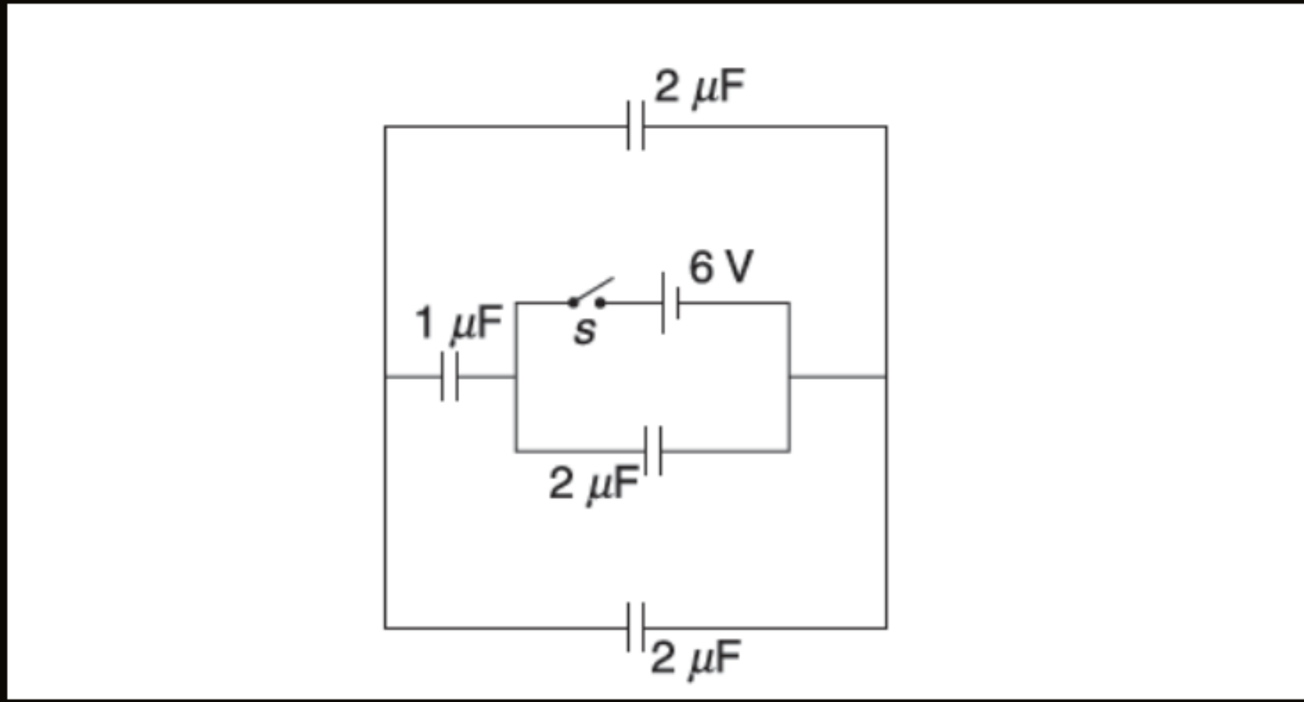
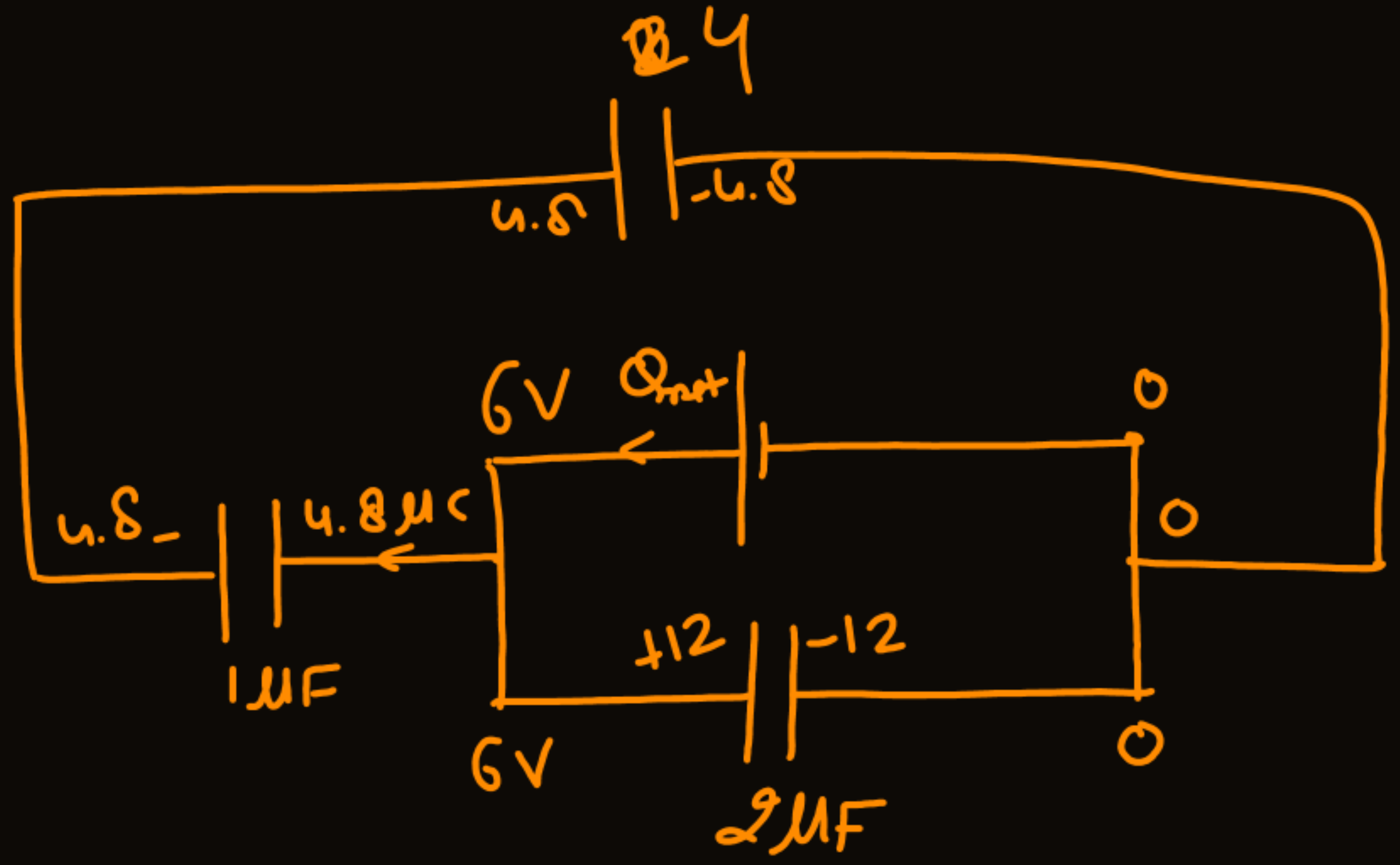
Q. 13: Find charge supplied by the cell after the switch is closed.





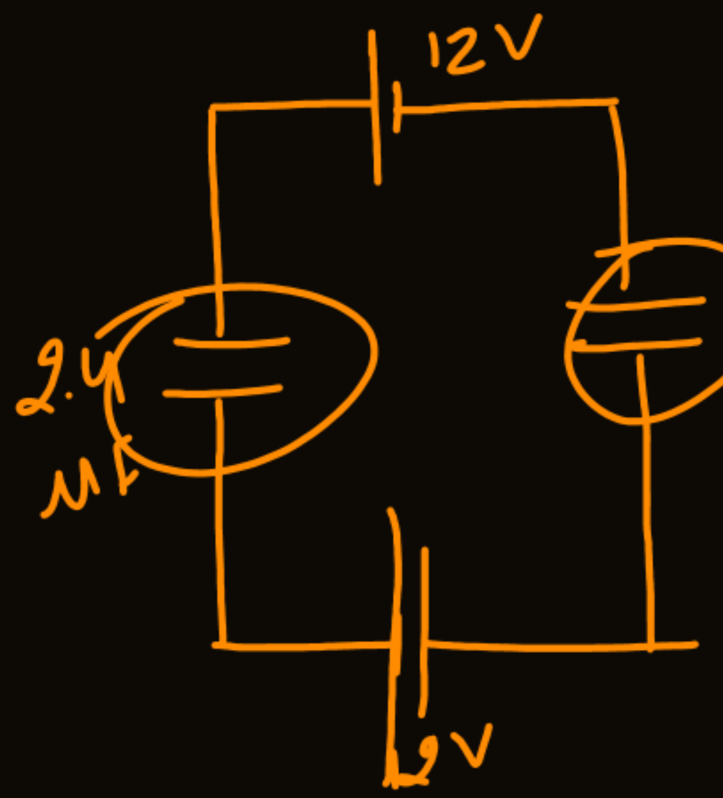
$$C_{eq} = \frac{4 \times 1}{4+1} = \frac{4}{5}\,\mu F$$

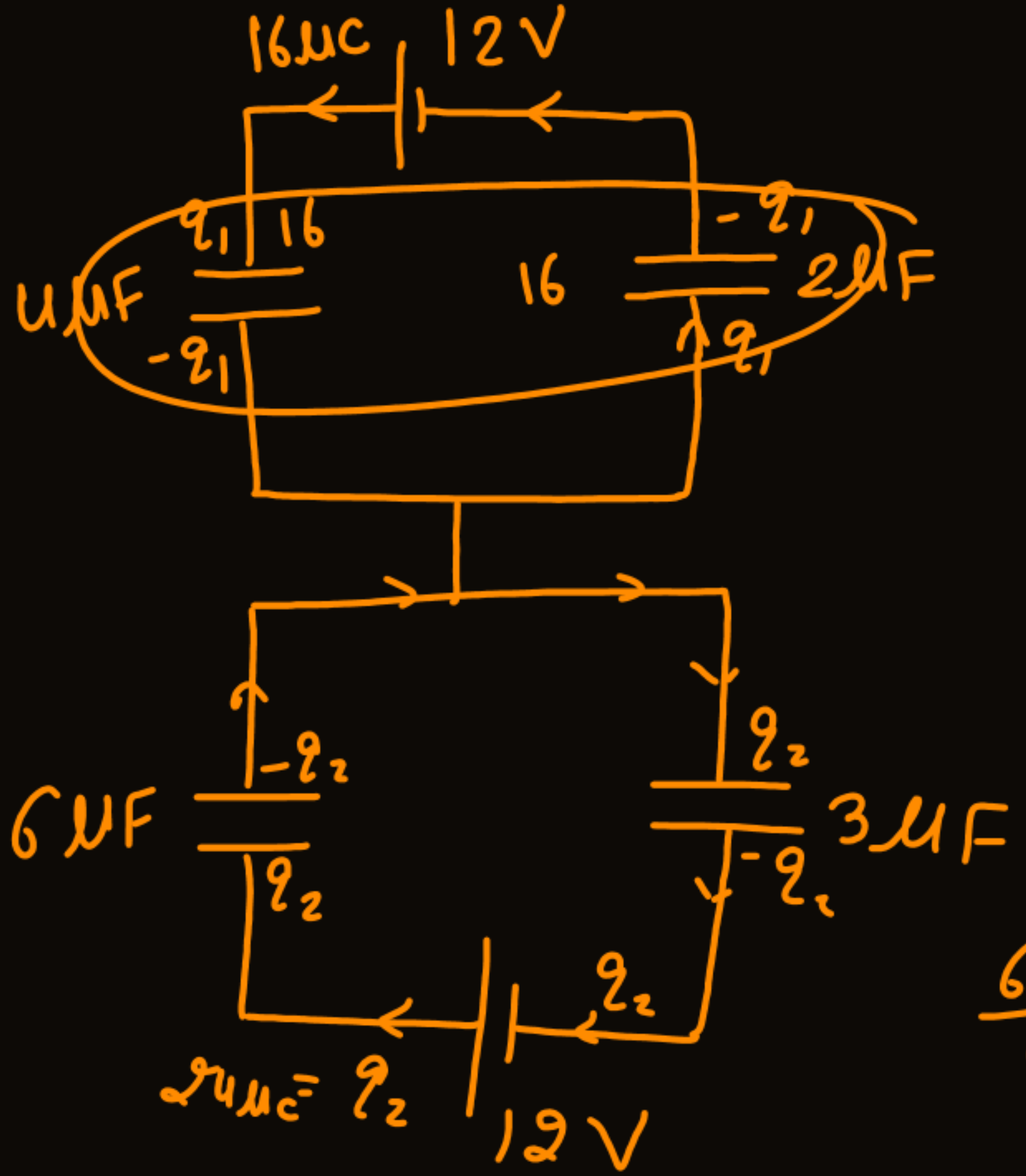
$$Q = C_{eq} V = \frac{4}{5}\,\mu F \times 6V$$

$$= \frac{24}{5}\,\mu C = \underline{4.8\,\mu C}$$

$$Q_{net} = 12 + 4.8$$

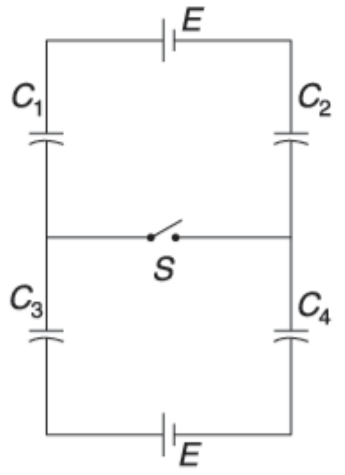
$$= \underline{16.8\,\mu C}$$





Q. 18: In the circuit shown in Figure $E = 12$ V, $C_1 = 4\ \mu F$, $C_2 = 2\ \mu$F, $C_3 = 6\ \mu$F and $C_4 = 3\ \mu F$.

Find the heat produced in the circuit after switch S is shorted.



$E_q = 0$

$q = 0$

$\frac{4\times 2}{6_{3}} = \frac{4}{3}\mu F$

$q_1 = 12V \times \frac{4}{3}\mu F$

$= 16\mu F$

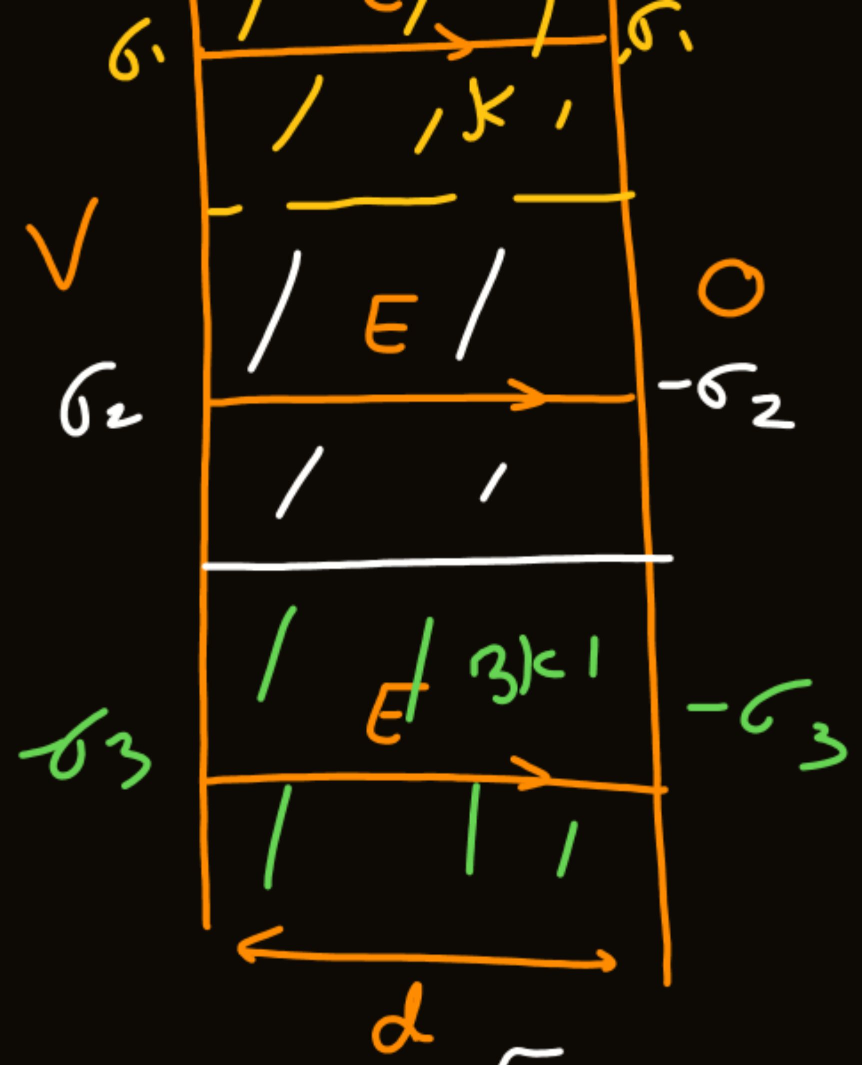
$\frac{6\times 3}{9} = 2\mu F$

$q_2 = 2\times 12 = 24\mu C$

$W_B = \Delta U + Heat$

$12\times 16\mu J + 12\times 24\mu J = \frac{(16)^2}{2\times 4/3}\mu J + \frac{(24)^2}{2\times 2} + Heat$

$Heat = 240\mu J$

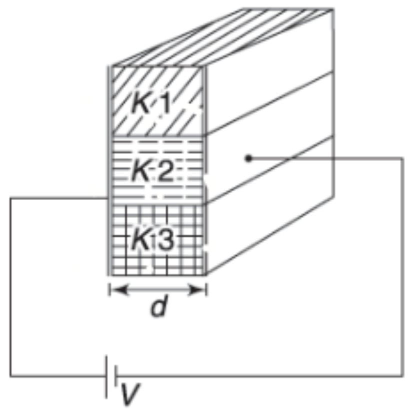


$d \qquad \varepsilon_0 K \qquad \varepsilon_0 2K \qquad \varepsilon_0 3K$

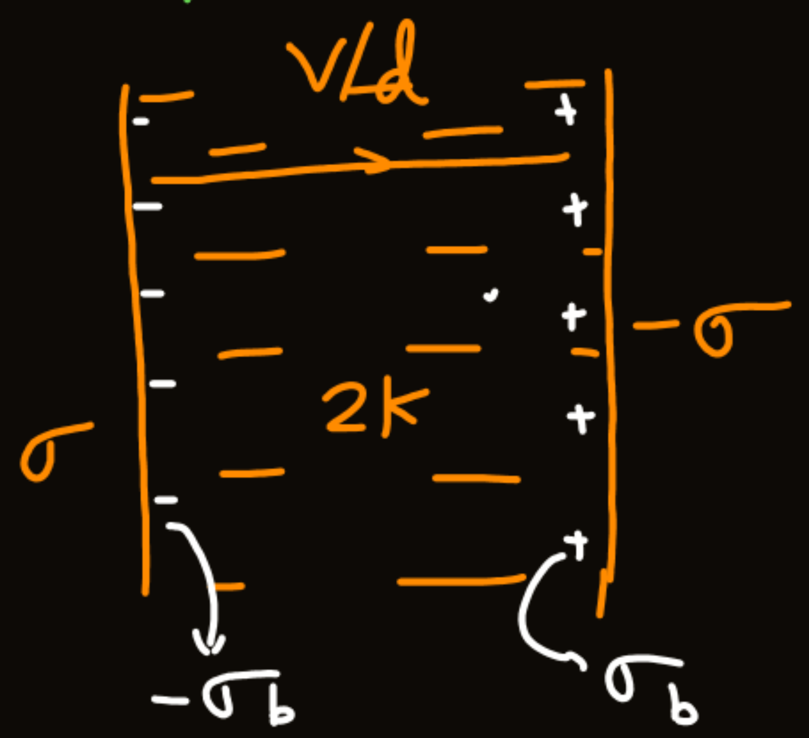
density on the surface of the capacitor plate.

(b) Calculate the surface charge density of bound (induced) charge on the middle dielectric.

(c) If the three dielectrics occupy equal volume between the plates, calculate the capacitance of the capacitor.



$\sigma_1 : \sigma_2 : \sigma_3 = 1 : 2 : 3$



$$\frac{E_0}{2K} = \frac{V}{d}$$

$$E_0 = \frac{2KV}{d}$$

$$\frac{\sigma}{\varepsilon_0} = E_0 \ (\longrightarrow)$$

$$\frac{\sigma_b}{\varepsilon_0} = E' \ (\longleftarrow)$$

$$E = E_0 - E' = \frac{E_0}{(2K)} = \frac{V}{d} \Rightarrow \frac{2KV}{d} - E' = \frac{V}{d}$$

$$E' = (2K-1)\frac{V}{d} = \frac{\sigma_b}{\varepsilon_0}$$

$$\boxed{\sigma_b = (2K-1)\frac{V}{d}\varepsilon_0}$$

$$C_{eq} = C_1 + C_2 + C_3$$

$$= \frac{k\frac{A}{3}\varepsilon_0}{d} + \frac{2k\frac{A}{3}\varepsilon_0}{d} + \frac{3k\frac{A}{3}\varepsilon_0}{d}$$

$$= \frac{kA\varepsilon_0}{3d}\left\{1+2+3\right\}$$

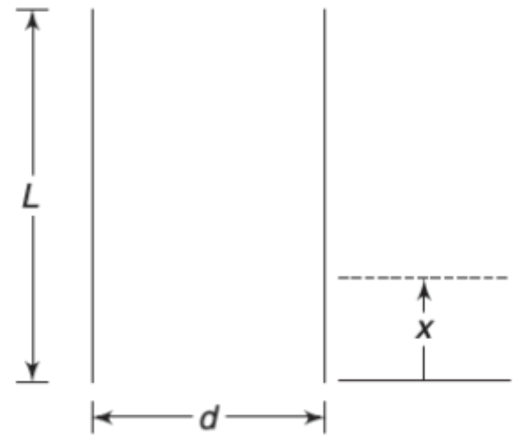
$$= \frac{2kA\varepsilon_0}{d}$$

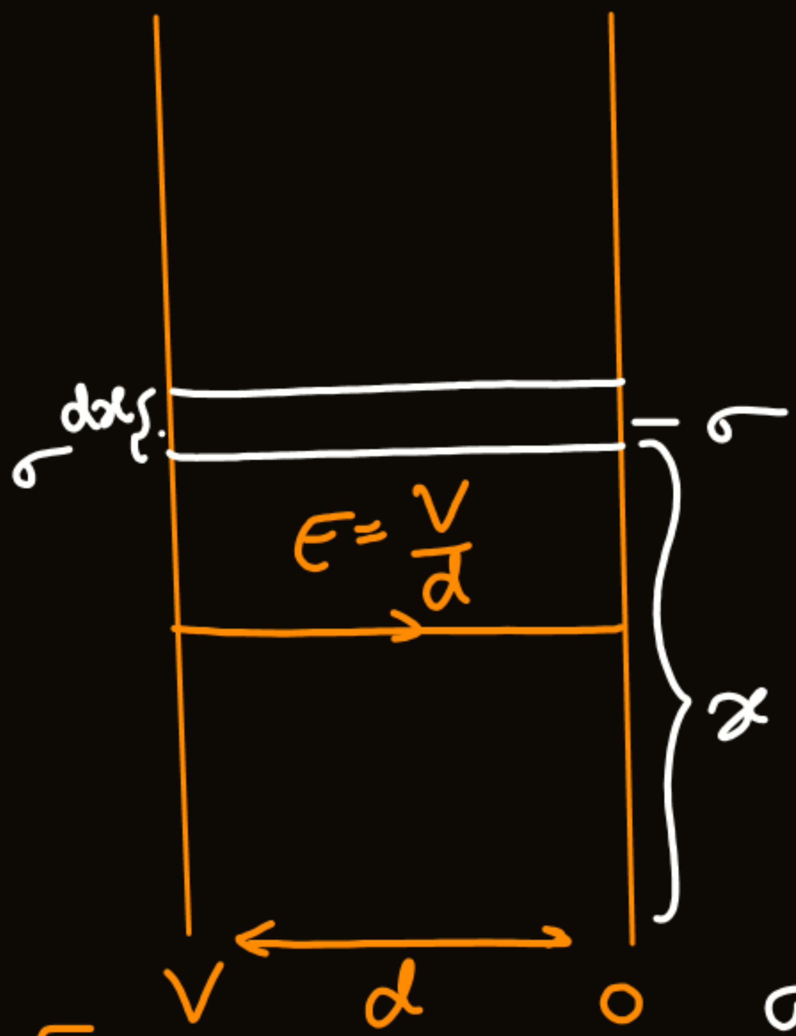
Q. 22: A parallel plate capacitor has square plates of side length L. Plates are kept vertical at separation d between them. The space between the plates is filled with a dielectric whose dielectric constant (K) changes with height (x) from the lower edge of the plates as $K = e^{\beta x}$ where β is a positive constant. A potential difference of V is applied across the capacitor plates.

(i) Plot the variation of surface charge density (σ) on the positive plate of the capacitor versus x.

(ii) Plot the variation of electric field between the plates as a function of x.

(iii) Calculate the capacitance of the capacitor.





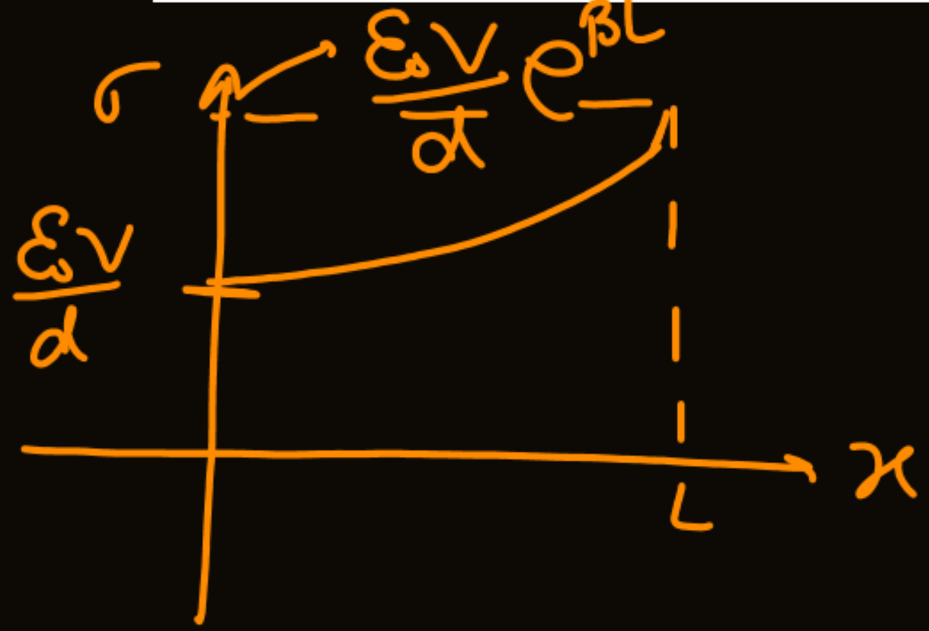
$$dC = \frac{e^{\beta x} L\,dx\,\varepsilon_0}{d}$$

$$dq = (dC)V$$

$$= \frac{e^{\beta x} L\,dx\,\varepsilon_0 V}{d}$$

$$\sigma = \frac{dq}{dA} = \frac{e^{\beta x} L\,\cancel{dx}\,\varepsilon_0 V}{d\,L\,\cancel{dx}}$$

$$\boxed{\sigma = \frac{\varepsilon_0 V}{d} e^{\beta x}}$$



$$C = \int dC = \frac{L\varepsilon_0}{d}\int_0^L e^{\beta x}\,dx$$

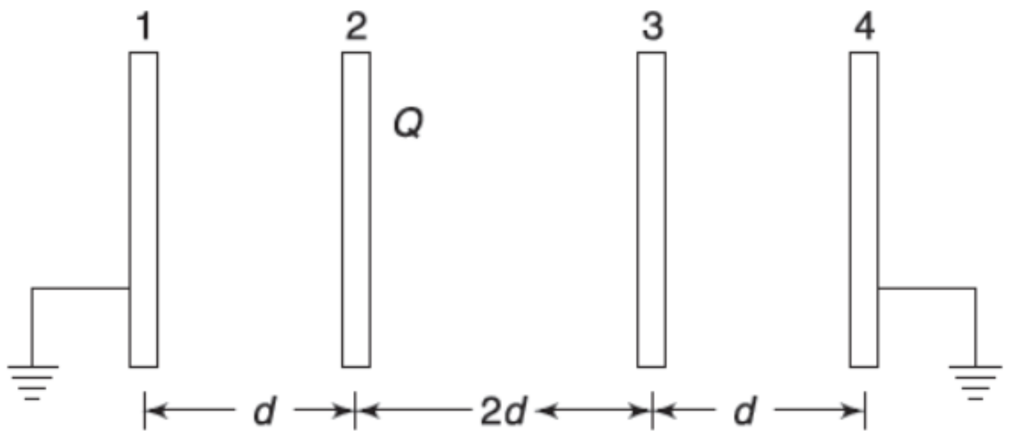
$$= \frac{L\varepsilon_0}{d}\left(\frac{e^{\beta x}}{\beta}\right)_0^L$$

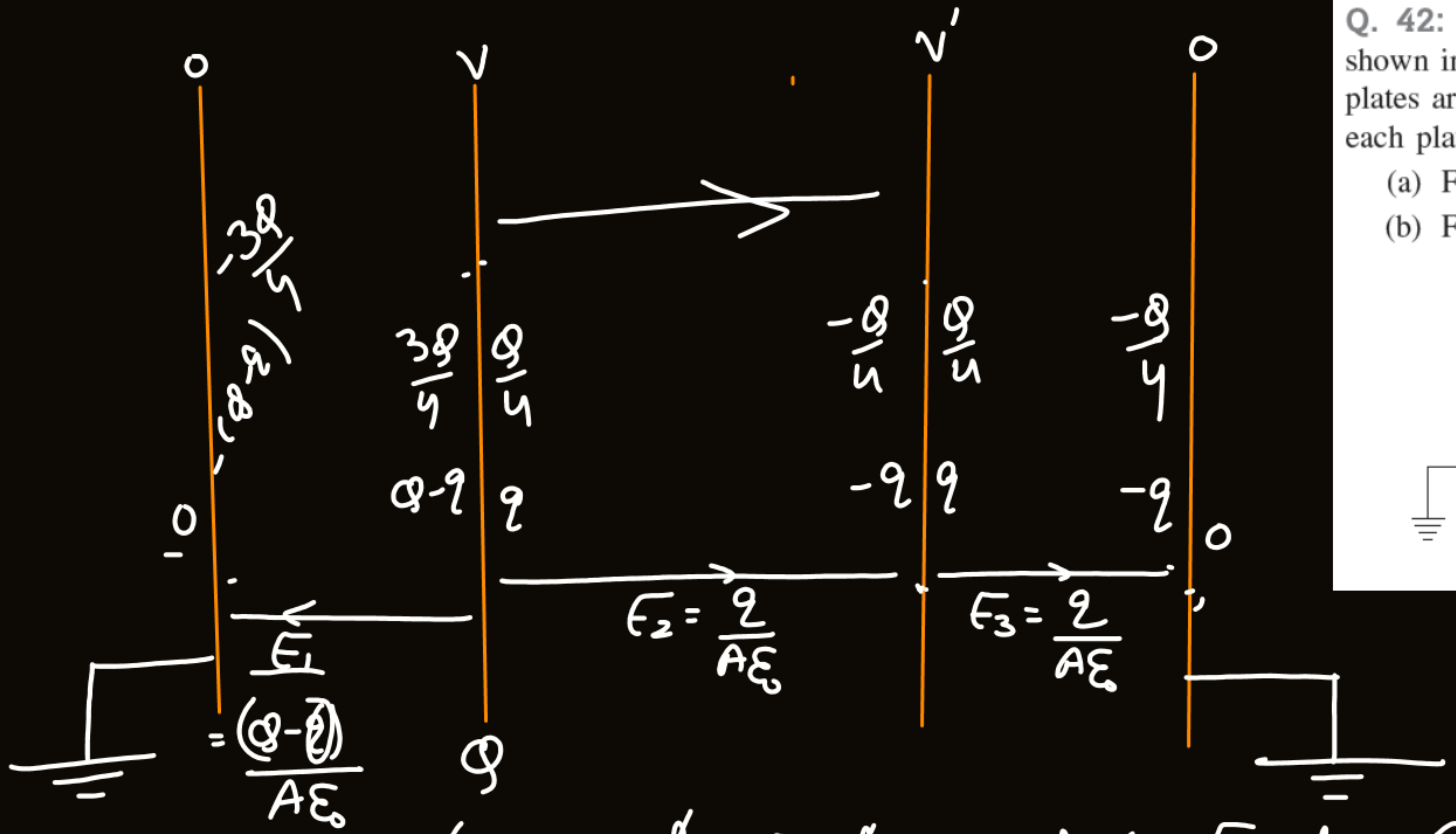
$$\boxed{C = \frac{L\varepsilon_0}{\beta d}\left\{e^{\beta L} - 1\right\}}$$

Q. 42: Four large identical metallic plates are placed as shown in the Figure. Plate 2 is given a charge Q. All other plates are neutral. Now plates 1 and 4 are earthed. Area of each plate is A.

(a) Find charge appearing on right side of plate 3.

(b) Find potential difference between plates 1 and 2.





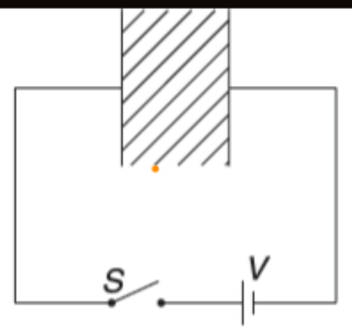
$$E_1 d = E_2\, 2d + E_3 d$$

$$Q - q = 2q + q$$

$$q = Q/4$$

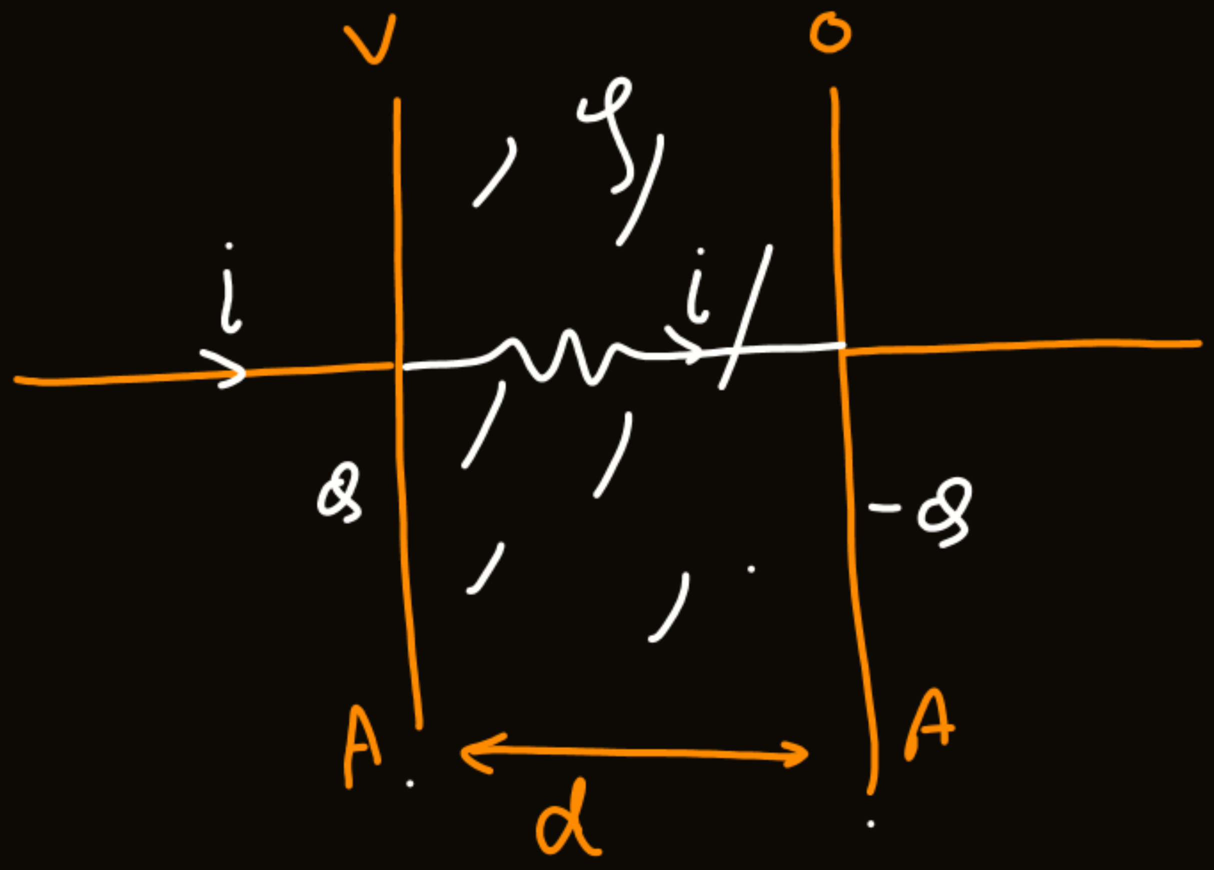
$$V = E_1 d = \frac{3Q}{4A\varepsilon_0} d$$

Q. 44: A parallel plate capacitor has plate area A and separation between the plates equal to d. A material of dielectric constant K and resistivity ρ is filled between the plates. The switch is closed to connect the capacitor to a cell of emf V.



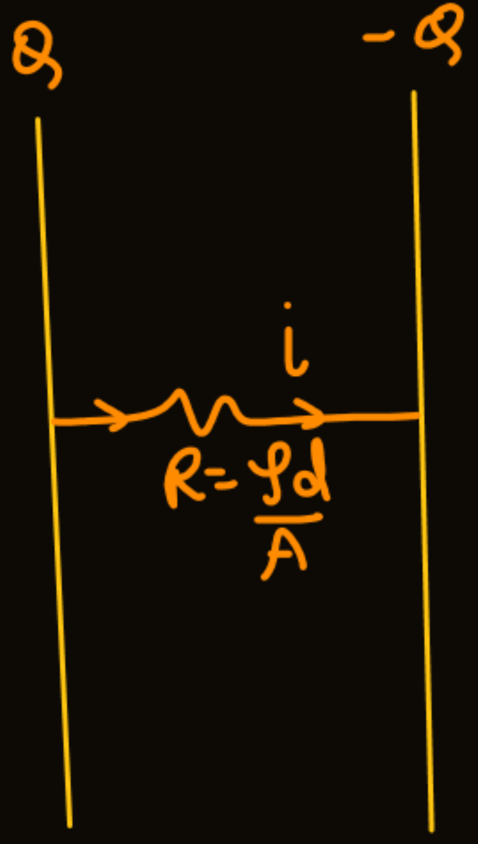
(a) Write the steady state current in the circuit and charge on the capacitor.

(b) When the circuit is in steady state, switch s is opened (at $t = 0$). Write charge on the capacitor as function of time (t) after this.

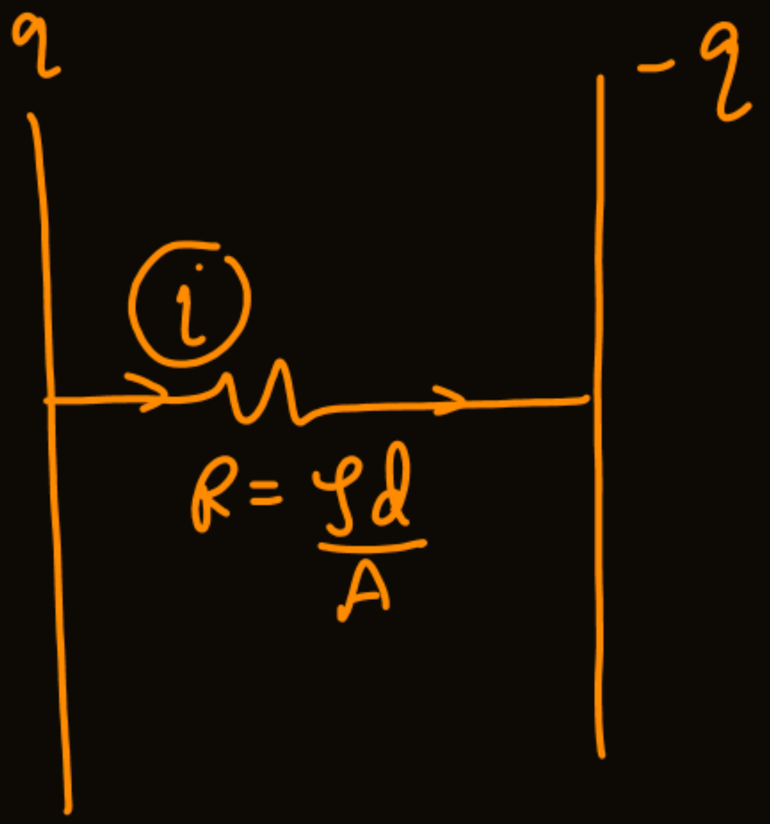


$$i = \frac{VA}{\rho d}$$

$$\boxed{q = CV = \frac{KA\varepsilon_0}{d}V}$$



$t = 0$



$$i = \frac{qA}{C\rho d} = \frac{q\cancel{A}\cancel{d}}{KA\varepsilon_0 \rho \cancel{d}}$$

$$i = \frac{q}{K\varepsilon_0\rho} = -\frac{dq}{dt}$$

$$-\int_0^t \frac{dt}{K\varepsilon_0\rho} = \int_Q^q \frac{dq}{q}$$

$$q = Q e^{-\frac{t}{K\varepsilon_0\rho}}$$

$$= \frac{KA\varepsilon_0 V}{d} e^{-\frac{t}{K\varepsilon_0\rho}}$$

$$i = \frac{AV}{\rho d} e^{-\frac{t}{K\varepsilon_0\rho}} \rightarrow \text{Leakage current}$$

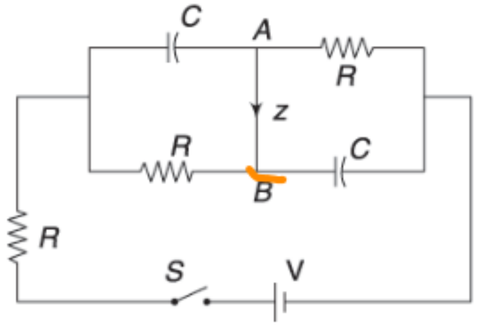
Q. 104: In the circuit shown in the fig, the switch 'S' is closed at time $t = 0$. The current in branch AB is represented by z and is taken to be positive when it is from A to B.

(a) Write the value of z immediately after the switch is closed.

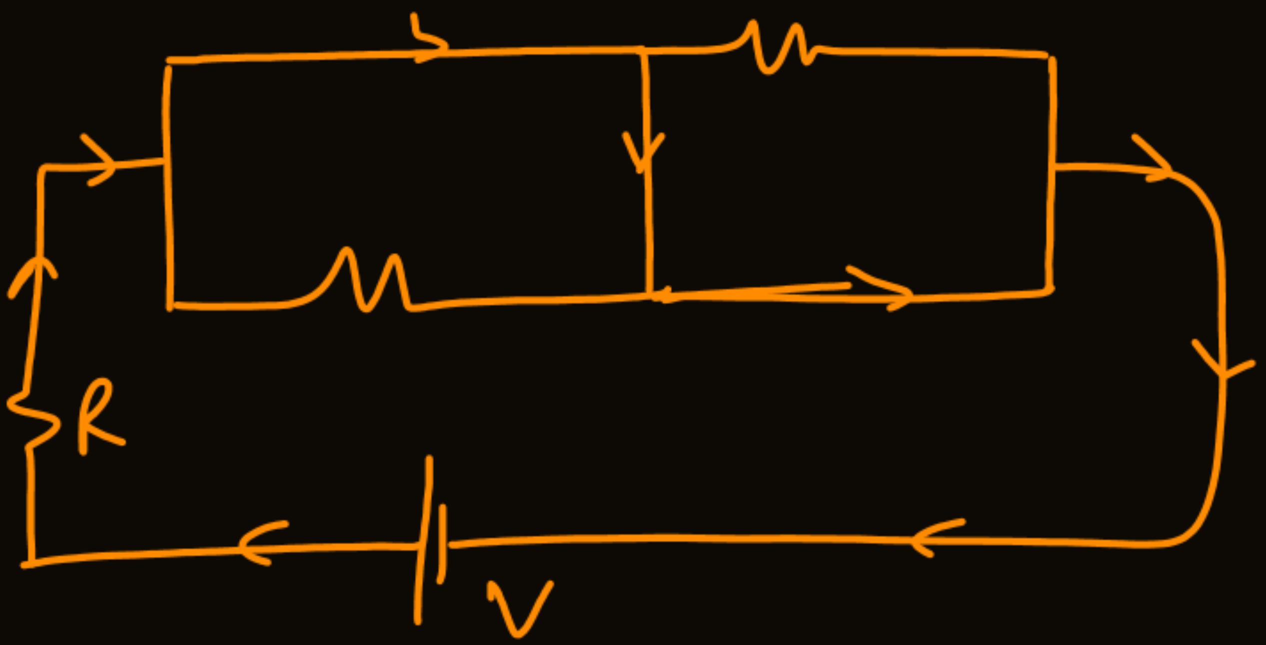
(b) Write the value of z infinite time after the switch is closed.

(c) Write z as a function of time (t) and plot the variation of z with time.

(d) At what time t_0 the current z becomes zero?



(a)



$R_{eq} = R \Rightarrow i = \frac{V}{R}$

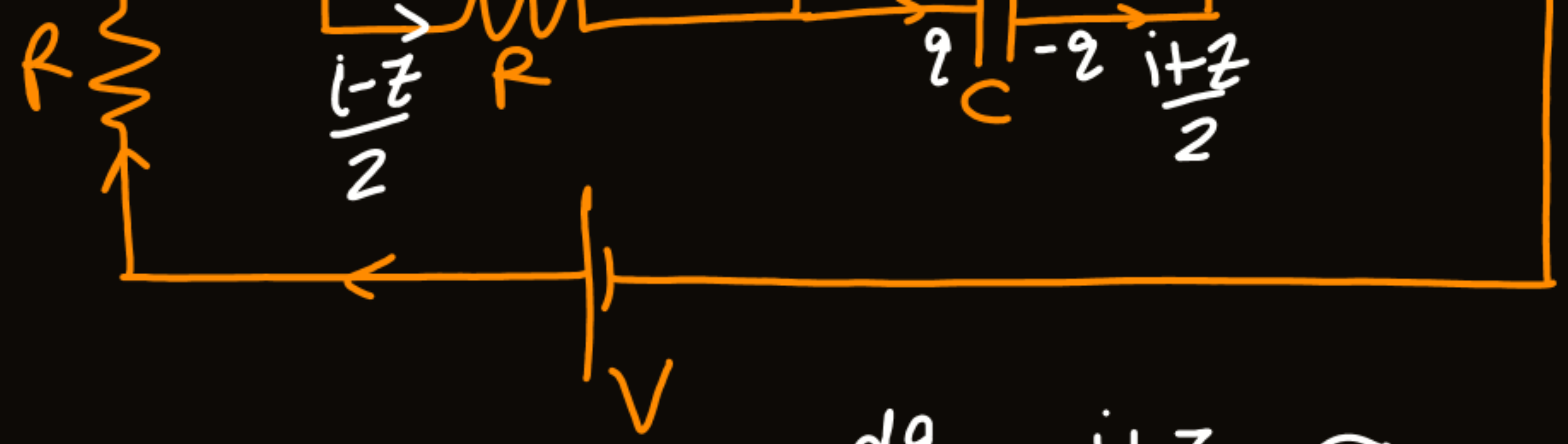
$z = \frac{V}{R}$

(b)

$R_{eq} = 3R$

$i = \frac{V}{3R}$

$\boxed{z = -V/3R}$



$$V = iR + (i-z)R$$

$$V = 2iR - zR \quad -(1)$$

$$\frac{dq}{dt} = \frac{i+z}{2} \quad -(2)$$

$$V - iR - \frac{q \times 2}{C} \quad -(3)$$

$$V - \left\{\frac{V}{2} + \frac{zR}{2}\right\} = \frac{2q}{C} = \frac{V - zR}{2}$$

$$\frac{2}{C}\frac{dq}{dt} = -\frac{R}{2}\frac{dz}{dt}$$

$$\frac{2}{C}\cdot\frac{i+z}{\cancel{2}} = -\frac{R}{\cancel{2}}\frac{dz}{dt}$$

$$\frac{V}{2R} + \frac{z}{2} + z = -\frac{RC}{2}\frac{dz}{dt}$$

$$\frac{V}{\cancel{2}R} + \frac{3z}{\cancel{2}} = -\frac{RC}{\cancel{2}}\frac{dz}{dt}$$

$$-\int_0^t \frac{dt}{RC} = \int_{V/R}^{z} \frac{dz}{\frac{V}{R} + 3z}$$

$$-\frac{t}{RC} = \frac{1}{3}\ln\left(\frac{V}{R}+3Z\right)\Bigg|_{\frac{V}{R}}^{Z}$$

$$t=0 \Rightarrow Z=\frac{V}{R}$$

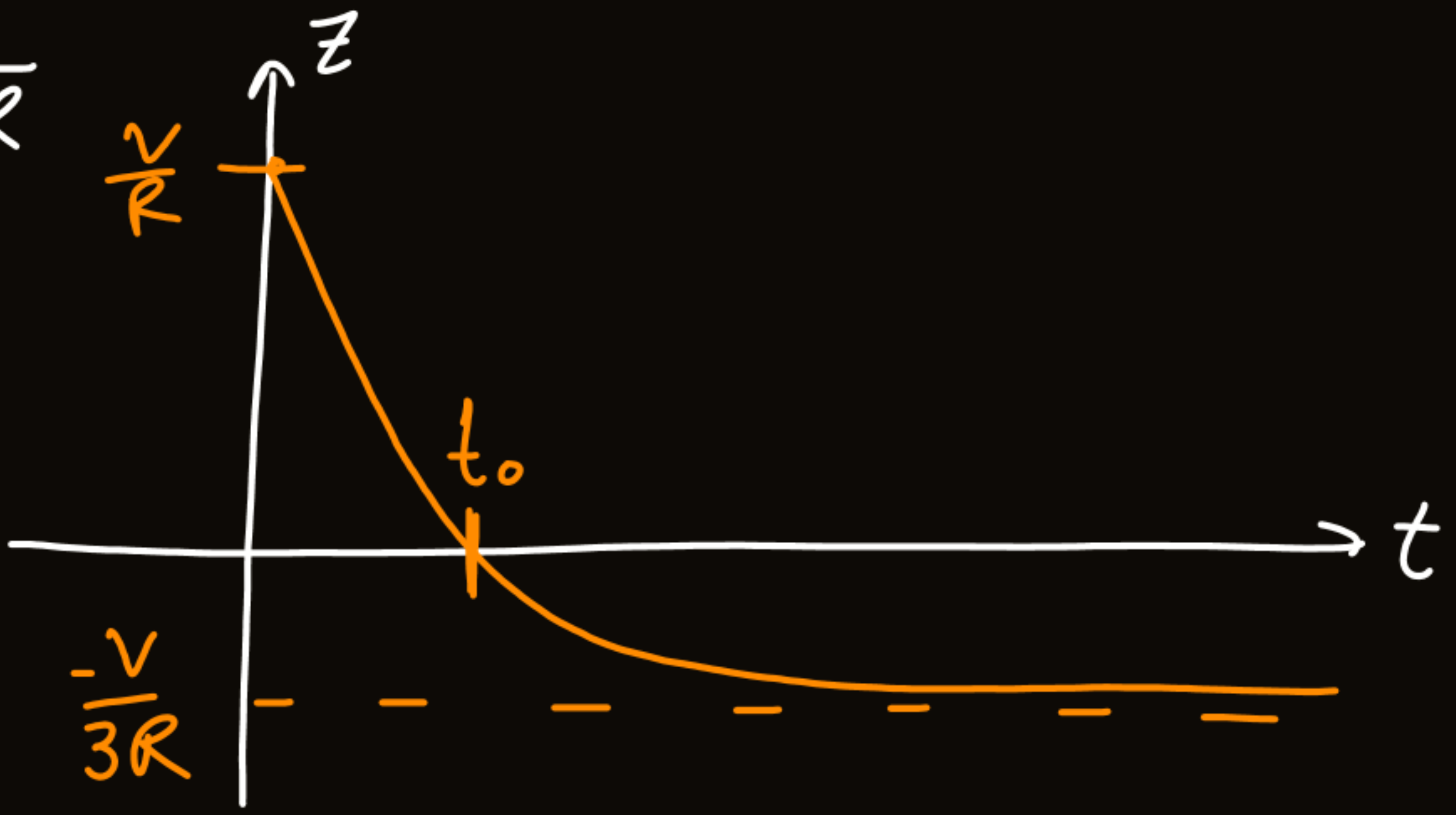
$$t\to\infty \quad Z=\frac{-V}{3R}$$

$$-\frac{3t}{RC} = \ln\frac{\frac{V}{R}+3Z}{\frac{V}{R}+\frac{3V}{R}} = \ln\frac{\frac{V}{R}+3Z}{4V/R}$$

$$e^{-\frac{3t}{RC}} = \frac{1}{4}+\frac{3Z}{(4V/R)}$$

$$\frac{3Z}{4V/R} = \left(e^{-\frac{3t}{RC}}-\frac{1}{4}\right)$$

$$\boxed{Z = \frac{4V}{3R}\left\{e^{-\frac{3t}{RC}}-\frac{1}{4}\right\}}$$



$$e^{-\frac{3t}{RC}} = \ln\frac{1}{4}$$

$$\frac{3t}{RC} = \ln 4 = 2\ln 2 \Rightarrow \boxed{t_0 = \frac{2RC}{3}\ln 2}$$