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MATHEMATICS

TARGET : JEE- Advanced 2021

CAPS-7

Determinants and System of Equations

ANSWER KEY OF CAPS-7

1.	(B)	**2.**	(C)	**3.**	(A)	**4.**	(D)	**5.**	(B)
6.	(A)	**7.**	(A)	**8.**	(C)	**9.**	(D)	**10.**	(ABC)
11.	(A)	**12.**	(D)	**13.**	(A)	**14.**	(B)	**15.**	(D)
16.	(A)	**17.**	(9)	**18.**	(4)	**19.**	(2)	**20.**	(3)
21.	(2)	**22.**	(50)	**23.**	(1)	**24.**	(6)	**25.**	(3)
26.	(1)	**27.**	(21)	**28.**	(A) P, Q, T ; (B) S; (C) P, R ; (D) R]			**31.**	(1)

SCQ (Single Correct Type) :

1. The number of positive integral solutions (x, y, z) of the equation

$$\begin{vmatrix} x^3+1 & x^2y & x^2z \\ xy^2 & y^3+1 & y^2z \\ xz^2 & yz^2 & z^3+1 \end{vmatrix} = 11 \text{ is\_\_\_}$$

(A) 0 (B) 3 (C) 6 (D) 12

Ans. **(B)**

Sol. det = $(1 + x^3 + y^3 + z^3) = 11$

(2, 1, 1) (1, 2, 1) (1, 1, 2)

2. If x = a, y = b, z = c is a solution of the system of linear equations x + 8y + 7z = 0, 9x + 2y + 3z= 0, x + y + z = 0 such that the point (a, b, c) lies on the plane x + 2y + z = 6, then 2a + b + c equals :

(A) –1 (B) 0 (C) 1 (D) 2

Ans. **(C)**

Sol.
$$\left.\begin{matrix} x+8y+7z=0 \\ 9x+2y+3z=0 \\ x+y+z=0 \end{matrix}\right\} \begin{matrix} 7y+6z=0 \\ \\ 7y+z=0 \end{matrix}$$

$$x=\lambda \mid y=\frac{-6(-7\lambda)}{7} \mid z=-7\lambda$$

$x=\lambda$ $y = 6\lambda$ $z = -7\lambda$

$\lambda+12\lambda-7\lambda=6$ $2\lambda+6\lambda-7\lambda$

$6\lambda=6$ $=2\lambda$

$\lambda=1$ $= 2$

3. Let $A = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ such that |A| = 0. If a, b, c are distinct, then the sum of the coordinates of the fixed point through which the line ax + by + c = 0 passes is

(A) 2 (B) 3 (C) 4 (D) 5

Ans. **(A)**

Sol. A upon simplification (a + b + c) (a – b) (b – c) (c – a)

The only way this can be 0 is (a + b + c) = 0

fixed point is (1, 1)

4. If a system of the equation $(\alpha + 1)^3 x + (\alpha + 2)^3 y - (\alpha + 3)^3 = 0$ and $(\alpha + 1) x + (\alpha + 2) y - (\alpha + 3) = 0$, x + y – 1 = 0 is consistent, then the value(s) of α is/are

(A) 1 (B) 0 (C) –3 (D) –2

Ans. **(D)**

Sol. For the system of three equation and two variables (3 lines in XY plane)

If system is consistent then either all three lines are concurrent or all three lines are identical

If lines are concurrent

$$\begin{vmatrix} (\alpha+1)^3 & (\alpha+2)^3 & -(\alpha+3)^3 \\ (\alpha+1) & (\alpha+2) & -(\alpha+3) \\ 1 & 1 & -1 \end{vmatrix} = 0$$

$\Rightarrow \alpha = -2$

Check at α = –2 (as three lines can be parallel also on putting det = 0)

Then, –x –1 = 0

and x + y – 1 = 0

x = –1, y = 2 (lines are concurrent)

Now check for remaining values if all three lines are coincident

If α = 1

Then, 8x + 27y – 64 = 0

2x + 3y – 4 = 0

x + y – 1 = 0

Three lines are forming a triangle

So, the system of equations is inconsistent

Similarly, for α = 0 and α = – 3

We can prove that the given system of equations is inconsistent.

5. If a, b, c are complex numbers and $z = \begin{vmatrix} 0 & -b & -c \\ \bar{b} & 0 & -a \\ \bar{c} & \bar{a} & 0 \end{vmatrix}$ is

(A) purely real (B) purely imaginary (C) 0 (D) none of these

Ans. **(B)**

Sol. $z = \begin{vmatrix} 0 & -b & -c \\ \bar{b} & 0 & -a \\ \bar{c} & \bar{a} & 0 \end{vmatrix} = ba\bar{c} - c\overline{ab}$ Let $z_1 = ba\bar{c}$

$= z_1 - \bar{z}_1 = 2\text{Im}(z_1)$ Purely Imaginary.

6. If $f(x) = \log_{10}x$ and $g(x) = e^{i\pi x}$ and $h(x) = \begin{vmatrix} f(x)g(x) & [f(x)]^{g(x)} & 1 \\ f(x^2)g(x^2) & [f(x^2)]^{g(x^2)} & 0 \\ f(x^3)g(x^3) & [f(x^3)]^{g(x^3)} & 1 \end{vmatrix}$, then the value of h(10) is

(A) 0 (B) 2 (C) 1 (D) 4

Ans. **(A)**

Sol. $f(x) = \log_{10}x$, $g(x) = e^{i\pi x} = \cos\pi x + i \sin\pi x$

$f(10) = 1 \qquad g(10) = 1$

$h(10) = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 0 \\ 3 & 3 & 1 \end{vmatrix} = 0$

7. Let S be the set of all real values of 'a' for which the following system of linear equations

ax+2y+5z=1

2x+y+3z=1

3y+7z=1

is consistent. Then the set S is :

(A) equal to R (B) equal to R–{1} (C) equal to {1} (D) an empty set

Ans. **(A)**

Sol. ax + 2y + 5z = 1

2x + y + 3z = 1

3y + 7z = 1

$\Delta \begin{vmatrix} a & 2 & 5 \\ 2 & 1 & 3 \\ 0 & 3 & 7 \end{vmatrix} = a(-2) - 2(14) + 5(6) = -2a + 2 = -2(a - 1)$

$\Delta_x \begin{vmatrix} 1 & 2 & 5 \\ 1 & 1 & 3 \\ 1 & 3 & 7 \end{vmatrix} = 1(-2) - 2(4) + 5(2) = 0$

$\Delta_y \begin{vmatrix} a & 1 & 5 \\ 2 & 1 & 3 \\ 0 & 1 & 7 \end{vmatrix} = a(4) - 1(14) + 5(2) = 4a - 4$

$\Delta_z \begin{vmatrix} a & 2 & 1 \\ 2 & 1 & 1 \\ 0 & 3 & 1 \end{vmatrix} = a(-2) - 2(2) + 1(6) = -2a + 2 = -2(a - 1)$

system is consistent

either $a \neq 1 \Rightarrow$ unique solution

now, if a = 1, system of equation becomes

$x + 2y + 5z = 1$

$2x + y + 3z = 1$

$3y + 7z = 1$

i.e. there are only two equations

$x + 2y + 5z = 1$

$3y + 7z = 1$

which are not parallel

system of equation is consistent

8. Let α, β, γ are the real roots of the equation $x^3 + ax^2 + bx + c = 0$ (a, b, c $\in$ R and a $\neq$ 0).

If the system of equations (in u, v and w) given by

$\alpha u + \beta v + \gamma w = 0$

$\beta u + \gamma v + \alpha w = 0$

$\gamma u + \alpha v + \beta w = 0$

has non-trivial solutions, then a^2 equals

(A) b (B) 2b (C) 3b (D) 4b

Ans. **(C)**

Sol. We have

$x^3 + ax^2 + bx + c = 0 \; \begin{cases} \alpha \\ \beta \\ \gamma \end{cases}$

$\therefore \quad \alpha + \beta + \gamma = -a$

$\alpha\beta + \beta\gamma + \gamma\alpha = b$

Since the given system of equations has non-trivial solutions, so

$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$

$\Rightarrow \quad Q^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma = 0$

$\Rightarrow \quad (\alpha + \beta + \gamma)[\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha] = 0$

$\Rightarrow \quad (\alpha + \beta + \gamma)[(\alpha + \beta + \gamma)^2 - 3(\alpha\beta + \beta\gamma + \gamma\alpha)] = 0$

$\Rightarrow \quad -a[a^2 - 3b] = 0$

Hence $a^2 = 3b$ (as $a \neq 0$)]

9. For a unique value of p and q, the system of equations given by

$x + y + z = 6$

$x + 2y + 3z = 14$

$2x + 5y + pz = q$

has infinitely many solutions, then the value of (p+q) is equal to

(A) 14 (B) 24 (C) 34 (D) 44

Ans. **(D)**

Sol. $x + y + z = 6$ .....(1)

$x + 2y + 3z = 14$ .....(2)

$2x + 5y + pz = q$ .....(3)

$\therefore$ (1) and (2) $\Rightarrow$ $y + 2z = 8$

$\therefore$ $y = 8 - 2z$

$\Rightarrow x = 6 - 8 + 2z - z = -2 + z$

$\Rightarrow x = z - 2$

On putting x and y in (3), we get

$2(z-2) + 5(8 - 2z) + pz = q$ $\Rightarrow$ $2z - 4 + 40 - 10z + pz = q$

$\Rightarrow (p-8)z = (q-36)$

$\therefore$ For infinite solution p = 8 and q = 36

$\Rightarrow$ p + q = 44 ans.

Alternative :

For infinitely many solutions, $\Delta = 0$ and also $\Delta_1 = 0 = \Delta_2 = \Delta_3$.

Now, $\Delta = 0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 5 & p \end{vmatrix} = 0$

$\Rightarrow 1(2p-15) - 1(p-6) + 1(5-4) = 0$

$\Rightarrow 2p - 15 - p + 6 + 1 = 0$ $\Rightarrow p = 8$

Now, $\Delta_3 = 0$

$\Rightarrow \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 14 \\ 2 & 5 & q \end{vmatrix} = 0$ $\Rightarrow 1\,(2q - 70) - 1\,(q - 28) + 6\,(5 - 4) = 0$ $\Rightarrow 2q - 70 - q + 28 + 6 = 0$

$\Rightarrow$ $q - 70 \div 34 = 0$ $\Rightarrow$ $q = 36$

Note that, $\Delta_1 = 0$ and $\Delta_2 = 0$

Hence, (p + q) = 8 + 36 = 44 Ans.

MCQ (One or more than one correct) :

10. Consider the system of equations

$$ax_1 + x_2 + x_3 = 1$$
$$x_1 + ax_2 + x_3 = 1$$
$$x_1 + x_2 + ax_3 = 1$$

then :

(A) if a = 2, then the system has unique solution.

(B) if a = 1, then the system has infinite solution.

(C) if a = –2, then the system has no solution.

(D) if a = 2, then the system has infinite solution.

Ans. **(A,B,C)**

Sol. Whe have, $\Delta = \begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix} = a^3 - 3a + 2 = (a-1)^2(a+2)$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix} = a^2 - 2a + 1 = (a-1)^2$$

$$\Delta_2 = \begin{vmatrix} a & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & a \end{vmatrix} = a^2 - 2a + 1 = (a-1)^2$$

$$\Delta_3 = \begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & 1 \end{vmatrix} = a^2 - 2a + 1 = (a-1)^2$$

Clearly, if a = 2, then $\Delta \neq 0 \Rightarrow$ system is consistent with unique solution

If a = 1, then all three equation are identical, so system is consistent with infinite solutions.

If a = – 2, then $\Delta = 0$ but $\Delta_1 \neq 0$, $\Delta_2 \neq 0$, $\Delta_3 \neq 0$,

$\Rightarrow$ system is inconsisten.

Comprehension Type Question:

Comprehension # 1

For $\alpha, \beta, \gamma, \theta \in R$. Let

$$A_\theta(\alpha, \beta, \gamma) = \begin{vmatrix} \cos(\alpha+\theta) & \sin(\alpha+\theta) & 1 \\ \cos(\beta+\theta) & \sin(\beta+\theta) & 1 \\ \cos(\gamma+\theta) & \sin(\gamma+\theta) & 1 \end{vmatrix}$$

11. If a = $A_{\pi/2}(\alpha, \beta, \gamma)$, b = $A_{\pi/3}(\alpha, \beta, \gamma)$. Which of the following is true

(A) a = b (B) a < b (C) a > b (D) 2a = b

Ans. **(A)**

12. $A_\theta^2 + A_\phi^2 - 2(A_{\theta+\phi})^2$ equals

(A) $-2A_\theta A_\phi$ (B) $A_\theta + A_\phi$ (C) $A_\theta - A_\phi$ (D) None of these

Ans. (D)

13. If α, β, γ are fixed, then $y = A_x(\alpha, \beta, \gamma)$ represents

(A) a straight line parallel to x-axis

(B) a straight line through the origin

(C) a parabola with vertex at origin

(D) None of these

Ans. (A)

Sol. $A_\theta(\alpha, \beta, \gamma) = \begin{vmatrix} \cos(\alpha+\theta) & \sin(\alpha+\theta) & 1 \\ \cos(\beta+\theta) & \sin(\beta+\theta) & 1 \\ \cos(\gamma+\theta) & \sin(\gamma+\theta) & 1 \end{vmatrix}$

$A_\theta(\alpha, \beta, \gamma) \Rightarrow \sin(\alpha-\beta)\sin(\beta-\gamma)\sin(\gamma-\alpha) = k$

$\Rightarrow$ which is independent of θ

(i) If $a = A_{\pi/2}(\alpha, \beta, \gamma)$ & $b = A_{\pi/3}(\alpha, \beta, \gamma)$

so $a = b$ (Independent of θ)

(ii) $A_\theta^2 + A_\phi^2 - 2(A_{\theta+\phi})^2 = k^2 + k^2 - 2k^2 = 0$

(iii) If α, β, γ are fixed then $y = A_x(\alpha, \beta, \gamma)$ = constant

which is a straight line parallel to x-axis.

Comprehension # 2

Consider the system of equations :

$ax + 4y + z = 0$

$2y + 3z - 1 = 0$

$3x - bz + 2 = 0$

Then

14. The given system of equations will have a unique solution if

(A) $ab = 15$ (B) $ab \neq 15$ (C) $ab = 5$ (D) $a \neq 5$

Ans. (B)

Sol. $\Delta = \begin{vmatrix} 9 & 4 & 1 \\ 0 & 2 & 3 \\ 3 & 0 & -6 \end{vmatrix}$

$\Delta = a(-2b) - 4(-9) + 1(-6) = -2ab + 30$

For unique sol.

$\Delta \neq 0 \quad \Rightarrow ab - 15 \neq 0 \; ab \neq 15$

15. The system of equations will have infinite solutions if

(A) $a = 3, b = 2$ (B) $a = 3, b = 4$ (C) $a = 5, b = 3$ (D) $a = 3, b = 5$