Ans. **(D)**

Sol. For infinite sol.

$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$

$$\Delta_x = \begin{vmatrix} 0 & 4 & 1 \\ 1 & 2 & 3 \\ -2 & 0 & -6 \end{vmatrix} = 0$$

$\Delta_x = -4(-b + 6) + 1(4) = 0$

$\Rightarrow -1 - b + 6 = 0$

$b = 5$ & $a = 3$

16. The given system of the equations will have no solution if

(A) $ab = 15, a \neq 3$ (B) $ab \neq 15, a \neq 3$ (C) $ab \neq 15, a = 3$ (D) None of these

Ans. **(A)**

Sol. For no. sol

$\Delta = 0$ but $\Delta_x \neq 0$

$\Rightarrow ab = 15, a \neq 3$

Numerical based Questions :

17. If $\begin{vmatrix} 1 & x & x^2 \\ x & x^2 & 1 \\ x^2 & 1 & x \end{vmatrix} = 3$ then the value of $\begin{vmatrix} x^3-1 & 0 & x-x^4 \\ 0 & x-x^4 & x^3-1 \\ x-x^4 & x^3-1 & 0 \end{vmatrix}$ is ___.

Ans. **(9)**

Sol. $D_c = D^2 = 9$

$$\Rightarrow D_c = \begin{vmatrix} x^3-1 & 0 & x-x^4 \\ 0 & x-x^4 & x^3-1 \\ x-x^4 & x^3-1 & 0 \end{vmatrix}$$

18. If $D = \begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$ = then find the units digit of $\dfrac{D}{(10!)^3}$

Ans. **(4)**

Sol. $$(\underline{|10})^3 \times 11 \times 11 \times 12 \times \begin{vmatrix} 1 & 1 & 1 \\ 11 & 12 & 13 \\ 132 & 13\times 12 & 14\times 13 \end{vmatrix}$$

$C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$(\underline{|10})^3 \times 11 \times 11 \times 12 \times \begin{vmatrix} 1 & 0 & 0 \\ 11 & 1 & 2 \\ 121 & 24 & 50 \end{vmatrix}$$

$= = 2904 \times (\lfloor 10)^3$

So, $\dfrac{D}{(\lfloor 10)^3} = 2904$

so unit digit = 4

19. Total number of 2 × 2 determinants whose entries are from the set {–1, 0, 1} has value equal to– 1 is N. Then Sum of digits of N is :

Ans. **2**

Sol. $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = -1$

ad – bc = – 1

$\Rightarrow$ ad = –1, bc = 0 .... case (1)

ad = 0 bc = 1 .... case (2)

$\Rightarrow$ 2 × $2C_1$× $3C_1$ – 2 = 10

$2C_1$× $3C_1$× 2 = 12 – 2 = 10

n(A) = 20

20. If $x \neq 0, y \neq 0, z \neq 0$ and $\begin{vmatrix} 1+2x & 1+x & 1+x \\ 1+2y & 1+3y & 1+y \\ 1+2z & 1+2z & 1+4z \end{vmatrix} = 0$, then the value of $x^{-1} + y^{-1} + z^{-1} + 9$ is

Ans. **(3)**

Sol. $C_3 \to C_3 - C_2$, $C_2 \to C_2 - C_1$

$$\begin{vmatrix} 1+2x & -x & 0 \\ 1+2y & y & 2y \\ 1+2z & 0 & 2z \end{vmatrix} = 0$$

$\Rightarrow$ z(1 + 2x)yz + x [xz (1 + 2y) + xy (1+2z)] = 0

$\Rightarrow$ yz+ 2xyz + xz + 2xyz + xy + 2xyz = 0

$\Rightarrow \dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} + 9 = 3$

21. If $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} \geq 0$, where a, b, c $\in$ R^+ – {0}, then $\dfrac{a+b}{c}$ is

Ans. **2**

Sol. Let $\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$ Operate : $C_1 \to C_1 + C_2 + C_3$

$\Rightarrow \Delta = \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(a+b+a) & a+b & b+c \\ 2(a+b+c) & b+c & c+a \end{vmatrix} = 2(a + b + c) \begin{vmatrix} 1 & c+a & a+b \\ 1 & a+b & b+c \\ 1 & b+c & c+a \end{vmatrix}$ Operate : $R_2 \to R_2 - R_1$

; $R_3 \to R_3 - R_1$

$\Rightarrow D = 2(a + b + c) \begin{vmatrix} 1 & c+a & a+b \\ 0 & b-c & c-a \\ 0 & b-a & c-b \end{vmatrix}$

$= 2(a + b + c)\ 1 \begin{vmatrix} b-c & c-a \\ b-a & c-b \end{vmatrix}$. open w.r.t. R_1

$= 2(a + b + c) [(b - c) (c - b) - (c - a) (b - a)]=2(a + b + c) [bc - b^2 - c^2 + cb -(cb-ac-ab+a^2)]$

$= 2(a + b + c) (ab + bc + ca - a^2 - b^2 - c^2) \le 0$ $\Rightarrow a = b = c$

22. If a_1, a_2, a_3 , 5, 4, a_6, a_7, a_8, a_9 are in H.P. and $D = \begin{vmatrix} a_1 & a_2 & a_3 \\ 5 & 4 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$, then the value of 21D is

(Where [.] represents, the greatest integer function)

Ans. **50**

Sol. $D = \begin{vmatrix} a_1 & a_2 & a_3 \\ 5 & 4 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ $\Rightarrow \frac{1}{a_1}, \frac{1}{a_2}, \ldots\ldots, \frac{1}{5}, \frac{1}{4}, \ldots\ldots, \frac{1}{a_9}$ are in A.P.

$d = \frac{1}{4} - \frac{1}{5} = \frac{1}{20} \Rightarrow \frac{1}{5} = \frac{1}{a} + \frac{3}{20} \Rightarrow \frac{1}{a_1} = \frac{1}{20} \quad \therefore \frac{1}{a_n} = \frac{1}{a_1} + \frac{(n-1)}{20} = \frac{n}{20} \Rightarrow a_n = \frac{20}{n}$

Hence , $D = \begin{vmatrix} 20 & \frac{20}{2} & \frac{20}{3} \\ \frac{20}{4} & \frac{20}{5} & \frac{20}{6} \\ \frac{20}{7} & \frac{20}{8} & \frac{20}{9} \end{vmatrix} = \frac{(20)^3}{4 \times 7} \begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 1 & \frac{4}{5} & \frac{2}{3} \\ 1 & \frac{7}{8} & \frac{7}{9} \end{vmatrix}$

$R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$= \frac{(20)^3}{4 \times 7} \begin{vmatrix} 0 & \frac{-3}{10} & \frac{-1}{3} \\ 0 & \frac{-3}{40} & \frac{-1}{9} \\ 1 & \frac{7}{8} & \frac{7}{9} \end{vmatrix} = \frac{50}{21} \Rightarrow$ 21D = 50

23. The absolute value of a for which system of equations, $a^3x + (a + 1)^3y + (a + 2)^3z = 0$, $ax + (a + 1)\,y + (a + 2)\,z = 0$, $x + y + z = 0$, has a non–zero solution is:

Ans. **1**

Sol. $\begin{vmatrix} a^3 & (a+1)^3 & (a+2)^3 \\ a & (a+1) & (a+2) \\ 1 & 1 & 1 \end{vmatrix} = 0$ for non - zero solution

$C_2 \to C_2 - C_1,\ C_3 \to C_3 - C_1$

$$\begin{vmatrix} a^3 & (a+1)^3 - a^3 & (a+2)^3 - a^3 \\ a & 1 & 2 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$\Rightarrow 2((a + 1)^3 - a^3) - ((a + 2)^3 - a^3) = 0 \qquad \Rightarrow 2\,(3a^2 + 3a + 1) = 6a^2 + 12a + 8$

$\Rightarrow 6a + 2 = 12\,a + 8 \qquad \Rightarrow -6 = 6a \qquad \Rightarrow a = -1 \Rightarrow \quad |a| = 1$

24. Consider the system of equations

$x + y + z = 4$

$2x + y + 3z = 6$

$x + 2y + pz = q$

Let L denotes the value of p if the system of equations has no solution. and M denotes the value of q if the system of equations has infinite solutions.

Find the unit digit of $(L^2 + M^2)$.

Ans. **(6)**

Sol. $x + y + z = 4 \quad (1)$

$2x + y + 3z = 6 \qquad (2)$

$x + 2y + pz = q \qquad (3)$

Solving (1) and (2) $\Rightarrow x = 2 - 2z$

and $y = 2 + z$

Put in equation (3), we get

$pz = q - 6$

Hence, for unique solution $p \neq 0,\ q \in R$

for no solution we must have $p = 0,\ q \neq 6$

for infinite solution $p = 0$ and $q = 6$

$\therefore \qquad L = 0,\ M = 6$

$\Rightarrow \qquad L + M = 0 + 36 = 36$

So, unit digit = 6

25. If 3^n is a factor of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ {}^{n}C_1 & {}^{n+3}C_1 & {}^{n+6}C_1 \\ {}^{n}C_2 & {}^{n+3}C_2 & {}^{n+6}C_2 \end{vmatrix}$ then the maximum value of n is............

Ans. **(3)**

Sol. $\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ n & (n+3) & (n+6) \\ n(n-1) & (n+3)(n+2) & (n+6)(n+5) \end{vmatrix} = 27$

So n maximum = 3

26. Let the matrix A and B be defined as $A = \begin{bmatrix} 3 & 2 \\ 2 & \alpha \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 7 & 3 \end{bmatrix}$. If $\det(2A^9 B^{-1}) = -2$, then find the number of distinct possible real values of α.

Ans. **1**

Sol. We have

$\det(2A^9B^{-1}) = -2 \Rightarrow \frac{2^2|A|^9}{|B|} = -2 \Rightarrow 4\,(3\alpha - 4)^9 = -2\,(2) \Rightarrow (3\alpha-4)^9 = -1,$

which is possible when $3\alpha - 4 = -1 \Rightarrow 3\alpha = 3 \Rightarrow \alpha = 1$.

So, only one real value of α exists. **Ans.**

27. Let A_n and B_n be square matrices of order 3, which are defined as

$A_n = [a_{ij}]$ and $B_n = [b_{ij}]$ where $a_{ij} = \frac{2i+j}{3^{2n}}$ and $b_{ij} = \frac{3i-j}{2^{2n}}$ for all i and j, $1 \le i, j \le 3$.

If $\quad l = \lim\limits_{n\to\infty} \text{Tr.}\left(3A_1 + 3^2A_2 + 3^3A_3 + \ldots\ldots + 3^nA_n\right)$

and $\quad m = \lim\limits_{n\to\infty} \text{Tr.}\left(2B_1 + 2^2B_2 + 2^3B_3 + \ldots\ldots + 2^nB_n\right)$,

then find the value of $(l + m)$.

[**Note :** Tr. (P) denotes the trace of matrix P.]

Ans. **21**

Sol. $A_n = \left[\frac{2i+j}{3^{2n}}\right] = \frac{1}{3^{2n}}[2i+j]$

$3^n A_n = \frac{1}{3^n}[2i+j]$

$\text{Tr.}(3^nA_n) = \frac{1}{3^n}(3+6+9) = \frac{18}{3^n}\ \forall\, n \in N$

$\therefore\ \text{Tr.}(3A_1 + 3^2A_2 + \ldots\ldots + 3^nA_n) = \text{Tr.}(3A_1) + \text{Tr.}(3^2A_2) + \ldots\ldots.. + \text{Tr.}(3^nA_n)$

$= 6 + 2 + \frac{2}{3} + \ldots\ldots + \frac{18}{3^n}$

$\therefore\quad l = \lim\limits_{n\to\infty} \text{tr}\left(3A_1 + 3^2A_2 + \ldots\ldots. + 3^nA_n\right) = \lim\limits_{n\to\infty}\left(6 + 2 + \frac{2}{3} + \ldots\ldots. + \frac{18}{3^n}\right) = \frac{6}{1-\frac{1}{3}} = 9$

$|||^{ly}\quad \text{Tr.}(2^nB_n) = \frac{1}{2^n}(2+4+6) = \frac{12}{2^n}$

$\text{Tr.}(2^1B_1 + 2^2B_2 + 2^3B_3 + \ldots\ldots.+2^nB_n) = \text{tr}(2B_1) + \text{tr}(2^2B_2) + \ldots\ldots.+ \text{tr}(2^nB_n)$

$= \frac{12}{2} + \frac{12}{2^2} + \ldots\ldots\ldots + \frac{12}{2^n}$.

$\therefore\quad m = \lim\limits_{n\to\infty} \text{tr}\left(2B_1 + 2^2B_2 + \ldots\ldots + 2^nB_n\right) = \lim\limits_{n\to\infty}\left(\frac{12}{2} + \frac{12}{2^2} + \ldots\ldots + \frac{12}{2^n}\right) = \frac{6}{1-\frac{1}{2}} = 12$

$\therefore\quad l + m = 9 + 12 = 21$. **Ans.**

Matrix Match Type :

28. Consider a square matrix A of order 2 which has its elements as 0,1,2 and 4. Let N denote the number of such matrices.

Column - A	**Column - B**	
(A) Possible non-negative value of det(A) is	(P)	2
(B) Sum of values of determinants corresponding to N matrices is	(Q)	4
(C) If absolute value of (det(A)) is least, then possible value of $\lvert adj(adj(adj\ A)) \rvert$	(R)	–2
(D) If det (A) is algebraically least, then possible value of det($4A^{-1}$) is	(S)	0
	(T)	8

Ans. (A) P, Q, T ; (B) S; (C) P, R ; (D) R]

Sol. Here 24 matrices are possible.

Values of determinants corresponding to these matrices are as follows :

$$\begin{vmatrix} 1 & 0 \\ 4 & 2 \end{vmatrix} = 2 \text{ (4 matrices)}, \quad \begin{vmatrix} 1 & 0 \\ 2 & 4 \end{vmatrix} = 4 \text{ (4 matrices)}, \quad \begin{vmatrix} 2 & 0 \\ 1 & 4 \end{vmatrix} = 8 \text{ (4 matrices)}$$

And 12 more matrices are there, values of whose determinants are –2, –4, –8.

(A) Possible non-negative values of det. (A) are 2, 4, 8.

(B) Sum of these 24 determinants is 0.

(C) Mod. (det(A)) is least $\therefore$ | A | = ± 2

$$\Rightarrow |\text{adj (adj (adj (A))}| = |A|^{(n-1)^3} = \pm 2$$

(D) Least value of det.(A) is –8

$$\text{Now, } |4A^{-1}| = 16\,\frac{1}{|A|} = \frac{16}{-8} = -2]$$

Subjective Type Questions:

29. If the system of equations x = cy + bz, y = az + cx and z = bx + ay has a non-zero solution and at least one of a, b, c is a proper fraction, prove that $a^2 + b^2 + c^2 < 3$ and abc > – 1.

Sol. We have given that system has non-trival solution

$$\Rightarrow \begin{vmatrix} 1 & -c & -b \\ -c & 1 & -a \\ -b & -a & 1 \end{vmatrix} = 0$$

$\Rightarrow$ $1(1 - a^2) + c(- c - ab) - b(ac + b) = 0$

$\Rightarrow$ $1 - 2abc - a^2 - b^2 - c^2 = 0$

$\Rightarrow$ $(a + bc)^2 = (1 - b^2)(1 - c^2)$

Similarly $(b + ac)^2 = (1 - a^2)(1 - b^2)$, $(c + ab)^2 = (1 - a^2)(1 - b^2)$

Hence $(1 - a^2)$, $(1 + b^2)$, $(1 - c^2)$

All have same sign. Since atleast one of them is

$1 - a^2 > 0$, $1 - b^2 > 0$, $1 - c^2 > 0$

$\Rightarrow$ $a^2 + b^2 + c^2 < 3$ $\Rightarrow$ $1 - 2abc < 3$ $\Rightarrow$ abc > – 1

30. $\Delta = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$, Show that minimum value of Δ is $27a^2b^2$. Given ab is constant.

Sol. $\Delta = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$,

Applying $C_1 - bC_3$ and $C_2 + aC_3$

$$\Delta = \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b(1+a^2+b^2) & -a(1+a^2+b^2) & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b \end{vmatrix}$$

Applying $R_3 - bR_1$

$$\Delta = (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ 0 & -a & 1-a^2+b^2 \end{vmatrix}$$

$$= (1+a^2+b^2)^2(1+a^2+b^2) = (1+a^2+b^2)^3$$

Now both a^2 and b^2 are positive

Therefore applying AM $\geq$ GM.

$$\frac{1+a^2+b^2}{3} \geq \sqrt[3]{a^2b^2} \qquad \Rightarrow \Delta \geq 27a^2b^2$$

31. If t is real and $\lambda = \dfrac{t^2-3t+4}{t^2+3t+4}$, then find number of solutions of the system of equations $3x - y + 4z = 3$, $x + 2y - 3z = -2$, $6x + 5y + \lambda z = -3$ for a particular value of λ.

Sol. We have $\lambda = \dfrac{t^2-3t+4}{t^2+3t+4}$

$\Rightarrow$ $(\lambda - 1)\, t^2 + 3\,(\lambda + 1)\, t + 4\,(\lambda - 1) = 0$

As $t \in R$, so

$9\,(\lambda + 1)^2 - 16\,(\lambda - 1)^2 \geq 0$

$\Rightarrow$ $(7 - \lambda)\,(7\lambda - 1) \geq 0$ $\Rightarrow$ $\dfrac{1}{7} \leq \lambda \leq 7$

Now, $D = \begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & \lambda \end{vmatrix} = 7\,(\lambda + 5) \neq 0 \;\forall\; \lambda \in \left[\dfrac{1}{7}, 7\right]$

Hence, the given system has unique solution.