Q. 17: Electric pot(e)ntial in a 3 dimensional space is given by $V = \left(\frac{1}{x} + \frac{1}{y} + \frac{2}{z}\right)$ volt where x, y and z are in meter. A particle has charge $q = 10^{-12}$ C and mass $m = 10^{-9}$ g and is constrained to move in xy plane. Find the initial acceleration of the particle if it is released at (1, 1, 1) m.

$$E_x = -\frac{\partial V}{\partial x} = +\frac{1}{x^2} \qquad E_y = -\frac{\partial V}{\partial y} = +\frac{1}{y^2}$$

$$E_z = +\frac{2}{z^2}$$

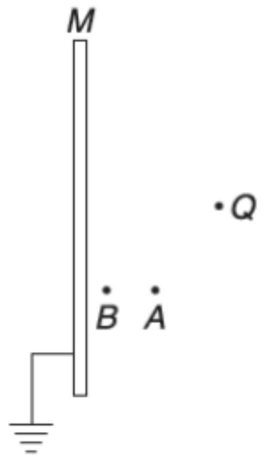
$$\vec{E} = (+\hat{i} + \hat{j} + 2\hat{k})\frac{N}{C}$$

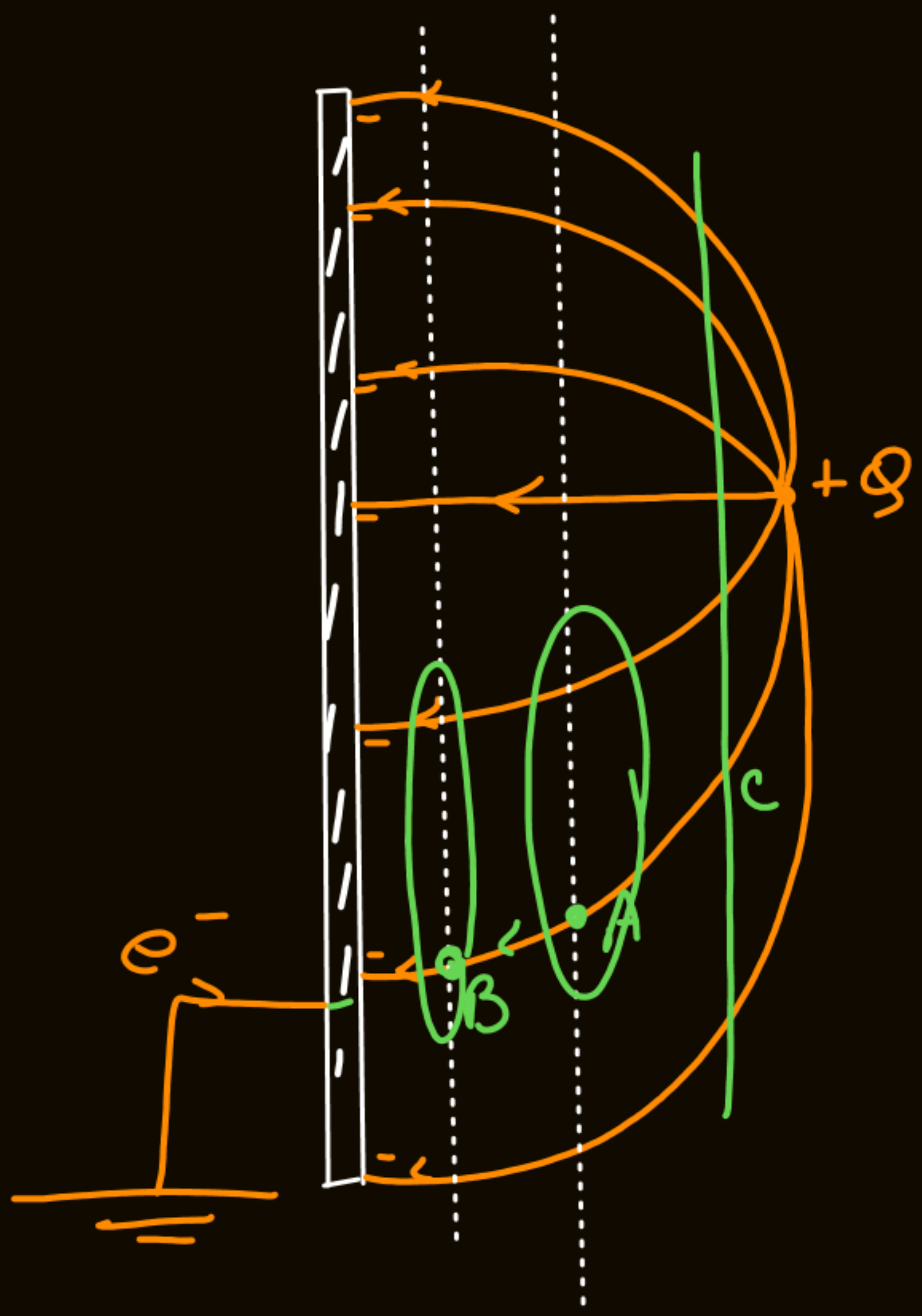
$$q\vec{E} = 10^{-12}\{\hat{i} + \hat{j} + 2\hat{k}\}\,N$$

$$\vec{F} = (\hat{i} + \hat{j}) \times 10^{-12}\,N = 10^{-12}\,kg\,\vec{a}$$

$$\vec{a} = (\hat{i} + \hat{j})\,m/s^2$$

Q. 18: A metal plate M is grounded. A point charge $+Q$ is placed in front of it. Consider two points A and B as shown in fig. At which point (A or B) is the electric field stronger? At which point is the potential higher?





$E_B < E_A$

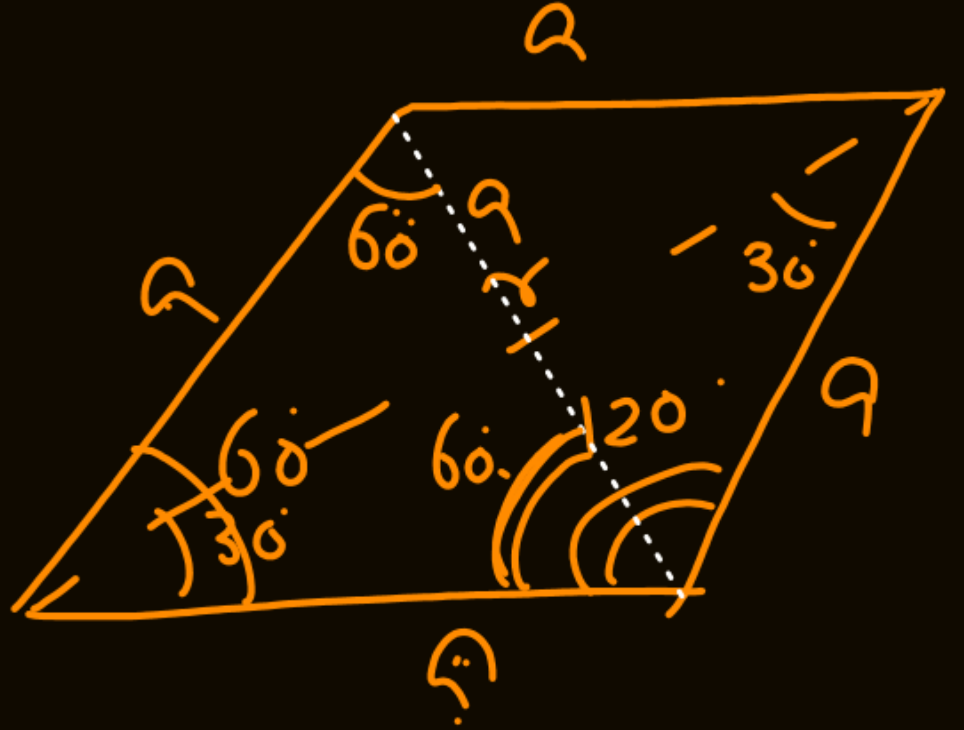
$V_B < V_A$

$$W_{elect} = U_i - U_f$$

$$W_{ext} = U_f - U_i =$$

$$U_i = 0$$

$$U_f = \frac{k(q)(-q)}{a} + \frac{kq \cdot q}{\sqrt{3}a} = \frac{kq^2}{a}\left\{\frac{1}{\sqrt{3}} - 1\right\}$$

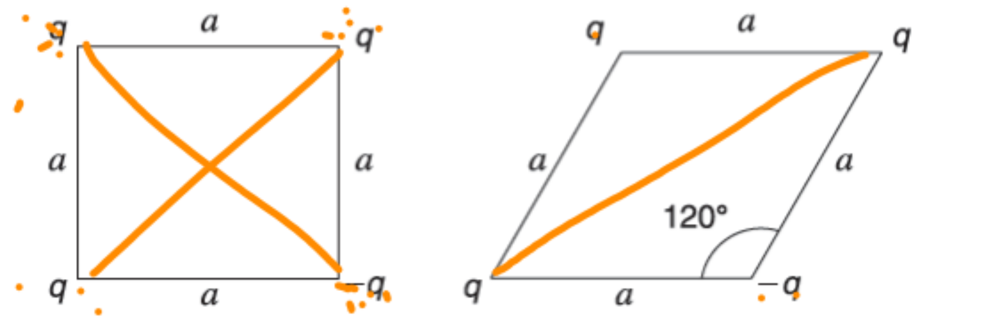


$$^4C_2 = \frac{4 \times 3}{2} = 6$$

$$\frac{r \times 2}{\sqrt{3}} = \frac{a \times 2}{1}$$

$$r = \sqrt{3}a$$

Q. 21: Four point charges q, q, q and $-q$ are placed at the vertices of a square of side length **a**. The configuration is changed and the charge are positioned at the vertices of a rhombus of side length **a** with $-q$ charge at the vertex where angle is 120°. Find the work done by the external agent in changing the configuration.

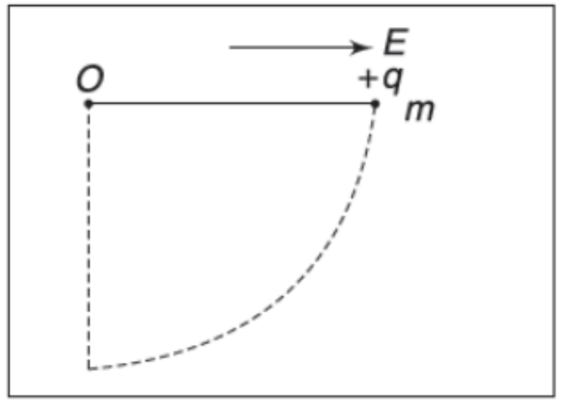


WET $\quad W_g + W_T + W_e = K_f - K_i$

Q. 25: There is a uniform horizontal electric field of strength E in a region. A pendulum bob is pulled to make the string horizontal and released. The bob has mass m and charge q.

(a) Find the maximum angle (θ_0) that the bob swings before coming to rest momentarily.

(b) Find E if the bob comes to rest when the string is vertical.

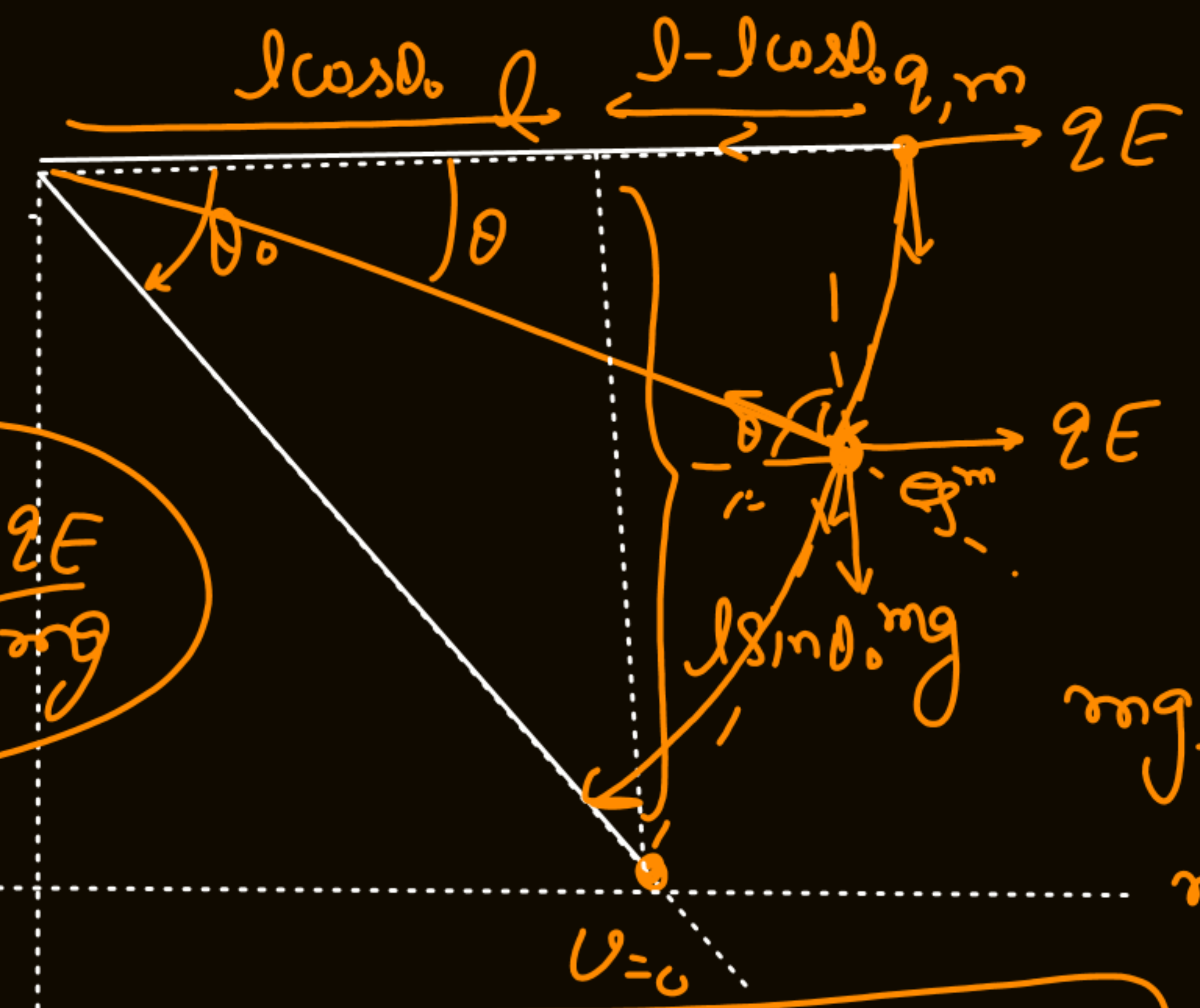


$T\sin\theta = mg$

$T\cos\theta = qE$

$\tan\theta = \frac{qE}{mg}$

$\theta = 2\tan^{-1}\frac{qE}{mg}$



$mg\,l\sin\theta_0 + 0 - qE\,l(1-\cos\theta_0) = 0 - 0$

$mg\,l\sin\frac{\theta_0}{2}\cos\frac{\theta_0}{2} = qE\,l\sin^2\frac{\theta_0}{2}$

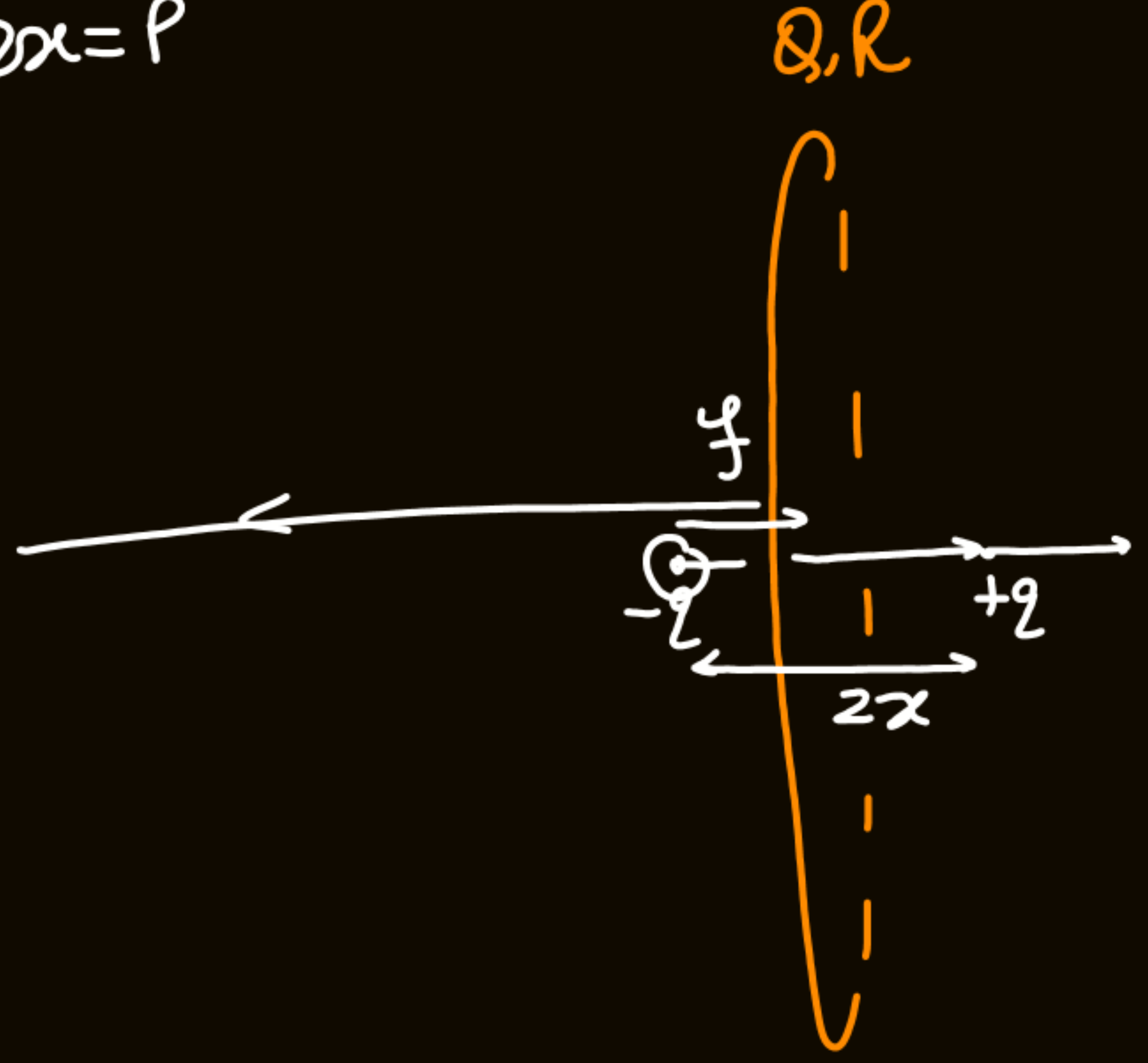
$\tan\frac{\theta_0}{2} = \frac{mg}{qE}$

$\theta_0 = 2\tan^{-1}\frac{mg}{qE}$

Q. 30: Short electric dipole of dipole moment P is placed at the centre of a ring of radius R having charge Q uniformly distributed on its circumference. The dipole moment vector is along the axis of the ring. Find force on the dipole due to the ring.

P

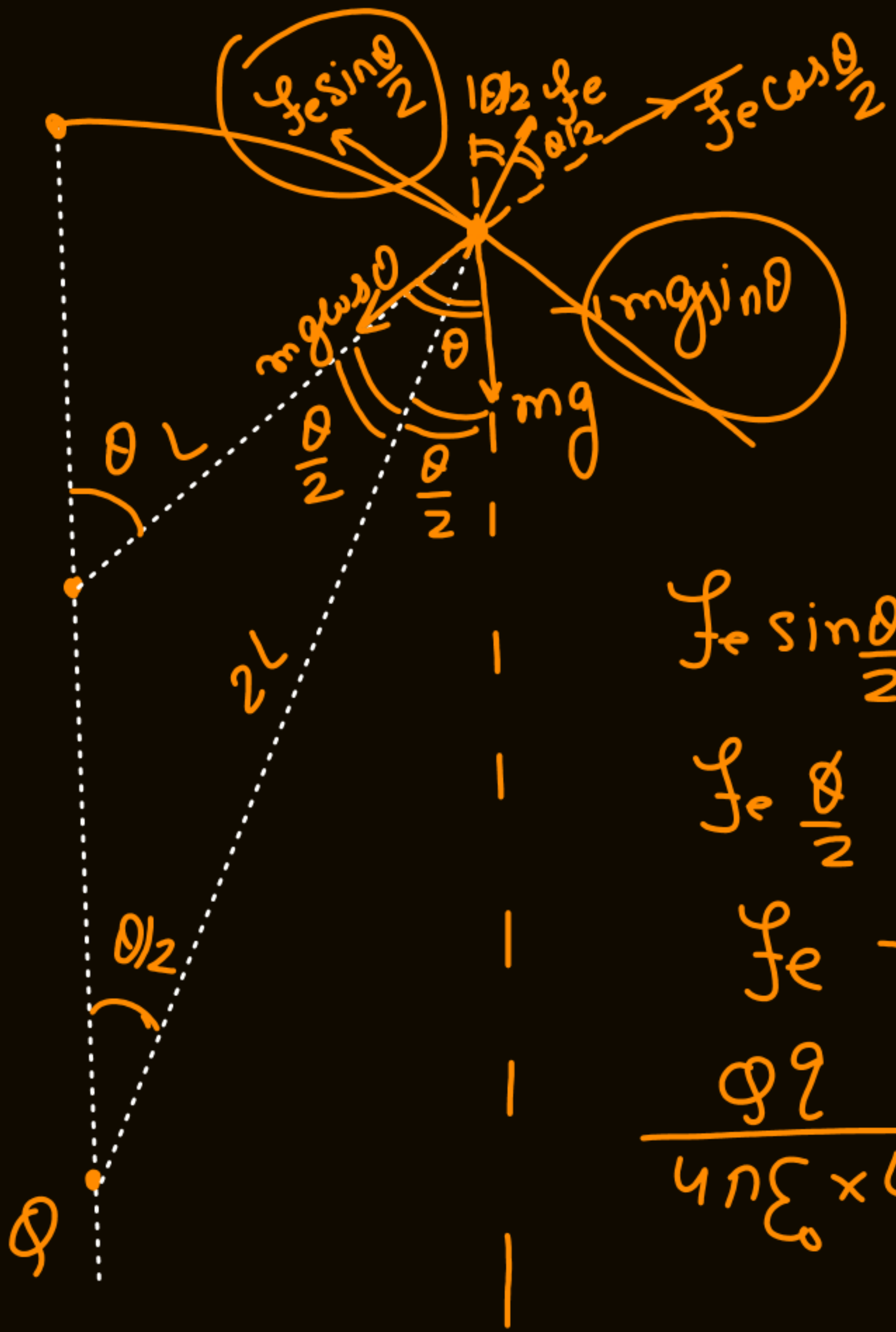
$q\,2x = P$



$$f = \frac{q k Q x}{R^3}$$

$$2f = f_{net} = \frac{2kQxq}{R^3} = \frac{kQP}{R^3}$$

$$\frac{kP}{R^3}\int dQ = \frac{kP}{R^3} Q$$



Q. 33: A particle of mass m and charge q is attached to a light insulating thread of length L. The other end of the thread is secured at point O. Exactly below point O, there is a small ball having charge Q fixed on an insulating horizontal surface. The particle remains in equilibrium vertically above the ball with the string taut. Distance of the ball from point O is L. Find the minimum value of Q for which the particle will be in a stable equilibrium for any gentle horizontal push given to it.

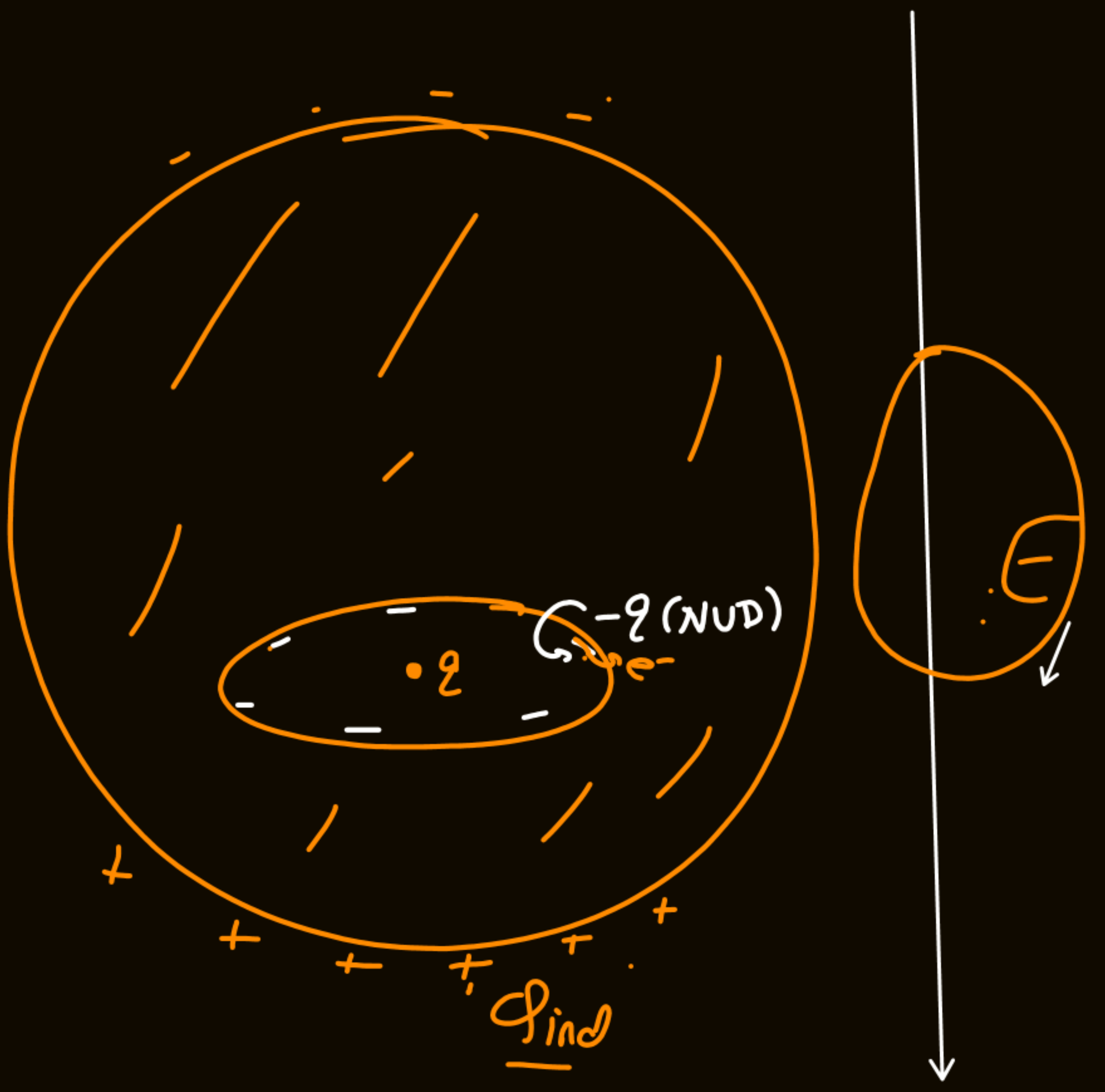
$$F_e \sin\frac{\theta}{2} > mg\sin\theta$$

$$F_e \frac{\theta}{2} > mg\,\theta$$

$$F_e > 2mg$$

$$\frac{Qq}{4\pi\varepsilon_0 \times 4L^2} > 2mg$$

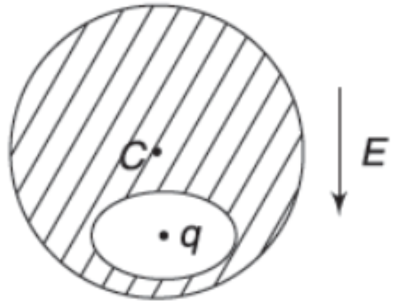
$$Q > \frac{32 mg \pi \varepsilon_0 L^2}{q}$$

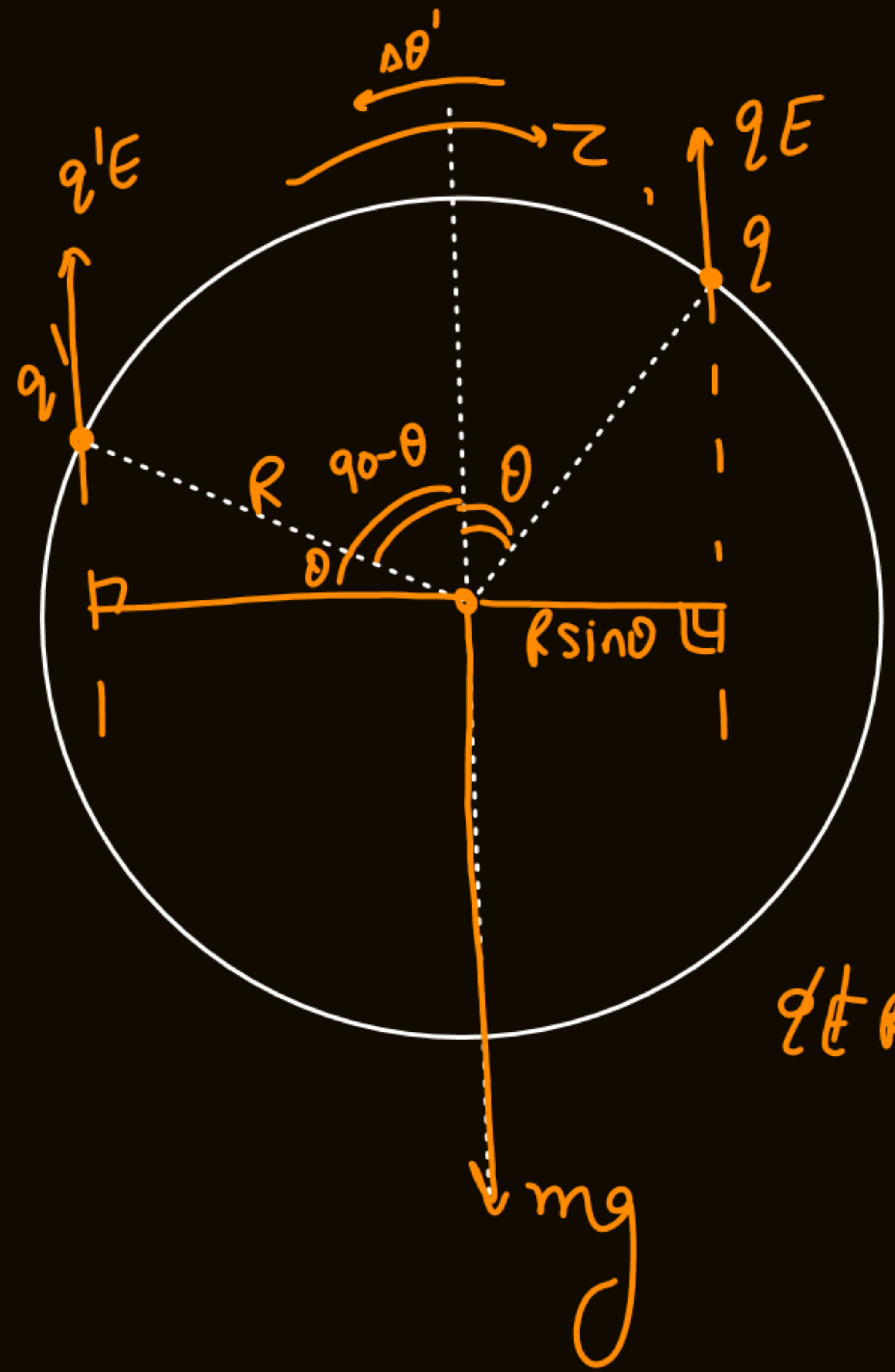


Q. 42: A neutral spherical conductor has a cavity. A point charge q is located inside it. It is in equilibrium. An external electric field (E) is switched on that is directed parallel to the line joining the centre of the sphere to the point charge.

(a) What is the direction of acceleration of the charge particle inside the cavity after E is switched on.

(b) How is the induced charge on the wall of the cavity affected due to the external field.

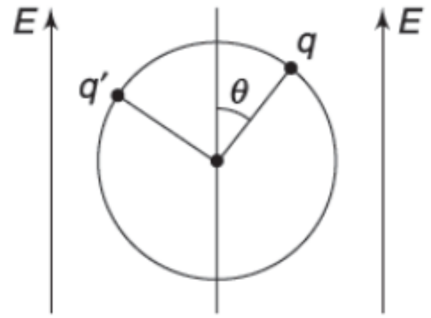




Q. 55: A uniform non conducting ring has mass m and radius R. Two point charges q and q' are fixed on its circumference at a separation of $\sqrt{2}R$. The ring remains in equilibrium in air with its plane vertical in a region where exists a uniform vertically upward electric field E. Given $E = \dfrac{4mg}{7q}$.

(a) Find angle θ in equilibrium position (see figure).

(b) The ring is given a small rotation in the plane of the figure and released. Will it perform oscillations?



$$qE + q'E = mg$$

$$(q+q')\frac{4mg}{7q} = mg$$

$$q' = \frac{7q}{4} - q$$

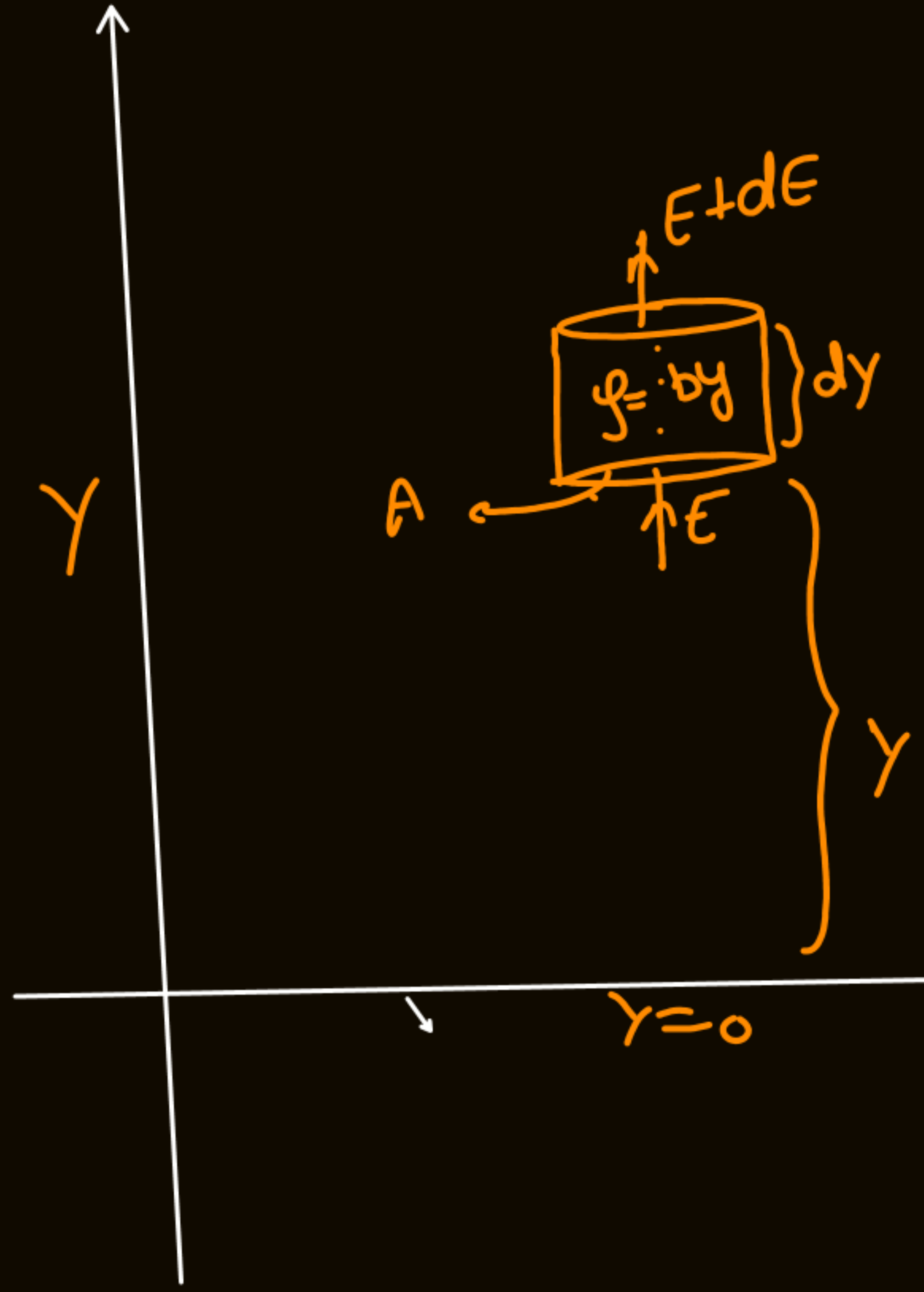
$$= \frac{3q}{4}$$

$$qE\,R\sin\theta = \frac{3q}{4}E\,R\cos\theta$$

$$\tan\theta = 3/4$$

$$\boxed{\theta = 37^\circ}$$

Q. 59: In an insulating medium (dielectric constant = 1) the charge density varies with y Co-ordinate as $\rho = by$, Where b is a positive constant. The electric field is zero at $y = 0$ and everywhere else it is along y direction. Calculate the electric field as a function of y.



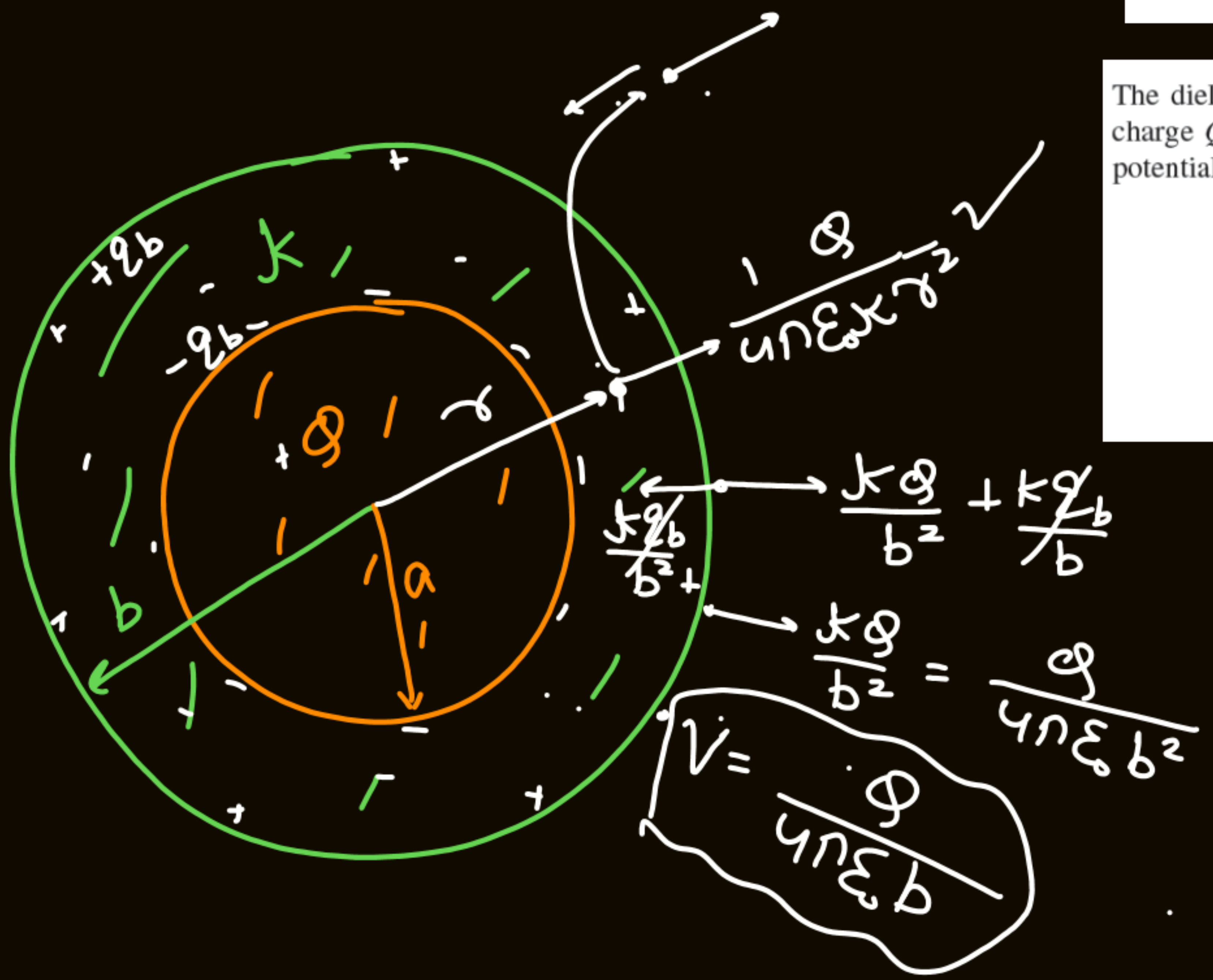
$$\oint = (E+dE)A - EA$$

$$= dE \cdot A = (by)\, A\, dy$$

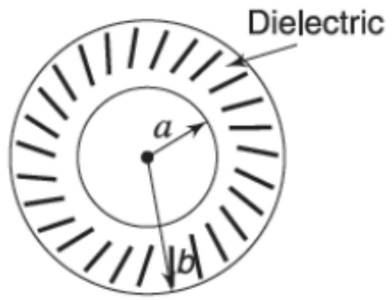
$$\int_0^E dE = \int_0^y by\, dy$$

$$E = \frac{by^2}{2}$$

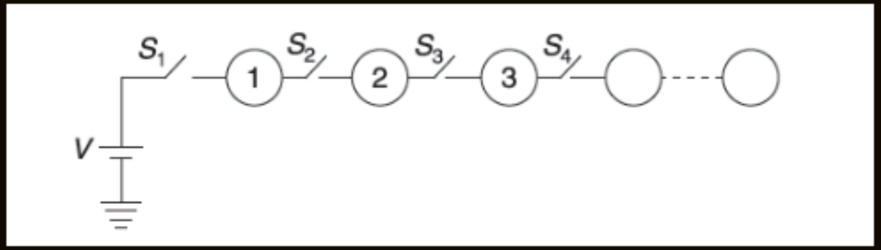
Q. 88: Conducting ball of radius a is surrounded by a layer of dielectric having inner radius a and outer radius b.



The dielectric constant is K. The conducting ball is given a charge Q. Write the magnitude of electric field and electric potential at the outer surface of the dielectric.



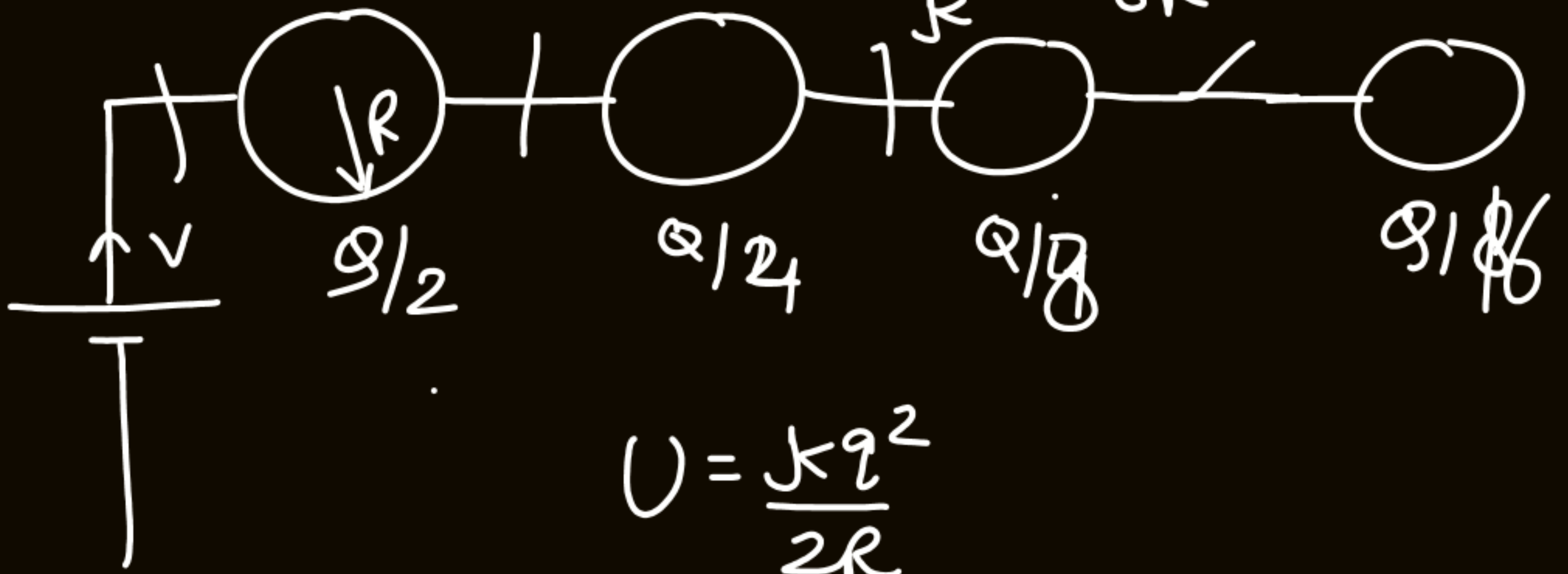
Q. 105: Large number of identical conducting spheres have been laid as shown in figure. Radius of each sphere is R and all of them are uncharged. Switch S_1 is closed to connect sphere 1 to the positive terminal of a V volt cell whose other terminal is grounded. After some time switch S_1 is opened and S_2 is closed. Thereafter, S_2 is opened and S_3 is closed, next S_3 is opened and S_4 is closed. The process is continued till the last switch is closed. Consider the cell and spheres to be your system and calculate the loss in energy of the system in the entire process.



$$W_b = V\left(\frac{RV}{k}\right) = \frac{RV^2}{k}$$

$$W_b = \Delta U + \text{Heat}$$

$$\frac{RV^2}{k} = \frac{k}{6R}\left(\frac{RV}{k}\right)^2 + \text{Heat}$$



$$\text{Heat} = \frac{5RV^2}{6k}$$

$$\frac{kQ}{R} = V$$

$$Q = \frac{RV}{k}$$

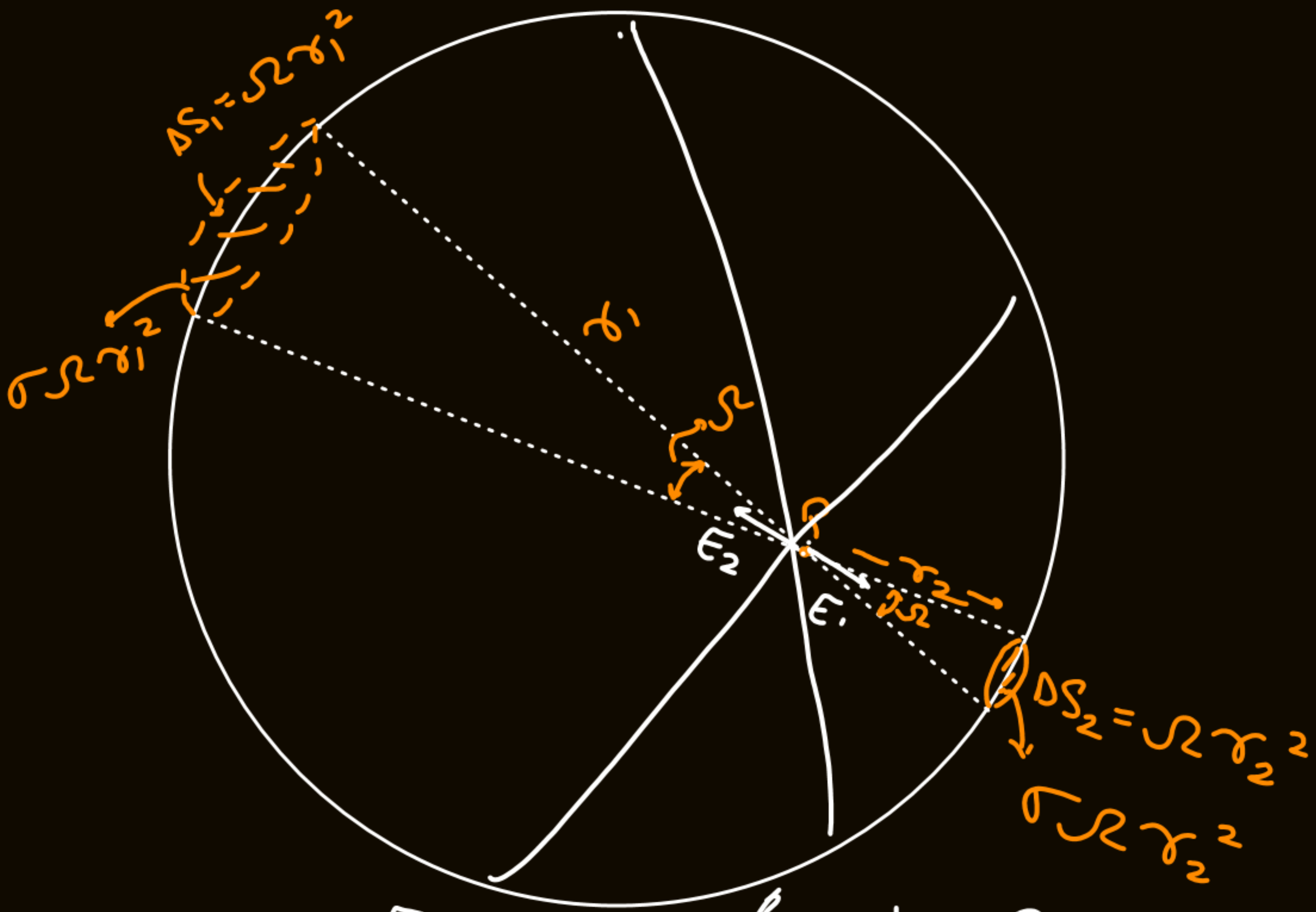
$$U = \frac{kq^2}{2R}$$

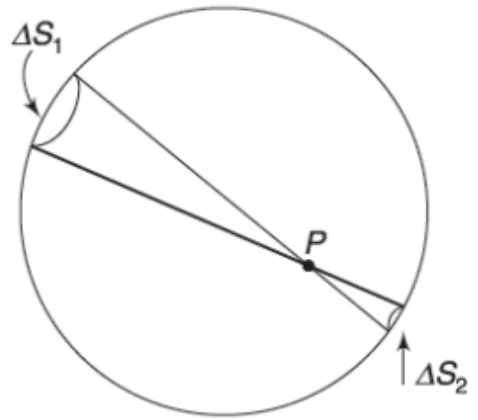
$$U = \frac{k(Q/2)^2}{2R} + \frac{k(Q/4)^2}{2R} + \frac{k(Q/8)^2}{2R} + \frac{k(Q/16)^2}{2R} + \dots$$

$$= \frac{kQ^2}{8R}\left\{1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots\right\} = \frac{kQ^2}{8R}\cdot\frac{1}{1-1/4}$$

$$= \frac{kQ^2}{8R} \times \frac{4}{3} = \frac{kQ^2}{6R}$$

Q. 111: Consider a uniformly charged spherical shell. Two cones having same semi vertical angle, and their common apex at P, intercept the shell. The intercepts have area $\Delta S1$ and $\Delta S2$. For a cone of very small angle, $\Delta S1$ and $\Delta S2$ will be very small and charge on them can be regarded as point charge for the purpose of writing electric field at point P. Prove that the charge on $\Delta S1$ and $\Delta S2$ produce equal and opposite field at P. Hence, argue that field at all points inside the uniformly charged spherical shell is zero.





$$E_1 = \frac{k\sigma\Omega r_1^2}{r_1^2} = k\sigma\Omega$$

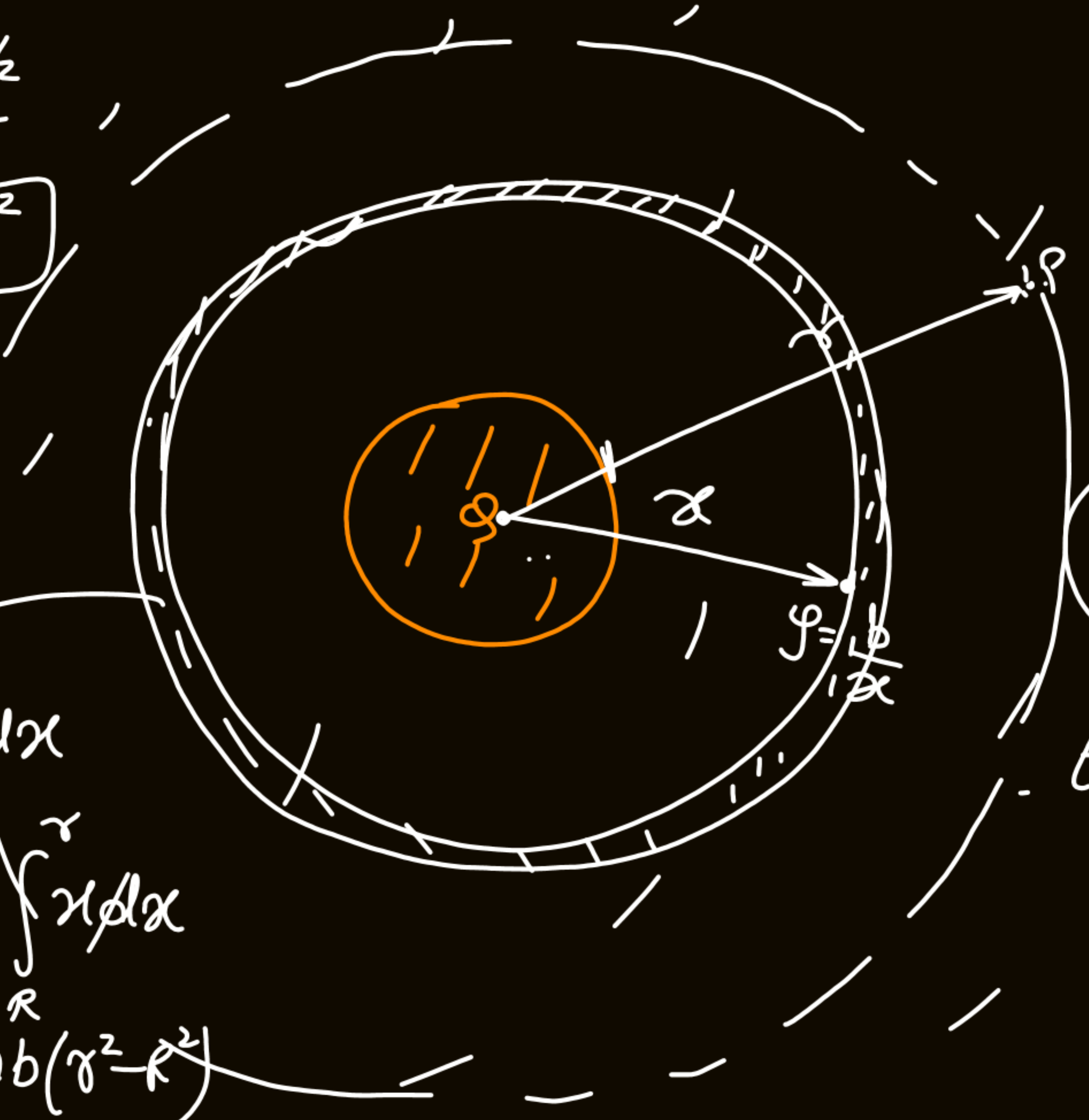
$$E_2 = \frac{k\sigma\Omega r_2^2}{r_2^2} = k\sigma\Omega$$

Q. 123: A ball of radius R has a uniformly distributed charge Q. The surrounding space of the ball is filled with a volume charge density $\rho = \frac{b}{r}$, where b is a constant and r is the distance from the centre of the ball.

It was found that the magnitude of electric field outside the ball is independent of distance r.

(a) Find the value of Q.

(b) Find the magnitude of electric field outside the ball.



$$\frac{Q}{\pi\varepsilon_0} = \frac{2bR^2}{\varepsilon_0}$$

$$Q = 2\pi b R^2$$

$$\frac{b}{x} 4\pi x^2 dx$$

$$Q' = 4\pi b \int_R^r x\, dx = 2\pi b (r^2 - R^2)$$

$$Q = \oint \vec{E}\cdot d\vec{s} = E\pi r^2 = \frac{\{Q + 2\pi b(r^2 - R^2)\}}{\varepsilon_0}$$

$$E = \frac{Q}{\pi r^2 \varepsilon_0} + \frac{2b}{\varepsilon_0} - \frac{bR^2}{r^2\varepsilon_0}$$

$$= \left(\frac{Q}{\pi\varepsilon_0} - \frac{bR^2}{\varepsilon_0}\right)\frac{1}{r^2} + \frac{2b}{\varepsilon_0}$$

$$E = \frac{2b}{\varepsilon_0}$$