competishun *Power of real gurus*

MATHEMATICS

TARGET : JEE- Advanced 2021

CAPS-8

Complex Number

ANSWER KEY OF CAPS-8

1.	(D)	**2.**	(D)	**3.**	(D)	**4.**	(D)	**5.**	(D)
6.	(C)	**7.**	(ABCD)	**8.**	(AB)	**9.**	(ACD)	**10.**	(ABCD)
11.	(3)	**12.**	(3)	**13.**	(3)	**14.**	(0)	**15.**	(0)
16.	(99)	**17.**	(7)	**18.**	(32)	**19.**	(13)	**20.**	(25)
21.	(1)	**22.**	(16)	**23.**	(A)	**24.**	$\left|(z_1 - z_2)\right| \in \left[3\sqrt{2}, 13\sqrt{2}\right]$		
25.	$-\omega$ or $-\omega^2$								

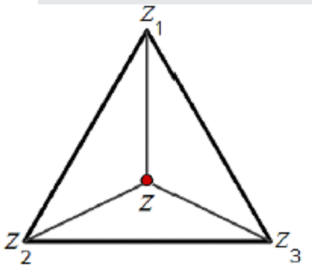
SCQ (Single Correct Type) :

1. The least value of $|z-3-4i|^2 + |z+2-7i|^2 + |z-5+2i|^2$ occurs when

(A) 1 + 3i (B) 3 + 3i (C) 3 + 4i (D) none of these

Ans. (D)

Sol.



Let z = x + iy

Then $|z-3-4i|^2 + |z+2-7i|^2 + |z-5+2i|^2$

$= 3\left(x^2+y^2-4x-9y\right)+107 = 3\left\{(x-2)^2+(y-3)^2\right\}+68$

The least value occurs when x = 2 and y = 3

z = 2 + 3i

Alternatively, $|Z-Z_1|^2 + |Z-Z_2|^2 + |Z-Z_3|^2$ will be minimum

when z be the centroid of the triangle formed by the points Z_1, Z_2 and Z_3

$\Rightarrow z = \dfrac{Z_1+Z_2+Z_3}{3}$

Here $z_1 = 3 + 4i$, $z_2 = -2 + 7i$, $z_3 = 5 - 2i$ $\Rightarrow z = 2 + 3i$

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2. $|z-3i+4|+|z-ki+4|=k$

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) 2

Ans. **(D)**

Sol. $|z-(-4+3i)|+|z-(-4+ki)|=k$

$\therefore$ PS + PS' = 2a represents an ellipse if SS' < 2a

$\Rightarrow |3-k|<k \quad \Rightarrow k>\frac{3}{2}$

3. If $|z|^2+\bar{A}z^2+A\bar{z}^2+B\bar{z}+\bar{B}z+c=0$ represents a pair of intersecting lines with angle of intersection θ then the value of |A| is

(A) $\tan\theta$ (B) $\cos\theta$ (C) $\sec\theta$ (D) $\frac{\sec\theta}{2}$

Ans. **(D)**

Sol. $z = x + iy$, $A = \alpha + i\beta \quad B = \gamma + \delta i$

$x^2 + y^2 + 2\alpha(x^2 - y^2) + 4xy\beta + \alpha x - \beta y + c = 0.$

$(1 + 2\alpha)x^2 + (1 - 2\alpha)y^2 + (4\beta)xy + \alpha x - \beta y + c = 0$

$\tan\theta = \frac{2\sqrt{4\beta^2-(1-4\alpha^2)}}{2}$

$\tan\theta = \sqrt{4\beta^2+4\alpha^2-1} = \sqrt{4|A|^2-1}$

$4|A|^2 = 1 + \tan^2\theta$

$|A| = \frac{\sec\theta}{2}$

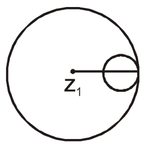
4. If $|z - z_1| = |z_1|$ and $|z - z_2| = |z_2|$ be the of two circles if the two circles touch each other then

(A) $\mathrm{Re}(z_1 z_2) = 0$ (B) $\mathrm{Re}\left(\frac{z_1}{z_2}\right) = 0$ (C) $I_m(z_1 z_2) = 0$ (D) $I_m\left(\frac{z_1}{z_2}\right) = 0$

Ans. **(D)**

Sol. $|z_1 - z_2| = ||z_1| - |z_2||$ or $|z_1 - z_2| = |z_1| + |z_2|$

$\therefore \ \mathrm{Arg}\left(\frac{z_1}{z_2}\right) = 0, \pm\pi$



$\mathrm{Im}\left(\frac{z_1}{z_2}\right) = 0$

5. If $|z_1| = 2$, $|z_2| = 3$, $|z_3| = 4$ and $|2z_1 + 3z_2 + 4z_3| = 4$, then absolute value of $8z_2z_3 + 27z_3z_1 + 64z_1z_2$ equals

(A) 24 (B) 48 (C) 72 (D) 96

Ans. **(D)**

Sol. $8\, z_2\, z_3 + 27 z_3\, z_1 + 64\, z_1\, z_2 = 2.\bar{z}_1\, z_1\, z_2\, z_3 + 3.\bar{z}_2\, z_2\, z_3\, z_1 + 4\, \bar{z}_3\, z_1\, z_2\, z_3$

$(\because 4 = z_1\bar{z}_1, 9 = z_2\bar{z}_2, 16 = \bar{z}_3 z_3)$

$= (2\bar{z}_1 + 3\bar{z}_2 + 4\bar{z}_3)\,(z_1\, z_2\, z_3) = \overline{(2z_1 + 3z_2 + 4z_3)}\,(z_1\, z_2\, z_3)$

So absolute value = 4. 2. 3. 4 = 96

6. If z = x + i y then the equation of a straight line Ax + By + C = 0 where A, B, C $\in$ R, can be written on the complex plane in the form $\bar{a}z + a\bar{z} + 2C = 0$ where 'a' is equal to :

(A) $\frac{(A + iB)}{2}$ (B) $\frac{A - iB}{2}$ (C) A + i B (D) none

Ans. **(C)**

Sol. $A\left(\frac{z+\bar{z}}{2}\right) + B\left(\frac{z-\bar{z}}{2i}\right) + C = 0$

$\Rightarrow z\left[\frac{A}{2} + \frac{B}{2i}\right] + \bar{z}\left(\frac{A}{2} - \frac{B}{2i}\right) + C = 0 \Rightarrow z(A - iB) + \bar{z}(A + iB) + 2C = 0$

MCQ (One or more than one correct) :

7. If from a point P, representing the complex number z_1, on the curve $|z| = 2$, two tangents are drawn to the curve $|z| = 1$, meeting the curve at points $Q(z_2)$ and $Q(z_3)$, then _____.

(A) the complex number $\frac{z_1 + z_2 + z_3}{3}$ will lie on the curve $|z| = 1$

(B) $\left(\frac{4}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right)\left(\frac{4}{\bar{z}_1} + \frac{1}{\bar{z}_2} + \frac{1}{\bar{z}_3}\right) = 9$

(C) $\arg\left(\frac{z_2}{z_3}\right) = \frac{2\pi}{3}$

(D) orthocenter and circumcenter of ΔPQR will coincide

Ans. **(ABCD)**

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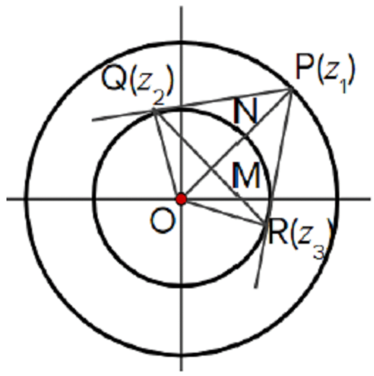

Sol.

Since OQ = 1 and OP = 2,

so sin ($\angle$OPQ) = $\frac{1}{2}$ and hence $\angle$QPR = $\frac{\pi}{3}$

Then, ΔPQR is equilateral

Also, OM $\perp$ QR

Then from ΔOMQ, OM = $\frac{1}{2}$

Then centroid of ΔPQR lies on |z| = 1

As PQR is an equilateral triangle,

so orthocentre, circumcentre and centroid will coincide.

Now, $\left|\frac{z_1+z_2+z_3}{3}\right| = 1 \Rightarrow |z_1+z_2+z_3|^2 = 9$

$\Rightarrow (z_1+z_2+z_3)(\bar{z}_1+\bar{z}_2+\bar{z}_3) = 9 \quad \Rightarrow \left(\frac{4}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right)\left(\frac{4}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right) = 9$

and $\angle$QOP = 120º

8. If z_1, z_2, z_3 are any three roots of the equation $z^6 = (z+1)^6$, , then arg $\left(\frac{z_1-z_3}{z_2-z_3}\right)$ can be equal to ______.

(A) 0 (B) π (C) $\frac{\pi}{4}$ (D) $-\frac{\pi}{4}$

Ans. **(AB)**

Sol. $z^6 = (z + 1)^6$

$\Rightarrow |z^6| = |(z+1)^6|$

$\Rightarrow |z| = |z+1|$

$\therefore$ z lies on the line $x = -\frac{1}{2}$

$\therefore$ Roots of z^6 = (z + 1) are collinear

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9. Z_1, Z_2 are two complex numbers satisfying $i|z_1|^2 z_2 - |z_2|^2 z_1 = z_1 - iz_2$. Then which of the following is/are correct.

(A) $\text{Re}\left(\frac{z_1}{z_2}\right) = 0$ (B) $I_m\left(\frac{z_1}{z_2}\right) = 0$

(C) $z_1\bar{z}_2 + \bar{z}_1 z_2 = 0$ (D) $|z_1||z_2| = 1$ or $|z_1| = |z_2|$

Ans. **(ACD)**

Sol. $iz_2\left(1+|z_1|^2\right) = z_1\left(1+|z_2|^2\right)$

or, $\frac{z_1}{z_2}$ is purely imaginary

$\Rightarrow \text{Re}\left(\frac{z_1}{z_2}\right) = 0$

Also, $\frac{z_1}{z_2} = \frac{-\bar{z}_1}{\bar{z}_2}$ i.e. $z_1\bar{z}_2 + z_2\bar{z}_1 = 0$

As $i|z_1|^2 z_2 - |z_2|^2 z_1 = z_1 - iz_2$

or, $iz_1\bar{z}_1 z_2 - z_2\bar{z}_2 z_1 = z_1 - iz_2$

or, $\left(\bar{z}_1 z_2 + i\right)\left(iz_1 + z_2\right) = 0$

$\Rightarrow$ either $\bar{z}_1 z_2 = -i$ or $iz_1 + z_2 = 0$

$|z_1||z_2| = 1$ or $|z_1| = |z_2|$

10. The curve represented by $z = \frac{3}{2+\cos\theta + i\sin\theta}$, $\theta \in [0, 2\pi)$

(A) never meets the imaginary axis (B) meets the real axis in exactly two points

(C) has maximum value of |z| as 3 (D) has minimum value of |z| as 1

Ans. **(ABCD)**

Sol. $z = \frac{3}{2+\cos\theta + i\sin\theta} = \frac{3(2+\cos\theta - i\sin\theta)}{(2+\cos\theta)^2 + \sin^2\theta} = \frac{3.(2+\cos\theta - i\sin\theta)}{5+4\cos\theta}$

For imaginary axis, real part = 0 i.e. $2 + \cos\theta = 0$ which is not possible, so curve never meets the imaginary axis

For real axis Im z = 0 $\Rightarrow \sin\theta = 0 \Rightarrow \theta = 0, \pi \in [0, 2\pi)$, so curve meets the real axis in two points.

$|z| = 3.\frac{\sqrt{(2+\cos\theta)^2 + (\sin\theta)^2}}{5+4\cos\theta} = 3(5 + 4\cos\theta)^{-1/2} \Rightarrow |z|_{max} = 3, |z|_{min} = 1$

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Numerical based Questions :

11. Let z_1, z_2, z_3 be three complex number such that $|z_1| = |z_2| = |z_3| = 1$ and $\frac{z_1^2}{z_2 z_3} + \frac{z_2^2}{z_1 z_3} + \frac{z_3^2}{z_1 z_2} + 1 = 0$, then sum of possible values of $|z_1 + z_2 + z_3|$ is _____.

Ans. **(3)**

Sol. $\frac{z_1^2}{z_2 z_3} + \frac{z_2^2}{z_1 z_3} + \frac{z_3^2}{z_1 z_2} + 1 = 0$

$\Rightarrow z_1^3 + z_2^3 + z_3^3 + z_1 z_2 z_3 = 0$

$\Rightarrow (z_1 + z_2 + z_3)\left(z_1^2 + z_2^2 + z_3^2 - z_1 z_2 - z_3 z_1 - z_2 z_3\right) = -4 z_1 z_2 z_3$

$\Rightarrow (z_1 + z_2 + z_3)\left((z_1 + z_2 + z_3)^2 - 3 z_1 z_2 z_3 (\bar{z}_1 + \bar{z}_2 + \bar{z}_3)\right) = -4 z_1 z_2 z_3$

$\Rightarrow (z_1 + z_2 + z_3)^3 = z_1 z_2 z_3 \left(-4 + 3|z_1 + z_2 + z_3|^2\right)$

$\Rightarrow |z_1 + z_2 + z_3|^3 = \left|3|z_1 + z_2 + z_3|^2 - 4\right|$

$\Rightarrow |z_1 + z_2 + z_3| = 1$ or 2

12. If the vertices of a triangle ABC, A (z_1), B(z_2) and C(z_3) lie on the circle $|z - 3| = 16$, such that $z_1 + z_2 + z_3 = 9$, then the value of $\frac{z_1 \tan A + z_2 \tan B + z_3 \tan C}{\tan A \tan B \tan C}$ is equal to _____.

Ans. **(3)**

Sol. $\frac{z_1 + z_2 + z_3}{3} = 3$

Clearly centroid coincides with circumcentre, hence ΔABC is equilateral

13. If ω is a non-real complex root of $z^{28} = 1$ and such that $|\omega + 1|$ is maximum and $x = \frac{1}{2}\left|\omega - \frac{1}{\omega}\right|$, then $8x^4 + 4x^3 - 8x^2 - 3x + 4$ is equal to _____.

Ans. **(3)**

Sol. $\omega = e^{\frac{i2\pi}{28}}$

$x = \frac{1}{2}|\omega - \bar{\omega}| = \frac{1}{2}\left|2\sin\frac{\pi}{14}\right| = \sin\frac{\pi}{14}$

If $\theta = \frac{\pi}{14} \Rightarrow 3\theta + 4\theta = \frac{\pi}{2}$

$\sin 3\theta = \cos 4\theta$

$3x - 4x^3 = 2\cos^2 2\theta - 1 = 2(1 - 2\sin^2\theta)^2 - 1$

$3x - 4x^3 = 2\left(1 - 2x^2\right)^2 - 1 \quad = 2\left(4x^4 - 4x^2 + 1\right) - 1 \quad = 8x^4 - 8x^2 + 1$

$8x^4 + 4x^3 - 8x^2 - 3x + 1 = 0$

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14. Let z_1, z_2, z_3 be complex numbers such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3|$ then $z_1^2 + z_2^2 + z_3^2$ is ______.

Ans. **(0)**

Sol. $z_1^2 + z_2^2 + z_3^2 = (z_1 + z_2 + z_3)^2 - 2(z_1z_2 + z_2z_3 + z_2z_1)$

$= -2z_1z_2z_3\left(\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right) = -2z_1z_2z_3(\bar{z}_1 + \bar{z}_2 + \bar{z}_3) = 0$

15. Let 1 $\alpha_1, \alpha_2, \alpha_3, ...\alpha_{10}$ be the eleven 11th roots of unity. Let $\lambda = \sum_{r=1}^{10} r(\alpha_r + \alpha_{11-r})$. The value of $\frac{\lambda + 11}{11}$ equals

Ans. **(0)**

Sol. $\sum_{r=1}^{10} r(\alpha_r + \alpha_{11-r})$

$\lambda = 1(\alpha_1 + \alpha_{10}) + 2(\alpha_2 + \alpha_9) + 3(\alpha_3 + \alpha_8) + 4(\alpha_4 + \alpha_7) + 5(\alpha_2 + \alpha_6)$

$+6(\alpha_6 + \alpha_5) + 7(\alpha_7 + \alpha_4) + 8(\alpha_8 + \alpha_3) + 9(\alpha_9 + \alpha_2) + 10(\alpha_{10} + \alpha_1)$

$= 11(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8 + \alpha_9 + \alpha_{10}) = (-1) = -11$

$\Rightarrow \frac{\lambda + 11}{11} = 0$

16. If a and b are positive integer such that N = $(a + ib)^3 - 107i$ is a positive integer then find the value of $\frac{N}{2}$

Ans. 99

Sol. N = $(a + ib)^3 - 107i$

= $(a^3 - 3ab^2) + i[3a^2b - b^3] - 107i$ = Positive integer

$\therefore$ $3a^2b - b^3 - 107 = 0$

$b(3a^2 - b^2) = 107$

b = 1 $3a^2 - b^2 = 107$ 107 is prime

$\Rightarrow$ a = 6 or b = 107 $3a^2 - (107)^2 = 1$

a is not integer not possible

$\therefore$ a = 6 b = 1

N = 216 – 3 × 6 = 216 – 18 = 198.

17. Let z,w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and arg zw = π. If Re(z) < 0 and principal arg z = $\frac{a\pi}{b}$ then find the value of a + b. (where a & b are co-prime natural numbers)

Ans. 7

Sol. Since, $\bar{z} + i\bar{w} = 0 \Rightarrow \ = -i\bar{w} \Rightarrow \quad z = iw \Rightarrow w = -iz$

Also, $\arg(zw) = \pi \quad \Rightarrow \quad \arg(-iz^2) = \pi$

$\Rightarrow \arg(-i) + 2\arg(z) = \pi \quad \Rightarrow \quad -\dfrac{\pi}{2} + 2\arg(z) = \pi \qquad \left[\because \arg(-i) = -\dfrac{\pi}{2}\right]$

$\Rightarrow 2\arg(z) = \dfrac{3\pi}{2} \quad \Rightarrow \quad \arg(z) = \dfrac{3\pi}{4}$.

18. Let z is a complex number satisfying the equation, $z^3 - (3 + i)z + m + 2i = 0$, where $m \in R$. Suppose the equation has a real root α, then find the value of $\alpha^4 + m^4$

Ans. **32**

Sol. $z = \alpha \qquad \alpha \in R$

$\alpha^3 - (3 + i)\alpha + m + 2i = 0$

$\alpha^3 - 3\alpha + m = 0 \qquad \& \qquad -\alpha + 2 = 0$

$\alpha = 2$

$8 - 6 + m = 0$

$\Rightarrow \qquad m = -2$

$\alpha^4 + m^4 = 32$

19. If $x = 9^{1/3}\, 9^{1/9}\, 9^{1/27} \ldots\infty$, $y = 4^{1/3}\, 4^{-1/9}\, 4^{1/27} \ldots \infty$, and $z = \sum_{r=1}^{\infty}(1+i)^{-r}$ and principal argument of P = (x + yz) is $-\tan^{-1}\left(\dfrac{\sqrt{a}}{b}\right)$ then determine $a^2 + b^2$. (where a & b are co-prime natural numbers)

Ans. **13**

Sol. $x = 9^{\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots} = 9^{\frac{\frac{1}{3}}{1-\frac{1}{3}}} = 9^{\frac{1}{2}} = 3$; $y = 4^{\frac{1}{3}-\frac{1}{9}+\frac{1}{27}+\ldots} = 4^{\frac{\frac{1}{3}}{1+\frac{1}{3}}} = 4^{\frac{1}{4}} = \sqrt{2}$

$z = \sum_{r=1}^{\infty}(1+i)^{-r} = \dfrac{1}{1+i} + \dfrac{1}{(1+i)^2} + \dfrac{1}{(1+i)^2} + \ldots$

$= \dfrac{\frac{1}{1+i}}{1-\frac{1}{1+i}} = \dfrac{1}{i} = -i$

Let $P = x + yz = 3 - i\sqrt{2}$ (fourth quadrant). Then, $\arg P = -\tan^{-1}\left(\dfrac{\sqrt{2}}{3}\right) = -\tan^{-1}\left(\dfrac{\sqrt{a}}{b}\right)$

Hence a = 2, b = 3

20. $z_1, z_2 \in c$ and $z_1^2 + z_2^2 \in R$,

$z_1(z_1^2 - 3z_2^2) = 2$, $z_2(3z_1^2 - z_2^2) = 11$

If $z_1^2 + z_2^2 = \lambda$ then determine λ^2

Ans. **25**

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Sol. $z_1(z_1^2 - 3z_2^2) = 2$ ...(1)

$z_2(3z_1^2 - z_2^2) = 11$ ...(2)

Now (1) + i(2)

$(z_1 + iz_2)^3 = 2 + 11i$ ...(3)

(i) – i(2)

$(z_1 - iz_2)^3 = 2 - 11i$ ...(4)

Multiply (3) & (4) we get

$(z_1^2 + z_2^2)^3 = 125$

$z_1^2 + z_2^2 = 5 = \lambda$

21. Let $|z| = 2$ and $w = \dfrac{z+1}{z-1}$ where $z, w \in C$ (where C is the set of complex numbers), then find product of maximum and minimum value of $|w|$.

Ans. **1**

Sol. Let $z = a + ib$

Given $|z| = 2 \Rightarrow a^2 + b^2 = 4 \Rightarrow a, b \in [-2, 2]$

Now $w = \dfrac{(a+1)+ib}{(a-1)+ib} \Rightarrow |w| = \sqrt{\dfrac{(a+1)^2+b^2}{(a-1)^2+b^2}} = \sqrt{\dfrac{a^2+b^2+2a+1}{a^2+b^2-2a+1}} = \sqrt{\dfrac{5+2a}{5-2a}}$

$|w|_{max} = \sqrt{\dfrac{5+4}{1}} = 3$ (when a = 2) ; $|w|_{min} = \sqrt{\dfrac{5+4}{9}} = \dfrac{1}{3}$ (when a = –2)

Hence, required product is 1.

22. If ω and ω^2 are the non-real cube roots of unity and $a, b, c \in R$ such that $\dfrac{1}{a+\omega} + \dfrac{1}{b+\omega} + \dfrac{1}{c+\omega} = 2\omega^2$ and $\dfrac{1}{a+\omega^2} + \dfrac{1}{b+\omega^2} + \dfrac{1}{c+\omega^2} = 2\omega$. If $\dfrac{1}{a+1} + \dfrac{1}{b+1} + \dfrac{1}{c+1} = \lambda$ then determine λ^4

Ans. **16**

Sol. Now consider the equation

$\dfrac{1}{a+x} + \dfrac{1}{b+x} + \dfrac{1}{c+x} = \dfrac{2}{x}$ ...(1)

Take LCM and solve we get

$x^3 + (ab + bc + ac)x - 2abc = 0$

Now roots of this cubic are ω, ω^2

let another root be α

$\alpha + \omega + \omega^2 = 0$

$\alpha = 1$

hence put x = 1 in the equation (1) we get

$\dfrac{1}{a+1} + \dfrac{1}{b+1} + \dfrac{1}{c+1} = 2 = \lambda$

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Matrix Match Type :

23. Match the following:

Consider the two circles in the complex plane

$C_1 : \left|\frac{z+i}{z-1}\right| = 2$

$C_2 : z\bar{z} - (3+4i)z - (3-4i)\bar{z} + 9 = 0$

Let (α_1, β_1) be the centre of C_1 and (α_2, β_2) be that of C_2

Column-I	**Column-II**
(A) $2\alpha_1 + \beta_1$ equals	(p) 11
(B) $\alpha_2 - 2\beta_2$ euqals	(q) 3
(C) The radius of circle C_1 equals	(r) 4
(D) The radius of circle C_2 equals	(s) $\frac{2\sqrt{2}}{3}$
	(t) 2

Options:

(A) A → q; B → p; C → s; D → r;

(B) A → q; B → r; C → s; D → p;

(C) A → q; B → s; C → p; D → r;

(D) A → p; B → q; C → s; D → r;

Ans. (A)

Sol. $C_1 \to 3x^2 + 3y^2 - 8x - 2y + 3 = 0$ is a circle with radius $\frac{2\sqrt{2}}{3}$

$C_2 \to x^2 + y^2 - 6x + 8y + 9 = 0$ is a circle with radius 4

$(\alpha_1, \beta_1) = \left(\frac{4}{3}, \frac{1}{3}\right), (\alpha_2, \beta_2) = (3, -4)$

Subjective based Questions :

24. If z_1 and z_2 are the two complex numbers satisfying $|z - 3 - 4i| = 8$ and $\text{Arg}\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2}$ then find the range of the values of $|z_1 - z_2|$.

Ans. $|(z_1 - z_2)| \in \left[3\sqrt{2}, 13\sqrt{2}\right]$

Sol. $|z - 3 - 4i| = 8$

$||z - 3 - 4i| - |3 + 4i|| \le |(z - 3 - 4i) + (3 + 4i)| \le |z - 3 - 4i| + |3 + 4i|$

$\Rightarrow |8 - 5| \le |z| \le 8 + 5 \qquad \Rightarrow \quad 3 \le |z| \le 13 \qquad$ ....(1)

Now, z_1 and a_2 follows $|z - 3 - 4i| = 8$

and $\operatorname{Arg}\left(\dfrac{z_1}{z_2}\right) = \dfrac{\pi}{2} \quad \Rightarrow \dfrac{z_1}{z_2} = i$

So, $|z_1 - z_2| = |z_2(1-i)| = \sqrt{2}\,|z_2|$ .... (2)

from (1) and (2)

$|z_2| \in [3, 13] \quad \Rightarrow |(z_1 - z_2)| \in \left[3\sqrt{2}, 13\sqrt{2}\right]$

25. Let a, b, c be distinct complex numbers such that $\dfrac{a}{1-b} = \dfrac{b}{1-c} = \dfrac{c}{1-a} = k$, (a, b, c $\neq$ 1). Find the value of k.

Ans. $-\omega$ or $-\omega^2$

Sol. $\dfrac{a}{1-b} = \dfrac{b}{1-c} = \dfrac{c}{1-a} = k$

$a = k - bk \qquad b = k - kc \qquad c = k - ak$

$\Rightarrow \quad a = k - (k^2 - k^2c)$

$a = k - (k^2 - (k^3 - ak^3)]$

$a = k - k^2 + k^3 - ak^3$

$a(k^3 + 1) = k[k^2 - k + 1]$

$(ak + a - k)(k^2 - k + 1) = 0$

$k = \dfrac{-a}{a-1}$ or $k^2 - k + 1 = 0$

$k = -\omega, -\omega^2$ and $k = \dfrac{a}{1-a}$ is not possible. Hence Proved.

Power of real gurus