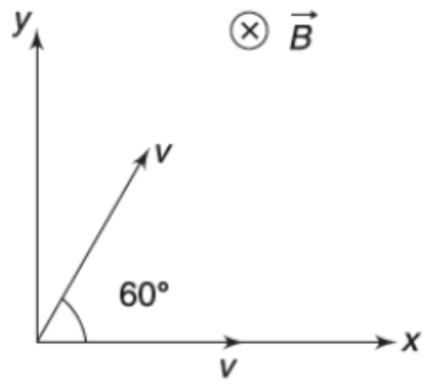
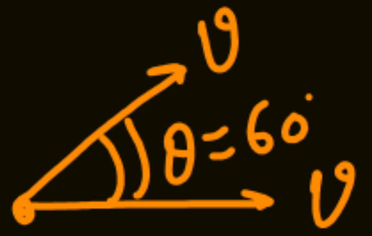
Q. 9: Two identical charged particles are projected simultaneously from origin in xy plane. Each particle has charge q and mass m and has been projected with velocity v as shown in the figure. There exists a uniform magnetic field B in negative z direction.

(i) Find $\vec{v}_1\cdot\vec{v}_2$ at time t where $\vec{v}_1$ and $\vec{v}_2$ are velocities of the particles at time t.

(ii) Find $\vec{r}_1\cdot\vec{r}_2$ at time $t=\frac{\pi m}{qB}$ where $\vec{r}_1$ and $\vec{r}_2$ are the position vectors of the two particles.



(1)



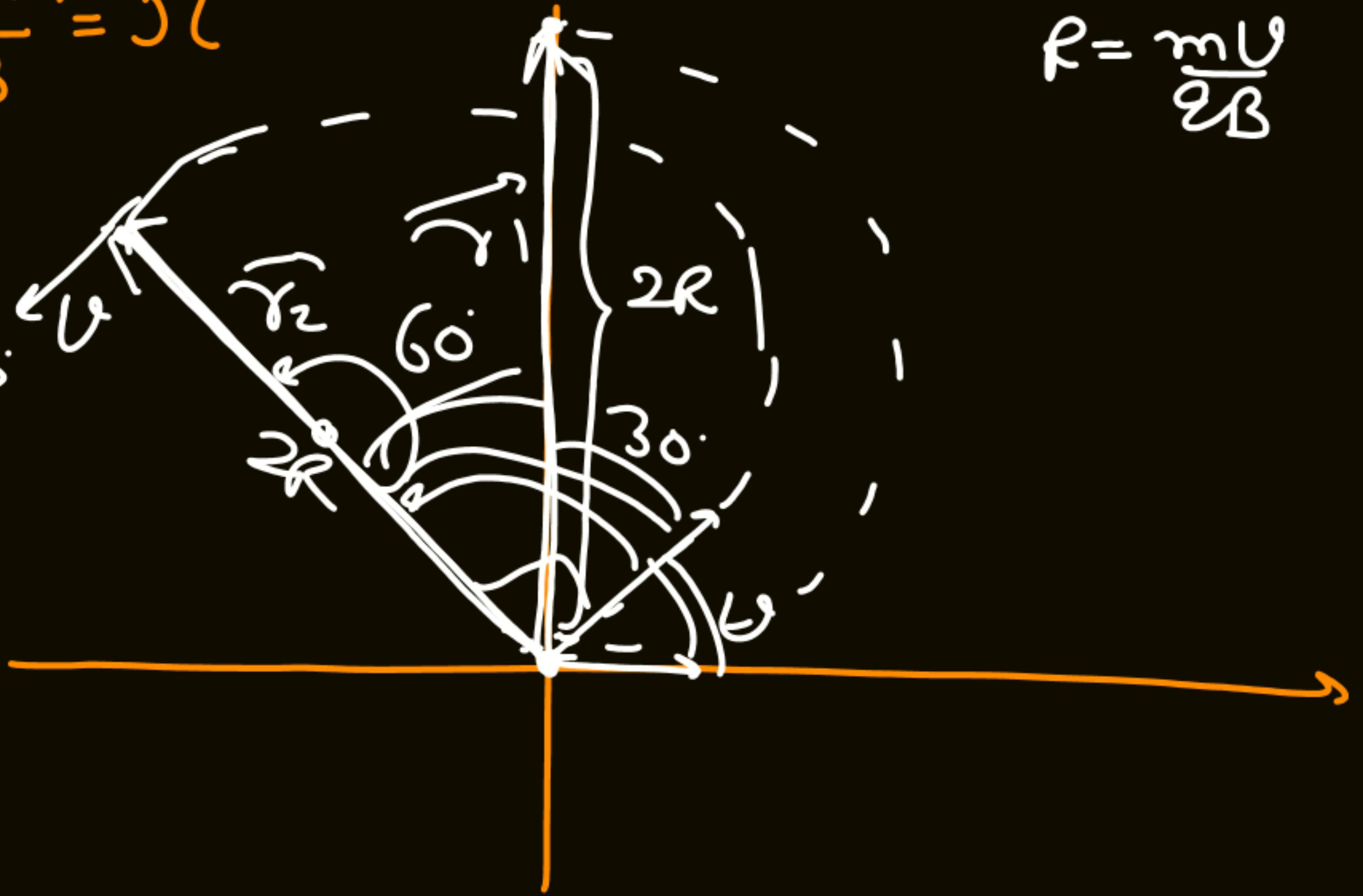
$$\vec{v}_1\cdot\vec{v}_2 = v_1 v_2\cos\theta = v\cdot v\cos 60^\circ = \frac{v^2}{2}$$

(2)

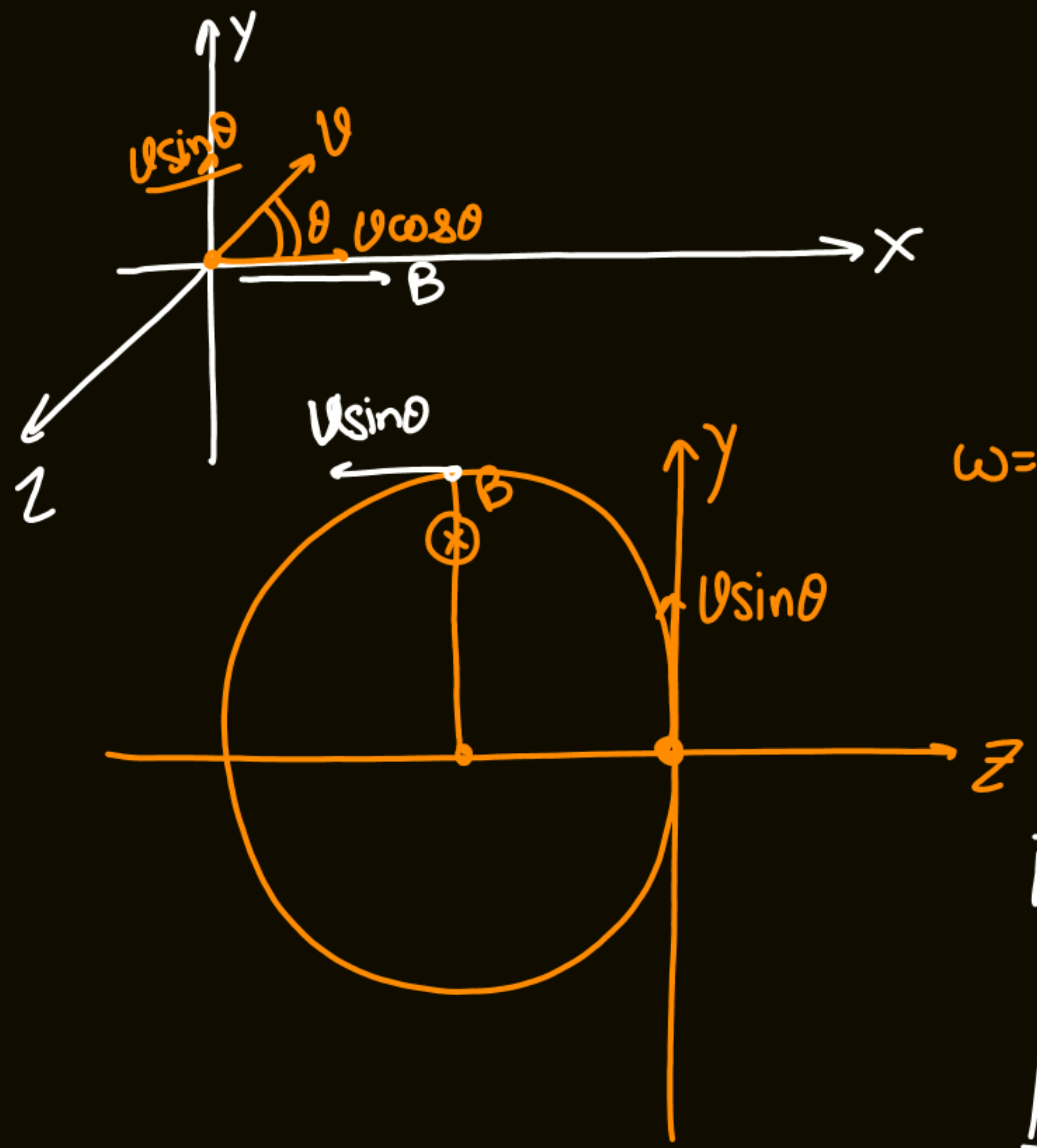
$$\theta=\omega t = \frac{qB}{m}\cdot\frac{\pi m}{qB} = \pi$$

$$R=\frac{mv}{qB}$$

$$\vec{r}_1\cdot\vec{r}_2 = r_1 r_2\cos\theta = (2R)(2R)\cos 60^\circ = 2R^2 = 2\left(\frac{mv}{qB}\right)^2 = \frac{2m^2v^2}{q^2B^2}$$



Q. 11: A particle having mass m and charge q is projected with a velocity v making an angle θ with the direction of a uniform magnetic field B. Calculate the magnitude of change in velocity of the particle after time $t = \frac{\pi m}{2qB}$.



$$\vec{v}_1 = v\cos\theta\,\hat{i} + v\sin\theta\,\hat{j}$$

$$f_x = 0 \quad a_x = 0 \quad v_x = u\cos\theta$$

$$\omega = \frac{qB}{m} \qquad \theta = \omega t = \frac{qB}{m}\cdot\frac{\pi m}{qB} = \frac{\pi}{2}$$

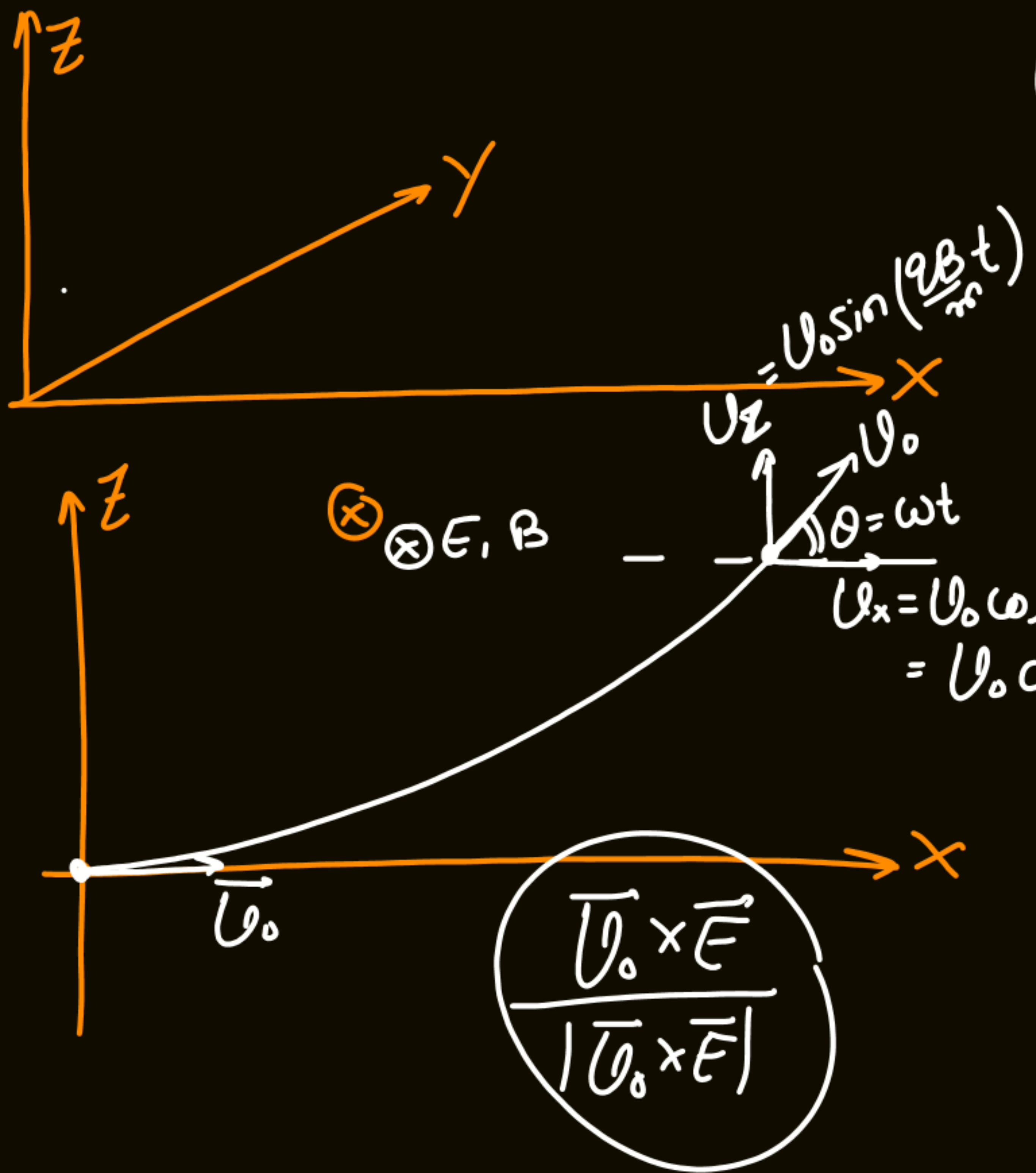
$$\vec{v}_f = u\cos\theta\,\hat{i} - u\sin\theta\,\hat{k}$$

$$\vec{\Delta v} = \vec{v}_f - \vec{v}_1 = -u\sin\theta\,\hat{k} - u\sin\theta\,\hat{j}$$

$$\boxed{|\vec{\Delta v}| = \sqrt{2}\,v\sin\theta}$$

Q. 19: A particle of mass m and charge q is moving in a region where uniform electric field $\vec{E}$ and and uniform magnetic field $\vec{B}$ are present. It is given that $\frac{\vec{E}}{|\vec{E}|} = \frac{\vec{B}}{|\vec{B}|}$. At time $t = 0$, velocity of the particle is $\vec{v}_0$ and $\vec{v}_0 \cdot \vec{E} = 0$. Write the velocity of the particle at time t.

$$v_y = 0 + \frac{qE}{m} t$$



$$\vec{v} = v_0 \cos\left(\frac{qB}{m} t\right) \hat{j} + v_0 \sin\left(\frac{qB}{m} t\right) \hat{k} + \frac{qE}{m} t \, \hat{j}$$

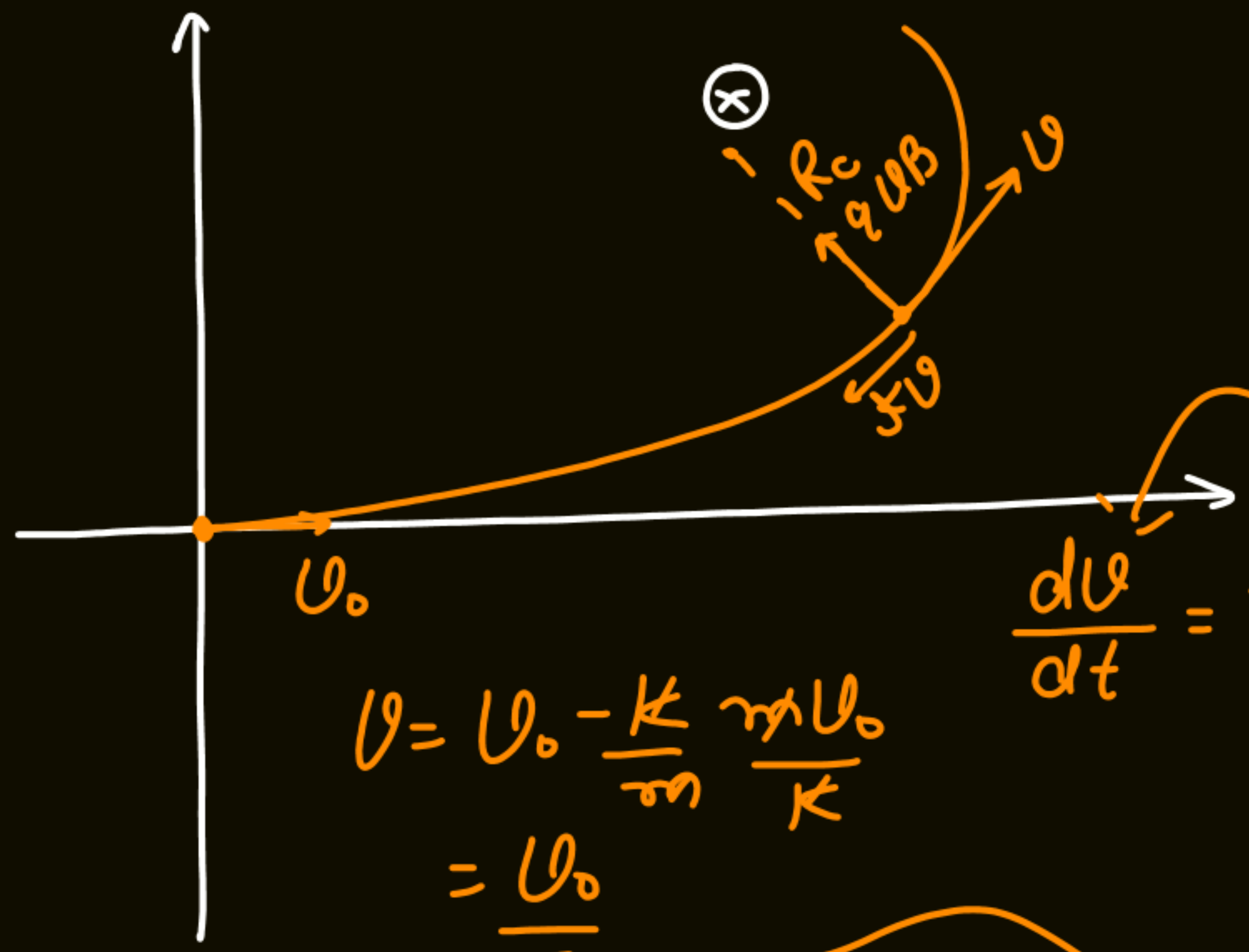
$$\vec{v} = \cos\left(\frac{qB}{m} t\right) \vec{v}_0 + v_0 \sin\left(\frac{qB}{m} t\right) \frac{\vec{v}_0 \times \vec{E}}{|\vec{v}_0 \times \vec{E}|} + \frac{q\cancel{E}}{m} t \frac{\vec{E}}{\cancel{E}}$$

Q. 20: A particle of mass m and charge q is projected into a region having a uniform magnetic field B_0. Initial velocity (v_0) of the particle is perpendicular to the magnetic field. Apart from the magnetic force the particle faces a frictional force which has a magnitude of $f = kv$ where v is instantaneous speed and k is a positive constant.

(a) Find the radius of curvature of the path of the particle after it has travelled through a distance of

$$x_0 = \frac{mv_0}{2k}.$$

(b) Plot the variation of radius of curvature of the path of the particle with time (t).



$$\downarrow R = \frac{mv}{qB} \downarrow$$

$$R_c = \frac{mv}{qB}$$

$$\frac{dv}{dt} = -\frac{kv}{m} = v\frac{dv}{dx}$$

$$-\frac{k}{m}\int_0^x dx = \int_{v_0}^{v} dv$$

$$-\frac{k}{m}x = v - v_0$$

$$v = v_0 - \frac{k}{m}\frac{mv_0}{k}$$

$$= \frac{v_0}{2}$$

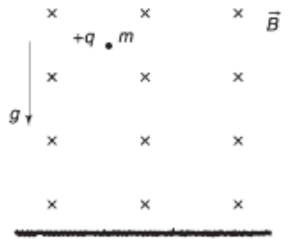
$$R_c = \frac{mv_0/2}{qB} \Rightarrow \boxed{R_c = \frac{mv_0}{2qB}}$$

$$\int_{v_0}^{v}\frac{dv}{v} = -\frac{k}{m}\int_0^t dt$$

$$\boxed{v = v_0 e^{-\frac{k}{m}t}}$$

$$\boxed{R_c = \frac{mv_0 e^{-\frac{kt}{m}}}{qB}}$$

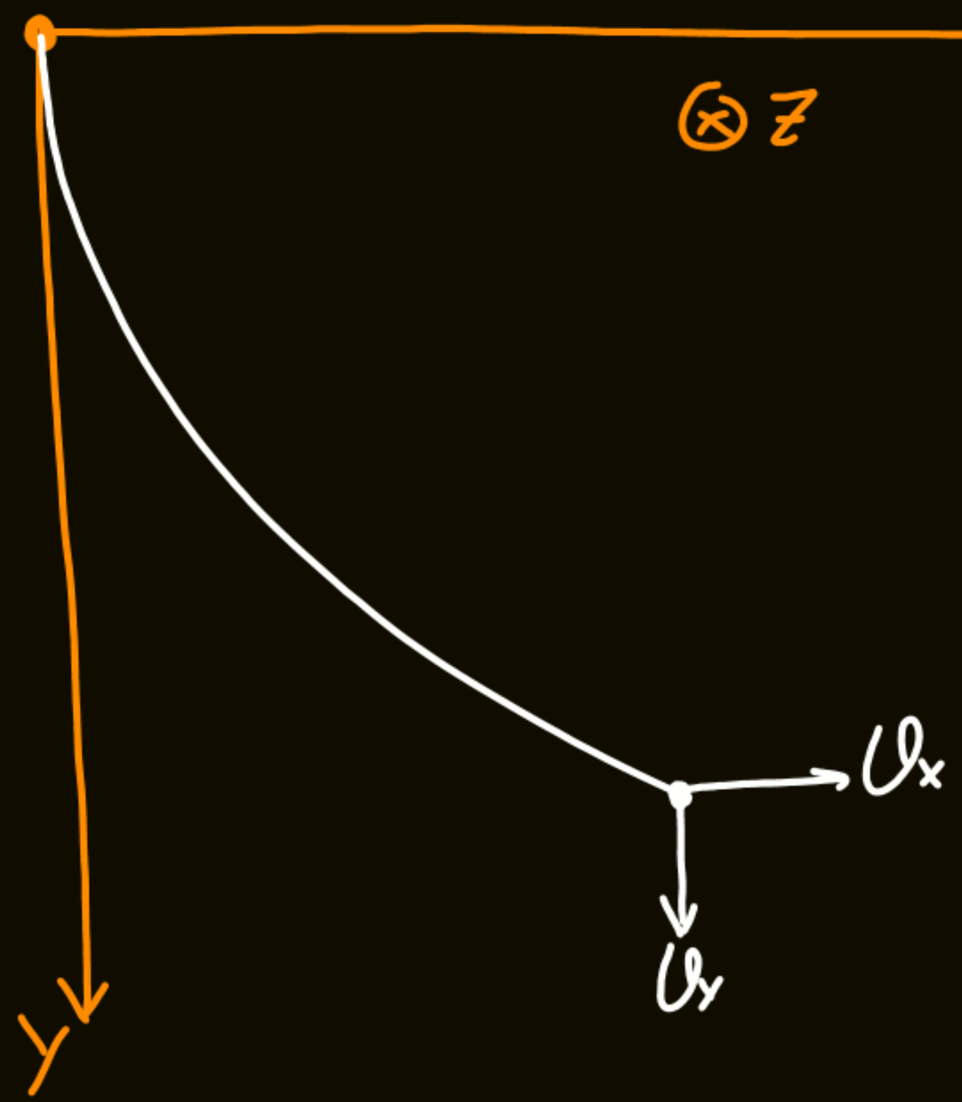
Q. 31. A particle having charge q and mass m is dropped from a large height from the ground. There exists a uniform horizontal magnetic field B in the entire space as shown in the fig. Assume that the acceleration due to gravity remains constant over the entire height involved.



(a) Argue qualitatively that the particle will touch a maximum depth and then start climbing up.

(b) Find the speed of the particle at the moment it starts climbing up.

(c) At what depth from the starting point does the particle starts climbing up?



$$\vec{B} = B\hat{k}$$

$$\vec{F} = \vec{F}_m + \vec{F}_g$$

$$= q\{v_x\hat{i} + v_y\hat{j}\} \times B\hat{k} + mg\hat{j}$$

$$= (mg - qBv_x)\hat{j} + \underline{qv_yB}\,\hat{i}$$

$$a_x = \frac{qBv_y}{m} = \frac{dv_x}{dt}$$

$$a_y = \frac{mg - qBv_x}{m} = \frac{dv_y}{dt} \Rightarrow -\frac{qB}{m}\frac{dv_x}{dt} = \frac{d^2v_y}{dt^2}$$

$$\frac{d^2v_y}{dt^2} = -\frac{q^2B^2}{m^2}v_y$$

$$\frac{d^2v_y}{dt^2} + \frac{q^2B^2}{m^2}v_y = 0$$

$$\boxed{v_y = A\sin\left(\frac{qB}{m}t + \theta\right)}$$

$t = 0 \quad v_y = 0 = A\sin(\theta) \Rightarrow \theta = 0$

$$v_y = A\sin\left(\frac{qB}{m}t\right)$$

$t = 0 \Rightarrow a_y = g \qquad \frac{dv_y}{dt} = \frac{AqB}{m}\cos\frac{qB}{m}t$

$$g = \frac{AqB}{m} \times 1$$

$$\boxed{A = \frac{mg}{qB}}$$

$$\therefore \boxed{v_y = \frac{mg}{qB}\sin\left(\frac{qB}{m}t\right)}$$

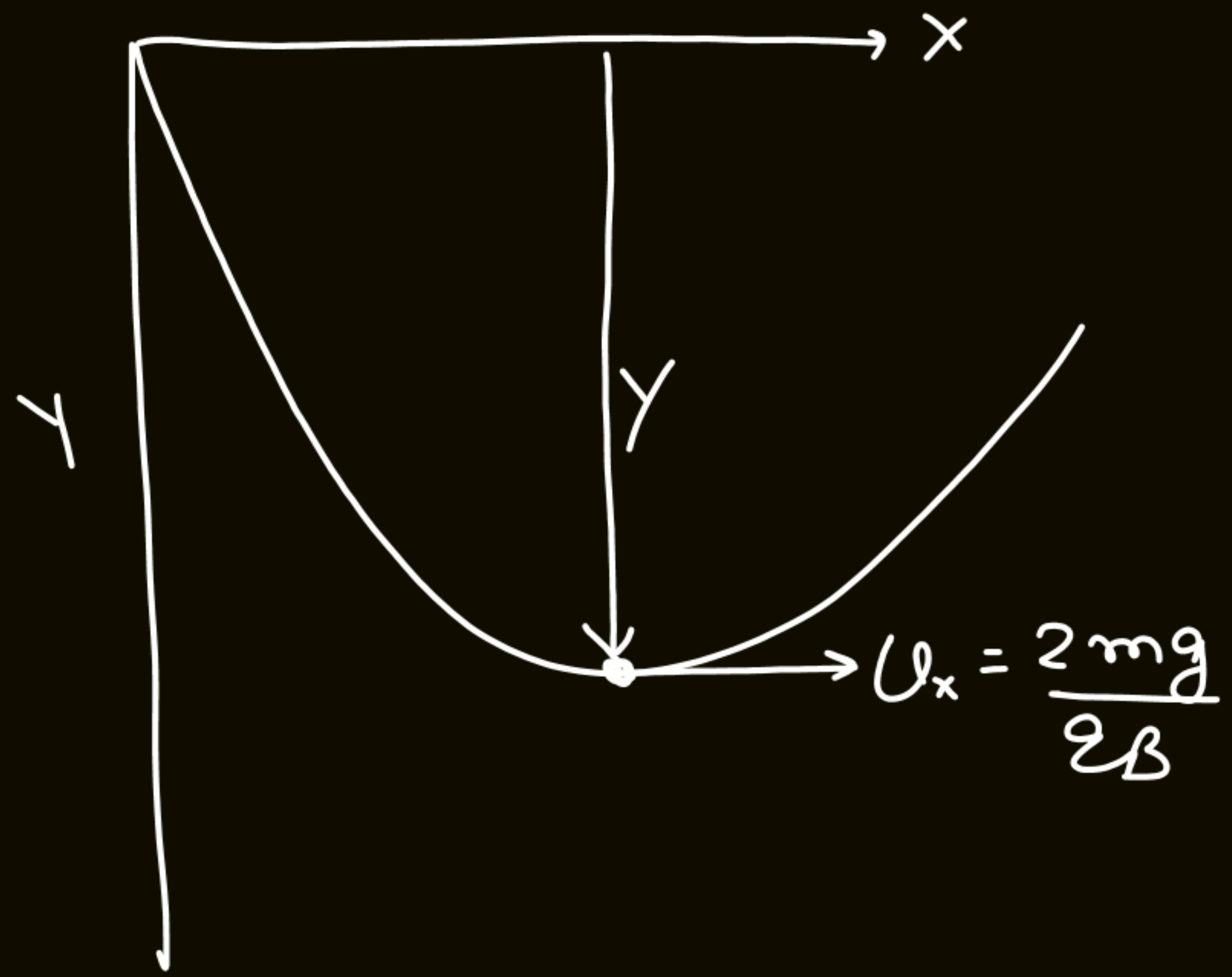
$v_y = 0 \qquad \frac{qB}{m}t = \pi \Rightarrow \boxed{t = \frac{\pi m}{qB}}$

$$\frac{dv_x}{dt} = \frac{qB}{m}\frac{mg}{qB}\sin\left(\frac{qB}{m}t\right)$$

$$\int_0^{v_x} dv_x = g\int_0^t \sin\left(\frac{qB}{m}t\right)dt$$

$$\boxed{v_x = \frac{gm}{qB}\left(1 - \cos\frac{qB}{m}t\right)}$$

$$v_x = \frac{mg}{qB}\{1 + 1\} = \frac{2mg}{qB} = v_{net}$$



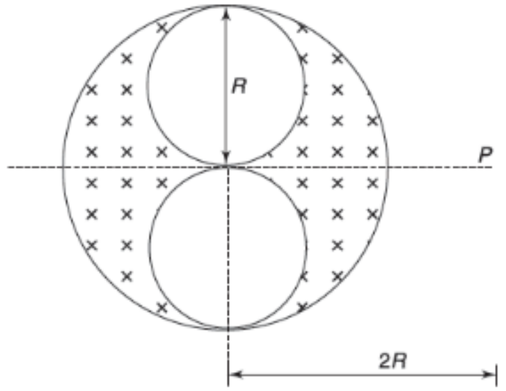
WET

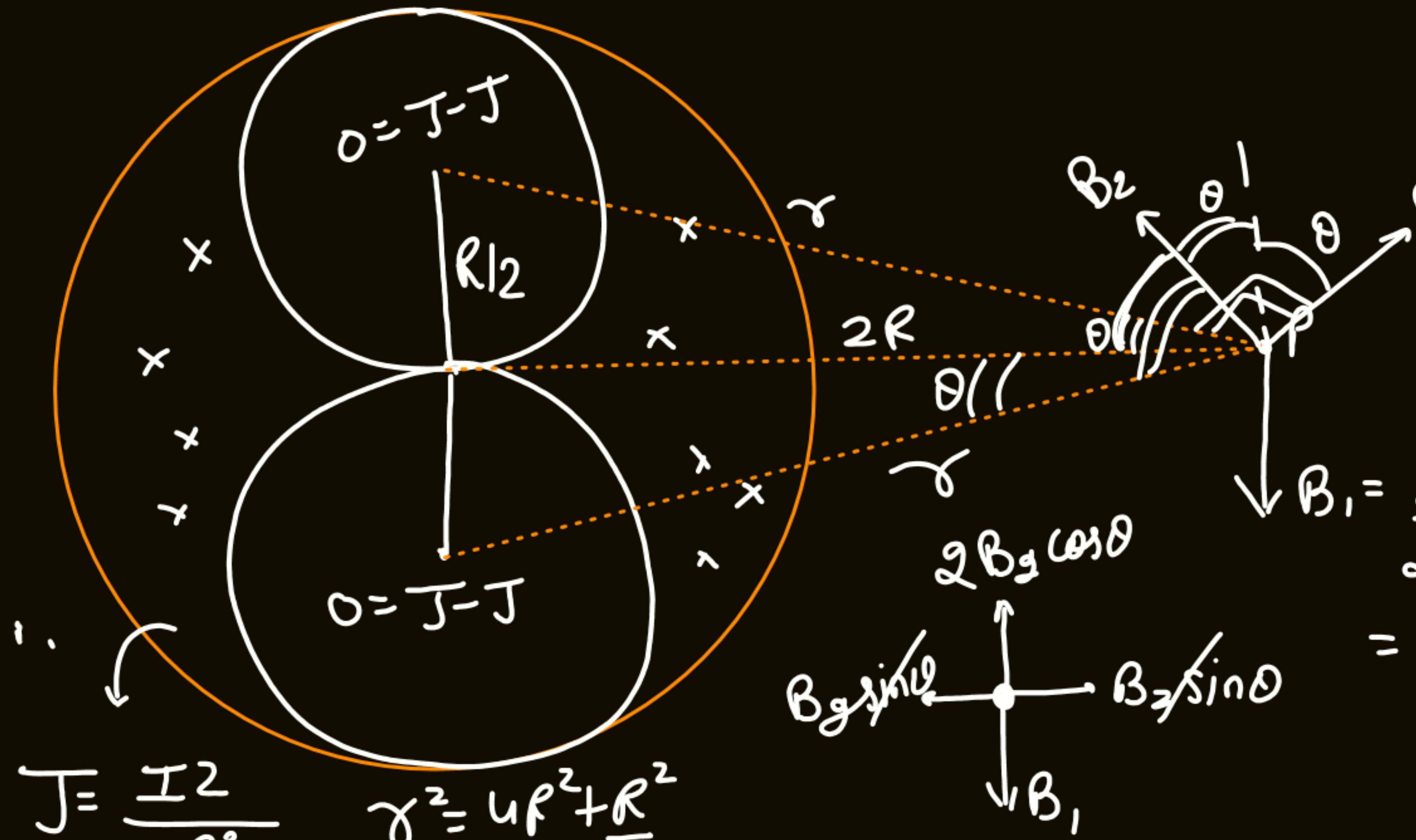
$$W_g + W_m = K_f - K_i$$

$$mgy + 0 = \frac{1}{2} m \frac{4m^2g^2}{q^2B^2} - 0$$

$$Y = \frac{2m^2g}{q^2B^2}$$

Q. 14: A long cylindrical conductor of radius R has two cylindrical cavities of diameter R through its entire length, as shown in the figure. There is a current I through the conductor distributed uniformly in its entire cross section (apart from the cavity region). Find magnetic field at point P at a distance $r = 2R$ from the axis of the conductor (see figure).





$$B_2 = \frac{\mu_0 i'}{2\pi r} = \frac{\mu_0 I}{2\pi r 2}$$

$$B_1 = \frac{\mu_0 i}{2\pi(2R)} = \frac{\mu_0 I}{2\pi R}$$

$$i = J\pi R^2 = 2I$$

$$i' = \frac{2I}{\pi R^2}\cdot\frac{\pi R^2}{4} = \frac{I}{2}$$

$$J = \frac{I2}{\pi R^2}$$

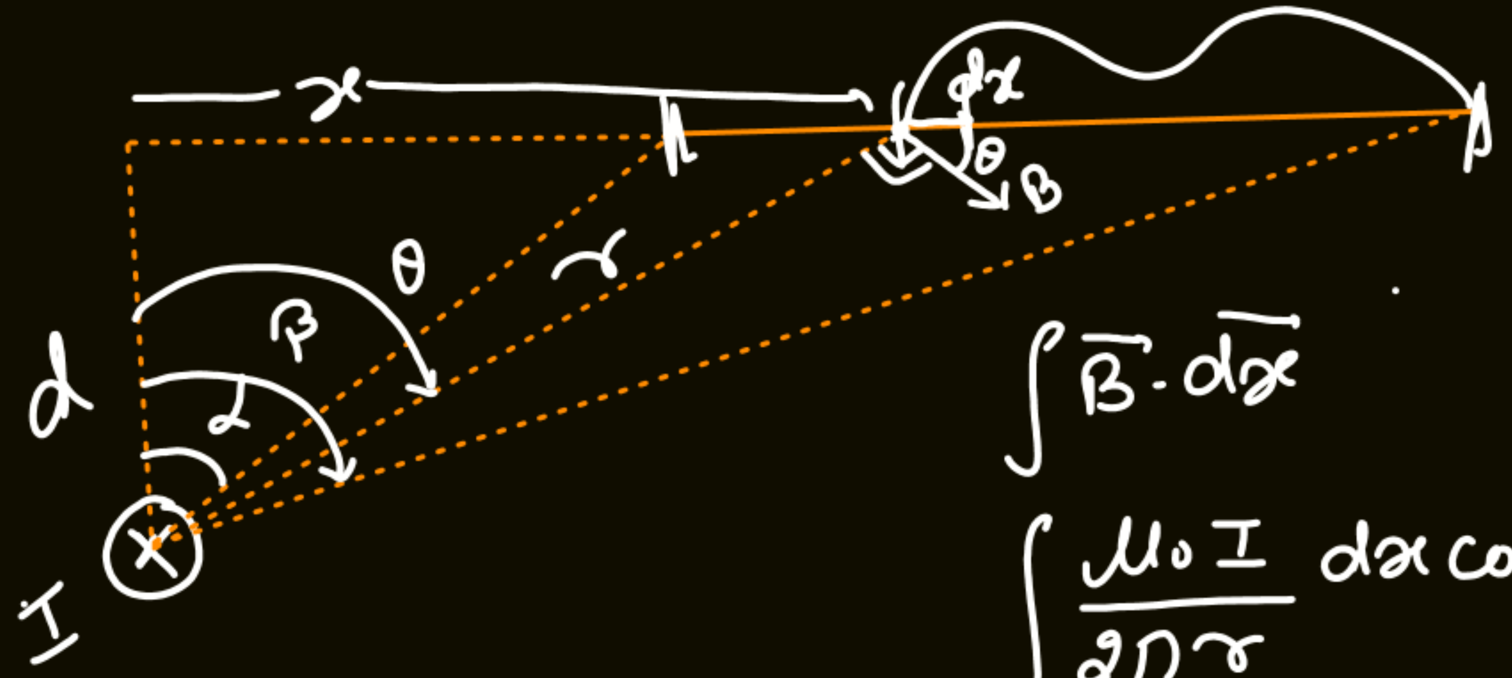
$$r^2 = 4R^2 + \frac{R^2}{4} = \frac{17R^2}{4}$$

$$B_{net} = B_1 - 2B_2\cos\theta = \frac{\mu_0 I}{2\pi R} - 2\times\frac{\mu_0 I}{4\pi r}\times\frac{2R}{r}$$

$$B_{net} = \frac{\mu_0 I}{2\pi R} - \frac{\mu_0 I\, 4 \cancel{R}}{\pi\, 17 R^{\cancel{2}}} = \frac{\mu_0 I}{\pi R}\left\{\frac{1}{2} - \frac{4}{17}\right\}$$

$$\boxed{B_{net} = \frac{\mu_0 I}{\pi R}\cdot\frac{9}{34}}$$

@



$\beta - \alpha = \theta$

$\cos\theta = \dfrac{d}{r}$

$\boxed{r = d\sec\theta}$

$x = d\tan\theta$

$dx = d\sec^2\theta\, d\theta$

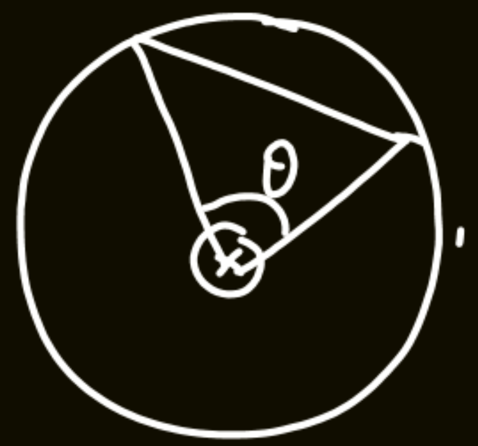
$\int \vec{B}\cdot\vec{dx}$

$\int \dfrac{\mu_0 I}{2\pi r}\, dx\cos\theta$

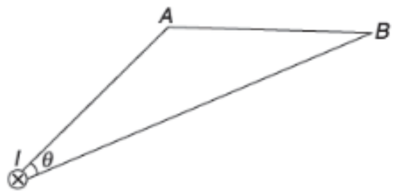
$\int \dfrac{\mu_0 I}{2\pi d\cancel{\sec\theta}}\, d\,\cancel{\sec^2\theta}\, d\theta\, \cancel{\cos\theta}$

$= \dfrac{\mu_0 I}{2\pi}\int_{\alpha}^{\beta} d\theta = \dfrac{\mu_0 I}{2\pi}(\beta - \alpha)$

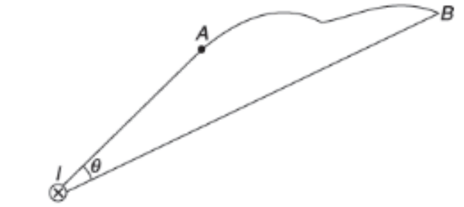
$= \dfrac{\mu_0 I\,\theta}{2\pi}$



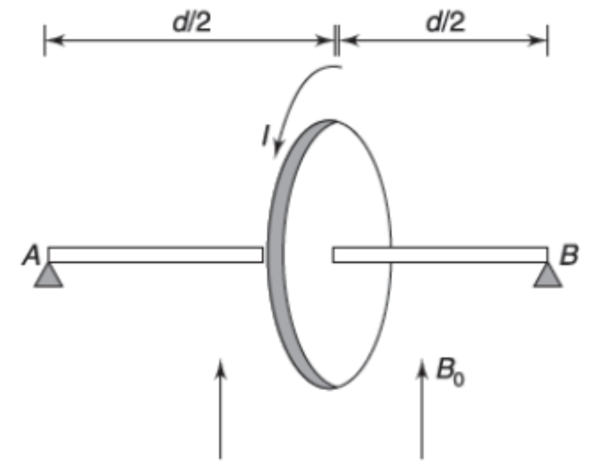
(a) A long straight wire carries a current I into the plane of the figure. AB is a straight line in the plane of the figure subtending an angle θ at the point of intersection of the wire with the plane. Find (by integration) the line integral of magnetic field along the line AB.

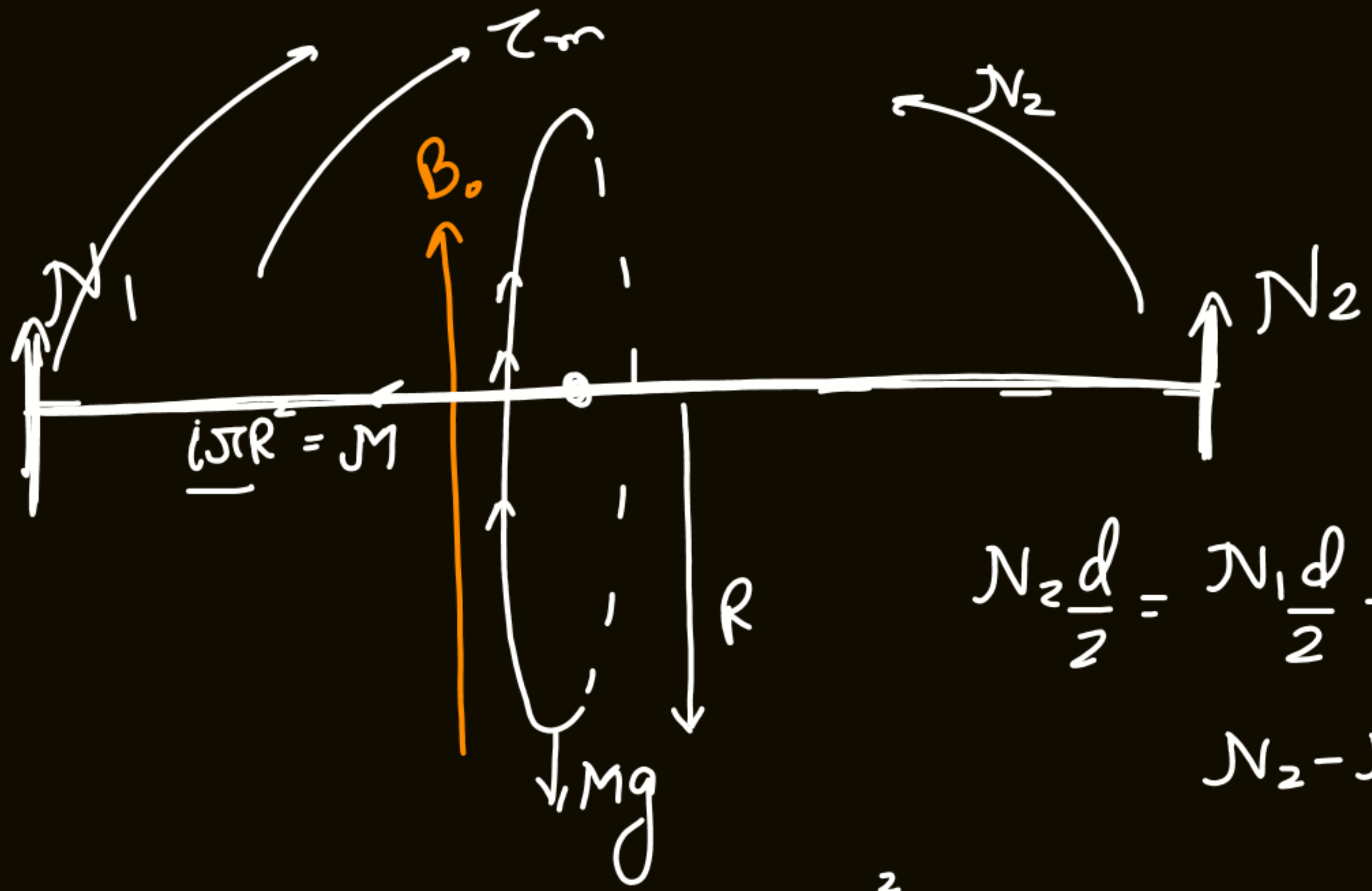


(b) In the last problem the straight line AB is replaced with a curved line AB as shown in figure. Can you calculate the line integral of magnetic field B along this curved line? If yes, what is its value?



Q. 40: A wooden disc of mass M and radius R has a single loop of wire wound on its circumference. It is mounted on a massless rod of length d. The ends of the rod are supported at its ends so that the rod is horizontal and disc is vertical. A uniform magnetic field B_0 exists in vertically upward direction. When a current I is given to the wire one end of the rod leaves the support. Find least value of I.





$$N_1 + N_2 = Mg \quad -(1)$$

$$N_2 \frac{d}{2} = N_1 \frac{d}{2} + i\pi R^2 B_0$$

$$N_2 - N_1 = \frac{2B_0 i\pi R^2}{d} \quad -(2)$$

$$N_2 = \frac{Mg}{2} + \frac{B_0 i\pi R^2}{d}$$

$$N_1 = \frac{Mg}{2} - \frac{B_0 i\pi R^2}{d}$$

$$N_1 = 0 \Rightarrow \frac{Mg}{2} = \frac{B_0 \pi R^2 i}{d}$$

$$\boxed{i = \frac{Mgd}{2B_0 \pi R^2}}$$

Q. 45: A light freely deformable conducting wire with insulation has its two ends (A and C) fixed to the ceiling. The two vertical parts of the wire are close to each other. A load of mass m is attached to the middle of the wire. The entire region has a uniform horizontal magnetic field B directed out of the plane of the figure. Prove that the two parts of the wire take the shape of circular arcs when a current I is passed through the wire. Neglect the magnetic interaction between the two parts of the wire.