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MATHEMATICS

TARGET : JEE- Advanced 2021

CAPS-6

Matrices & Determinant

ANSWER KEY OF CAPS-6

1.	(D)	**2.**	(D)	**3.**	(C)	**4.**	(A)	**5.**	(A)
6.	(A)	**7.**	(D)	**8.**	(A)	**9.**	(ABC)	**10.**	(AB)
11.	(ABC)	**12.**	(BC)	**13.**	(BC)	**14.**	(AC)	**15.**	(A)
16.	(D)	**17.**	(A)	**18.**	(B)	**19.**	(ACD)	**20.**	(AC)
21.	(ABCD)	**22.**	(4.00)	**23.**	(6.00)	**24.**	(A)	**25.**	(A)

SCQ (Single Correct Type) :

1. Let $a = \lim\limits_{x\to 1} \dfrac{x}{\ln x} - \dfrac{1}{x\ln x}$; $b = \lim\limits_{x\to 0} \dfrac{x^3-16x}{4x+x^2}$; $c = \lim\limits_{x\to 0} \dfrac{\ln(1+\sin x)}{x}$ and

$d = \lim\limits_{x\to -1} \dfrac{(x+1)^3}{3\left(\sin(x+1)-(x+1)\right)}$, then the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

(A) Idempotent (B) Involutary (C) Non singular (D) Nilpotent

Ans. **(D)**

Sol. $a = +2$; $b = -4$; $c = 1$; $d = -2$

Let $A = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$

now $\begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ = null matrix

hence A is nilpotent $\Rightarrow$ (D)

note that any matrix of the form $\begin{bmatrix} a & -a^2 \\ 1 & -a \end{bmatrix}$ is a nilpotent]

2. A is a 2 × 2 matrix such that $A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $A^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Find the sum of the elements of A.

(A) –1 (B) 0 (C) 2 (D) 5

Ans. **(D)**

Sol. $A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ ....(1)

and $A^2\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$ ....(2)

Let A be given by $A=\begin{bmatrix}a & b\\c & d\end{bmatrix}$.

The first equation gives

$a-b=-1$ ....(3) and $c-d=2$ ....(4)

For second equation, $A^2\begin{bmatrix}1\\-1\end{bmatrix}=A\left(A\begin{bmatrix}1\\-1\end{bmatrix}\right)=A\left(\begin{bmatrix}-1\\2\end{bmatrix}\right)=\begin{bmatrix}1\\0\end{bmatrix}$.

This gives $-a+2b=1$ ....(5) and $-c+2d=0$ ....(6)

(3) + (5) $\Rightarrow$ $b=0$ and $a=-1$

(4) + (6) $\Rightarrow$ $d=2$ and $c=4$

so the sum $a+b+c+d=5$ Ans.]

3. Matrix A satisfies $A^2=2A-I$ where I is the identity matrix then for $n\geq 2$, A^n is equal to $(n\in N)$

(A) $nA-I$ (B) $2^{n-1}A-(n-1)I$ (C) $nA-(n-1)I$ (D) $2^{n-1}A-I$

Ans. **(C)**

Sol. $A^2=2A-I$ $\Rightarrow$ $A^3=2A^2-IA$

$=2(2A-I)-A$ $(A^2=2A-I)$

$A^3=3A-2I$

$A^4=3A^2-2A$

$=3(2A-I)-2A$ $(A^2=2A-I)$

$A^4=4A-3I$

$A^5=5A-4I$

$A^n=nA-(n-1)I$

4. If $A=\begin{bmatrix}0 & 1 & 2\\1 & 2 & 3\\3 & a & 1\end{bmatrix}$, $A^{-1}=\begin{bmatrix}1/2 & -1/2 & 1/2\\-4 & 3 & c\\5/2 & -3/2 & 1/2\end{bmatrix}$, then

(A) $a=1, c=-1$ (B) $a=2, c=-\frac{1}{2}$ (C) $a=-1, c=1$ (D) $a=\frac{1}{2}$, $c=\frac{1}{2}$

Ans. **(A)**

Sol. $AA^{-1}=I$ $\Rightarrow$ $R_2C_3=0$

$\frac{1}{2}+2c+\frac{3}{2}=0$

$2c=-2\Rightarrow$ $c=-1$

also $R_3C_2 = 0$

$$-\frac{3}{2} + 3a - \frac{3}{2} = 0$$

$3a = 3 \Rightarrow \quad a = 1$

hence a = 1; c = – 1]

5. The number of solution of the matrix equation $X^2 = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$ is

(A) more than 2 (B) 2 (C) 1 (D) 0

Ans. (A)

Sol. Let $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$X^2 = \begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix}$$

$a^2 + bc = 1$

$ab + bd = 1 \Rightarrow b(a + d) = 1$

$ac + cd = 2 \Rightarrow c(a + d) = 2$

$\Rightarrow \quad \frac{b}{c} = \frac{1}{2} \quad \Rightarrow \quad c = 2b$

$bc + d^2 = 3 \Rightarrow (d^2 - a^2) = 2 \Rightarrow (d - a)(a + d) = 2$

$d - a = 2b$ (using $bc = 1 - a^2$)

$a + d = 1/b$

$2d = 2b + 1/b \qquad 2a = 1/b - 2b$

$d = b + 1/2b \qquad a = 1/2b - b$

$c = 2b$

$$\left(b^2 + \frac{1}{4b^2} + 1\right) + 2b^2 = 3 \Rightarrow 3b^2 + \frac{1}{4b^2} = 2$$

$$3x + \frac{1}{4x} = 2 \Rightarrow b = \pm\frac{1}{\sqrt{6}} \quad \text{or} \quad b = \pm\frac{1}{\sqrt{2}}$$

Matrices are $\begin{pmatrix} 0 & 1/\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{pmatrix}; \begin{pmatrix} 0 & -1/\sqrt{2} \\ -\sqrt{2} & -\sqrt{2} \end{pmatrix}; \begin{pmatrix} 2/\sqrt{6} & -1/\sqrt{6} \\ 2/\sqrt{6} & 4/\sqrt{6} \end{pmatrix}$

6. 'A' is a 3×3 matrix with entries from the set $\{-1,0,1\}$. The probability that 'A' is neither symmetric nor skew symmetric is

(A) $\dfrac{3^9 - 3^6 - 3^3 + 1}{3^9}$ (B) $\dfrac{3^9 - 3^6 - 3^3}{3^9}$ (C) $\dfrac{3^9 - 1}{3^{10}}$ (D) $\dfrac{3^9 - 3^3 + 1}{3^9}$

Ans. **(A)**

Sol. Total number of matrices that can be formed is 3^9

Let A = $[A_y]_{3\times3}$ where $a_y \in \{-1, 0, 1\}$

If 'A' is symmetric then $a_y = a_y$ $\forall$ i, j

If 'A' is skew symmetric then $a_y = -a_y$ $\forall$ i, j

7. $A = \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$ and MA = A^{2m}, m$\in$N for some matrix M, then which one of the following is correct?

(A) $M = \begin{bmatrix} a^{2m} & b^{2m} \\ b^{2m} & -a^{2m} \end{bmatrix}$ (B) $M = (a^2 + b^2)^m \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

(C) $M = (a^m + b^m)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) $M = (a^2 + b^2)^{m-1} \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$

Ans. **(D)**

Sol. $A^{2m} = \begin{bmatrix} (a^2 + b^2)^m & 0 \\ 0 & (a^2 = b^2)^m \end{bmatrix}$

$= (a^2 + b^2)^m \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$MA = A^{2m}$

$MAA^{-1} = A^{2m} A^{-1}$

$M = A^{2m} A^{-1}$

$M = A^{2m-2} A$

$= (a^2 + b^2)^{m-1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$

$= (a^2 + b^2)^{m-1} \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$

8. Let 'A' and 'B' be two given matrices such that AB = A and BA = B, then A^2 B^2 equals

(A) A (B) B (C) I (D) 0

Ans. **(A)**

Sol. AB = A, BA = B, A^2 B^2 = A (AB) B = A (AB) = A^2 = A (BA) = AB = A

MCQ (One or more than one correct) :

9. P is a non-singular matrix and A, B are two matrices such that $B = P^{-1}AP$ then the true statements among the following are

(A) A is invertible iff B is invertible

(B) $B^n = P^{-1} A^n P \forall n \in N$

(C) $\forall \lambda \in R$, $B - \lambda I = P^{-1}(A - \lambda I)P$ (I is the identity matrix)

(D) A,B are both singular matrices

Ans. **(ABC)**

Sol. Conceptual

10. If A and B are respectively a symmetric and a skew symmetric matrix such that AB = BA then

(A) $(A - B)^{-1}(A + B)$ is orthogonal matrix when $(A - B)$ is non-singular

(B) $(A + B)^{-1}(A - B)$ is orthogonal matrix when $(A + B)$ is non-singular

(C) $\det[(A - B)^{-1}(A + B)] = 1$ and $\det[(A + B)^{-1}(A - B)] = -1$

(D) $\det[(A - B)^{-1}(A + B)] = -1$ and $\det[(A + B)^{-1}(A - B)] = 1$

Ans. **(AB)**

Sol. $A^T = A$

$B^T = -B$

Now, $(A - B)^{-1}(A + B)[(A - B)^{-1}(A + B)]^T$

$= (A - B)^{-1}(A + B)(A + B)^T((A - B)^T)^{-1}$

$= (A - B)^{-1}(A + B)(A^T + B^T)^T(A^T - B^T)^{-1}$

$= (A - B)^{-1}(A + B)(A - B)(A + B)^{-1}$

$= (A - B)^{-1}(A - B)(A + B)(A + B)^{-1} = I$

$\Rightarrow (A - B)^{-1}(A + B)$ is orthogonal matrix

$(A + B)^{-1}(A - B) = ((A - B)^{-1}(A + B))^{-1}$ and if X is an orthogonal matrix than so is X^{-1}

For orthogonal matrix X, $\det X = \pm 1$

But As $\det[(A - B)^{-1}(A + B)(A + B)^{-1}(A - B)]$

$= \det I = 1$

or, $\det[(A - B)^{-1}(A + B)].\det[(A + B)^{-1}(A - B)] = 1$

Either both will be 1 or −1

11. If $A = \begin{bmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{bmatrix}$ (where $\alpha_2 \neq \beta_1$ and $\alpha_1, \alpha_2, \beta_1, \beta_2$ are non-zero) satisfies the equation $x^2 + k = 0$, then

(A) trace A = 0 (B) $\alpha_1 \beta_2 < 0$ (C) det A = k (D) det A = – k

Ans. **(ABC)**

Sol. $A^2 + kI = 0$

$$\begin{bmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{bmatrix}\begin{bmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{bmatrix} = \begin{bmatrix} -k & 0 \\ 0 & -k \end{bmatrix}$$

$\alpha_1^2 + \alpha_2\beta_1 = -k$ ......(i)

$\alpha_1\alpha_2 + \alpha_2\beta_2 = 0$ ......(ii)

$\alpha_1\beta_1 + \beta_1\beta_2 = 0$ ......(iii)

$\alpha_2\beta_2 + \beta_2^2 = -k$ ......(iv)

(i) – (iv)

$\alpha_1^2 - \beta_1^2 = 0$

$\alpha_1 = \pm\beta_2$ ......(A)

From (ii) & (iii)

$\alpha_1\alpha_2 + \alpha_2\beta_2 = \alpha_1\beta_1 + \beta_1\beta_2$

$\Rightarrow \alpha_1(\alpha_1 - \beta_2) + \beta_2(\alpha_2 - \beta_1) = 0$

$\Rightarrow (\alpha_1 + \beta_2)(\alpha_2 - \beta_1) = 0$

$\Rightarrow \alpha_1 + \beta_2 = 0$ ....(B)

(As $a_2 \neq \beta_1$)

From (A) & (B)

$\alpha_1 + \beta_2 = 0$ trace A = 0

$\det(A^2) = k^2$

$(\det A)^2 = k^2$

$\det A = \pm k$

12. Let A is a $n \times n$ matrix in which diagonal elements are 1,2,3,.....,n (i.e., $a_{11} = 1$, $a_{22} = 2$, $a_{33} = 3$, ...$a_{ii} = i$,...$a_{nn} = n$) and all other elements are equal to 'n ' then

(A) A_n is singular for all 'n '

(B) A_n is non-singular for all 'n '

(C) det .A_5 =120

(D) det . $A_n = 0$

Ans. **(BC)**

Sol. $A_n \begin{pmatrix} 1 & n & n\ldots & n \\ n & 2 & n\ldots & n \\ n & n & 3\ldots & n \\ n & n & n\ldots & n \end{pmatrix}$

$\therefore$ A_n is nonsiongular for all 'n'

$|A_n| = (-1)^{n+1}.n!$

13. If the matrix A and B are of 3 × 3 and $(I - AB)^{-1}A$ is invertible, then which of the following statements is/are correct?

(A) $I - BA$ is not invertible

(B) $I - BA$ is invertible

(C) $I - BA$ has for its inverse $I + B(I - AB)^{-1}A$

(D) $I - BA$ has for its inverse $I + A(I - BA)^{-1}B$

Ans. **(BC)**

Sol. $(I - AB)^{-1}$ exists

Consider the equation $(I - BA)\,X = I$

i.e., $X - I = BAX$

$-A\,(X - I) = (I - AB - I)AX$

$\therefore\ X - I = BAX = B\,(I - AB)^{-1}A$

$X = I + B\,(I - AB)^{-1}\,A$

14. If A, B are two square matrices of same order such that A + B = AB and I is identity matrix of order same as that of A, B, then

(A) AB = BA (B) $|A - I| = 0$ (C) $|B - I| \neq 0$ (D) $|A - B| = 0$

Ans. **(AC)**

Sol. $A + B = AB$

$\Rightarrow I - (A + B - AB) = I$ $\Rightarrow (I - A)\,(I - B) = I$ $\Rightarrow |I - A|.\,|I - B| = 1$

Also, $(I - B)\,(I - A) = I$

$\Rightarrow I - B - A + BA = I$

as A + B = B + A & A + B = AB

$\Rightarrow AB = BA$ $\Rightarrow$ (a) and (c) are correct

Comprehension Type Question:

Omprehension # 1

A Pythagorean triple is triplet of positive integers (a, b, c) such that $a^2 + b^2 = c^2$. Define the matrices A, B and C by

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix},\ B = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -2 \\ 2 & 2 & 3 \end{bmatrix} \text{ and } C = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

15. If we write Pythagorean triples (a, b, c) in matrix form as [a, b, c] then which of the following matrix product is not a Pythagorean triplet?

(A) [3,4,5]A (B) [3,4,5]B (C) [3,4,5]C (D) None of these

Ans. **(A)**

Sol. $\underset{1\times 3}{[3\ \ 4\ \ 5]}\underset{3\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix}} = \underset{\text{Not a Pythagoren triplet}}{[21\ \ 20\ \ 32]}$

|||ly $[3\ \ 4\ \ 5]\,[B] = [5\ \ 12\ \ 13]$

and $[3\ \ 4\ \ 5]\,[C] = [15\ \ 8\ \ 17]$

hence question no.(i) answer is (A)

16. Which one of the following does not hold good?

(A) A^{-1} = adj. A
(B) $(AB)^{-1}$ = adj. (AB)
(C) $(BC)^{-1}$ = adj. (BC)
(D) $(ABC)^{-1} \neq$ adj. (ABC)

Ans. **(D)**

Sol. det. A = 1 ; det. B = 1, det. C = 1 (verify)

det. (AB) = (det. A) (det. B) = (1) (1) = 1

det. (BC) = (det. B) (det. C) = (1) (1) = 1

det. (CA) = (det. C) (det. A) = (1) (1) = 1

det. (ABC) = (det. A) (det. B) (det. C) = 1

17. Tr $(A + B^T + 3C)$ equals

(A) 17 (B) 15 (C) 19 (D) 18

Ans. **(A)**

Sol. Tr $(A + B^T + 3C)$ = = 5 + 3 + 9 = 17

Omprehension # 2

Consider the matrix function

$$A(x) = \begin{bmatrix} \cos^{-1}x & \sin^{-1}x & \operatorname{cosec}^{-1}x \\ \sin^{-1}x & \sec^{-1}x & \tan^{-1}x \\ \operatorname{cosec}^{-1}x & \tan^{-1}x & \cot^{-1}x \end{bmatrix}$$

and $B = A^{-1}$. Also, det.$(A(x))$ denotes the determinant of square matrix A(x).

18. Which of the following statement(s) is(are) correct?

(A) $A(-x) = A(x)$.
(B) $A(x) + A(-x) = \pi I_3$
(C) $A(-x) = -A(x)$.
(D) $A(x) + A(-x) = -\pi I_3$

[**Note :** I_3 denotes an identity matrix of order 3.]

Ans. **(B)**

Sol. $A(-x) = \begin{bmatrix} \pi - \cos^{-1}x & -\sin^{-1}x & -\text{cosec}^{-1}x \\ -\sin^{-1}x & \pi - \sec^{-1}x & -\tan^{-1}x \\ -\text{cosec}^{-1}x & -\tan^{-1}x & \pi - \cot^{-1}x \end{bmatrix} = -A(x) + \pi I_{3 \times 3}$

$\therefore\ A(x) + A(-x) = \pi I_3$

19. Which of the following statement(s) is (are) correct?

(A) A(x) is a symmetric matrix.
(B) A(x) is a skew symmetric matrix.
(C) Maximum value of det.(A(x)) equals $\frac{\pi^3}{8}$
(D) Minimum value of det.(A(x)) equals $\frac{\pi^3}{16}$.

Ans. **(ACD)**

Sol. A(x) is symmetric matrix as $A^T = A$.

Now, $x \in \{-1, 1\}$

$\therefore\quad |A(-1)| = \frac{\pi^3}{8} = M$ and $|A(1)| = \frac{\pi^3}{16} = m$.

20. Which of the following statement(s) is(are) correct?

(A) det.(A(x)) is a continuous function in its domain but not differentiable in its domain.
(B) det.(A(x)) is a continuous and differentiable function in its domain.
(C) det.(A(x)) is a bounded function.
(D) det.(A(x)) is one-one and odd function.

Ans. **(AC)**

Sol. $\det.(A(x))$ is a point function.

$\therefore\quad \det.(A(x))$ is a continuous function but not differentiable function in its domain.

Also, $\det.(A(x)) \in \left\{\frac{\pi^3}{8}, \frac{\pi^3}{16}\right\} \Rightarrow$ bounded function.

Note : (i) $\det.(A(x))$ is a one-one function.

(ii) $\det.(A(x))$ is not odd function because $|A(-1)| + |A(1)| \neq 0$

21. Which of the following statement(s) is(are) correct?

(A) If a = det. (B) + det. (B^2) + det. (B^3) + ... ∞, then minimum value of a equals $\frac{8}{\pi^3 - 8}$.

(B) If b = det. adj. (B) + det. adj. (B^2) + det. adj. (B^3) + ... ∞, then maximum value of b is $\frac{256}{\pi^6 - 256}$.

(C) If a = det. (B) + det. (B^2) + det. (B^3) + ... ∞, then maximum value of a equals $\frac{16}{\pi^3 - 16}$.

(D) If b = det. adj. (B) + det. adj. (B^2) + det. adj. (B^3) + ... ∞, then minimum value of b is $\frac{64}{\pi^6 - 64}$.

Ans. **(ABCD)**

Sol. Given $B = A^{-1} \Rightarrow |B| = \frac{1}{|A|}$

Now, a = det. (B) + det. (B^2) + det. (B^3) + ... $\infty = \frac{|B|}{1-|B|} = \frac{\frac{1}{|A|}}{1-\frac{1}{|A|}} = \frac{1}{|A|-1}$

$\therefore \quad a_{max} = \frac{16}{\pi^3 - 16}$ and $a_{min} = \frac{8}{\pi^3 - 8}$

$b = |\text{adj}(B)| + |(\text{adj } B)^2| + |(\text{adj } B)^3| + \ldots\ldots\infty$

$b = |B|^2 + |B|^4 + |B|^6 + \ldots\ldots\infty = \frac{|B|^2}{1-|B|^2} = \frac{1}{|A|^2 - 1} \quad \left(\text{As, adj}(B) = |B|^2\right)$

therefore, $b_{max} = \frac{256}{\pi^6 - 256}$, $b_{min} = \frac{64}{\pi^6 - 64}$.

Numerical based Questions :

22. If $(\alpha\beta)(\delta\beta) = \gamma\gamma\gamma$ such that $\alpha, \beta, \delta, \gamma$ represents a number from 1 to 9 & $\alpha, \beta, \delta, \gamma$ are all different digits & $\alpha\beta$, $\delta\beta$ are two digit numbers & $\gamma\gamma\gamma$ is a three digit number, $\alpha > \delta$, and the trace of the matrix

$$A = \begin{bmatrix} \alpha & 1 & 2 & 0 \\ 0 & \beta & 1 & 1 \\ 0 & 0 & \gamma & 3 \\ 1 & 1 & 0 & \gamma \end{bmatrix}$$ is a, then $\frac{a}{7}$ is equal to

Ans. **(4.00)**

Sol. $\gamma\gamma\gamma = \gamma\,(111)$

$= \gamma.3.37 = (\alpha\beta)\,(\gamma\beta)$

Clearly, γ is

Clearly, γ is equal to the last digit of β^2. Now 37 must divide one of the $\alpha\beta$ or $\gamma\beta$

Let it be $\alpha\beta$ then $\alpha\beta$ = 37 or 74 (if γ = 8, then γ .3.37 = 12.74)

$\alpha\beta$ = 74 is not possible because

$$\left.\begin{array}{l}(\alpha\beta)(\gamma\beta) \geq 74.14 = 1036 \\ \text{But } \gamma(111) = 12.74\end{array}\right\} \text{for } \gamma = 8$$

$\therefore\ \alpha\beta = 37 \Rightarrow \delta\beta = 27 \Rightarrow \beta = 7, \alpha = 3, \delta = 2\ \&\ \gamma = 9$

$\therefore\ \alpha + \beta + \gamma + \delta = 21$

23. Let A be the set of all 3×3 symmetric matrices all of whose entries either 0 or 1. Five of these entries are 1 and four of them are 0. If n is the number of such matrices, then n/2 is

Ans. **(6.00)**

Sol. We have five entries 1 and the remaining four entries are 0

Since the matrix is symmetric, we must have an even number of zeros for $i \neq j$

Hence, we have two cases:

(i) Two entries in the diagonal are zero

We can select two places from three (in diagonal) in ${}^3C_2$ ways

Now we have to select elements for the upper triangle

For the upper triangle, we have three places of which one entry is 0 and two entries are 1

One place from three can be selected in ${}^3C_1$ ways

Hence, the number of matrices is ${}^3C_2 \times {}^3C_1 = 9$

(ii) If all the entries in the principal diagonal is 1, we have two 0 and one 1 in the upper triangle

Hence, the number of matrices is 3

Thus, total matrices = 12

Matrix Match Type :

24. **Match the following:**

For 3×3 matrix, a_{ij} represents the elements of i^{th} row and j^{th} column.

Column-I		Column-II	
A	$a_{ij} + a_{jk} + a_{ki} = 0$, then	p	This represents a symmetric matrix
B	$a_{ij} - a_{jk} - a_{ki} = 0$, then	q	This represents a skew symmetric matrix
C	$(-1)^{i+j} a_{ij} + (-1)^{j+k} a_{jk} + (-1)^{k+i} a_{ki} = 0$, then	r	This represents a matrix whose all diagonal elements are zeros
		s	Value of determinant is zero

Code :

(A) A → (q, r, s); B → (p, q, r, s); C → (q, r, s)

(B) A → (q, s); B → (p, q, r, s); C → (q, r, s)

(C) A → (q, r, s); B → (p, q, s); C → (q, r, s)

(D) A → (q, r, s); B → (p, q, r, s); C → (q, s)

Ans. **(A)**

Sol. (A) $a_{11} = a_{22} = a_{33} = 0$ & $a_{12} + a_{21} = 0$ & $a_{31} + a_{13} = 0$ & $a_{23} + a_{32} = 0$

(B) $i = j = k \Rightarrow a_{11} = a_{22} = a_{33} = 0$

(1) $i = 1, j = 2, k = 1$ then $a_{12} = a_{21}$

(2) $i = 1, j = 3, k = 1$ then $a_{13} = a_{31}$

(3) $i = 1, j = 3, k = 2$ then $a_{23} = a_{32}$

(C) & (D) are by similar argument

25. **Match the following:**

Column-I		Column-II	
A	If $|A| = 2$ then $|2A^{-1}| =$ (where A is a matrix of order 3)	p	1
B	If $|A| = \frac{1}{8}$ then $|\text{adj}(\text{adj}\, 2A)| =$ (where A is matrix of order 3)	q	4
C	If $(A + B)^2 = A^2 + B^2$ and $|A| = 2$ & $|B| =$ (where A and B are matrices of odd order)	r	Does not exist
D	$|A_{2\times2}| = 2$, $|B_{3\times3}| = 3$ and $|C_{4\times4}| = 4$ & $|ABC| =$	s	0

Code:

(A) A → q; B → p; C → s; D → r

(B) A → q; B → r; C → p; D → s

(C) A → q; B → r; C → r; D → s

(D) A → q; B → q; C → s; D → r

Ans. **(A)**

Sol. Conceptual