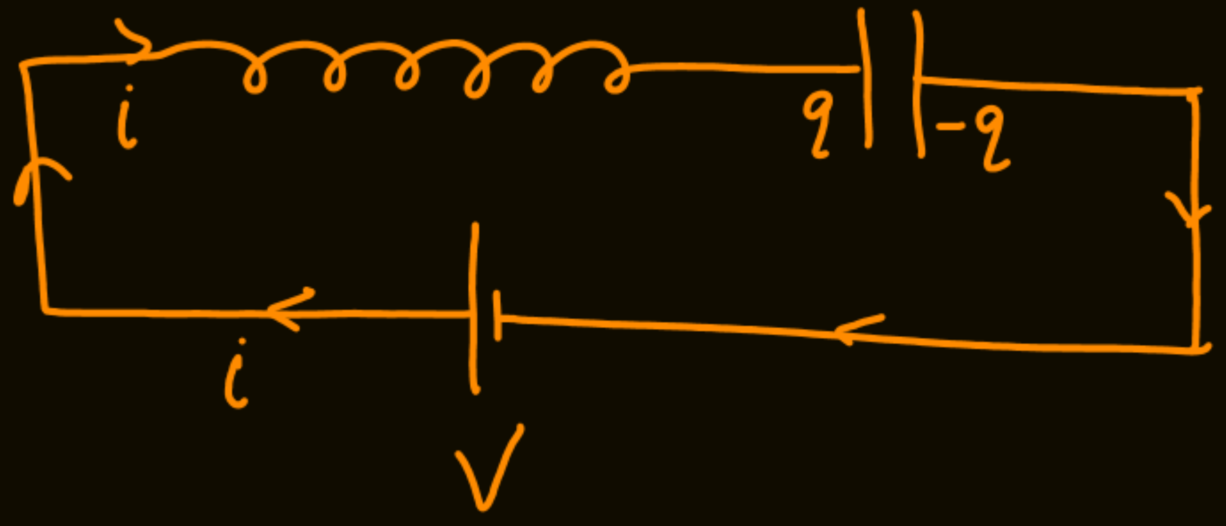
$$x = \frac{mg}{k} + x' = \frac{mg}{k}\left\{1 - \cos\omega t\right\}$$

Q.85: In the circuit shown in the figure, switch S is closed at time $t = 0$.

(a) Write current in the circuit and charge on capacitor as a function of time. Draw the graphical plot for the same.

(b) Find maximum charge on the capacitor. What is potential difference across the inductor when charge on the capacitor is maximum?





$$i = \frac{dq}{dt} \quad \text{—(1)}$$

$$V - L\frac{di}{dt} - \frac{q}{C} = 0$$

$$-L\frac{d^2 i}{dt^2} - \frac{i}{C} = 0$$

$$\boxed{\frac{d^2 i}{dt^2} + \frac{1}{LC} i = 0}$$

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

$$i = i_0 \sin(\omega t + \theta)$$

$t = 0 \quad i = 0 \Rightarrow \theta = 0.$

$$\boxed{i = i_0 \sin\omega t} = \frac{dq}{dt}$$

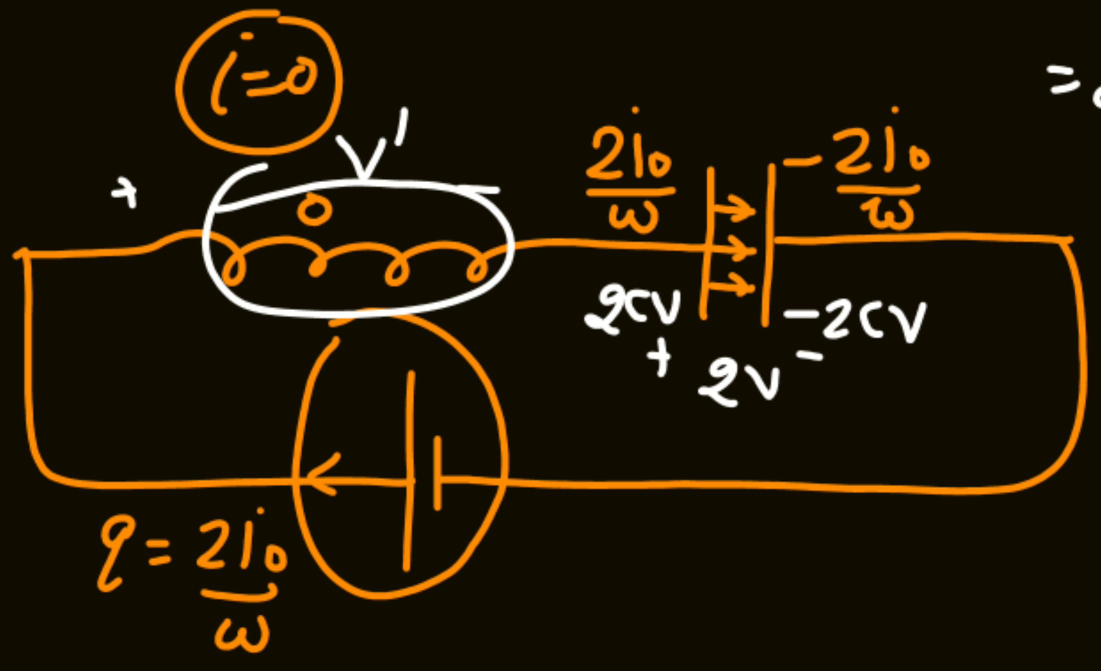
$$\int_0^q dq = i_0 \int_0^t \sin\omega t \, dt$$

$$\boxed{q = \frac{i_0}{\omega}\{1 - \cos\omega t\}}$$

$\cos\omega t = -1 \qquad q_{max} = \frac{2i_0}{\omega} = 2CV$

$$V - V' - 2V = 0$$

$$\boxed{V' = V}$$



$$W_b = \Delta U$$

$$V\cancel{\left(\frac{2i_0}{\omega}\right)} = \frac{\left(\frac{2i_0}{\omega}\right)^{\cancel{2}}}{2C}$$

$$2CV = \frac{\cancel{2}i_0}{\omega}$$

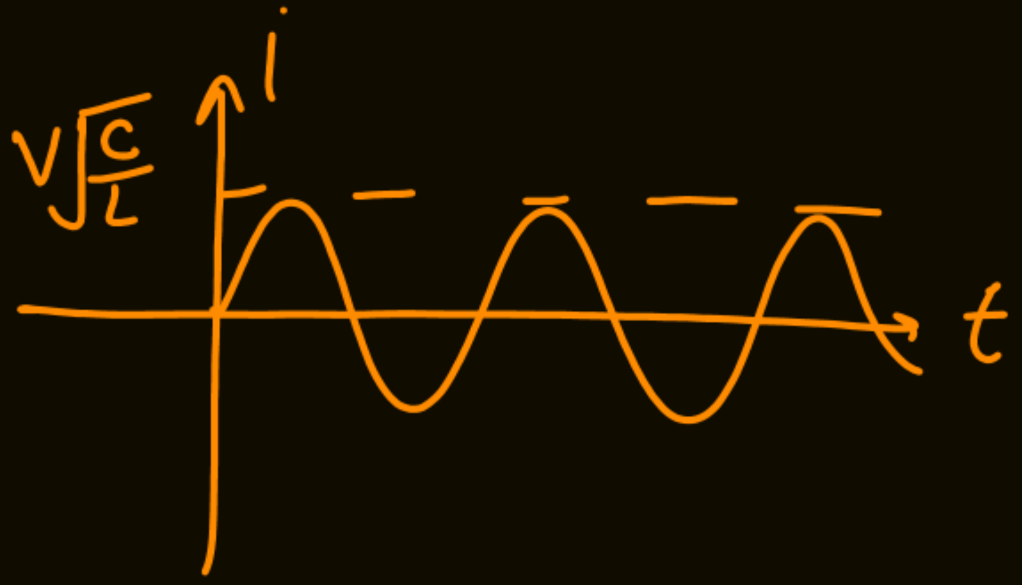
$$\boxed{i_0 = CV\omega}$$

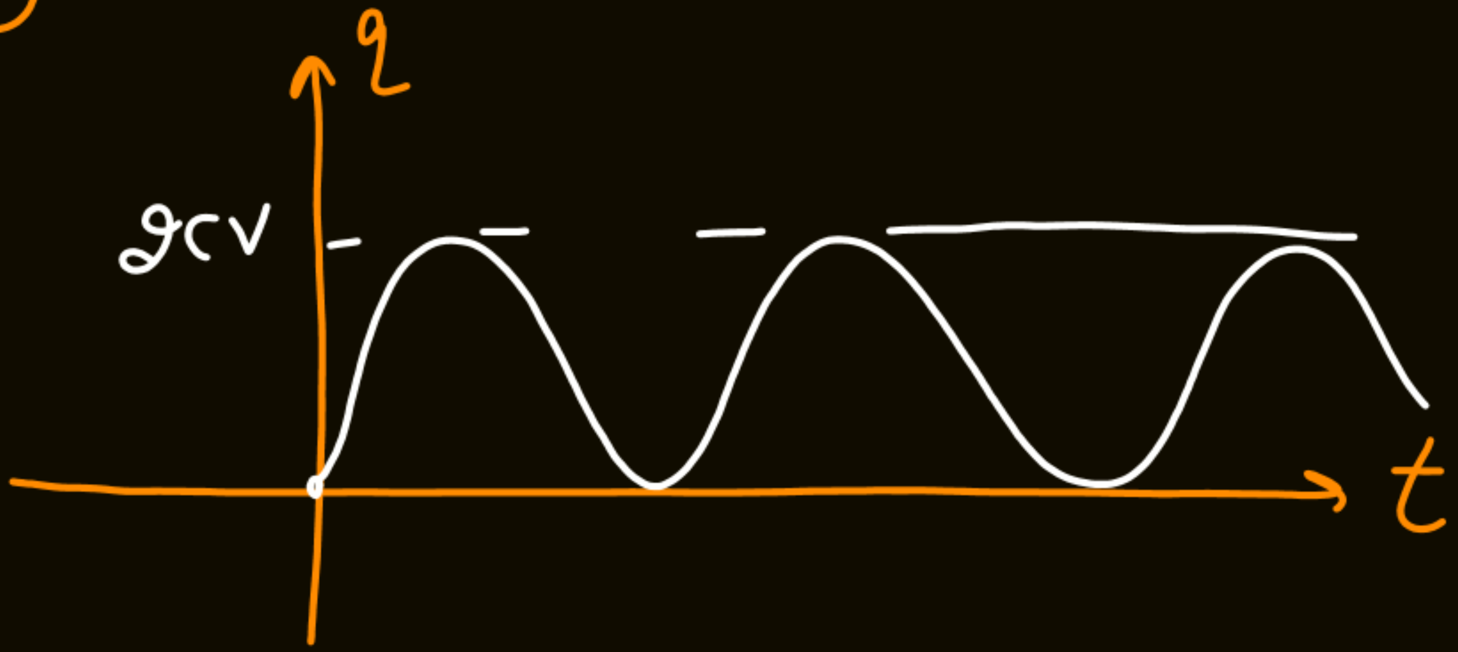
$$i = CV\omega \sin\omega t$$

$$= \frac{CV}{\sqrt{LC}} \sin \frac{1}{\sqrt{LC}} t$$

$$\boxed{i = V\sqrt{\frac{C}{L}} \sin \frac{t}{\sqrt{LC}}}$$

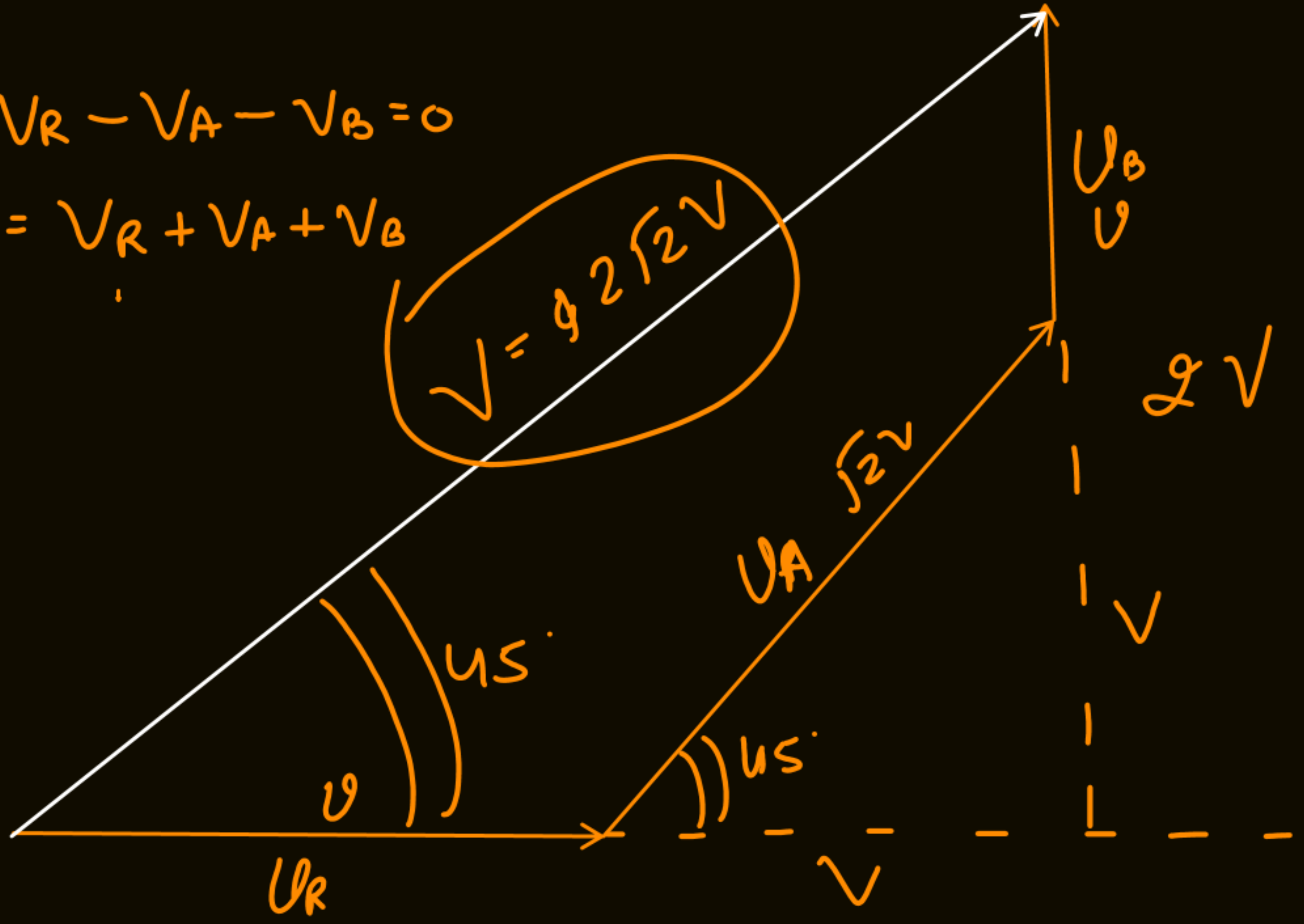
$$q = \frac{i_0}{\omega}(1 - \cos\omega t) = CV\left\{1 - \cos\frac{t}{\sqrt{LC}}\right\}$$





$$V - V_R - V_A - V_B = 0$$

$$V = V_R + V_A + V_B$$

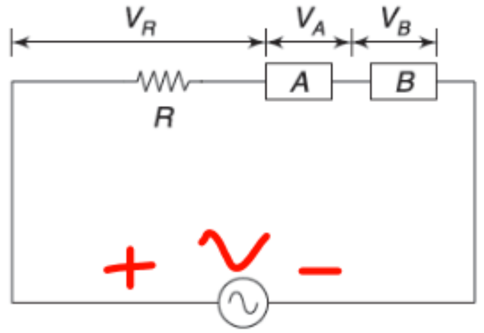


Q. 8: In the circuit shown in the Figure, the voltage across resistance R, box A and box B are represented as

$$v_R = V\sin(\omega t),\ v_A = \sqrt{2}\,V\sin\left(\omega t + \frac{\pi}{4}\right) \text{ and}$$

$$v_B = V\sin\left(\omega t + \frac{\pi}{2}\right)$$

Find the phase difference between current and the applied voltage.



$$V = 2\sqrt{2}V\sin\left(\omega t + \frac{\pi}{4}\right)$$

$2V$

$$\underline{\theta = 45^\circ}$$

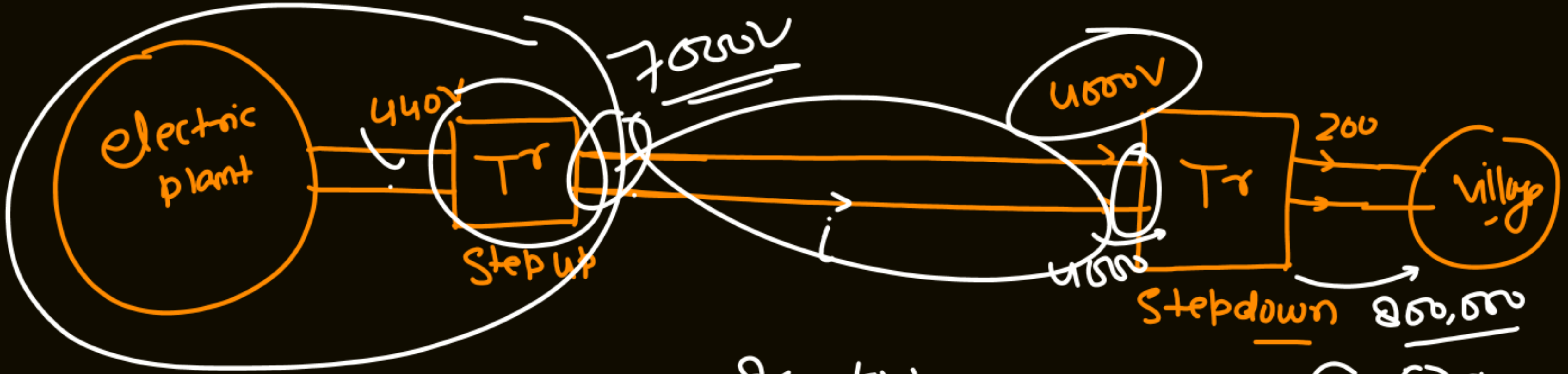
Q. 23: A village with a demand of 800 kW electric power at 220 V is located 30 km from an electric plant generating power at 440 V. The resistance of the two wire line carrying power is 0.25 Ω/km. The village gets power from the line through a 4000 V – 220 V step down transformer at a sub-station in the village. Assume negligible power loss in the transformers.

(a) Estimate the power loss in form of heat in the transmission line.

(b) How much power must the plant supply?

(c) What is the input and output voltage of step-up transformer at the plant ?

(d) What difference will it make if the village receives power through a 40,000 V – 220 V step down transformer



$60 \times 0.25 = 15\,\Omega$

$P = i^2 R$

$= 40000 \times 15$

$= 600{,}000$

$= 600\text{kW}$ (a)

800kW
600kW
1400kW

$iR = 200 \times 15$

$= 3000V$

$P = Vi$

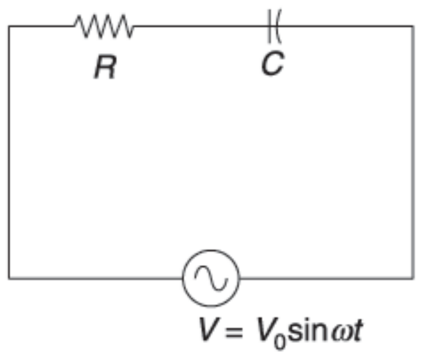
$i = \dfrac{800{,}000}{4000} = 200\text{ Amp.}$

Q. 24: A resistance R and a capacitor having capacitance C are connected to an alternating source having emf $v = V_0 \sin(\omega t)$. It is given that $\omega = \frac{1}{\sqrt{3}RC}$

(a) Plot the variation of power supplied by the source as a function of time. Mark the maximum and minimum values of power in the graph.

(b) How does the plot change if capacitor is removed and only R remains connected to the source?

(c) Plot the graph when only C remains connected to source and R is removed.



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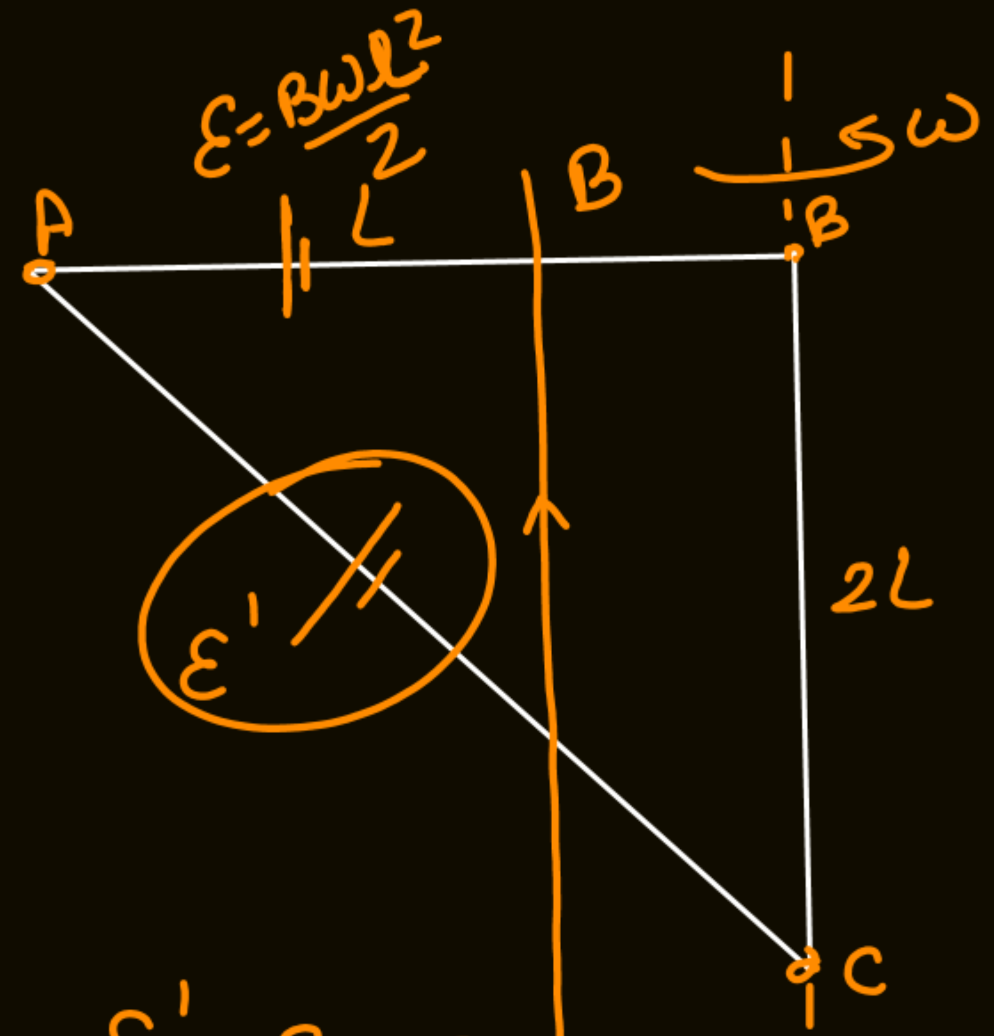

$\theta = 90^\circ$

$\phi = 0 = \text{const.}$

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$\mathcal{E}_{net} = \mathcal{E}' - \mathcal{E} = 0$

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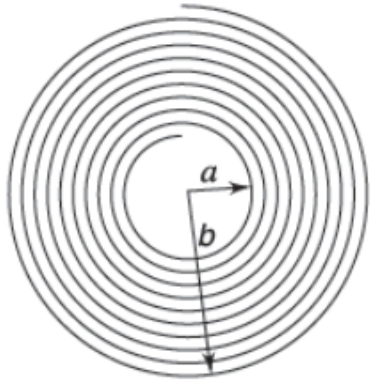

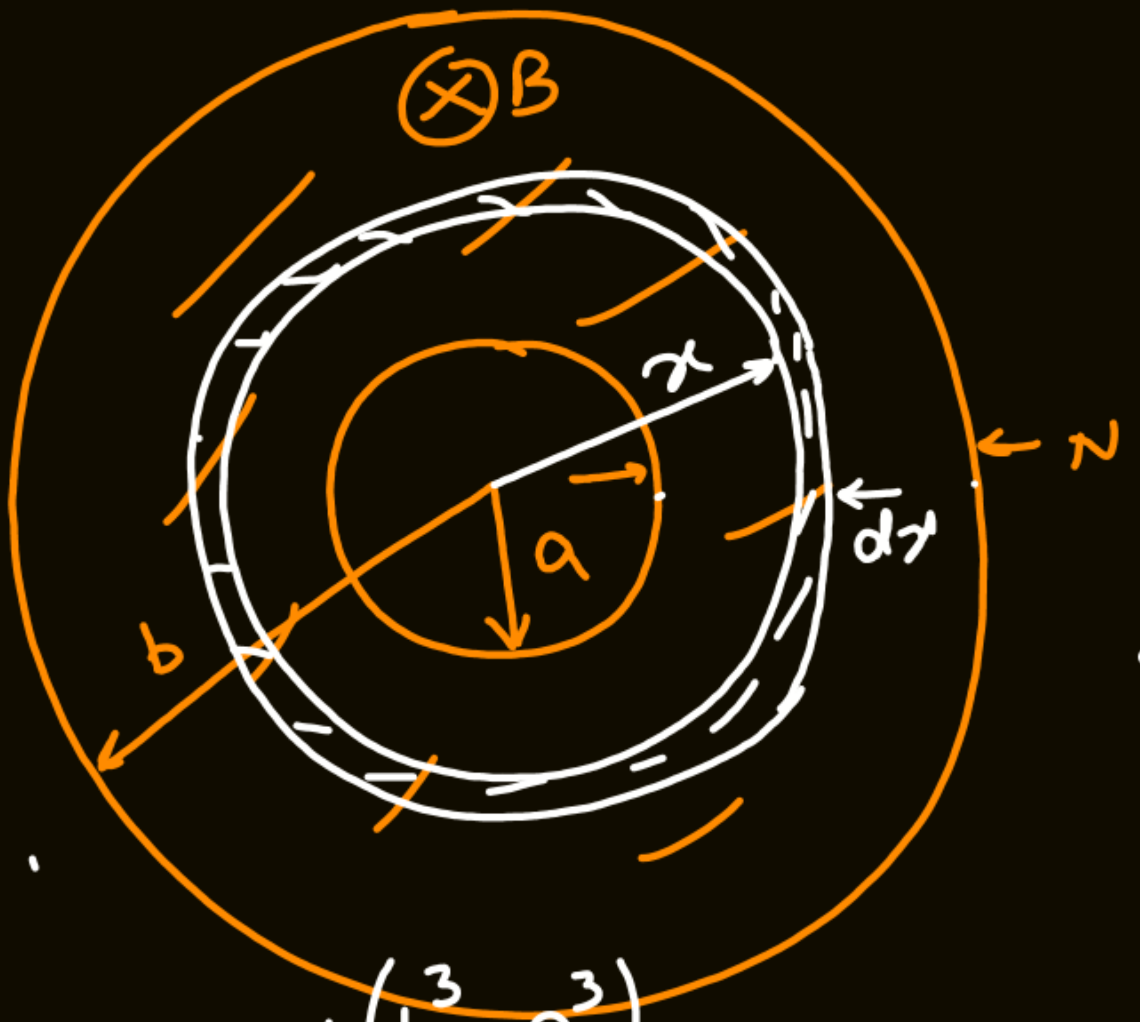

$b-a \rightarrow N$

$dx \rightarrow \frac{N}{b-a}dx$

$$\phi = \int_a^b (\pi x^2 B)\frac{N}{b-a}dx$$

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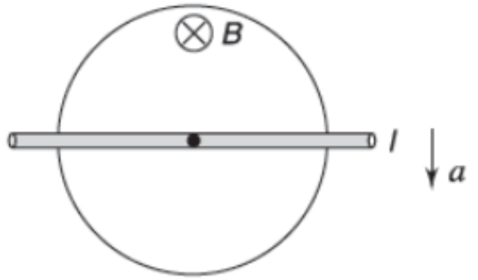
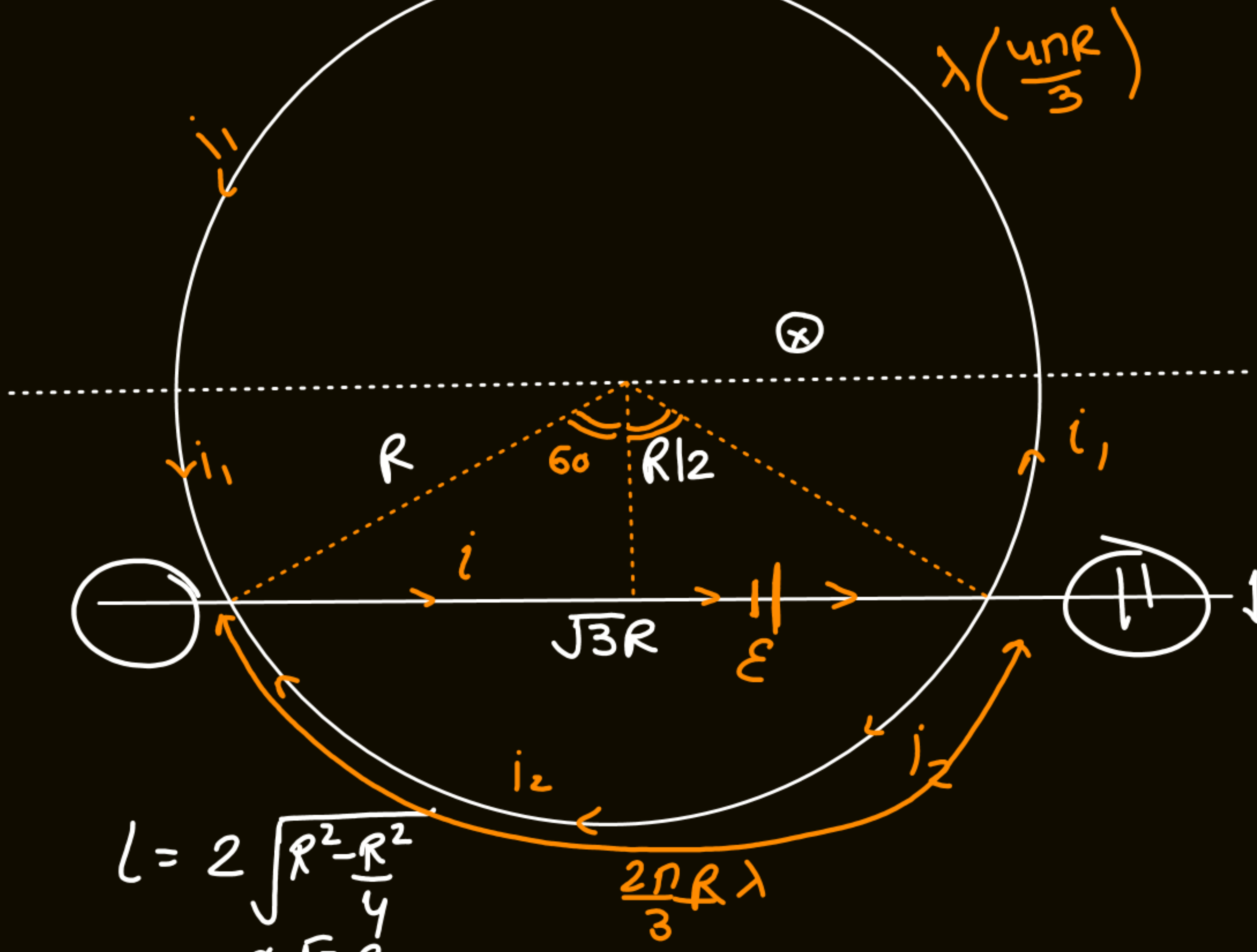


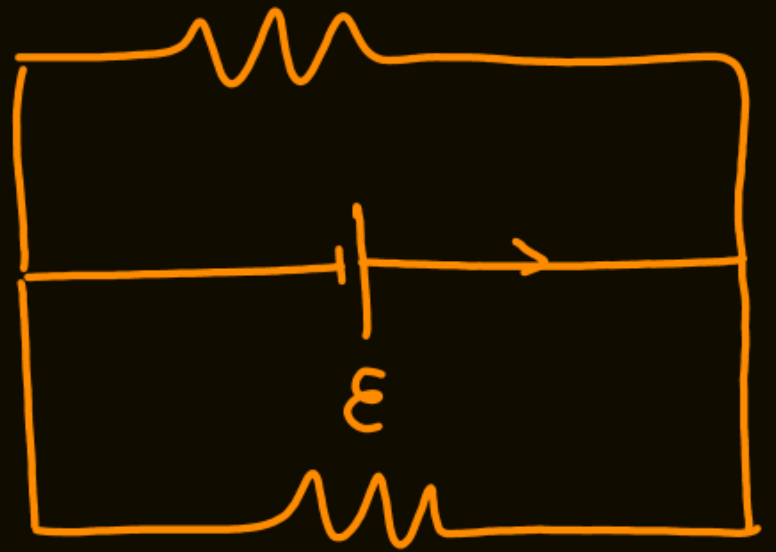


$$l = 2\sqrt{R^2 - \frac{R^2}{4}} = \cancel{2}\frac{\sqrt{3}R}{\cancel{2}}$$

$$v = \sqrt{2 \times a \, R/2} = \sqrt{aR}$$

$$\mathcal{E} = vBl = \sqrt{aR} \cdot B\sqrt{3}R$$

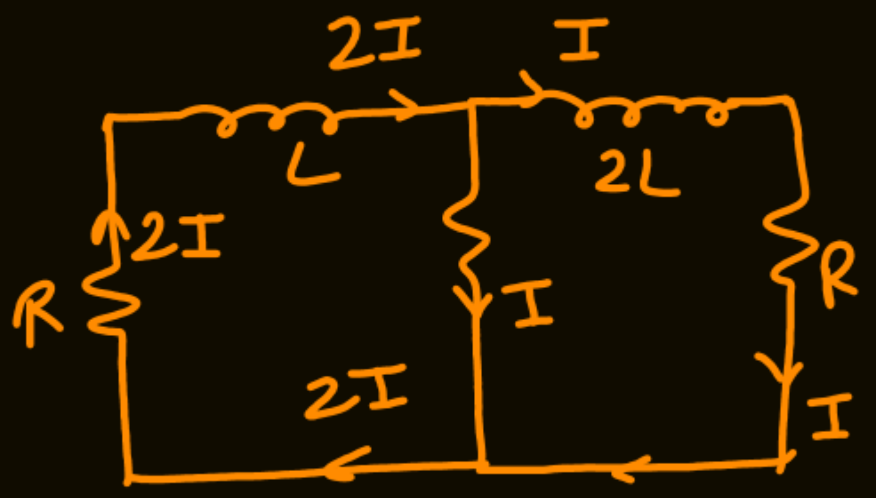
$$\boxed{\mathcal{E} = \sqrt{3a}\, B R^{3/2}}$$



$$r_{eq} = \frac{\frac{2\pi R\lambda}{3} \times \frac{\overset{2}{\cancel{4\pi R\lambda}}}{\cancel{3}}}{3\,\frac{\cancel{6\pi R\lambda}}{\cancel{3}}} = \frac{4\pi R\lambda}{9}$$

$$i = \frac{\varepsilon}{r_{eq}} = \frac{\sqrt{3a}\,B R^{3/2} g}{4\pi R\lambda}$$

$$= \frac{\sqrt{3aR}\;B\,g}{4\pi\lambda}$$



$$\frac{1}{2}L(2I)^2 + \frac{1}{2}\times 2LI^2$$
$$= 3LI^2$$

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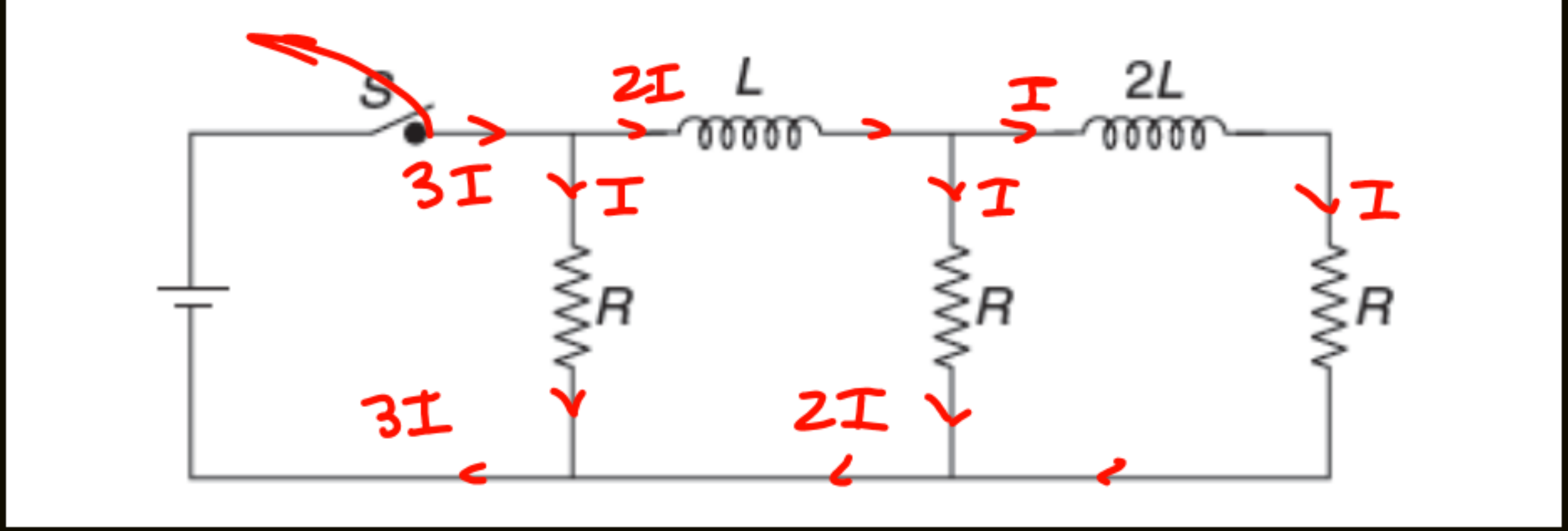

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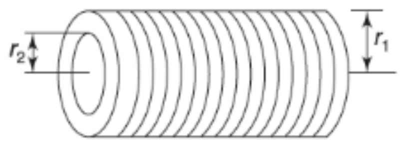


$$B = \mu_0 n_1 i$$

$$\phi = (\mu_0 n_1 i)\pi r_2^2 n_2 l$$

$$\phi = \underbrace{(\mu_0 n_1 n_2 \pi r_2^2 l)}_{M} kt$$

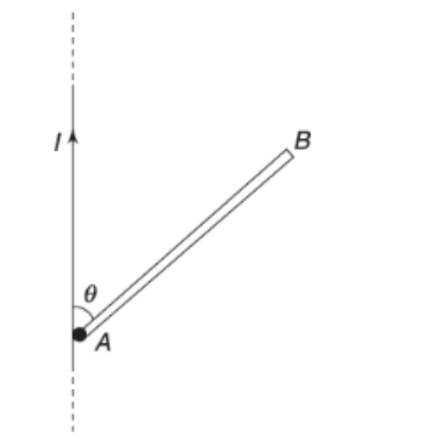
$$\varepsilon = \frac{d\phi}{dt} = \mu_0 n_1 n_2 \pi r_2^2 l k$$

$$i = i_0\left\{1 - e^{-\frac{Rt}{L}}\right\}$$

$$i = \frac{\varepsilon}{R}\left\{1 - e^{-\frac{Rt}{L}}\right\}$$

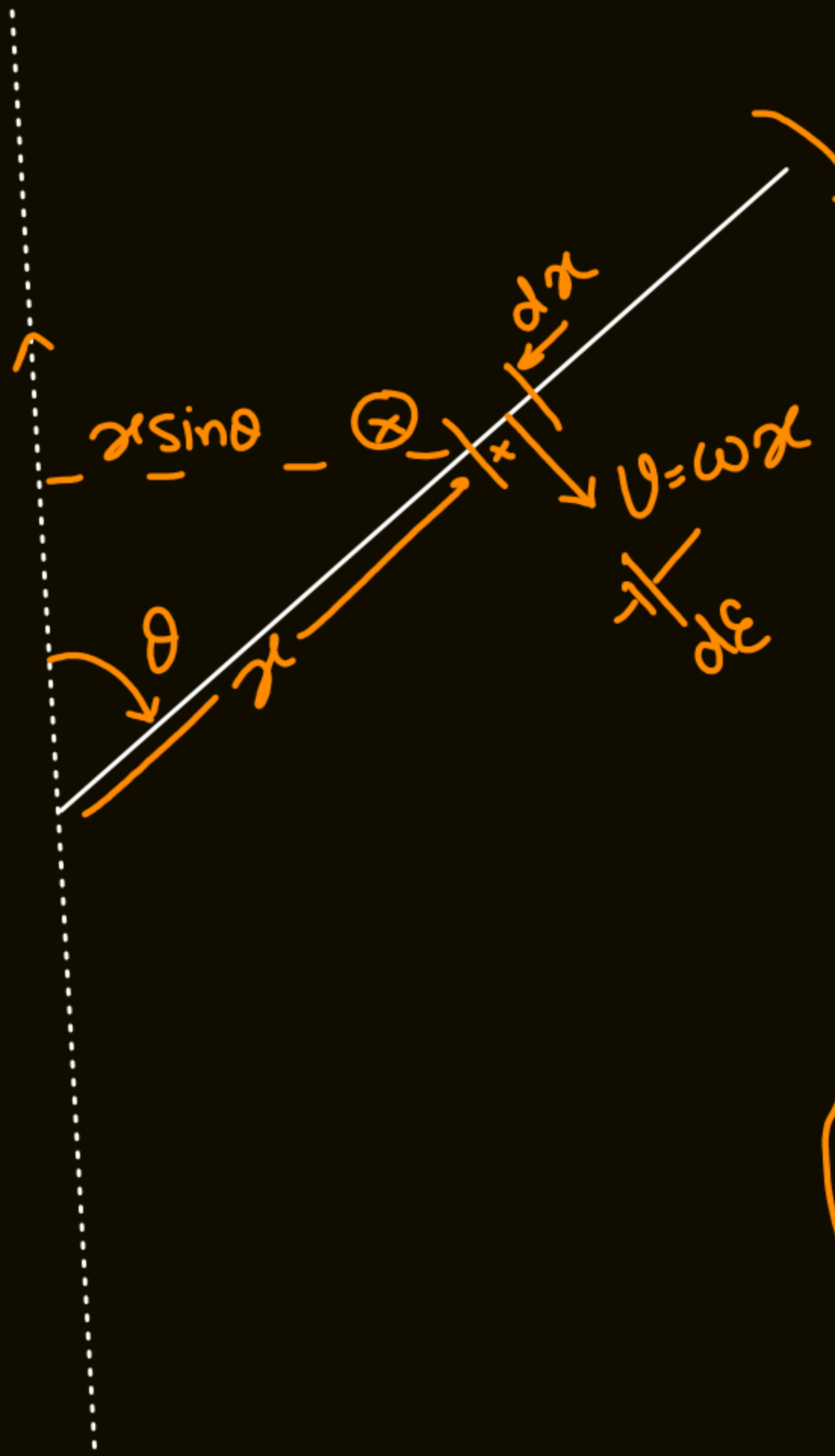
$$\boxed{L = \mu_0 n_2^2 \pi r_2^2 l}$$

equilibrium. Calculate the emf between the ends o
when it has rotated through an angle θ (see Figure



$$mg\frac{L}{2}(1-\cos\theta) = \frac{1}{2}\frac{mL^2}{3}\omega^2$$

$$\omega = \sqrt{\frac{3g}{L}(1-\cos\theta)}$$



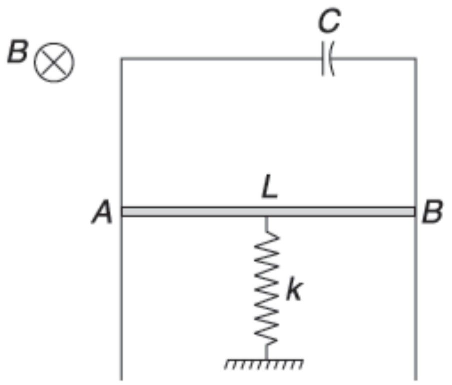
$$B = \frac{\mu_0 I}{2\pi x\sin\theta}$$

$$\mathcal{E} = \int d\mathcal{E} = \int \omega \cancel{x} \frac{\mu_0 I}{2\pi \cancel{x}\sin\theta}\,dx$$

$$= \frac{\omega\mu_0 I}{2\pi\sin\theta}\int_0^L dx$$

$$\boxed{\mathcal{E} = \frac{\omega\mu_0 I L}{2\pi\sin\theta}}$$

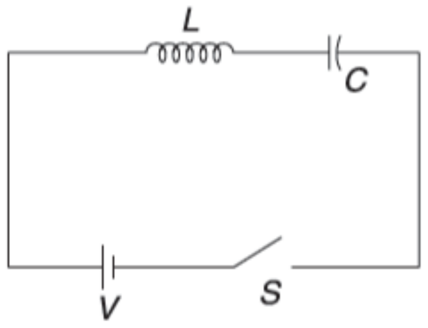
Q. 45: Two metal bars are fixed vertically and are connected on top by a capacitor of capacitance C. A sliding conductor AB can slide freely on the two bars. Length of conductor AB is L and its mass is m. It is connected to a vertical spring of force constant k. The conductor AB is released at time $t = 0$, from a position where the spring is relaxed. Taking initial position of the conductor as origin and downward direction as positive x axis, write the x co-ordinate of the conductor as a function of time. The entire space has a uniform horizontal magnetic field B. Neglect resistance and inductance of the circuit and assume that the bar AB always remains horizontal.



Q.85: In the circuit shown in the figure, switch S is closed at time $t = 0$.

(a) Write current in the circuit and charge on capacitor as a function of time. Draw the graphical plot for the same.

(b) Find maximum charge on the capacitor. What is potential difference across the inductor when charge on the capacitor is maximum?

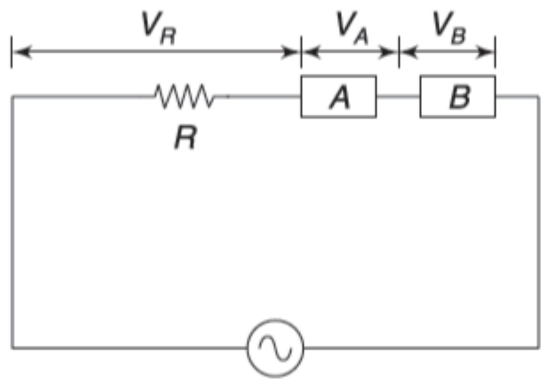


Q. 8: In the circuit shown in the Figure, the voltage across resistance R, box A and box B are represented as

$$v_R = V\sin(\omega t),\ v_A = \sqrt{2}\,V\sin\left(\omega t + \frac{\pi}{4}\right) \text{ and}$$

$$v_B = V\sin\left(\omega t + \frac{\pi}{2}\right)$$

Find the phase difference between current and the applied voltage.



Q. 23: A village with a demand of 800 kW electric power at 220 V is located 30 km from an electric plant generating power at 440 V. The resistance of the two wire line carrying power is 0.25 Ω/km. The village gets power from the line through a 4000 V – 220 V step down transformer at a sub-station in the village. Assume negligible power loss in the transformers.

(a) Estimate the power loss in form of heat in the transmission line.

(b) How much power must the plant supply?

(c) What is the input and output voltage of step-up transformer at the plant ?

(d) What difference will it make if the village receives power through a 40,000 V – 220 V step down transformer

Q. 24: A resistance R and a capacitor having capacitance C are connected to an alternating source having emf $v = V_0 \sin(\omega t)$. It is given that $\omega = \dfrac{1}{\sqrt{3}RC}$

(a) Plot the variation of power supplied by the source as a function of time. Mark the maximum and minimum values of power in the graph.

(b) How does the plot change if capacitor is removed and only R remains connected to the source?

(c) Plot the graph when only C remains connected to source and R is removed.