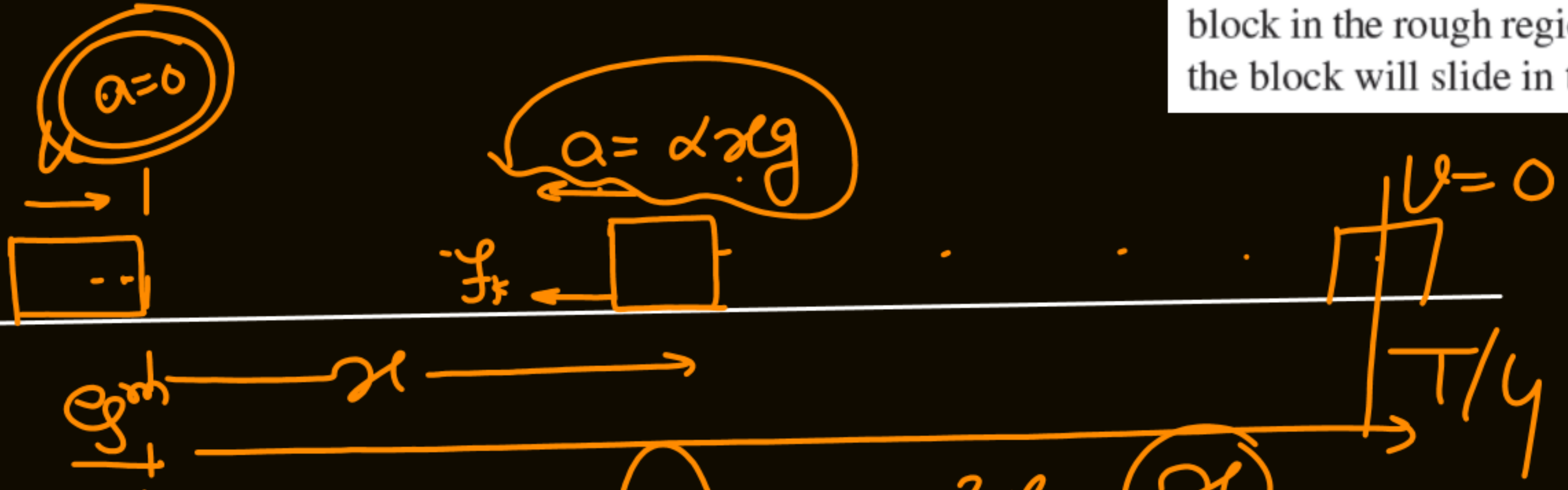
A block of mass m is moving along positive x direction on a smooth horizontal surface with velocity u. It enters a rough horizontal region at $x = 0$. The coefficient of friction in this rough region varies according to $\mu = ax$, where 'a' is a positive constant and x is displacement of the block in the rough region. Find the time for which the block will slide in this rough region.

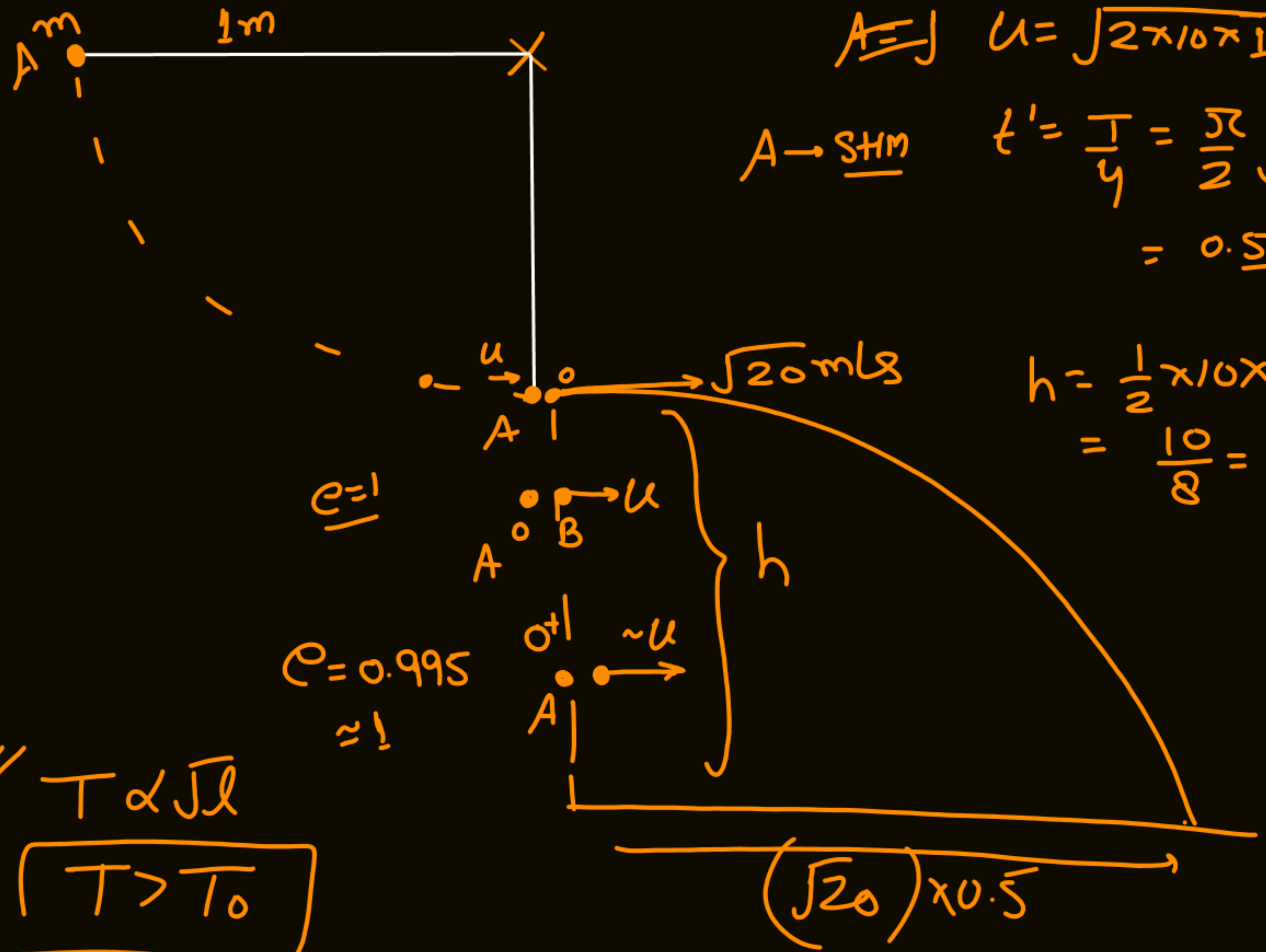
$$f_k = \mu mg = \alpha x mg = ma \qquad a = \alpha x g$$



$$a = \alpha g x = \omega^2 x$$

$$\alpha g \rightarrow \omega^2 \qquad \omega = \sqrt{\alpha g}$$

$$t = \frac{T}{4} = \frac{1}{4} \times \frac{2\pi}{\omega} = \frac{\pi}{2\sqrt{\alpha g}}$$



A→J $u = \sqrt{2 \times 10 \times 1} = \sqrt{20}$

A → SHM $t' = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{l}{g}}$

$= 0.5$ sec

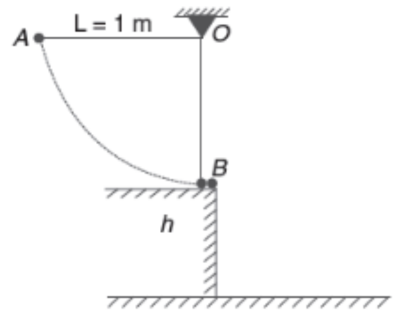
$h = \frac{1}{2} \times 10 \times (0.5)^2$

$= \frac{10}{8} = 1.25$ m

✓ $T \propto \sqrt{l}$

$\boxed{T > T_0}$

(i) A small steel ball (B) is at rest on the edge of a table of height h. Another identical steel ball (A) is tied to a light string of length $L = 1.0$ m and is released from the position shown so that it swings like a pendulum. At the lowest position of its path it hits the ball B which is at rest. Ball B flies off the table and hits the ground in time t. After collision the ball A keeps moving for a time t' before coming to rest for the first time. Find the value of h if $t = t'$. Collision between the balls is head on and coefficient of restitution is $e = 0.995$.

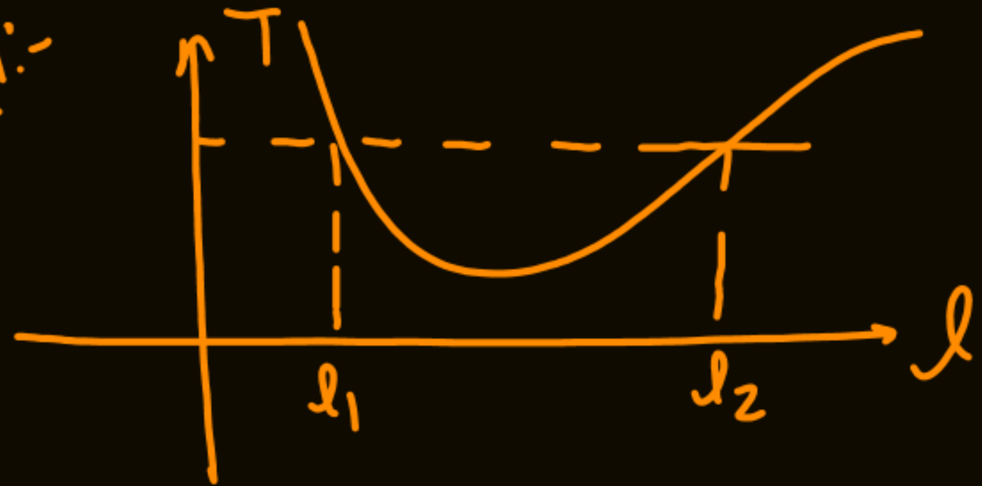


(ii) A pendulum has a particle of mass m attached to a massless rod of length L. The rod is released from a position where it makes an angle $\theta_0 \left(< \frac{\pi}{2}\right)$ with the vertical. The time period of oscillation is observed to be T_0. Another similar pendulum has a rod of length $2L$. Time period of this pendulum when released from position θ_0 is T. Which is larger T or T_0?

A rigid body is to be suspended like a physical pendulum so as to have a time period of $T = 0.2\pi$ second for small amplitude oscillations. The minimum distance of the point of suspension from the centre of mass of the body is $l_1 = \cancel{0.4}\,m$ 4cm to get this time period. Find the maximum distance (l_2) of a point of suspension from the centre of mass of the body so as to get the same time period. $[g = 10\ m/s^2]$

4cm = 0.04m

Sol:-



$$T = 2\pi\sqrt{\frac{I_H}{mgl}}$$

$$0.2\pi = 2\cancel{\pi}\sqrt{\frac{I_{com} + ml^2}{mgl}}$$

$$0.01\,mgl = I_{com} + ml^2$$

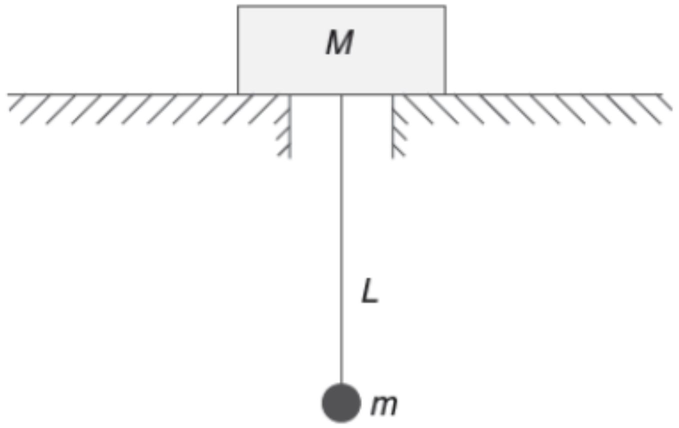
$$1\;l^2 - 0.01\,g\,l + \frac{I_{com}}{m} = 0 \;\to\; l_1,\ l_2$$

$$l_1 + l_2 = 0.01\,g = 0.01 \times 10 = 0.1$$

$$0.04\,m + l_2 = 0.1$$

$$\boxed{l_2 = 0.06\,m}$$

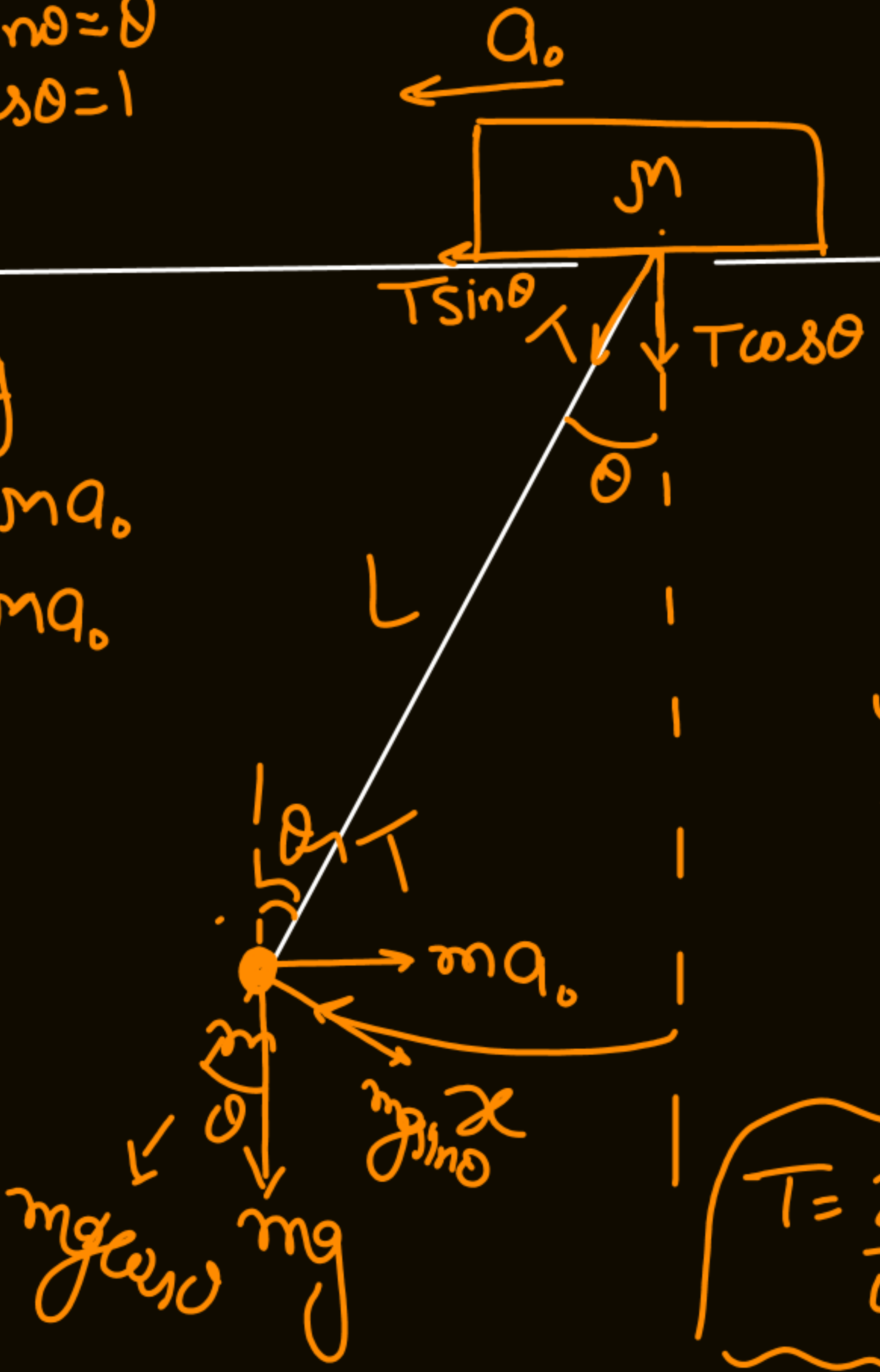
A block of mass M rests on a smooth horizontal table. There is a small gap in the table under the block through which a pendulum has been attached to the block. The bob of the simple pendulum has mass m and length of the pendulum is L. The pendulum is set into small oscillations in the vertical plane of the figure. Calculate its time period. The table does not interfere with the motion of the string.



$\theta = \frac{x}{L}$

$\sin\theta \simeq \theta$

$\cos\theta = 1$



$T \simeq mg$

$T\sin\theta = Ma_0$

$mg\theta = Ma_0$

$\Omega = \frac{d\theta}{dt}$

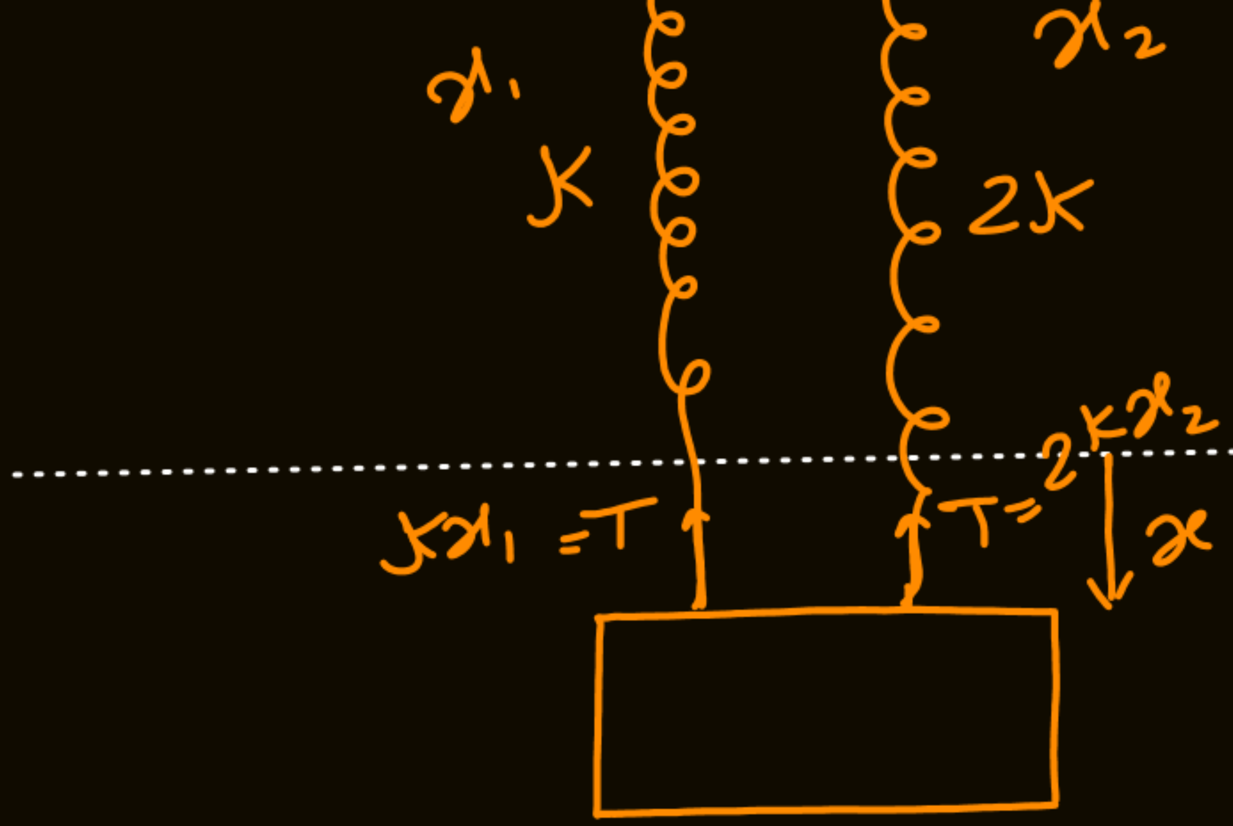
$F_{Restoring} = ma_0 + mg\theta$

$F_{Restoring} = \frac{m^2 g}{M}\theta + mg\theta$

$F = \frac{(m^2 + Mm)gx}{ML} = ma$

$\boxed{a = \left(\frac{M+m}{ML}\right)gx}$ ω^2

$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{ML}{(M+m)g}}}$

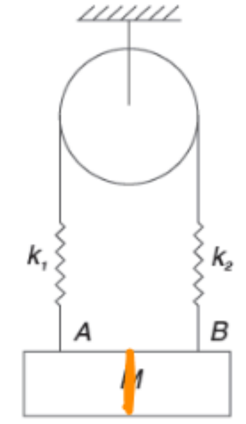


$x_1 + x_2 = 2x$ — (2)

$2x_2 + x_2 = 2x$

$$\boxed{x_2 = \frac{2x}{3} \qquad x_1 = \frac{4x}{3}}$$

of the block (represented by line AB) always remains horizontal.



$a = \frac{8K}{3M} x$ → ω^2

$$F_{Restoring} = 2T = 2Kx_1 = 2K \times \frac{4x}{3} = \frac{8Kx}{3} = Ma$$

$$\omega = \sqrt{\frac{8K}{3M}}$$

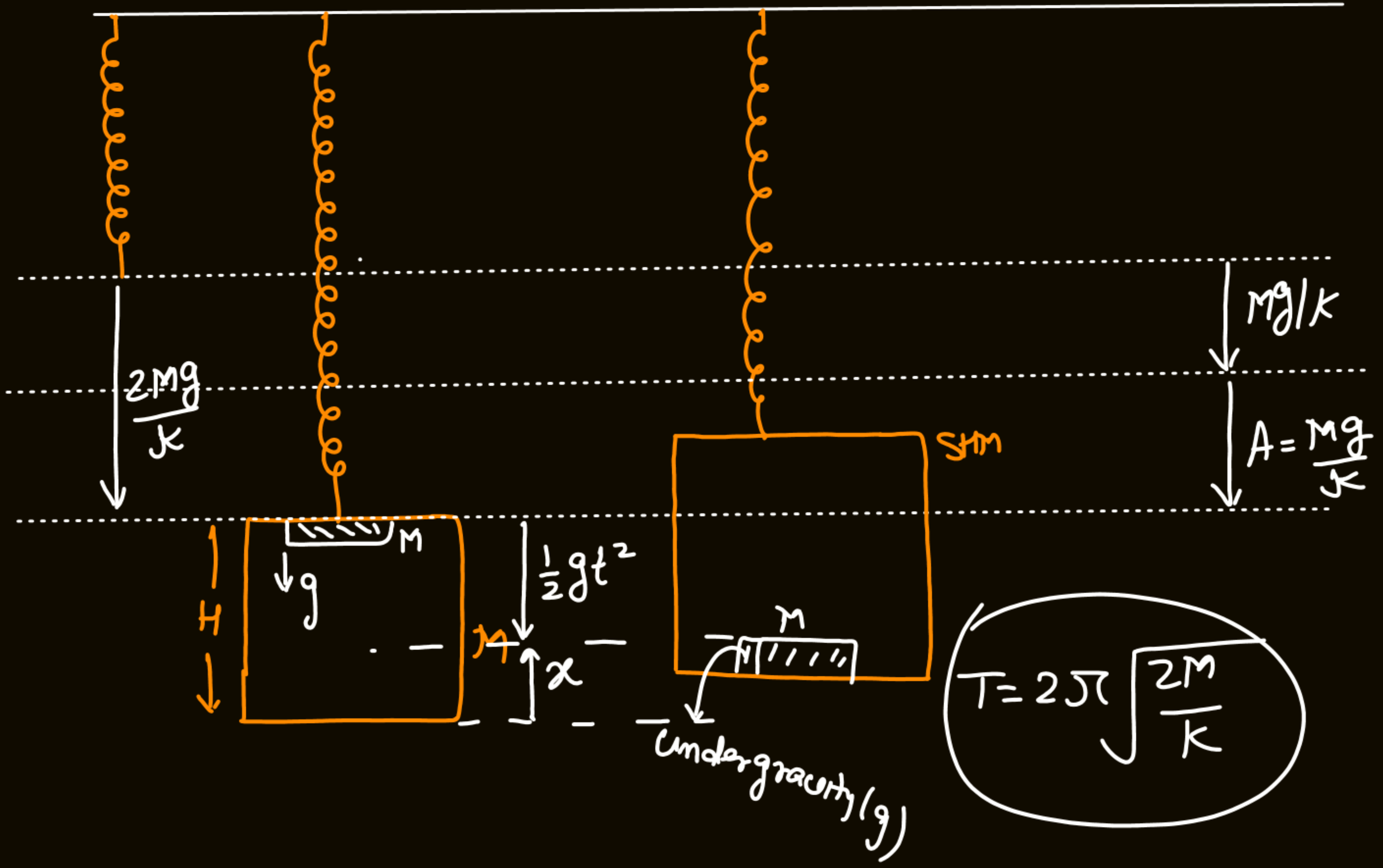
$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3M}{8K}}}$$

$$T = 2\pi\sqrt{\frac{\mu}{K_{eq}}}$$

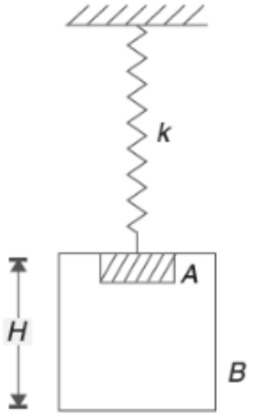
$$= 2\pi\sqrt{\frac{M \times 3}{4 \times 2K}}$$

$$K_{eq} = \frac{k_1 k_2}{k_1 + k_2} = \frac{K \times 2K}{3K} = \frac{2K}{3}$$

$$\mu = \frac{\frac{M}{2} \times \frac{M}{2}}{M} = \frac{M}{4}$$



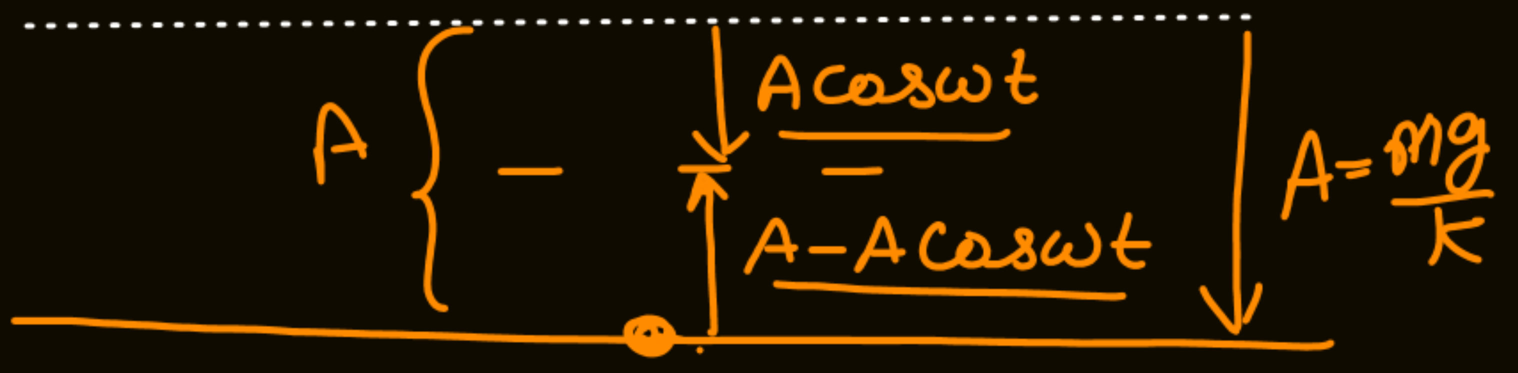
A box B of mass M hangs from an ideal spring of force constant k. A small particle, also of mass M, is stuck to the ceiling of the box and the system is in equilibrium. The particle gets detached from the ceiling and falls to strike the floor of the box. It takes time 't' for the particle to hit the floor after it gets detached from the ceiling. The particle, on hitting the floor, sticks to it and the system thereafter oscillates with a time period T. Find the height H of the box if it is given that $t = \frac{T}{6\sqrt{2}}$. Assume that the floor and ceiling of the box always remain horizontal.



$$\frac{1}{2}gt^2 + x = H$$

$$\frac{1}{2}g\left(\frac{T}{6\sqrt{2}}\right)^2 + \frac{mg}{k}\left(1-\cos\sqrt{\frac{k}{m}}\cdot\frac{T}{6\sqrt{2}}\right) = H$$

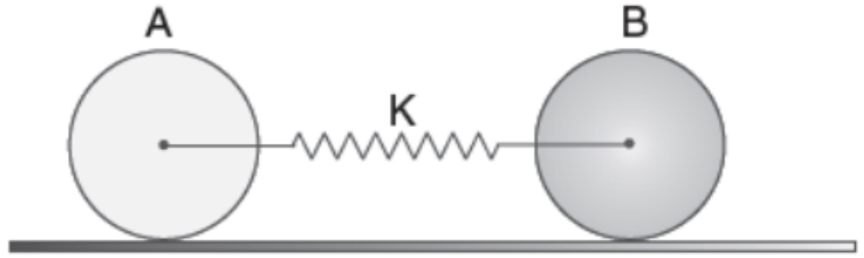
$$\frac{1}{2}g\,\frac{8\pi^2 m}{72k} + \frac{mg}{k}\left\{1-\cos\sqrt{\frac{k}{m}}\,\frac{2\pi}{6\sqrt{2}}\sqrt{\frac{2m}{k}}\right\}$$

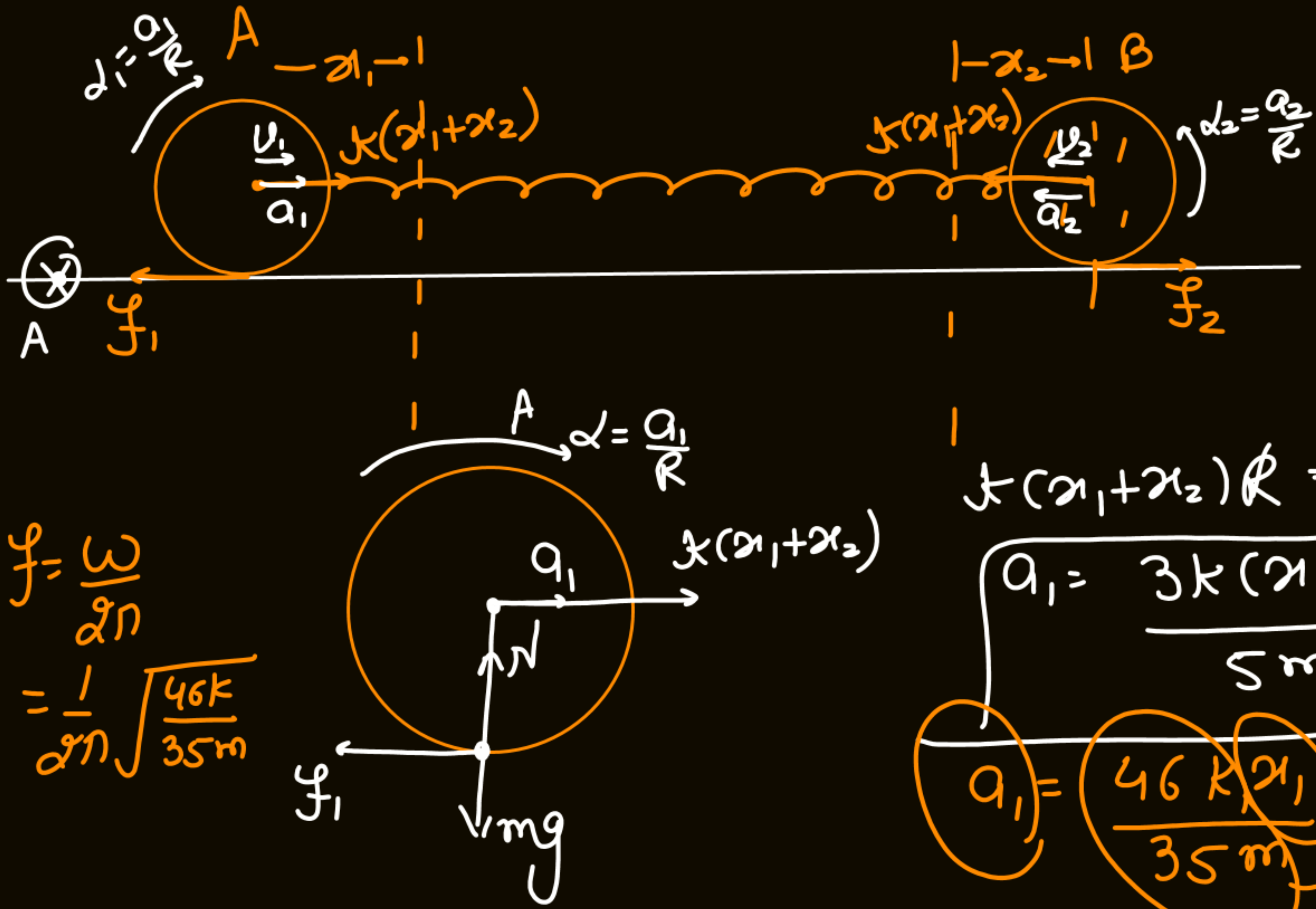


$$\frac{m\pi^2 g}{18k} + \frac{mg}{k}\left\{1-\frac{1}{2}\right\}$$

$$\boxed{\frac{m\pi^2 g}{18k} + \frac{mg}{2k}}$$

Two spheres A and B of the same mass m and the same radius are placed on a rough horizontal surface. A is a uniform hollow sphere and B is uniform solid sphere. They are tied centrally to a light spring of spring constant k as shown in figure. A and B are released when the extension in the spring is x_0. Friction is sufficient and the spheres do not slip on the surface. Find the frequency and amplitude of SHM of the sphere A.



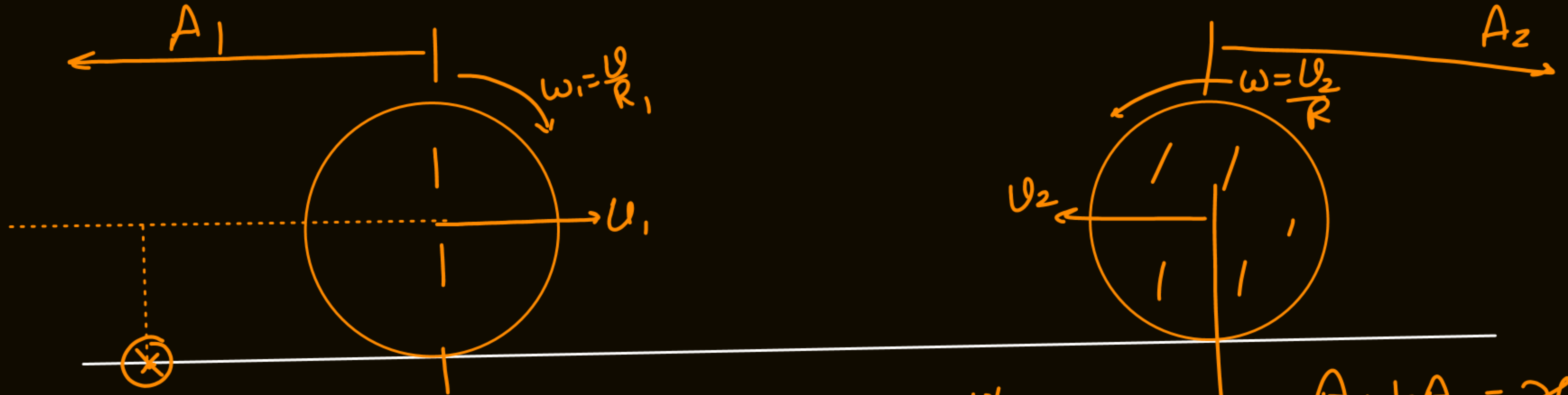


$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{46k}{35m}}$$

$$k(x_1+x_2)R = \left(\frac{5}{3}mR^2\right)\frac{a_1}{R}$$

$$\boxed{a_1 = \frac{3k(x_1+x_2)}{5m}} = \frac{3k}{5m}\left(x_1 + \frac{25x_1}{21}\right)$$

$$a_1 = \frac{46k\,x_1}{35m} \rightarrow \omega^2 \qquad \omega = \sqrt{\frac{46k}{35m}}$$



$$A_1 + A_2 = x_0 \quad \text{—(1)}$$

$$\cancel{m} v_1 \cancel{R} + \frac{2}{3} \cancel{m} R^{\cancel{2}} \frac{v_1}{\cancel{R}} = \cancel{m} v_2 \cancel{R} + \frac{2}{5} \cancel{m} R^{\cancel{2}} \frac{v_2}{\cancel{R}}$$

$$25 A_1 = 21 A_2 \quad \text{—(2)}$$

$$\frac{5}{3} v_1 = \frac{7}{5} v_2$$

$$25 v_1 = 21 v_2$$

$$\boxed{25 x_1 = 21 x_2}$$

$$A_2 = \frac{25 x_0}{46}$$

$$A_1 + \frac{25 A_1}{21} = x_0$$

$$\boxed{A_1 = \frac{21 x_0}{46}}$$